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Timor Sharabi

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1 Manhattan Distance Heuristic is Consistence

For a heuristic to be consistent, we must show that

$$h(n) \leq h(n') + c(n, n') \qquad \forall n, n'$$

in Manhattans distance we have 2 cases, after a move one less tile is out of position or one more tile is out of position, since we can move only the blank.

case 1: After a move, one more tile is out of position,$c(n, n') \geq 1$, therefore

$$\begin{aligned} h(n) &= h(n') - 1 \\ &\leq h(n') - 1 + c(n, n') \\ &\leq h(n') + c(n, n') \end{aligned}$$

case 2: After a move, one more tile is in position,$c(n, n') \geq 1$, therefore

$$\begin{aligned} h(n) &= h(n') + 1 \\ &\leq h(n') + c(n, n') \end{aligned}$$

2 Manhattan Distance Heuristic is Admissible

Moreover, we will prove that any heuristic which is consistent, must be admissible.

Definition 2.1. let $c(n, a, n')$ be the cost by moving from node n to node n' via operation a.

Assume h is a consistence heuristic function and let $k(n)$ be the cost of the cheapest path from n to the goal node.
We will prove by induction on the number of steps to the goal that $h(n) \leq k(n)$.
Base case: we already on the goal then we have more 0 steps from node n, then n is a goal. Hence,

$$h(n) = 0 \leq k(n).$$

Induction step:
If n is i steps away from the goal, there must exist some node n' that move one step from n, by operation a s.t. n' is on the optimal path from n to the goal (via operation a) and n' is $i-1$ steps away from the goal.
Therefore(by assuming h is consistence),

$$h(n) \leq c(n, a, n') + h(n')$$

But by the induction hypothesis, we know that $h(n') \leq k(n')$. Therefore,

$$h(n) \leq c(n, a, n') + k(n') = k(n)$$

Since n' is on the optimal path from n to the goal via operation a.