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Course Code	:	**BCS-012**
Course Title	:	**Basic Mathematics**
Assignment Number	:	**BCA(1)012/Assignment/2022-23**
Maximum Marks	:	**100**
Weightage	:	**25%**
Last Date of Submission	:	**31st October, 2022 (For July Session)** **15th April, 2023 (For January Session)**

Note: This assignment has 15 questions of 80 marks (Q.no.1 to 14 are of 5 marks each, Q15 carries 10 marks). Answer all the questions. Rest 20 marks are for viva voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation.

Q1. Solve the following system of equations by using Matrix Inverse Method.

1. $3x+4y+7z=14$
2. $2x-y+3z=4$
3. $2x+2y-3z=0$

Q2. Use principle of Mathematical Induction to prove that:
$\frac{1}{1*2}+\frac{1}{2*3}+\dots\dots\dots\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Q3. How many terms of G.P $\sqrt{3}$, 3 , $3\sqrt{3}$, ... Add up to 39

Q4. If $y = a.e^{mx} + b.e^{-mx}$, Prove that $d^2y/dx^2 = m^2 y$

Q5. For what value of 'k' the points $(-k + 1, 2k)$, $(k, 2 - 2k)$ and $(-4 - k, 6 - 2k)$ are collinear.

Q6. Evaluate $\int \frac{x\,dx}{[(x+1)(2x-1)]}$ and $\int \frac{dx}{(e^x-1)^2}$

Q7. If 1, w, w^2 are Cube Roots of unity show that $(1+w)^2 - (1+w)^3 + w^2 = 0$.

Q8. If α, β are roots of equation $2x^2-3x-5=0$ form a Quadratic equation whose roots are α^2, β^2

Q9. Solve the inequality $\frac{3}{5}(x-2) \le \frac{5}{3}(2-x)$ and graph the solution set.

Q10. A spherical ballon is being Inflated at the rate of 900 cm^3/sec. How fast is the Radius of the ballon Increasing when the Radius is15 cm.

Q11. Find the area bounded by the curves $x^2 = y$ and $y=x$.

Q12. Determine the values of x for which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing and for which it is decreasing.

Q13. Using integration, find length of the curve $y = 3 - x$ from (-1, 4) to (3, 0).

Q14. Show that the lines $\frac{x-5}{4}=\frac{y-7}{-4}=\frac{z-3}{-5}$ and $\frac{x-8}{4}=\frac{y-4}{-4}=\frac{z-5}{4}$ Intersect.

Q15. A manufacturer makes two types of furniture, chairs and tables. Both the products are processed on three machines A1, A2 and A3. Machine A1 requires 3 hours for a chair and 3 hours for a table, machine A2 requires 5 hours for a chair and 2 hours for a table and machine A3 requires 2 hours for a chair and 6 hours for a table. The maximum time available on machines A1, A2 and A3 is 36 hours, 50 hours and 60 hours respectively. Profits are \$ 20 per chair and \$ 30 per table. Formulate the above as a linear programming problem to maximize the profit and solve it.

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Q1 Solve the following System of equations by using Matrix inverse Method.

$1 \cdot 3x + 4y + 7z = 14 \quad 2 \cdot 2x - y + 3z = 4 \quad 3 \cdot 2x + 2y - 3z = 0$

Solution

Here $3x + 4y + 7z = 14$

$2x - y + 3z = 4$

$2x + 2y - 3z = 0$

Now converting given equations into matrix from

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

Now, $A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 2 & 2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and

$$B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$\therefore AX = B$

$\therefore X = A^{-1}B$

$$|A| = \begin{vmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 2 & 2 & -3 \end{vmatrix}$$

