

Stochastic Resonance in Neural Network, Noise Color Effects*

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ABSTRACT

Some systems cannot be predicted by classical theories and require the development of combined deterministic and stochastic theories that make use of noise for their dynamical prediction. Noise is not always an interfering signal which perturbs the system, it can also enhance its performance. This property can be observed through Stochastic Resonance (SR). To detect this phenomena it is necessary a system with bistable potential barrier with a threshold, the input of the system should be a weak periodic signal which amplitude is below threshold together with a stochastic signal. The behaviour of the SR is detected in a neural network and it is studied under noise color variations. Here it is found that Pink noise amplifies the sub-threshold input signal twenty times more in comparison to white noise. This could be evidence of the functionality of background noise in the brain, where neurons are naturally embedded in pink noise.

1. Introduction

In theoretical and experimental physics, it is common practice to do linear approximations to solve non-linear problems. These approximations have given satisfactory results, that until certain margin of error, theoretical analysis and linear numerical methods agree with observed experimental findings[10]. However, measurements are subject to the same linear approximations as numerical methods which require the validation of current models. Often, experimental data which does not match with known models, is considered to be subject to noise and eliminated from the average trend of the data. This deletion of the noisy data is done due to the complexity of noise and limited theoretical tools to analyze them. In addition, models and experimental devices are constrained to implicit linearizations.

Real physical phenomena is intrinsically non-linear and its dynamics modulated in a noisy environment. An example is the nervous system, a highly non-linear system which is embedded in a noisy biological environment dependent on multiple interactions, for example, channel opening which is only triggered by precise stimulus like voltage, pH or the binding of a ligand [13], its poisson like distribution allows coherence in the brain. The nervous system is also a detection device whose input signals are strongly immersed in a continuous noise, which comes from the external environment through the sensory neurons. The nervous system is acting as a decoder of noisy input output information by computing small signals embedded in fluctuating noisy conditions. Enhancement and amplification of those fluc-

tuations is a relevant candidate to explain perception [4]. Hence, it is interesting to study Stochastic Resonance (SR), a phenomena property of non-linear systems. SR enabled to have computation with small fluctuating signals. There is also evidence of the role of SR in the functioning of the brain for the detection of weak signals. The relevance in synchronisation and coherence in neural connections, exactly by the fact that the poissonian opening at the channels allows the detection of coherences. [8].

For the first time, the SR model was proposed and numerically stimulated by Roberto Benzi[3]. Moreover, Luca Galimaitoni studied the SR model and presented the detailed theory behind the stochastic resonance[8]. A special case of SR systems which is called *Ghost Stochastic Resonance* was discovered by Oscar Calvo[5]. It occurs for the maximum resonance frequency where the input energy disappears. SR is completely different from the resonance observed in linear systems. The most significant difference is the resonance frequency. The resonance frequency is obtained according to the periodic input signal in SR models, while it depends on the structural properties of the system in linear resonance models.

There are analytical limitations to the understanding of the dependence of the stochastic resonance, on the parameters of the periodic input perturbation, the noise and the physics of the system. The amount and dimensionality of these stochastic differential equations and non-linear equations becomes intractable and unsolvable. However, the experimental technique proposed in this research is a powerful tool to better understand and implement SR.

SR models can be exploited in many physical systems such as lasers, SQUID and neural networks. In SR models, adding noise to the input stimulus in the sub-threshold regime can enhance the information sensing process in sensory systems. This is a remarkable property of SR models [11]. One of the important applications of SR models based on this property,

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is for electro-optical devices to enhance their measurement for data acquisition from small signals, which are embedded in noise. An example of this application was done by Bruno Ando [2]. Another application of SR models is analysing them to enhance neural signals. A pioneer in this endeavour is Frank Moss [11], who showed that SR models agree with the neuron models and its properties. He expressed that the sources of noise can be important for coherence in the brain. Here we study the case of Pink noise, this choice of noise is motivated by the importance of the background noise of the brain, exhibiting a pink-noise-like power spectrum. Apart from the brain, pink noise is ubiquitous in physical and biological systems, see, e.g.,[12].

Non-linear systems are composed of three main components, i.e., a threshold value, a weak input signal to stimulate the system in the sub-threshold regime and a noise signal. These components are always found in nature and their interactions facilitate SR. According to the stochastic differential equations that model this phenomena, it can be concluded that SR depends on the subthreshold periodic input signal, the noise signal and the system structure; however, there is no analytical solution for SR models. Hence, numerical simulations are used to obtain the solution of these models. SR models have an intrinsic error. This error originates from selecting the threshold value and it can cause serious complications during measurements. Hence, this error should be reduced through optimising SR models [2]. The following methods are employed in order to optimising SR models:

- The Inter-Spike Interval Histogram (ISIH) analysis
- The Signal to Noise Ratio (SNR) analysis

They obtain the optimum points for SR models when the noise value is given. By using these optimum points for the SR model, the input signal is preserved and amplified. On the other hand, different types of noises, i.e., white and color noises can affect non-linear phenomena, differently. Thus, it is important to find the type and amplitude of the noise signal which maximises the performance of the SR model to amplify the input signal.

