6. (P-Si)

$S(x) = 1 - \frac{1}{x-2}$, $q(x) = 1 + \frac{1}{1-x^2}$

$S \circ q = S(q(x)) = 1 - \frac{1}{1+\frac{1}{1-x^2}-2} = 1 - \frac{1}{\frac{1}{1-x^2}-1} = 1 - \frac{1}{\frac{1-1+x^2}{1-x^2}} = 1 - \frac{1-x^2}{x^2} = \frac{2x^2-1}{x^2}$

$= 2 - \frac{1}{x^2}$

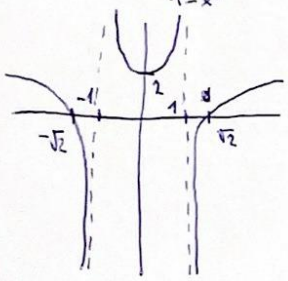
$D(S \circ q) = \mathbb{R} \setminus \{-1, 0, 1\}$

$q \circ S = q(S(x)) = 1 + \frac{1}{1-(1-\frac{1}{x-2})^2} = 1 + \frac{1}{1-1+\frac{2}{x-2}-\frac{1}{(x-2)^2}} = 1 + \frac{1}{\frac{2(x-2)-1}{(x-2)^2}} = 1 + \frac{(x-2)^2}{2(x-2)-1}$

$= 1 + \frac{(x-2)^2}{2x-5} = \frac{x^2-4x+4+2x-5}{2x-5} = \frac{x^2-2x-1}{2x-5}$

$D(q \circ S) = \mathbb{R} \setminus \{\frac{5}{2}, 2\}$

$q(x) = 1 + \frac{1}{1-x^2}$



$q(\langle -1, 1 \rangle) = \langle 2, \infty)$

$q^{-1}(\langle -3, -2 \rangle) = \langle -\frac{2\sqrt{3}}{3}, -\frac{\sqrt{5}}{2} \rangle \cup \langle \frac{\sqrt{5}}{2}, \frac{2\sqrt{3}}{3} \rangle$

$-3 = 1 + \frac{1}{1-x^2} \quad / \cdot (1-x^2)$

$-3 + 3x^2 = 1 - x^2 + 1$

$4x^2 = 5$

$x = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{2}$

$-2 = 1 + \frac{1}{1-x^2} \quad / \cdot (1-x^2)$

$-2 + 2x^2 = 1 - x^2 + 1$

$3x^2 = 4$

$x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

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