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$3(3-6)-4(-6-6)+7(4+2)$

$3(-3)-4(-12)+7(6)$

$-9+48+42$

81

Here $|A| = 81 \neq 0$

$\therefore A^{-1}$ is possible

$$Adj(A) = adj \begin{vmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 2 & 2 & -3 \end{vmatrix}$$

$$\begin{bmatrix} +\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} \\ -\begin{vmatrix} 4 & 7 \\ 2 & -3 \end{vmatrix} & +\begin{vmatrix} 3 & 7 \\ 2 & -3 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} \\ +\begin{vmatrix} 4 & 7 \\ -1 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & 7 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} \end{bmatrix}^T$$

$$\begin{bmatrix} +(3-6) & -(-6-6) & +(4+2) \\ -(-12-14) & +(-9-14) & -(6-8) \\ +(12+7) & -(9-14) & +(-3-8) \end{bmatrix}^T$$

$$\begin{bmatrix} -3 & 12 & 6 \\ 26 & -23 & 2 \\ 19 & 5 & -11 \end{bmatrix}^T$$

$$\begin{bmatrix} -3 & 26 & 19 \\ 12 & -23 & 5 \\ 6 & 2 & -11 \end{bmatrix}$$

Here, $X = A^{-1} B$

$$\therefore X = \frac{1}{81} \begin{bmatrix} -3 & 26 & 19 \\ 12 & -23 & 5 \\ 6 & 2 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$\frac{1}{81} \begin{bmatrix} -3 \times 14 + 26 \times 4 + 19 \times 0 \\ 12 \times 14 - 23 \times 4 + 5 \times 0 \\ 6 \times 14 + 2 \times 4 - 11 \times 0 \end{bmatrix}$$

$$\frac{1}{81} \begin{bmatrix} 62 \\ 76 \\ 92 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 62/81 \\ 76/81 \\ 92/81 \end{bmatrix}$$ Answer

Q2 Use Principle of mathematical Induction to prove that :

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \_\_\_ + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Solution

let; $P(n)$: $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \ldots + \frac{1}{n(n+1)}$

$= \frac{n}{n+1}$

Putting $n = 1$

LHS : $\frac{1}{1 \times 2} = \frac{1}{2}$

RHS $\frac{1}{1+1} = \frac{1}{2}$

$\therefore$ for $n=1$, LHS = RHS

Therefore, $P(n)$ is true for $n=1$

Now, assuming $P(K)$ is true for any value of K, where $K \in N$

i.e $P(K)$ $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \_\_\_ + \frac{1}{K(K+1)}$

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$$\frac{K}{K+1} \quad \text{—(1)}$$

Now, we shal prove that $P(K+1)$ is true whenever $P(K)$ is true.

LHS: $P(K+1) = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \text{---} + \frac{1}{(K+1)[(K+1)+]}$

$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \text{---} + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)}$$

$$\frac{K}{K+1} + \frac{1}{(K+1)(K+2)} \quad \text{[using equations (1)]}$$

$$\frac{K(K+2)+1}{(K+1)(K+2)}$$

$$\frac{K^2+2K+1}{(K+1)(K+2)}$$

$$\frac{(K+1)^2}{(K+1)(K+2)}$$

$$\frac{K+1}{K+2}$$

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RHS

$P(K+1) = \frac{K+1}{(K+1)+1}$

$\frac{K+1}{K+2}$

$\therefore$ for $n=K+1$, LHS = RHS

Hence by the principale of mathematical induction true for all $n \in N$ (Proved)

Q3 How many terms of G.P $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ addupto 39

Solution

Given

Sum of G.P. $= 39 + 13\sqrt{3}$

Where, $a = \sqrt{3}, r = \frac{3}{\sqrt{3}} = \sqrt{3}, n = ?$

By using the formula,

Sum of GP to n terms $S_n = \frac{a(r^n - 1)}{r-1}$ $(r > 1)$

$39 + 13\sqrt{3} = \frac{\sqrt{3}((\sqrt{3})^n - 1)}{\sqrt{3} - 1}$

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$$13(3+\sqrt{3})(\sqrt{3}-1) = \sqrt{3}((\sqrt{3})^n - 1)$$
$$13\sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1) = \sqrt{3}((\sqrt{3})^n - 1)$$
$$13(\sqrt{3}^2 - 1) = ((\sqrt{3})^n - 1)$$
$$13(3-1) = ((\sqrt{3})^n - 1)$$
$$26 = ((\sqrt{3})^n - 1)$$
$$(\sqrt{3})^n = 26+1 = 27 = 3^3 = (\sqrt{3})^6$$
$$\therefore n = 6$$