According to biological studies, SR plays an important role in the functioning of the brain for detecting weak input signals and synchronization of neural connections [9]. It is possible to see the phenomena in neurons, given that its function in the brain is the integration and information processing of electrochemical signals. This constant activity at the interior of the brain creates a background activity, due to the neuronal pulse and the signal transmission that causes fluctuations in the membrane potential creating a real source of noise [12].

One of the noise sources in the brain, is the complex variations of nonlinear chemical activity at the synaptic cleft, for example in the release of neurotransmitters which causes fluctuations in the neuronal potential [12] and is proposed as responsible for signal amplification in the SR model.

The purpose of this paper is to obtain an accurate SR model for neurons in the presence of different types of noises. This model can amplify the weak input signal efficiently for easier detection. Thus, the main contributions of this research are:

- Investigate if the non-linear behavior of neurons can be modelled by SR models. Then a SR model for an artificial neuron is employed in order to analyze conveying information of the weak input signal. In spite of using artificial neurons, our long term objective is analyzing biological neurons.
- Parameter optimization for the neuron model in order to amplify the weak input signal in the presence of different types of noises, i.e., white noise and color noise.
- Finally, it is shown in SR models, that pink noise is twenty times more optimal than white noise to amplify the weak input signal.

The rest of this paper is organized as follows. In Section 2, the theoretical background of the SR model of the required constraints are explained. Moreover, two methods for analyzing the output of the SR model are introduced. In Section 3, the detailed methodologies to develop the neural network architecture from electrical devices is presented together with the structure of the pink noise generator. In Section 4, the results of the power spectrum analysis and signal to noise ratio analysis are used to compare the efficiency of different types of noise. Finally, Section 5 contains the concluding remarks.

2. Theoretical background

The SR model can be developed through adding a weak periodic force, the variation of a bistable potential function, and a noise signal to the components of the Brownian motion model [1]. Hence, the SR model can be described as follows:

$$m \frac{d^2x}{dt^2} = F(t) - b \frac{dx}{dt} - \frac{dV(x)}{dx} + \xi(t), \quad (1)$$

where $F(t)$ is the periodic force, $\xi(t)$ is the noise signal and $V(x)$ is the bistable potential. For further information on the system (1), we also refer to [2], [7].

$$V(x) = -a \frac{x^2}{2} + b \frac{x^4}{2}, \quad (2)$$

where a and b are constants and they can modify the shape of potential as shown in Fig. 1. The constants a and b are related with the potential function characteristics as

$$x_m = \sqrt{\frac{a}{b}}, \quad \Delta V = \frac{a^2}{4b}, \quad (3)$$

where x_m and ΔV are shown in Fig. ???. A weak periodic signal as the input of the SR model is introduced. It is assumed that the amplitude of this signal is not sufficient to

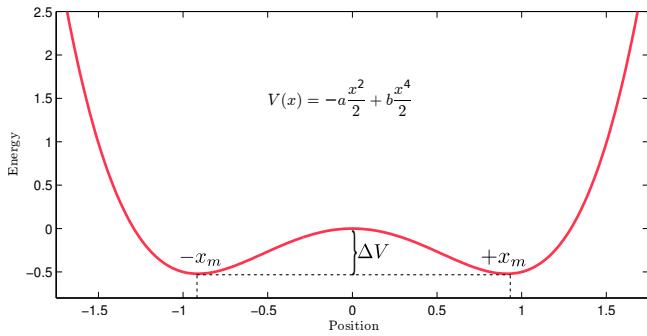


Figure 1: Bistable potential which is used in SR model.

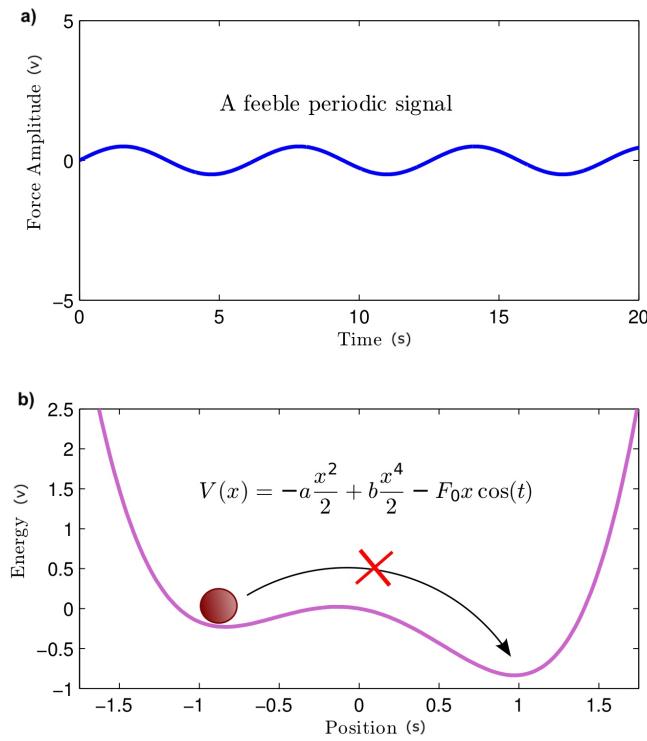


Figure 2: a) A weak periodic signal to move the particle. b) The amplitude of this signal is not sufficient to move particle from one side of the bistable potential to the other side.

