22. Searching a List

Topics:

- Linear Search
- Binary Search
- Measuring Execution Time
- The Divide and Conquer Framework

Search

Examples:

Is this song in that playlist?

Is this number in that phone book?

Is this name in that phone book?

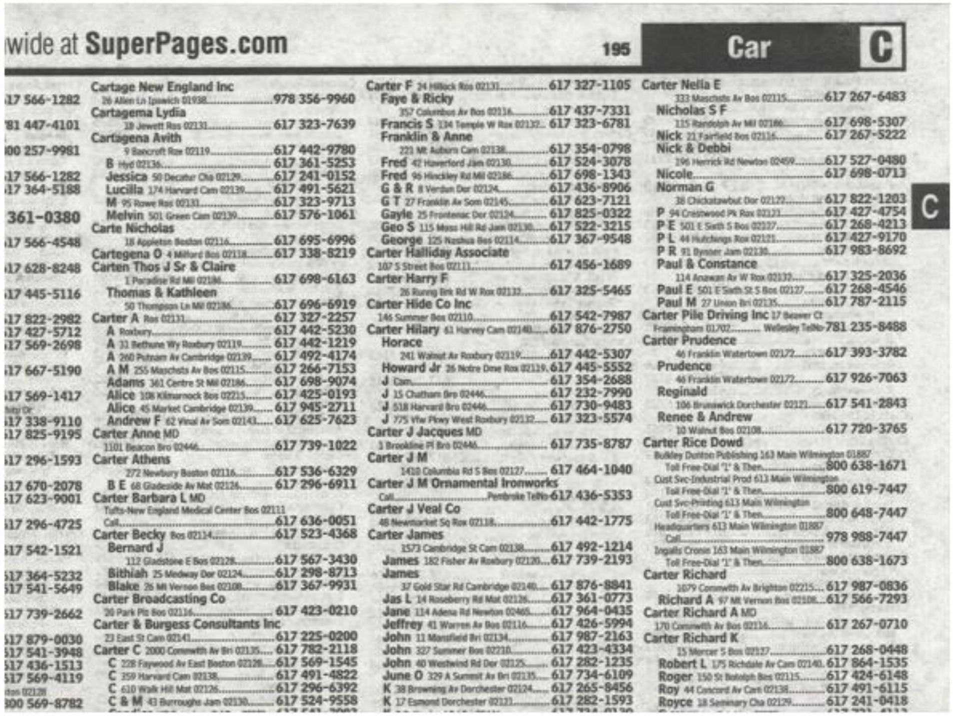
Is this fingerprint in that archive of fingerprints?

Is this photo in that yearbook?

More on Using Phone Books

The Manhatten phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a <u>tiny</u> , <u>tiny</u> fraction of those entries?



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Cartage New England Inc 26 Allen Ln Ipswich 01938	978 356-9960
Cartagema Lydia 18 Jewett Ros 02131	617 323-7639
Cartagena Avith 9 Bancroft Rox 02119	617 442-9780
B Hyd 02136	617 361-5253
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M 95 Rowe Ros 02131	617 323-9713
Melvin 501 Green Cam 02139	617 576-1061
Carte Nicholas 18 Appleton Boston 02116	617 695-6996
Cartegena O 4 Milford Bos 02118	617 338-8219
Carten Thos J Sr & Claire 1 Paradise Rd Mil 02186	617 698-6163
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A Roxbury	617 442-5230
A 31 Bethune Wy Roxbury 02119	617 442-1219
A 260 Putnam Av Cambridge 02139	617 492-4174
A M 255 Maschsts Av Bos 02115	617 266-7153
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Cust Svc-Printing 613 Main Wilmington Toll Free-Dial "1" & Then	800 648-7447
Headquarters 613 Main Wilmington 01887 Call	978 988-7447
Ingalls Cronin 163 Main Wilmington 01887 Toll Free-Dial "1" & Then	800 638-1673
Carter Richard 1079 Commwlth Av Brighton 02215	617 987-0836
Richard A 97 Mt Vernon Bos 02108	617 566-7293
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Roger 150 St Botolph Bos 02115	617 424-6148
Roy 44 Concord Av Cam 02138	617 491-6115
Royce 18 Seminary Cha 02129	617 241-0418

C

There must be a great search algorithm behind the scenes.