Hence, 6 terms are required to make a sum of $39 + 13\sqrt{3}$

Q4 If y = a. emx + b.e-mx, Prove that d2y/dx2 = m2y

Solution

$y = ae^{mx} + be^{-mx}$

Derivative with respect to (w.r.t) x, we have

$$\frac{dy}{dx} = \frac{d}{dx}(ae^{mx} + be^{-mx})$$

$$a\frac{d}{dx}(e^{mx}) + b\frac{d}{dx}(e^{-mx})$$

$$ame^{mx} - bme^{-mx}$$

$$m(ae^{mx} - be^{-mx})$$

$$\therefore \frac{dy}{dx} = m(ae^{mx} - be^{-mx})$$

Again, derivative w.r.t. x, we have

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$$\frac{d^2y}{dx^2} = m\frac{d}{dx}(ae^{mx} - be^{-mx})$$

$$m\left[a\frac{d}{dx}(e^{mx}) - b\frac{d}{dx}(e^{-mx})\right]$$

$$m(ame^{mx} + bme^{-mx})$$

$$m^2(ae^{mx} + be^{-mx})$$

$$m^2y \quad [\because y = ae^{mx} + be^{-mx})$$

$$\therefore \frac{d^2y}{dx^2} = m^2y \quad \text{[Hence proved)}$$

Q5 For What Value of K the points $(-K+1, 2K)$, $(K, 2-2K)$ and $(-4-K, 6-2K)$ are Collinear-

Solution :

Since $(-K+1, 2K)$, $(K, 2-2K)$ and $(-4-K, 6-2K)$ are Collinear:

$$\therefore \begin{vmatrix} -K+1 & 2K & 1 \\ K & 2-2K & 1 \\ -4-K & 6-2K & 1 \end{vmatrix} = 0$$

$$(-K+1)[(2-2K)-(6-2K)] - 2K[K-(-4-K)] + 1[K(6-2K) - (2-2K)(-4-K)] = 0$$

$$(-K+1)(2-2K-6+2K) - 2K(K+4+K) + 6K - 2K^2 + 8 + 2K - 8K - 2K^2) = 0$$

$$(-K+1)(-4) - 2K(2K+4) + (8-4K^2) = 0$$

$$4K - 4 - 4K^2 - 8K + 8 - 4K^2 = 0$$

$$-8K^2 - 4K + 4 = 0$$

$$-4(2K^2 + K - 1) = 0$$

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$$[2K(K+1)-(K+1)] = 0$$
$$(K+1)(2K-1) = 0$$
$$\therefore K = -1, \frac{1}{2} \quad \text{Ans}$$

Q6 Evaluate $\int \frac{x\,dx}{[(x+1)(2x-1)]}$ and $\int \frac{dx}{(ex-1)2}$

$$\int \frac{x\,dx}{(x+1)(2x-1)}$$
$$\frac{1}{3}\int \frac{3x\,dx}{(x+1)(2x-1)}$$
$$\frac{1}{3}\int \left[\frac{(2x-1)}{(x+1)(2x-1)} + \frac{(x+1)}{(x+1)(2x-1)}\right] dx$$
$$\frac{1}{3}\int \left(\frac{1}{x+1} + \frac{1}{2x-1}\right) dx$$
$$\frac{1}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{2x-1}$$
$$\frac{1}{3}\ln|x+1| + \frac{1}{3}\frac{\ln|2x-1|}{2} + C$$
$$\frac{1}{3}\ln|x+1| + \frac{1}{6}\ln|2x-1| + C \quad \text{Ans}$$

$$\int \frac{dx}{(e^x-1)^2}$$

let, $e^x - 1 = z$

Differentiating w.r.t. x, we have

$$e^x\,dx = dz$$

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$(z+1)\,dx = dz \quad [\because e^x - 1 = z]$