move particle from one side of the bistable potential to the other side as shown in Fig. 2. In addition to the weak periodic signal, a noise signal is added to the system. The noise signal enables the particle to cross from one side of the potential to the other side as shown in Fig. 3. Hence, the output of the system depends on the noise intensity. When the amplitude of the noise signal is low as in Fig. 3 (a), the rate of moving of the particle is low as shown in Fig. 3 (b). On the other hand, when the noise amplitude is high as Fig. 3 (e), the particle moves form one side to the other frequently. Thus, in these situations, the input signal is lost. However, when the noise intensity is optimal as in Fig. 3 (c), the crossing of a particle is synchronized with the weak periodic signal as shown in Fig. 3 (d). The SR model works

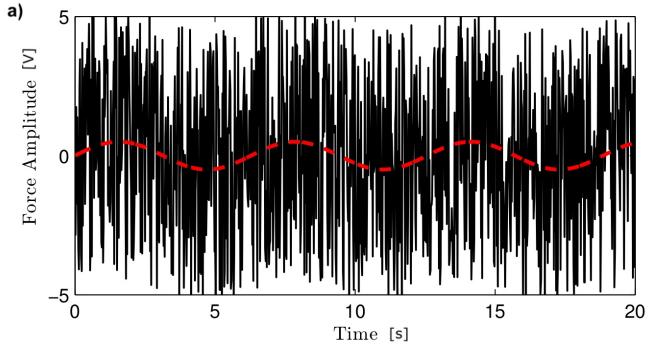


Figure 3: a) A noise signal is added to the weak signal. b) The particle can cross from one side of the bistable potential to the other side.

in this scenario. Variations of the number of received neurotransmitters and poissonian opening of ion channels, causes fluctuations in the membrane potential and creates a source of noise [12] which can be responsible for signal amplification in the SR model. The output signal of a neuron is a train of Dirac delta functions as shown in Fig. 4. One way to analyze the SR model in a neuron is measuring the inactivity intervals between the Dirac delta functions in the response of a neuron. Then, the collected responses from different neurons are saved for a histogram generation [12]. Fig. 6 shows the histogram of the time intervals. Variations of the noise signal modify the histogram. When the noise amplitude is low, the inactivity intervals are too long and the majority of intervals are longer than the signal's period. On the other hand, when the intensity of noise signal is high, the inactivity intervals are too short and the majority of intervals are shorter than the signal's period. However, when the amplitude of the noise signal is optimal, the inactivity intervals are equivalent to the times corresponding to the multiples of the input signal period as shown in Fig. 6. Thus, the SR model is designed based on the maximum value corresponding to the signal's period. The peak of the histogram with respect to the noise amplitude is studied, and thus, the optimum intensity of the noise signal for the SR model is obtained. This scheme is called ISIH analysis.

Another useful method to quantify a SR model is a SNR analysis which employs the Fourier transform. The Fourier transform of the output signal shows multiple peaks at the

