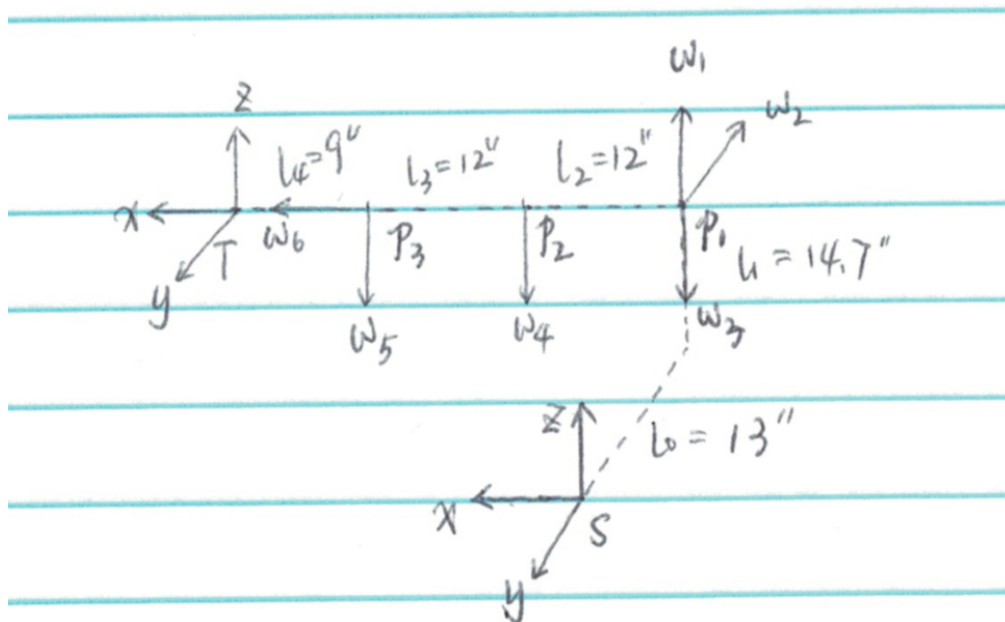


# Report of PA3: Jacobian

Peng Xu  
pxx37@case.edu

## 1. Graph of Frame



Graph of Frame

(As shown in the above graph, the number of the axes are renumbered based on 1.)

## 2. Theta List 1

$\theta = [0, \pi/3, 0, \pi/4, \pi/3, \pi/12]$

Spatial Jacobian:

Jst\_s =

-13.0000 14.7000 6.5000 6.5000 10.7426 36.1512

|        |         |         |         |         |          |
|--------|---------|---------|---------|---------|----------|
| 0      | 0       | 12.7306 | 24.7306 | 33.2159 | -1.9023  |
| 0      | 0       | 11.2583 | 11.2583 | 18.6068 | -12.6740 |
| 0      | 0       | 0.8660  | 0.8660  | 0.8660  | -0.1294  |
| 0      | -1.0000 | 0       | 0       | 0       | -0.9659  |
| 1.0000 | 0       | -0.5000 | -0.5000 | -0.5000 | -0.2241  |

SVD:

U =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.4524 | 0.8679  | -0.2008 | -0.0010 | -0.0018 | 0.0418  |
| -0.7942 | -0.2895 | 0.5335  | 0.0056  | 0.0164  | -0.0229 |
| -0.4043 | -0.4026 | -0.8184 | -0.0391 | 0.0223  | 0.0510  |
| -0.0261 | -0.0108 | -0.0247 | 0.0582  | -0.9733 | -0.2186 |
| 0.0059  | -0.0275 | 0.0236  | 0.6396  | -0.1321 | 0.7564  |
| 0.0183  | -0.0064 | 0.0638  | -0.7655 | -0.1854 | 0.6125  |

S =

|         |         |        |        |        |        |
|---------|---------|--------|--------|--------|--------|
| 52.2288 | 0       | 0      | 0      | 0      | 0      |
| 0       | 42.0495 | 0      | 0      | 0      | 0      |
| 0       | 0       | 6.4430 | 0      | 0      | 0      |
| 0       | 0       | 0      | 1.0136 | 0      | 0      |
| 0       | 0       | 0      | 0      | 0.4277 | 0      |
| 0       | 0       | 0      | 0      | 0      | 0.3160 |

V =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 0.1130  | -0.2685 | 0.4150  | -0.7429 | -0.3784 | 0.2186  |
| -0.1275 | 0.3041  | -0.4618 | -0.6449 | 0.2464  | -0.4489 |
| -0.3377 | -0.0614 | -0.5868 | 0.0580  | -0.7069 | 0.1865  |
| -0.5201 | -0.1440 | 0.4069  | 0.1249  | -0.2467 | -0.6832 |
| -0.7428 | -0.1852 | 0.0438  | -0.1152 | 0.4436  | 0.4494  |
| -0.1862 | 0.8813  | 0.3206  | -0.0036 | -0.1967 | 0.2173  |

