

## Assignment 2

### Trapezoidal decomposition, Delaunay triangulation

**Exercise 1.** Give an example of a collection of  $n$  segments and a bad insertion order for them, such that the incremental construction algorithm of the trapezoidal decomposition produces a graph  $G$  with many nodes—asymptotically as many as possible.

**Exercise 2.** We claimed that there exists a constant  $c$  such that, for every  $n$  and every collection of  $n$  segments, the probability that there will be a query point in the plane with query time larger than  $c \log n$  is at most  $O(1/n)$ . Finish the proof of this claim, by applying Chernoff's bound and the union bound.

**Exercise 3.** Write a computer program that reads as input a collection of  $n$  points and outputs its Delaunay triangulation, using the Randomized Incremental Construction (RIC) algorithm presented in class.

The program should read the input from a specified file. The file should contain  $2n + 1$  integers, where the first integer is  $n$ , and the remaining integers are the coordinates of the points  $x_1, y_1, \dots, x_n, y_n$  in this order. You can assume general position. You can also assume that all the points are contained in the triangle with vertices  $(0, 0)$ ,  $(10000, 0)$ ,  $(5000, 8660)$ , and that no circumcircle contains any of these points.

The Delaunay triangulation should be internally represented in a way that allows for efficient implementation of the algorithm. Provide a description of the representation you are using.

**Exercise 4.** Give an example of an  $n$ -point set such that, in the RIC Delaunay triangulation algorithm, the lookup of one of the points visits  $\Theta(n)$  nodes in expectation.