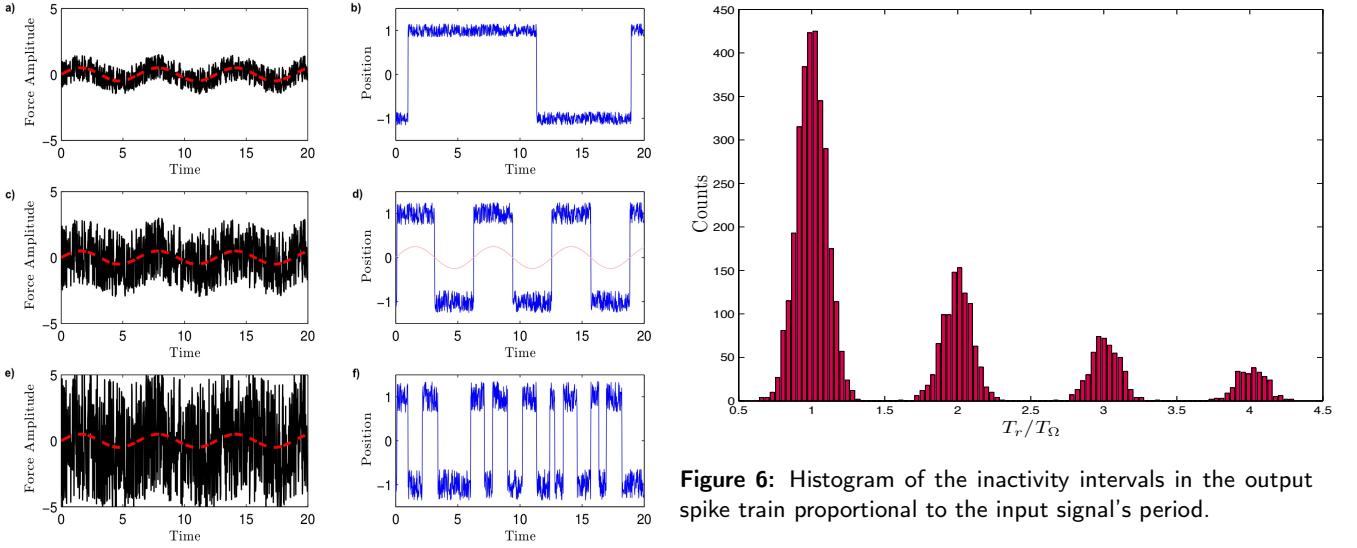


Figure 4: Response of the system: (a) Noise amplitude is weak, (b) the number of crosses from one side of the potential to the other side is low. (c) Noise amplitude is optimal, (d) the crossing from one side of the potential to the other side is synchronized with the weak input signal. (e) Noise amplitude is too high, (f) the crosses are frequent, and thus, the information of the input signal is lost.

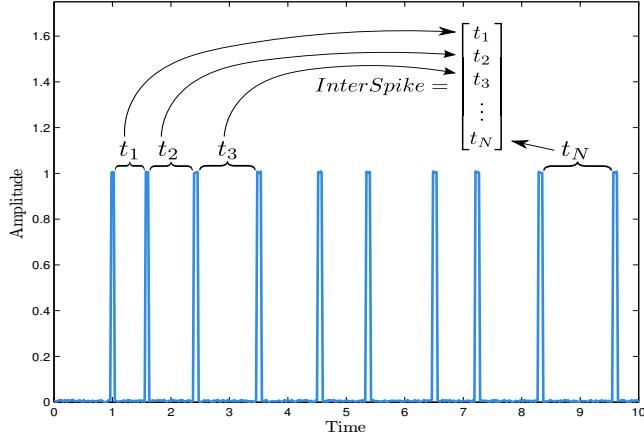


Figure 5: The output of an artificial neuron. The inactivity intervals are measured and saved to analyze.

multiples of the input signal's frequency values. In this method, the peak corresponding to the input frequency is integrated according to the expression in (4) [8] for different noise amplitudes. Then, the derived values are plotted with respect to the noise amplitude to obtain the optimum intensity of the noise signal for the SR model.

$$\text{SNR} = 2 \left[\lim_{\Delta\omega \rightarrow 0} \int_{\Omega-\Delta\omega}^{\Omega+\Delta\omega} S(\omega) d\omega \right] / S_N(\omega). \quad (4)$$

Figure 6: Histogram of the inactivity intervals in the output spike train proportional to the input signal's period.

3. Methodology

3.1. Artificial Neuron Circuit

In this subsection, the employed artificial neuron model for the SR analysis is described. The circuit of an artificial neuron is shown in Fig. 9, as is proposed and explained in detail at [5]. It works mainly based on a Schmitt Trigger monostable. This is a device with a non-dynamical threshold value which selects the input signals through comparison with this threshold value. Hence, when the input signal overcomes the threshold value, it generates a spike in the output.

The most relevant characteristic of the Schmidt Trigger is that its output can remember information about the input that was fed to them in the past, which is useful to simulate the behaviour of a neuron. On the other hand, the Schmitt Trigger as a bistable logic gate, can behave in a more unpredictable way and cannot be described with truth tables. The Schmitt Trigger device can simulate the nonlinear behavior of neurons in the sense that convey information through generating action potentials and setting a refractory period with feedback loop [1].

An action potential in a real neuron is a roughly 100 mV oscillation in the electrical potential across the cell membrane which lasts for about 1 ms, fig. ?? shows the waveform of an action potential. For a few milliseconds just after an action potential, it is impossible to generate another spike, this is called the refractory period as shown in Fig. ???. In addition to the Schmitt trigger device, there are operational amplifiers U1 and U2. Unit U1 is an operational amplifier which receives the input signal and amplifies it in order to feed the first monostable through pin 1B. Unit U2 generates a pulse with a duration of T_1 , when the input signal overcomes its intrinsic threshold. The value of T_1 is determined based on the resistance and the capacitance values. The output of unit U2 through pin 1Q simulates the output of the neuron. When the generated pulse by U2 enters in the descendant flank, the second monostable, i.e., unit U3 fires. It generates a pulse