Body Jacobian:

Jst\_b =

|          |         |          |          |         |         |
|----------|---------|----------|----------|---------|---------|
| -10.9917 | 0.0000  | 21.9834  | 10.3923  | 0.0000  | 0       |
| 1.8940   | 4.6991  | -11.4889 | -14.4889 | -8.6933 | 0       |
| -15.9094 | 17.5373 | 3.0784   | 3.8823   | 2.3294  | -0.0000 |
| -0.2241  | 0.9659  | 0        | 0        | 0       | 1.0000  |
| 0.9374   | 0.2500  | -0.2588  | -0.2588  | -0.2588 | 0       |
| 0.2665   | -0.0670 | -0.9659  | -0.9659  | -0.9659 | 0       |

SVD:

S =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.7546 | 0.1810  | 0.6304  | -0.0133 | -0.0101 | -0.0057 |
| 0.4899  | -0.4797 | 0.7233  | 0.0044  | -0.0060 | -0.0820 |
| -0.4338 | -0.8575 | -0.2727 | 0.0322  | -0.0250 | 0.0223  |
| -0.0076 | -0.0385 | -0.0110 | -0.9165 | 0.3978  | 0.0045  |
| 0.0222  | 0.0040  | 0.0045  | -0.3900 | -0.8998 | 0.1942  |
| 0.0423  | -0.0202 | 0.0697  | 0.0813  | 0.1770  | 0.9772  |

V =

|         |         |        |        |        |        |
|---------|---------|--------|--------|--------|--------|
| 33.9473 | 0       | 0      | 0      | 0      | 0      |
| 0       | 22.3214 | 0      | 0      | 0      | 0      |
| 0       | 0       | 9.9683 | 0      | 0      | 0      |
| 0       | 0       | 0      | 1.1420 | 0      | 0      |
| 0       | 0       | 0      | 0      | 0.6671 | 0      |
| 0       | 0       | 0      | 0      | 0      | 0.3369 |

D =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 0.4760  | 0.4817  | -0.1200 | -0.4337 | -0.5818 | -0.0210 |
| -0.1564 | -0.7762 | -0.1402 | -0.3533 | -0.4777 | -0.0172 |
| -0.6952 | 0.3077  | 0.4656  | -0.1947 | -0.2533 | -0.3212 |
| -0.4911 | 0.2473  | -0.5071 | -0.0483 | -0.0801 | 0.6571  |
| -0.1566 | 0.0982  | -0.7014 | 0.0517  | 0.0839  | -0.6813 |
| -0.0002 | -0.0017 | -0.0011 | -0.8026 | 0.5964  | 0.0134  |

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Conclusion:

All the singulars are non zeros which proves it is not a singular configuration.

### 3. Theta List 2

$\theta = [\pi/3, 0, -\pi/4, \pi/3, 0, \pi/6]$

Spatial Jacobian:

Jst\_s =

|          |         |         |         |         |          |
|----------|---------|---------|---------|---------|----------|
| -13.0000 | 7.3500  | 13.0000 | 1.4089  | -7.0764 | -10.3945 |
| 0        | 12.7306 | 0       | -3.1058 | 5.3795  | 10.3945  |
| 0        | 11.2583 | 0       | 0       | 0       | -1.1999  |
| 0        | 0.8660  | 0       | 0       | 0       | 0.7071   |
| 0        | -0.5000 | 0       | 0       | 0       | 0.7071   |
| 1.0000   | 0       | -1.0000 | -1.0000 | -1.0000 | 0        |

SVD:

U =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.9766 | 0.0281  | -0.2028 | -0.0656 | 0.0080  | -0.0000 |
| 0.1236  | 0.8791  | -0.4491 | -0.0663 | 0.0762  | -0.0000 |
| -0.1714 | 0.4726  | 0.8498  | 0.1201  | -0.0618 | 0.0828  |
| 0.0030  | 0.0539  | -0.0138 | -0.0718 | -0.7705 | -0.6309 |
| 0.0208  | -0.0066 | -0.1025 | -0.0716 | -0.6235 | 0.7714  |
| 0.0342  | -0.0069 | 0.1561  | -0.9832 | 0.0884  | -0.0000 |