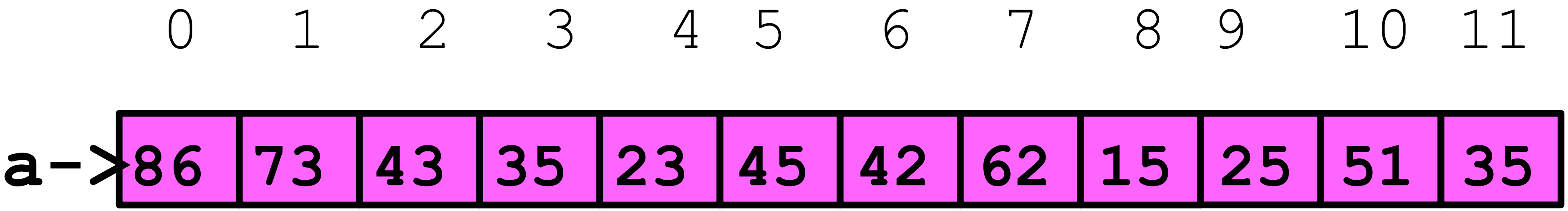
Linear Search

LinSearch: The Spec

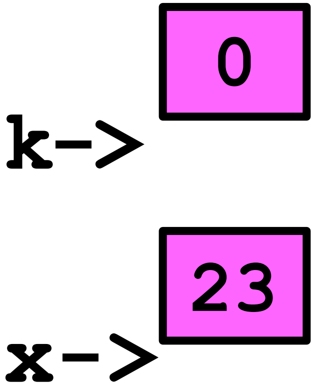
```
def LinSearch(x,a):
    """ Returns an int k with the
    property that a[k]==x is True.
    If no such k exists, then
    k==-1.

    PreC: a is a nonempty list of
    ints and x is an int.
    """
```

Could also apply the same ideas for searching a list of strings.

Linear Search



0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35



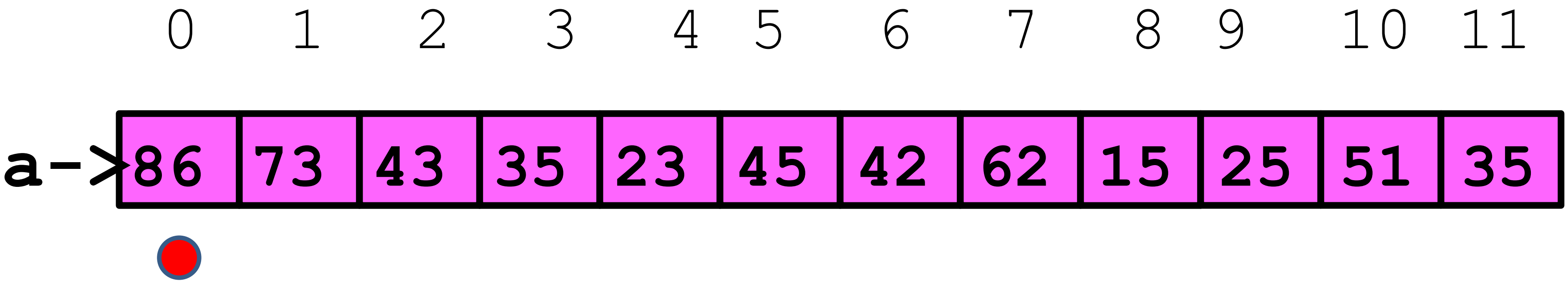
k-> 0

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Walk down the list looking for a match