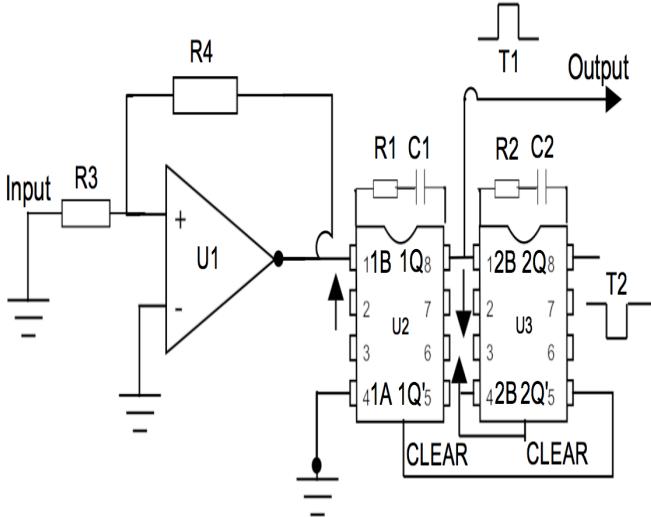


Figure 7: Artificial neuron circuit based on the monostable Schmitt Trigger.

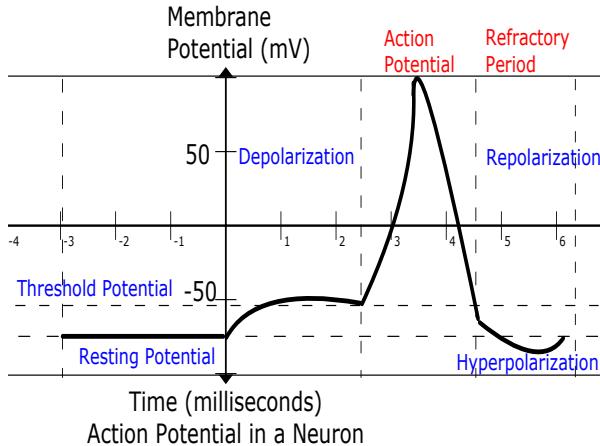


Figure 8: The waveform of an action potential. The neuron has a threshold. When the membrane potential overcomes this threshold value, the neuron generates an action potential [6].

with duration T_2 . The output of pin $2\bar{Q}$ of U3 works as the clear signal of the first monostable. Hence, U2 does not generate a new action potential in the descendant flank of every spike. This part of the circuit simulates the refractory period of a neuron. This cycle is repeated indefinitely until the stimulation signal is detected.

The experimental setup to analyse the SR model is shown in Fig. 10. A coherent signal generator is used to generate the periodic input signal. This signal is added to the noise signal generated by the noise source. The summation of these signals enters the artificial neuron circuit. This circuit generates an output signal when the input overcomes its intrinsic threshold. The output port is connected to an oscilloscope which saves the signal for later processing, in the experimental setup.

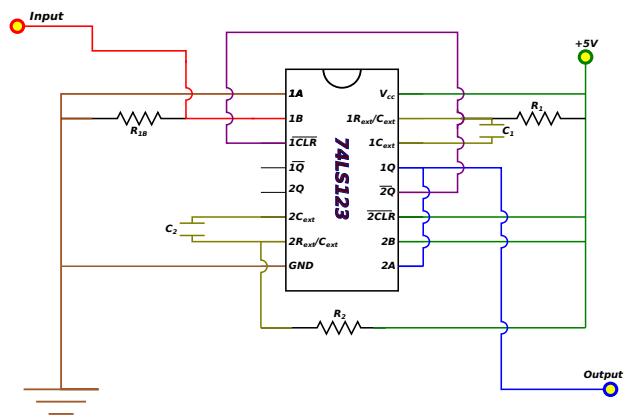


Figure 9: Schematic of the assembled artificial neuron circuit.

3.2. Pink Noise Generation

In this paper, the color noise is developed by using a pink noise generator. Pink noise is generated through coupling two filters as shown in Fig. 11. Next, a white noise is used as the input of these filters. Hence, they reduce high frequency components of white noise to produce pink noise. To evaluate the correct acquisition of pink noise, the power spectrum and the characteristic slope in the logarithmic scale are analyzed.

4. Results and Discussion

In this section, the results from the SR model analysis are obtained. The obtained results for pink noise and white noise are also compared with each other.

4.1. Power Spectrum Analysis

The Fourier transform is employed for the output signal of the circuit to obtain its power spectrum. It is worth to mention that the captured Fourier transforms by MATLAB are not shown here, since the numerical results are not as accurate as the obtained results through the spectrum analyzer. Hence, the obtained spectrum by the spectrum analyzer are used for our analysis in the following.

Fig.12 and Fig.13 show the power spectrum when white and pink noises are used, respectively. In these figures, the X axis corresponds to the frequency and the Y axis corresponds to the power spectrum amplitude. When the peak of the spectrum matches with the inputs signal frequency, it is possible to conclude that the input noise amplitude is optimal. Thus, the noise amplitude is modified to find the optimum values. It can be seen that the number of jumps in the power spectrum is corresponding to pink noise more than in the white noise case.

