DÚ 7 -(P-Si)

$f(x) = \frac{x^3}{(x+3)^2}$ $D_f = \mathbb{R} \setminus \{-3\}$

$f'(x) = \frac{(x^3)' \cdot (x+3)^2 - (x^3) \cdot ((x+3)^2)'}{(x+3)^4} = \frac{3x^2 \cdot (x+3)^2 - 2 \cdot x^3 \cdot (x+3)}{(x+3)^4} = \frac{(x+3)\left(3x^2 \cdot (x+3) - 2x^3\right)}{(x+3)^4} =$

$= \frac{3x^2(x+3) - 2x^3}{(x+3)^3} = \frac{3x^3 + 9x^2 - 2x^3}{(x+3)^3} = \frac{x^3 + 9x^2}{(x+3)^3}$

kdy první derivace neexistuje V ~~∞~~ se rovná 0

$x^3 + 9x^2 = 0$

$x^3 = -9x^2$

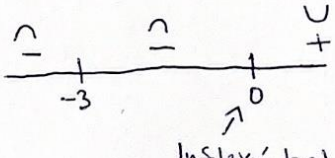
$x = -9$

Lokální maximum s hodnotou -20,25

$f''(x) = \left(\frac{x^3+9x^2}{(x+3)^3}\right)' = \frac{(x^3+9x^2)' \cdot (x+3)^3 - (x^3+9x^2) \cdot \left((x+3)^3\right)'}{(x+3)^6} = \frac{(3x^2+18x)(x+3)^2 - (x^3+9x^2) \cdot 3(x+3)^2}{(x+3)^6} =$

$= \frac{(x+3)^2 \cdot \left((3x^2+18x)(x+3) - (x^3+9x^2) \cdot 3\right)}{(x+3)^6} = \frac{(3x^2+18x)(x+3) - 3x^3 - 27x^2}{(x+3)^4} =$

$= \frac{3x^3 + 9x^2 + 18x^2 + 54x - 3x^3 - 27x^2}{(x+3)^4} = \frac{54x}{(x+3)^4}$



Inflexní bod

Asymptoty:

Vodorovné: $H_f = \mathbb{R}$, takže neexistuje

Svislé: $\lim_{x \to -3^-} \left(\frac{x^3}{(x+3)^2}\right) = \frac{-27}{\infty} = -\infty$

$\lim_{x \to -3^+} \left(\frac{x^3}{(x+3)^2}\right) = \frac{-27}{\infty} = -\infty$ } $x = -3$

Se směrnicí:

$A = \lim_{x \to \infty} \left(\frac{f(x)}{x}\right) = \lim_{x \to \infty} \left(\frac{\frac{x^3}{(x+3)^2}}{x}\right) = \lim_{x \to \infty} \left(\frac{x^2}{(x+3)^2}\right) = \lim_{x \to \infty} \left(\frac{x^2}{x^2+6x+9}\right) = \lim_{x \to \infty} \left(\frac{x^2}{x^2\left(1+\frac{6}{x}+\frac{9}{x^2}\right)}\right) =$