Linear Search



	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

k-> 0

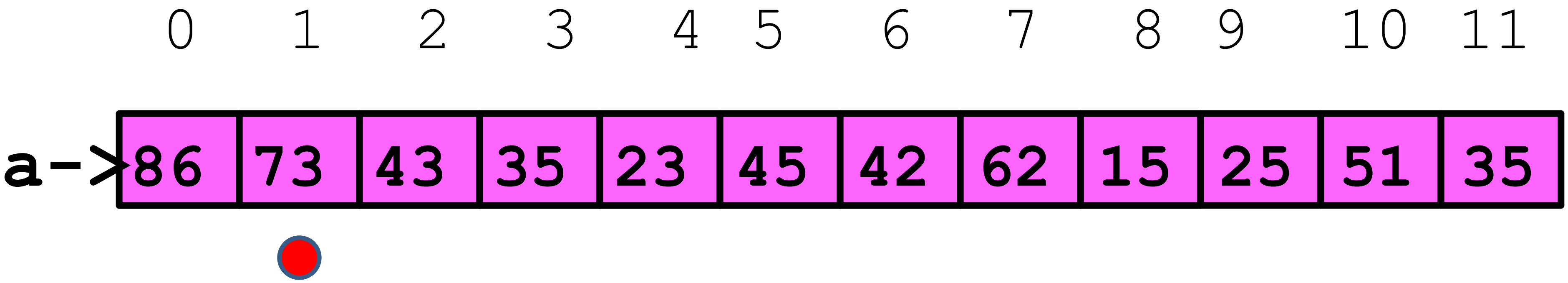
x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

Walk down the list looking for a match

Linear Search



0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35

k-> 1

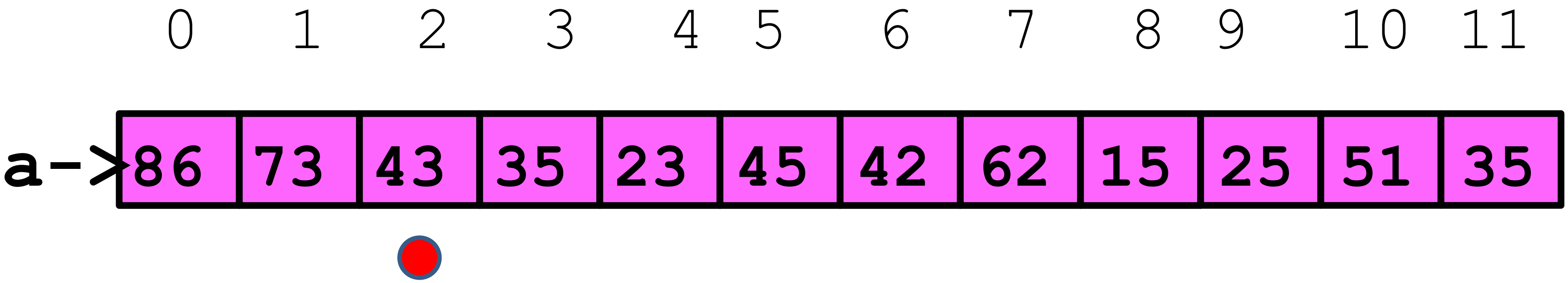
x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Nope