Through studying different power spectrums, it can be shown that the spectrum amplitude increases when the noise

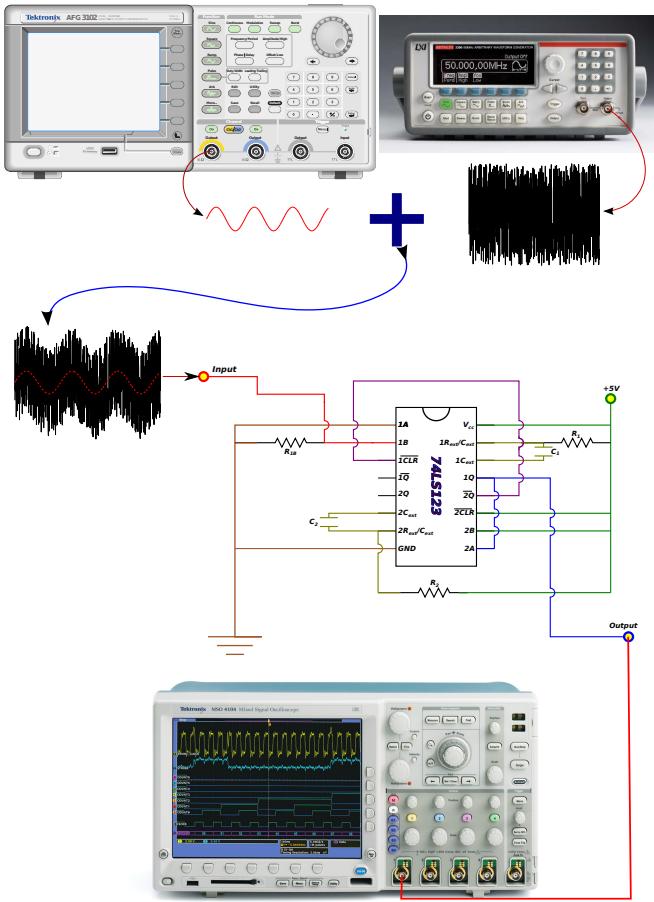


Figure 10: Experimental setup for the SR analysis. The upper left device is a coherent signal generator, the upper right one is a white noise generator, the circuit in the center is the artificial neuron and the bottom device is an oscilloscope which is used for data acquisition.

intensity increases. However, when the noise amplitude reaches a specific point, the power spectrum reaches a maximum value. After this point, the peak of the power spectrum starts to descend no matter how much the amplitude of pink noise increases.

4.2. Optimal SNR

It is necessary to repeat the power spectrum analysis for different noise amplitudes. It can be seen that a maximum peak is conserved. This peak is selected and it is integrated by using the expression in (4) for different values of noise amplitude. The obtained values were plotted with respect to the noise amplitude for white and pink noises in Fig. 14 and Fig. 15, respectively. These figures show the amplification factor for the input signal versus different amplitudes of white and pink noise, respectively. There is an important difference between white and pink noises. Although the measurements are made for the same voltages and under the same parameters, the maximum performance of the SR model is achieved for different optimum noise values. Furthermore, the range of noise values which maximize the SR performance for pink noise is wider than white noise. This

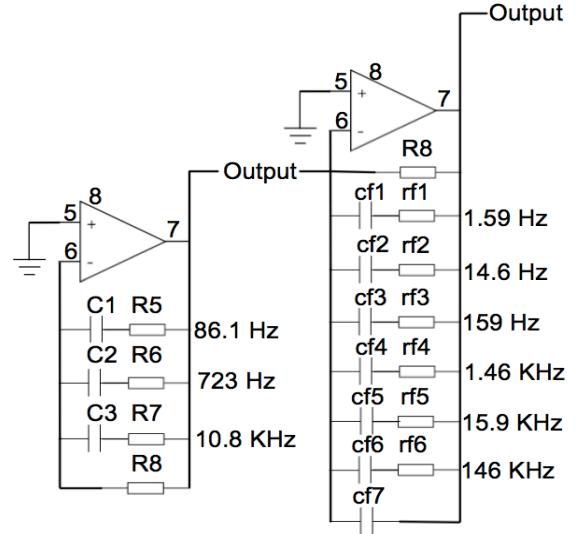


Figure 11: Structure of the filter for producing pink noise.

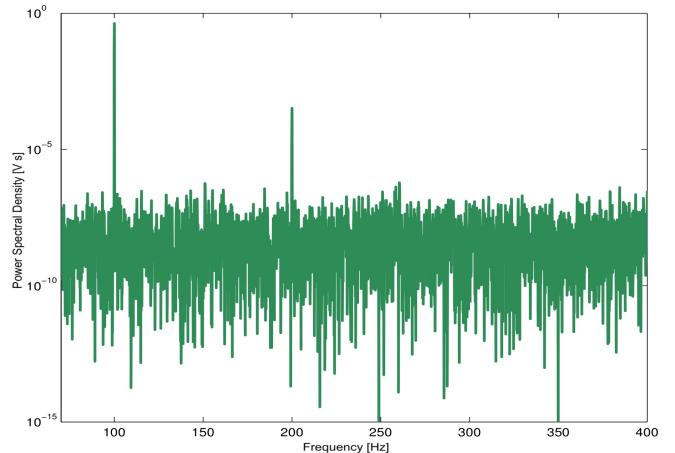


Figure 12: Power spectrum of the output signal in the presence of a white noise with 2500 mV amplitude.

range of values for white noise are sharper.