$\therefore dx = \frac{dz}{z+1}$

Now, on Subsituting, the intergral becomes

$$= \int \frac{dz}{z^2(z+1)}$$

$$\int \frac{z^2 - (z^2-1)}{z^2(z+1)}\,dz$$

$$\int \frac{z^2}{z^2(z+1)}\,dz - \int \frac{z^2-1}{z^2(z+1)}\,dz$$

$$\int \frac{dz}{z+1} - \int \frac{z-1}{z^2}\,dz$$

$$\int \frac{dz}{z+1} - \int \frac{z}{z^2}\,dz + \int \frac{dz}{z^2}$$

$$\int \frac{dz}{z+1} - \int \frac{dz}{z} + \int z^{-2}\,dz$$

$$\ln|z+1| - \ln|z| + \frac{z^{-2+1}}{-2+1} + c$$

$$\ln\left|\frac{z+1}{z}\right| - \frac{1}{z} + c$$

$$\ln\left|\frac{e^x}{e^x-1}\right| - \frac{1}{e^x-1} + c \quad \text{Ans}$$

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Q7 If 1, W, W_2 are cube roots of unity show that $(1+W)^2 - (1+W)3 + W2 = 0$

Solution
Since 1, w, w^2 are cube roots of unity
$\therefore W^3 = 1$ — (I)
and $1 + w + w^2 = 0$ (II)
Now; $(1+w)^2 - (1+w)^3 + w^2$
$w^4 + w^6 + w^2$
$(w^3).w + (w^3)^2 + w^2$
$1.w + (1)^2 + w^2$ [From equation (I)]
$= w + 1 + w^2$
$= 0$ [from equation (II)]
Hence Proved

Q8 If $a\beta$, are roots of equation $2x^2 - 3x - 5 = 0$ from a Quadratic equation whose roots are $a2$, $\beta 2$

Solution

Since a, β are root of equation $2x^2 - 3x - 5 = 0$
$\therefore$ Sum of root, $a + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$
$= -\frac{(-3)}{2} = \frac{3}{2}$

and Product of roots, $a\beta = \frac{\text{constant}}{\text{coefficient of } x^2}$

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$= \frac{(-5)}{2} = -\frac{5}{2}$

Now, $a^2 + \beta^2 = (a+\beta)^2 - 2a\beta$

$\left(\frac{3}{2}\right)^2 - 2\left(-\frac{5}{2}\right)$

$\frac{9}{4} + 5 = \frac{29}{4}$ —(I)

and $a^2 \times \beta^2 \Rightarrow (a\beta)^2 = \left(-\frac{5}{2}\right)^2 = \frac{25}{4}$ —(II)

By using the formula of quadratic equation ~~eq~~ we must have

The quadratic equation,

By using the formula of quadratic equation we must have

the quadratic equations.

$x^2 - (\text{Sum of roots})\, x + (\text{Product of roots}) = 0$

Since, the required equation have two roots which are a^2, β^2

$\therefore x^2 - (a^2 + \beta^2)\, x + (a^2 \times \beta^2) = 0$

$x^2 - (29/4)x + (25/4) = 0$ [from equation (I) and (II)]

$\therefore 4x^2 - 29x + 25 = 0$

Ans

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$$\frac{dV}{dt} = 900\ cm^3/s$$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 900$$

$$\frac{4}{3}\pi \frac{d}{dt}(r^3) = 900$$

$$\frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = 900$$

$$\frac{4}{3}\pi \times 3(15^2)\frac{dr}{dt} = 900 \quad [\because r = 15\ cm]$$

$$4 \times 15^2 \pi \cdot \frac{dr}{dt} = 900$$

$$\frac{dr}{dt} = \frac{900}{4 \times 15 \times 15 \times \pi}$$

$$\therefore \frac{dr}{dt} = \frac{1}{\pi}\ cm/s$$

Hence, the radius of the ballon incresing at the rate of $1/\pi$ cm/s when radius is 15 cm

Ans

Q9 Solve the inequality ~~x~~ $35(x-2) \leq 53(2-x)$ and graph the Solution Set.