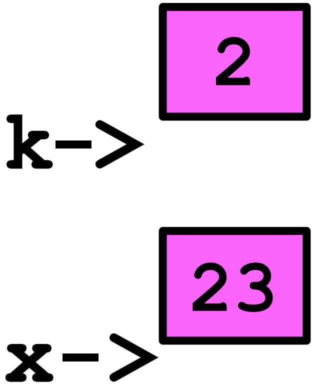
Walk down the list looking for a match

Linear Search



a->

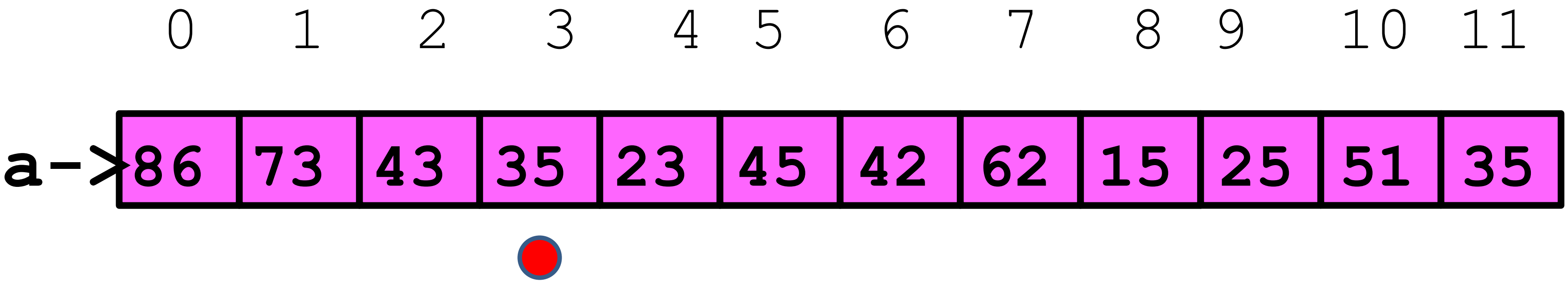
0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35



```
def  LinSearch(x,a):
   for k in range(len(a)):
       if x == a[k]:  Nope
          return k
   return -1
```

