PA2 Inverse Kinematics Report

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Overview

Use Subproblems to solve Inverse Kinematics problems.

The following files including a python program and a pdf report are attached.

- ./inverse_kinematics.py
- ./report.pdf

Test Results

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********** Test **********

Test Results:

theta_sol1 =  1.0000008438174133
theta_sol2 =  ([1.0000003992194313, 2.842726291263703], [-0.49999928623982914,
-3.048439232442923])
theta_sol3 =  [2.4870044319130766, 0.9999787910181026]
```

Main Results

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********** Main **********

Final Results:

Solution  1 :
[-3.0047537005118721, 2.0226153308037387, 2.2942539202820043, 5.1375119496773483,
0.17853578269327161, 4.7116745102606234]
Solution  2 :
[0.13683895307792115, -2.0186153308037391, -0.84733873330778897, 5.1375119496773483,
0.17853578269327161, 4.7116745102606234]
Solution  3 :
[0.13683895307792318, 1.1229773227860538, -1.1486009837552009, 5.1375119496773483,
0.96713757480896645, 1.5700818566708299]
Solution  4 :
[-3.0047537005118699, -1.1189773227860538, 1.992991669834592, 5.1375119496773483,
0.96713757480896645, 1.5700818566708299]
Solution  5 :
[-3.0047537005118712, 2.0226153308037391, 1.1486009837552016, 1.1456733575022375,
-0.96713757480896645, 4.7116745102606234]
Solution  6 :
[0.13683895307792185, -2.0186153308037396, -1.9929916698345913, 1.1456733575022375,
-0.96713757480896645, 4.7116745102606234]
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Solution  7 :
[0.13683895307792215, 1.1229773227860547, -2.2942539202820034, 1.1456733575022375,
-0.17853578269327089, 1.5700818566708299]
Solution  8 :
[-3.0047537005118712, -1.1189773227860547, 0.84733873330778975, 1.1456733575022375,
-0.17853578269327089, 1.5700818566708299]
```
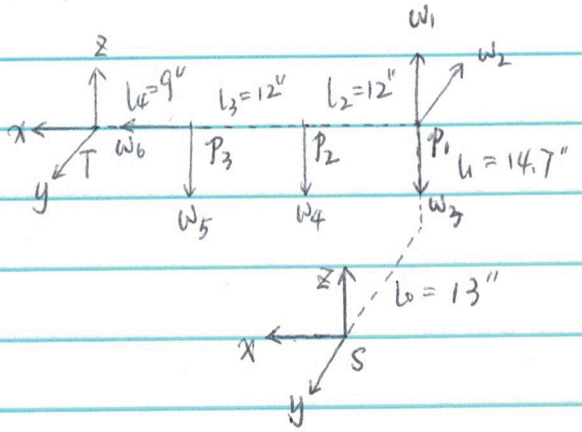

Maximum number of theoretical solutions

As shown above, the maximum number of theoretical solutions is 8.

This inverse kinematics problem was divided into 4 subproblems ---- Subproblem 3, Subproblem 2, Subproblem 2, and Subproblem 1, solved in sequence. The process is explained in the following two pages in Appendix.

Appendix



Note: As shown in the above graph, I renuber the exes from 1 to 6 for the convenience. $\omega_1, \omega_2, \cdots, \omega_6$ are associated with axis ①, ②, ⋯, ⑤

$$g_{st}(\theta) = e_1 e_2 e_3 e_4 e_5 e_6 \, g_{st}(0) = g_d$$

$$g_{st}(0) = \begin{bmatrix} & I & & l_2+l_3+l_4 \\ & & & -l_0 \\ & & & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \triangleq g_0$$

$$e_1 e_2 e_3 e_4 e_5 e_6 = g_d g_0^{-1} \triangleq g_1 \quad ①$$

$$e_{-6} e_{-5} e_{-4} e_{-3} e_{-2} e_{-1} = g_0 g_d^{-1} \triangleq g_2 \quad ②.$$

(1) Let p_1 be the intersection point of axis ξ_1, ξ_2 and ξ_3.

Apply ② to p_1.

$$e_{-6} e_{-5} e_{-4} p_1 = g_2 p_1 \quad ③$$

Let p_3 be the intersection point of axis ξ_5 and ξ_6

$$e_{-6} e_{-5} p_3 = p_3 \quad ④.$$

(3) − (4),

$$e_{-6} e_{-5} e_{-4} p_1 - e_{-6} e_{-5} p_3 = g_2 p_1 - p_3$$

$$\| e_{-6} e_{-5} (e_{-4} p_1 - p_3) \| = \| g_2 p_1 - p_3 \|$$

$$\| e_{-4} p_1 - p_3 \| = \| g_2 p_1 - p_3 \| \quad (5)$$

Apply SP3 to find θ_4.

(2). Let $p_4 = e_4 p_1$, so

$$e_{-6} e_{-5} p_4 = g_2 p_1 \quad (6)$$

Apply SP 2 to find θ_5, θ_6

(3). With $\theta_4, \theta_5, \theta_6$ known,

$$e_1 e_2 e_3 = g_1 e_{-6} e_{-5} e_{-4} = g_3 \quad (7)$$

Pick a point p_5 that is on ξ_3 but not on ξ_2.

$$e_1 e_2 p_5 = g_3 p_5 \quad (8)$$

Apply SP 2 to find θ_1 and θ_2.

(4). With (7) and θ_1 and θ_2.

$$e_3 = e_{-2} e_{-1} g_3 = g_4 \quad (9)$$

Pick a point p_6 that is not on ξ_3.

$$e_3 p_6 = g_4 p_6 \quad (10)$$

Apply SP 1 to find θ_3.