Given

$$\frac{3}{5}(x-2) \leq \frac{5}{3}(2-x)$$

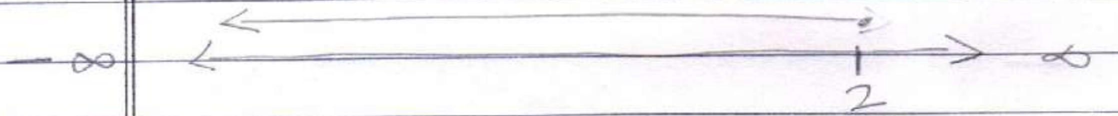
$$9(x-2) \leq 25(2-x)$$

$$9x - 18 \leq 50 - 25x$$

$$25x + 9x \leq 50 + 18$$

$$34x \leq 68$$

$$x \leq 2$$



$\therefore$ Solution Set, $x \in (-\infty, 2]$ Ans

10 A Spherical ballon is being inflated at the rate of $900\ cm^3/sec$. How fast is the radius of the ballon increasing when the radius is 15 cm.

The Volume of a Sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$ Rate of Change of Vol. (V) with respect to time (t) is given by,

$$\int_0^1 x\,dx - \int_0^1 x^2\,dx$$

$$\left[\frac{x^{1+1}}{1+1}\right]_0^1 - \left[\frac{x^{2+1}}{2+1}\right]_0^1$$

$$\frac{1}{2}\left[x^2\right]_0^1 - \frac{1}{3}\left[x^3\right]_0^1$$

$$\frac{1}{2}(1-0) - \frac{1}{3}(1-0)$$

$$\frac{1}{2} - \frac{1}{3}$$

$\frac{1}{6}$ Sq. unit Ans

Q12 Determine the value of x for which $f(x) = x4 - 8x3 + 22x2 - 24x + 21$ is increasing and for which it is ~~deter~~ ~~deter~~ decreasing.

Sol

Given:

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

Differentiating w.r.t. x, We have

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

$4(x^3 - 6x^2 + 11x - 6)$

$4(x-1)(x^2 - 5x + 6)$

$4(x-1)(x-3)(x-2)$

Finding Critical Point:

To get the Critical Points, we must have

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$f'(x) = 0$

$\therefore 4(x-1)(x-3)(x-2) = 0$

$\therefore x = 1, 2, 3$

Therefore, the Possible intervals are :
$(-\infty, 1)$ $(1, 2)$ $(2, 3)$, and $(3, \infty)$

Now, Checking the Sign.

Function	$(-\infty, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
$(x-1)$	$-ve$	$+ve$	$+ve$	$+ve$
$(x-2)$	$-ve$	$-ve$	$+ve$	$+ve$
$(x-3)$	$-ve$	$-ve$	$-ve$	$+ve$
$f'(x) = 4(x-1)(x-3)(x-2)$	$-ve$	$+ve$	$-ve$	$+ve$

from the above table, We have
$f'(x)$ is $+ve$ on the interval $(1, 2)$ and $(3, \infty)$
Hence, $f(x)$ is increasing on $(1, 2) \cup (3, \infty)$

Also we have
$f'(x)$ is $-ve$ on interval $(-\infty, 1)$ and $(2, 3)$
Hence, $f(x)$ is decreasing on $(-\infty, 1) \cup (2, 3)$

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Q13 Using intergration; find length of the curvey $y=3-x$ from $(-1,4)$ to $(3,0)$

A- Solution

$y = 3 - x$

Differentiating w.r.t. x we have

$\frac{dy}{dx} = -1$

$\therefore$ the required length of the curve $y=3-x$ is given by

$$\int_{-1}^{3} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

~~$\therefore$ the required length of the~~

$$\int_{-1}^{3} \sqrt{1+(-1)^2}\, dx \quad \left[\because \frac{dy}{dx} = -1\right]$$

$$\int_{-1}^{3} \sqrt{2}\, dx$$

$$\sqrt{2}\int_{-1}^{3} dx$$

$$\sqrt{2}\,[x]_{-1}^{3}$$

$$\sqrt{2}(3-(-1))$$

$$\sqrt{2}(3+1) \Rightarrow 4\sqrt{2} \text{ unit Ans}$$