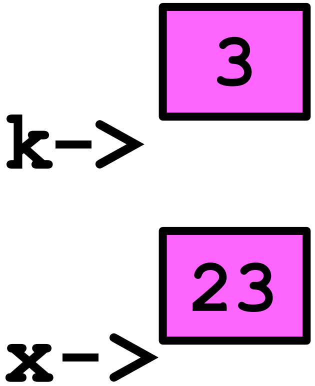
Walk down the list looking for a match

Linear Search



a->

0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35



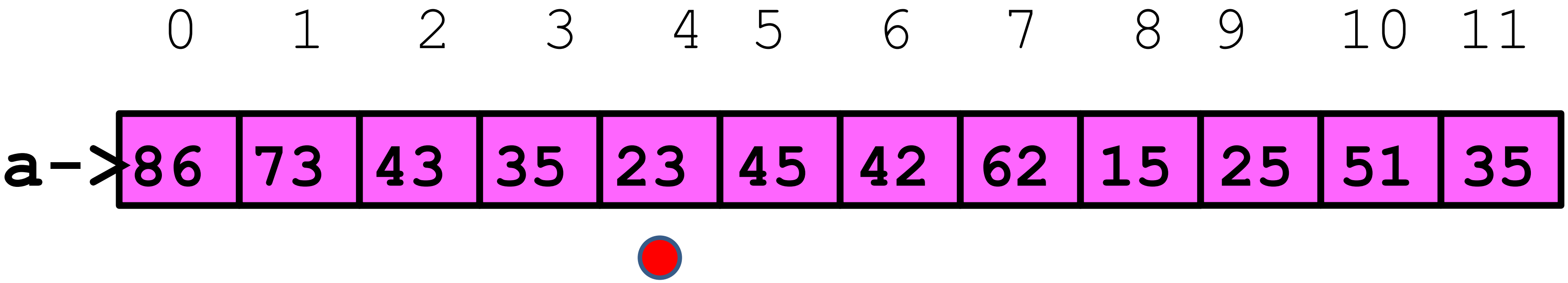
k-> 3

x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:  Nope
            return k
    return -1
```

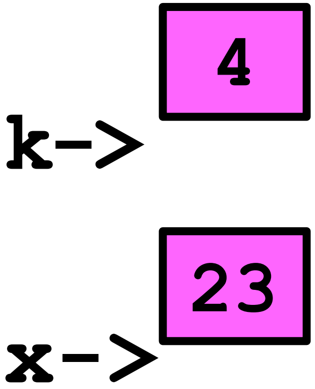
Walk down the list looking for a match

Linear Search



a->

0	1	2	3	4	5	6	7	8	9	10	11
86	73	43	35	23	45	42	62	15	25	51	35



k-> 4

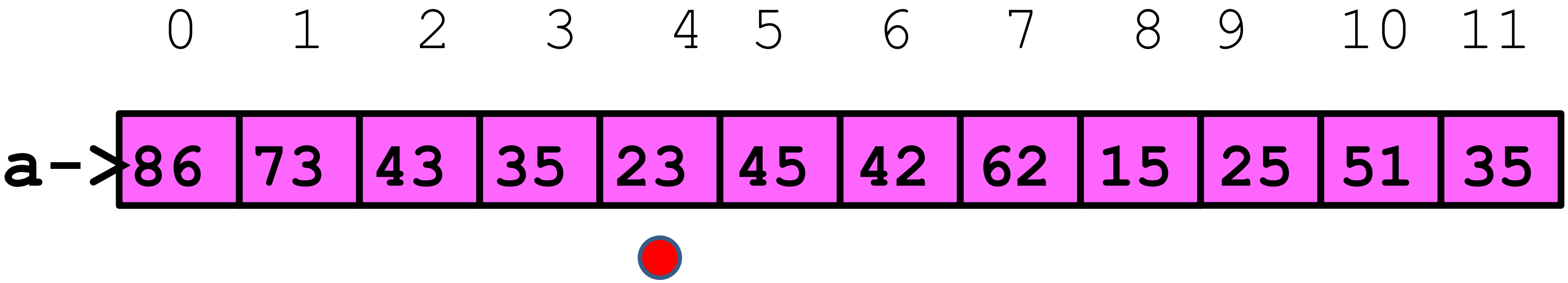
x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

Yup

Walk down the list looking for a match

Linear Search



	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

k-> 4

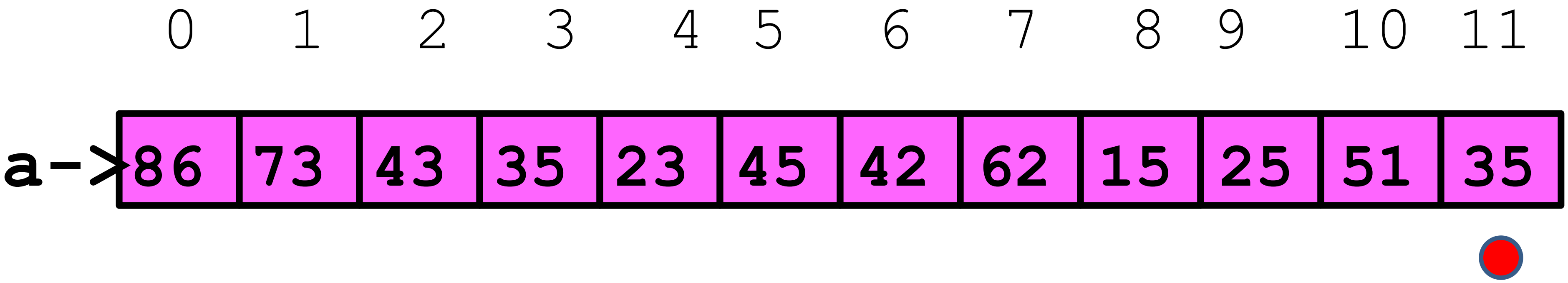
x-> 23

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

All done

Walk down the list looking for a match

Linear Search: No Match Case



	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35

k-> 11

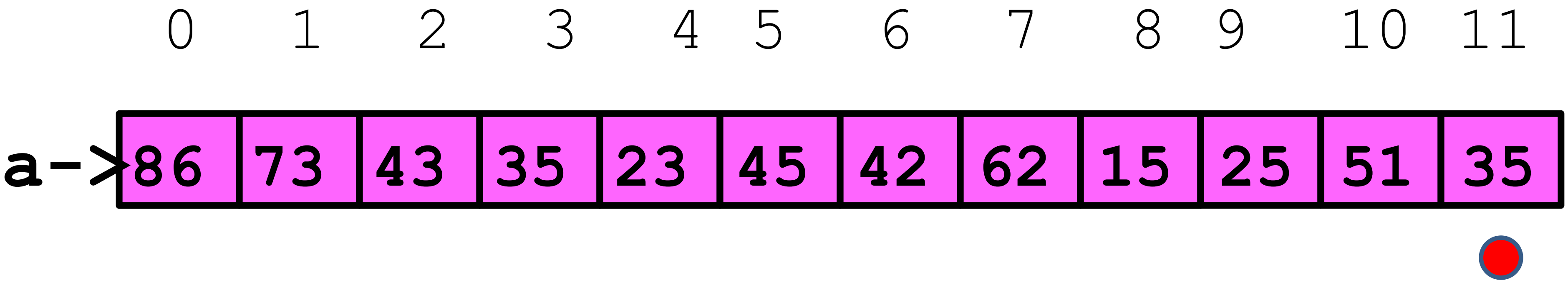
x-> 7