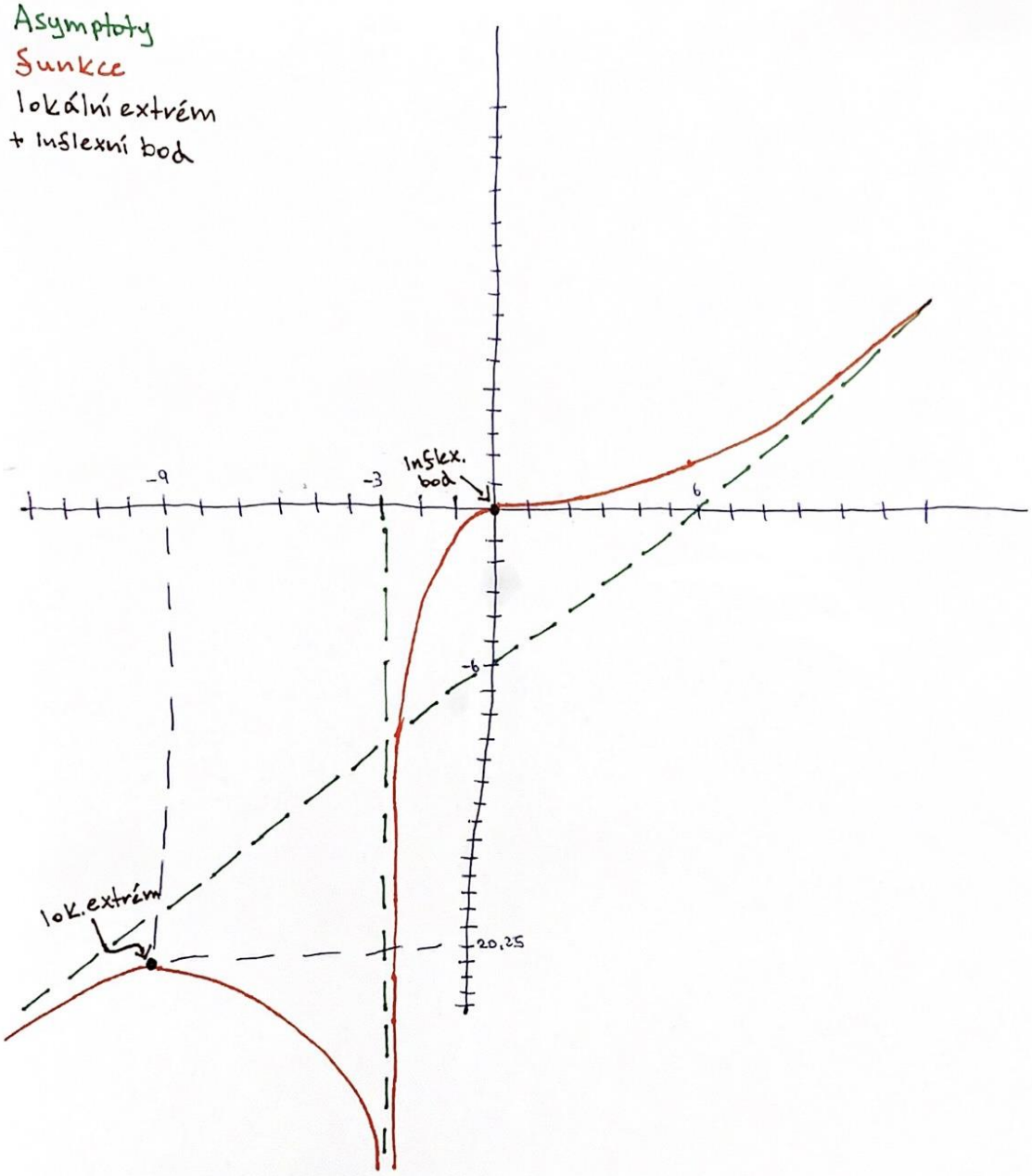
$= \lim_{x \to \infty} \left(\frac{1}{1+\frac{6}{x}+\frac{9}{x^2}}\right) = \frac{1}{1+0+0} = 1$ ← směrnice

$B = \lim_{x \to \infty} (f(x) - Ax) = \lim_{x \to \infty} \left(\frac{x^3}{(x+3)^2} - x\right) = \lim_{x \to \infty} \left(\frac{x^3 - (x^2+6x+9) \cdot x}{(x+3)^2}\right) =$

$= \lim_{x \to \infty} \left(\frac{x^3 - x^3 - 6x^2 - 9x}{(x+3)^2}\right) = \lim_{x \to \infty} \left(\frac{-6x^2-9x}{(x+3)^2}\right) = \lim_{x \to \infty} \left(\frac{-6x^2-9x}{x^2+6x+9}\right) = -\lim_{x \to \infty} \left(\frac{x^2\left(6+\frac{9}{x}\right)}{x^2\left(1+\frac{6}{x}+\frac{9}{x^2}\right)}\right) =$

$= -\lim_{x \to \infty} \left(\frac{6+\frac{9}{x}}{1+\frac{6}{x}+\frac{9}{x^2}}\right) = \frac{-6-0}{1+0+0} = -6$ $\quad y = x - 6$

Asymptoty
Funkce
lokální extrém
+ inflexní bod



Fiala