S =

|         |         |        |        |        |   |
|---------|---------|--------|--------|--------|---|
| 24.0207 | 0       | 0      | 0      | 0      | 0 |
| 0       | 19.4515 | 0      | 0      | 0      | 0 |
| 0       | 0       | 6.0795 | 0      | 0      | 0 |
| 0       | 0       | 0      | 1.5780 | 0      | 0 |
| 0       | 0       | 0      | 0      | 0.4590 | 0 |

0      0      0      0      0    0.0000

V =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 0.5300  | -0.0191 | 0.4592  | -0.0826 | -0.0327 | -0.7071 |
| -0.3140 | 0.8621  | 0.3947  | -0.0002 | -0.0494 | 0.0000  |
| -0.5300 | 0.0191  | -0.4592 | 0.0826  | 0.0327  | -0.7071 |
| -0.0747 | -0.1380 | 0.1568  | 0.6949  | -0.6841 | 0.0000  |
| 0.3140  | 0.2333  | -0.1870 | 0.6913  | 0.5780  | -0.0000 |
| 0.4853  | 0.4274  | -0.6024 | -0.1599 | -0.4397 | -0.0000 |

Body Jacobian:

Jst\_b =

|          |         |          |          |         |        |
|----------|---------|----------|----------|---------|--------|
| -10.3923 | -0.0000 | 10.3923  | 0.0000   | 0.0000  | 0      |
| 23.3827  | 14.3849 | -23.3827 | -18.1865 | -7.7942 | 0      |
| -13.5000 | 24.9153 | 13.5000  | 10.5000  | 4.5000  | 0.0000 |
| 0        | 0.2588  | 0        | 0        | 0       | 1.0000 |
| 0.5000   | -0.8365 | -0.5000  | -0.5000  | -0.5000 | 0      |
| 0.8660   | 0.4830  | -0.8660  | -0.8660  | -0.8660 | 0      |

SVD:

S =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.2793 | 0.0000  | -0.9587 | -0.0000 | 0.0533  | 0.0000  |
| 0.8308  | -0.4997 | -0.2392 | 0.0045  | 0.0497  | -0.0168 |
| -0.4797 | -0.8655 | 0.1381  | 0.0078  | -0.0287 | -0.0291 |
| -0.0000 | -0.0090 | 0.0000  | -1.0000 | 0.0000  | 0.0000  |
| 0.0202  | 0.0291  | -0.0336 | -0.0003 | -0.4985 | -0.8655 |
| 0.0349  | -0.0168 | -0.0581 | 0.0002  | -0.8634 | 0.4997  |

V =

|         |         |        |   |   |   |
|---------|---------|--------|---|---|---|
| 46.3345 | 0       | 0      | 0 | 0 | 0 |
| 0       | 28.7871 | 0      | 0 | 0 | 0 |
| 0       | 0       | 7.2692 | 0 | 0 | 0 |

|   |   |   |        |        |        |
|---|---|---|--------|--------|--------|
| 0 | 0 | 0 | 1.0000 | 0      | 0      |
| 0 | 0 | 0 | 0      | 0.5236 | 0      |
| 0 | 0 | 0 | 0      | 0      | 0.0000 |

D =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 0.6225  | -0.0000 | 0.3353  | -0.0000 | -0.0013 | -0.7071 |
| 0       | -1.0000 | 0.0000  | 0.0003  | -0.0000 | 0       |
| -0.6225 | -0.0000 | -0.3353 | -0.0000 | 0.0013  | -0.7071 |
| -0.4357 | 0.0000  | 0.8073  | 0.0000  | -0.3980 | -0.0000 |
| -0.1872 | -0.0000 | 0.3513  | 0.0000  | 0.9174  | -0.0000 |
| -0.0000 | -0.0003 | 0.0000  | -1.0000 | -0.0000 | 0.0000  |

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Conclusion:

There is one singular is zero which proves it is a singular configuration.

## 4. Theta List 3, my choice

$\theta = [\pi/3, \pi/2, 0, 0, 0, \pi/6]$

When picking up joint 2= $\pi/2$ , and joint 3, 4, 5 equal 0, then joint 1 and joint 6 are align, namely robot has singularity.