```
def  LinSearch(x,a):
    for k in range(len(a)):
        if x == a[k]:
            return k
    return -1
```

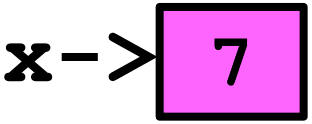
Nope

Walk down the list looking for a match

Linear Search: No Match Case



	0	1	2	3	4	5	6	7	8	9	10	11
a->	86	73	43	35	23	45	42	62	15	25	51	35
												●



x-> 7

```
def  LinSearch(x,a):
   for k in range(len(a)):
       if x == a[k]:
          return k
   return -1
```

Yup

Return -1 if no match

Linear Search: While Implementation

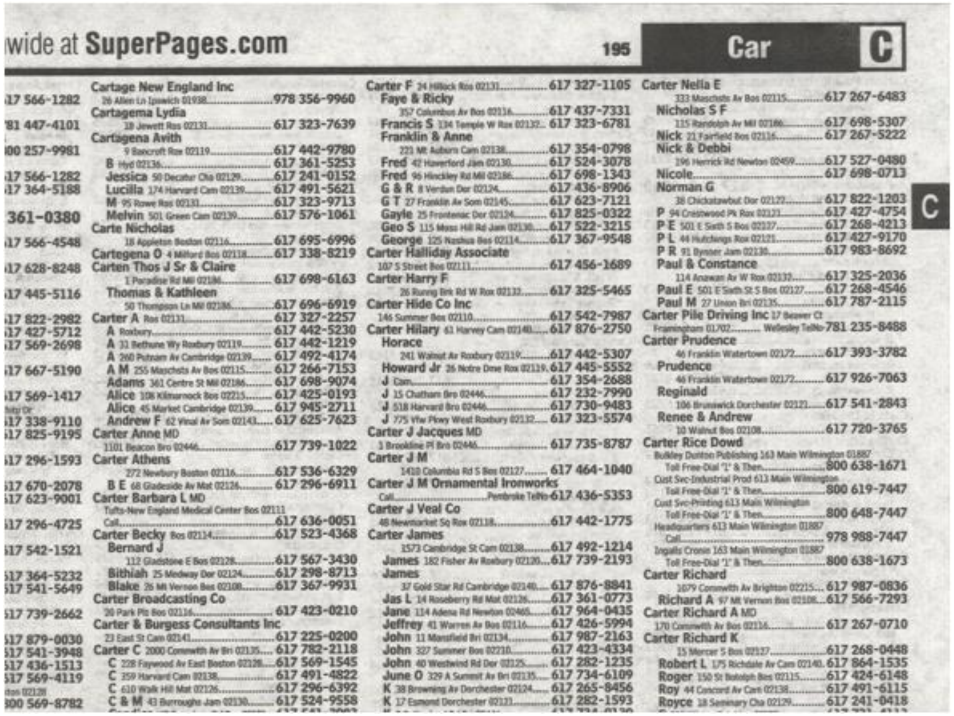
```
def  LinSearchW(x,a):
   k=0
   while k<len(a) and a[k]!=x:
       k+=1
   if k==len(a):
       return -1
   else:
       return k
```

Binary Search

Now we assume that the list
to be searched is sorted
from little to big.