4.3. Comparison between White and Pink Noises

Fig. 16 (a) and (b) show the power spectrum for white and pink noise, respectively. It can be seen that the contribution of the input signal frequency corresponding to pink noise, is 100 times less than the spectrum of white noise. It is a surprising finding according to Fig. 17. Although the contribution of pink noise in the input signal frequency is 100 times less than white noise amplification of the weak input signal for pink noise is 20 times bigger than for white noise, this is translated into a big saving in energy. This result is beneficial since numerous physical and biological phenomena are immersed in pink noise. Here, it is proved that their amplification factor is high due to their nature and intrinsic characteristics while requiring less power to achieve the enhancement. It is important to notice that theoretical models, numerical and computational, have used white noise since

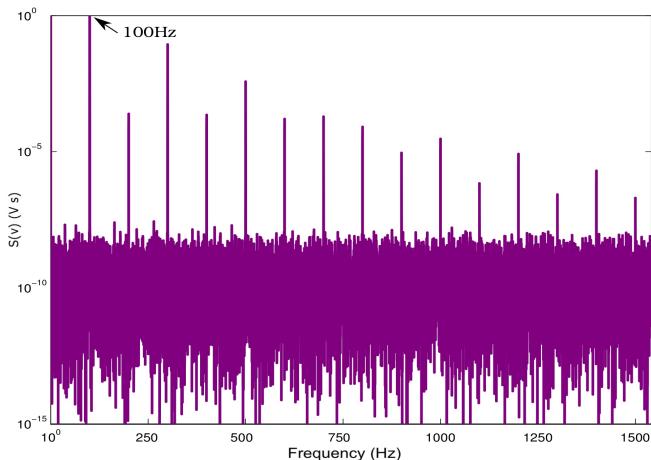


Figure 13: Power spectrum of the output signal in the presence of a pink noise with 2900 mV amplitude.

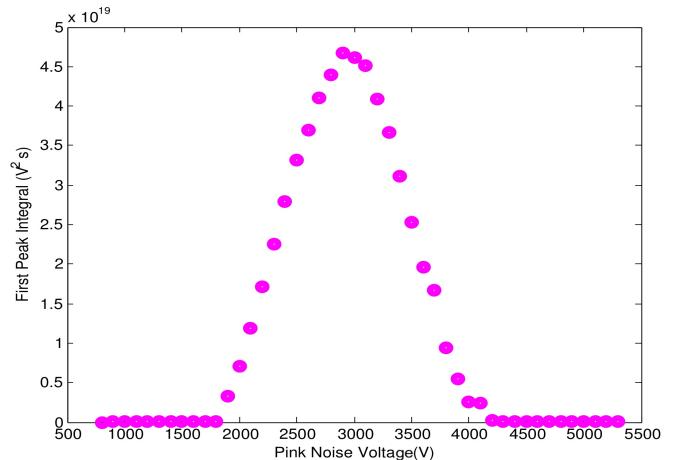


Figure 15: Signal to Noise Ratio analysis for pink noise whit an amplitude of 50 mV for the input periodic signal

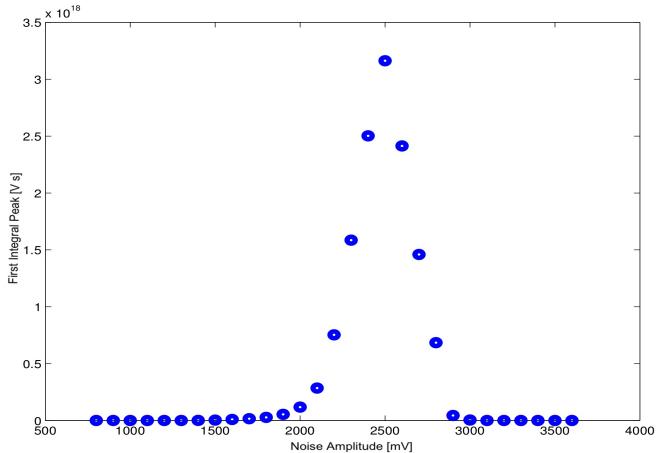


Figure 14: Signal to Noise Ratio analysis for white noise when the amplitude of input periodic signal is 50 mV.

there is a mathematical algorithm to generate it, while there is no mathematical algorithm that is able to generate pink noise. According to this finding, the importance of pink noise in order to improve the amplification factor is clear and it should be implemented in physical models to achieve higher accuracy. It is significant to mention that there is no bibliography which reported this relevant finding and here is the first report about it.