Spatial Jacobian:

Jst\_s =

|          |         |          |          |          |          |
|----------|---------|----------|----------|----------|----------|
| -13.0000 | 7.3500  | -12.7306 | -23.1229 | -33.5152 | -13.0000 |
| 0        | 12.7306 | 7.3500   | 13.3500  | 19.3500  | 0.0000   |
| 0        | 11.2583 | 6.5000   | 6.5000   | 6.5000   | 0.0000   |
| 0        | 0.8660  | 0.5000   | 0.5000   | 0.5000   | 0.0000   |
| 0        | -0.5000 | 0.8660   | 0.8660   | 0.8660   | 0.0000   |
| 1.0000   | 0       | -0.0000  | -0.0000  | -0.0000  | 1.0000   |

SVD:

U =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.8698 | -0.4776 | 0.1124  | 0.0147  | 0.0332  | 0.0380  |
| 0.4510  | -0.6753 | 0.5679  | 0.1019  | 0.0576  | 0.0657  |
| 0.1979  | -0.5595 | -0.7874 | -0.1111 | -0.1224 | -0.0234 |
| 0.0152  | -0.0430 | -0.0606 | -0.0085 | 0.7452  | -0.6624 |
| 0.0252  | 0.0117  | 0.0001  | -0.6703 | 0.4885  | 0.5580  |
| 0.0081  | 0.0292  | -0.2030 | 0.7264  | 0.4320  | 0.4935  |

S =

|         |         |        |        |        |        |
|---------|---------|--------|--------|--------|--------|
| 52.8645 | 0       | 0      | 0      | 0      | 0      |
| 0       | 20.6818 | 0      | 0      | 0      | 0      |
| 0       | 0       | 4.0488 | 0      | 0      | 0      |
| 0       | 0       | 0      | 1.2135 | 0      | 0      |
| 0       | 0       | 0      | 0      | 0.0000 | 0      |
| 0       | 0       | 0      | 0      | 0      | 0.0000 |

V =

|        |         |         |         |         |         |
|--------|---------|---------|---------|---------|---------|
| 0.2140 | 0.3016  | -0.4110 | 0.4408  | 0.3275  | -0.6267 |
| 0.0298 | -0.8921 | -0.2126 | 0.3977  | 0.0000  | -0.0000 |
| 0.2970 | -0.1224 | -0.5939 | -0.6143 | 0.3618  | 0.1891  |
| 0.5192 | -0.0784 | -0.0408 | -0.2365 | -0.7237 | -0.3781 |
| 0.7414 | -0.0343 | 0.5124  | 0.1413  | 0.3618  | 0.1891  |
| 0.2140 | 0.3016  | -0.4110 | 0.4408  | -0.3275 | 0.6267  |

Body Jacobian:

Jst\_b =

|         |         |          |          |         |        |
|---------|---------|----------|----------|---------|--------|
| 0       | 0.0000  | 0.0000   | 0.0000   | 0.0000  | 0      |
| 0.0000  | 16.5000 | -28.5788 | -18.1865 | -7.7942 | 0      |
| -0.0000 | 28.5788 | 16.5000  | 10.5000  | 4.5000  | 0.0000 |
| 1.0000  | 0       | 0        | 0        | 0       | 1.0000 |
| 0.0000  | -0.8660 | -0.5000  | -0.5000  | -0.5000 | 0      |

0.0000 0.5000 -0.8660 -0.8660 -0.8660 0

SVD:

S =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| 0.0000  | -0.0000 | 0       | -0.0000 | -0.4889 | -0.8723 |
| -0.8654 | -0.4998 | -0.0000 | -0.0339 | -0.0132 | 0.0074  |
| 0.4996  | -0.8656 | 0.0000  | 0.0195  | -0.0229 | 0.0128  |
| -0.0000 | -0.0000 | 1.0000  | 0.0000  | 0       | 0       |
| -0.0195 | 0.0262  | 0.0000  | 0.4996  | -0.7551 | 0.4232  |
| -0.0339 | -0.0151 | -0.0000 | 0.8654  | 0.4360  | -0.2444 |

V =

|         |         |        |        |        |        |
|---------|---------|--------|--------|--------|--------|
| 40.1680 | 0       | 0      | 0      | 0      | 0      |
| 0       | 33.0151 | 0      | 0      | 0      | 0      |
| 0       | 0       | 1.4142 | 0      | 0      | 0      |
| 0       | 0       | 0      | 0.7318 | 0      | 0      |
| 0       | 0       | 0      | 0      | 0.0000 | 0      |
| 0       | 0       | 0      | 0      | 0      | 0.0000 |

D =

|         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|
| -0.0000 | -0.0000 | 0.7071  | -0.0000 | 0.1147  | -0.6977 |
| -0.0000 | -1.0000 | -0.0000 | 0.0000  | 0.0000  | 0.0000  |
| 0.8219  | -0.0000 | 0.0000  | 0.3973  | 0.4028  | 0.0662  |
| 0.5234  | -0.0000 | 0.0000  | -0.2437 | -0.8057 | -0.1324 |
| 0.2249  | -0.0000 | 0.0000  | -0.8847 | 0.4028  | 0.0662  |
| 0.0000  | 0       | 0.7071  | 0.0000  | -0.1147 | 0.6977  |