5. Conclusion

In this paper, the SR model for an artificial neuron was implemented. Two schemes for the output signal analysis were used. The SNR analysis method which requires the integration of the maximum peaks is more accurate than the ISIH method since the ISIH analysis, its results are prone to numerical errors. Moreover, it is not recommended to use numerical methods to obtain the Fourier transform of the output signal, since there are complications for programming it under slight parameter variations, which change the results rapidly. However, using a spectrum analyser directly

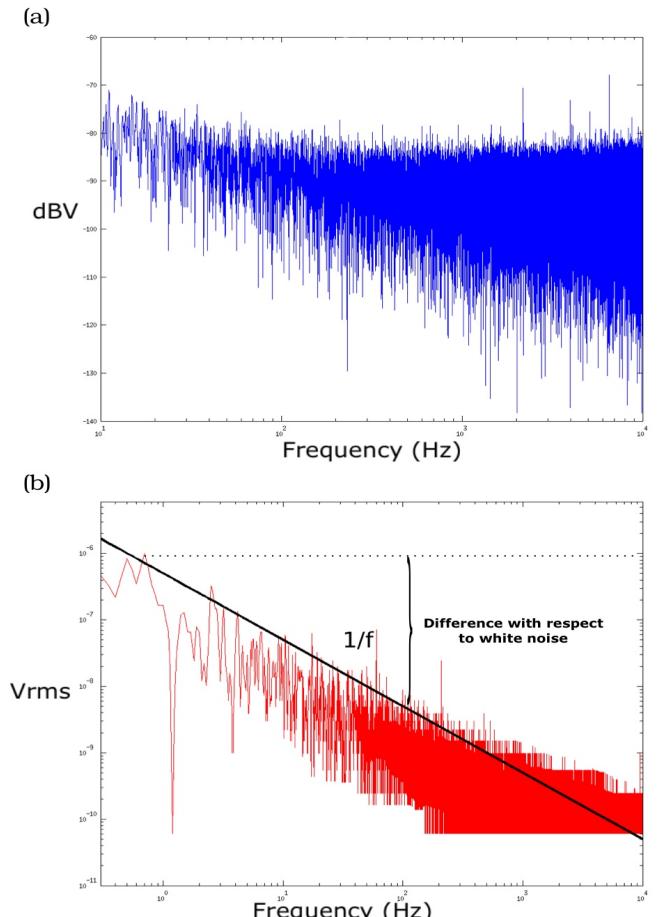


Figure 16: (a) White noise power spectrum, (b) Pink noise power spectrum.

is faster and more reliable.

It proved that the SR model amplifies the weak periodic input signal through noise. Thus, the SR model conserves the information of the input signals. It shows that the signal is embedded in a noisy environment with the contents of all fre-

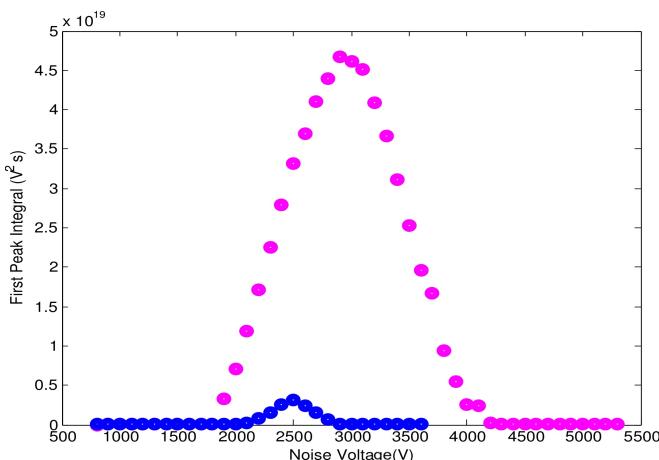


Figure 17: The contribution of pink noise to the input signal frequency is 100 times smaller than white noise, however, pink noise amplifies the weak input signal 20 times more than white noise.

quency values and with higher amplitudes than the desired signal. In the detection process, the frequency of the input signal and its multiples are retrieved. All measurements are replicated to validate the reliability of the results. It can be seen that the SR model can detect the input signal when the amplitude of this signal is decreased from 800 mv to 40 mv. Hence, it confirms that the SR model can conserve weak signals and enhance their amplitude. Moreover, the most important conclusion is that an artificial neuron can be modelled as a SR system in the presence of white or pink noise. Pink noise provides a higher performance in terms of amplification factor. Furthermore, pink noise has a wider range for optimum values, while white noise has a narrow optimum range. Hence, it is concluded that neurons are more sensitive to detect the signals which carry pink noise than signals with white noise or without noise. In addition to artificial neurons, the concluded results can be used in all types of systems with a threshold value and excitability behaviour where detection of weak signals embedded in a high amplitude noise is desirable. Since the noise of sensory neurons is limited by external and intrinsic neural noise, it is possible to add pink noise to the signals which are not detectable to retrieve by neurons. Thus, the neurons can detect weak signals creating an extra-sensorial capability in a natural way.

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