```
a = [10,20,40,60,90]

a = [‘brown’,’dog’,’fox’,‘lazy’,‘quick’,’the’]
```

Back to Using Phone Books



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Toll Free-Dial "1" & Then..........800 638-1673
Carter Richard
1079 Commwlth Av Brighton 02215..........617 987-0836
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C

The Ithaca phone book has 10,000+ entries.

The Manhatten phone book has 1,000,000+ entries. But it does not take 100 x longer to look something up. Why?

Key Idea: Repeated Halving

To find Derek Jeter's number…

```
B = phone book
while (B is longer than 1 page):
    1. P = middle page of B
    2. Let Q be the first name on P
    3. if  'Jeter" comes before Q:
           Rip away the 2nd half of B
       else:
           Rip away the 1st half of B.

Scan remaining page P line-by-line for 'Jeter'
```

What Happens to Phone Book Length?

Original:	3000 pages
After 1 rip:	1500 pages
After 2 rips:	750 pages
After 3 rips:	375 pages
After 4 rips:	188 pages
After 5 rips:	94 pages
After 12 rips:	1 page

Binary Search

The idea of repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with approximately $\log_2 n$ comparisons.

$\log_2 8 = 3$ $\log_2 64 = 6$ $\log_2 2$**k = k

What is $\log_2(n)$?

n	ceil($\log_2(n)$)
10	4
100	7
1000	10
10000	14
100000	17
1000000	20

BinSearch: The Spec

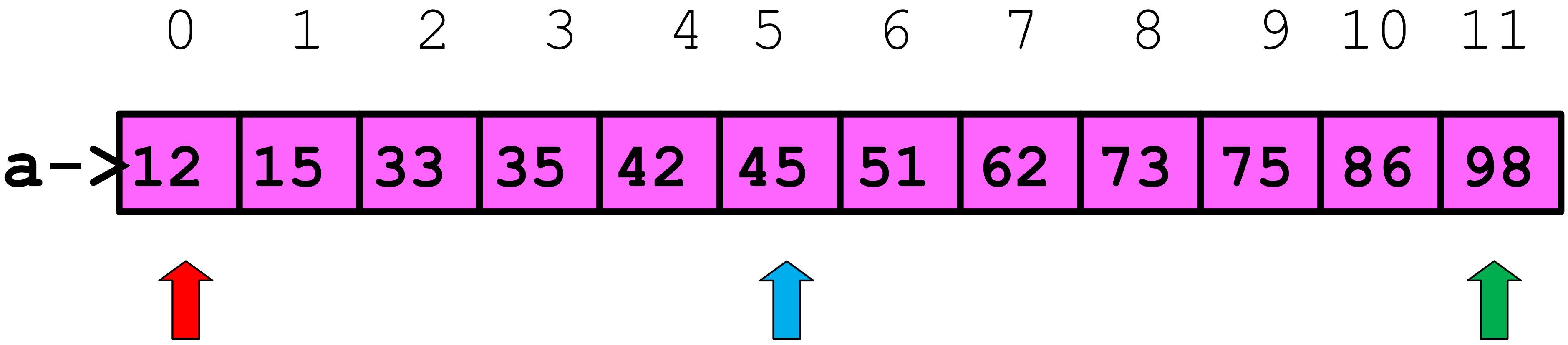
```
def BinSearch(x,a):
    """ Returns an int k with the
    property that a[k]==x is True.
    If no such k exists, then
    k==-1.

    PreC: a is a nonempty list of
    ints that is sorted from smallest
    to largest. x is an int.
    """
```

Example:
Does this List have an Element With Value Equal to 70?

0	1	2	3	4	5	6	7	8	9	10	11
12	**15**	**33**	**35**	**42**	**45**	**51**	**62**	**73**	**75**	**86**	**98**

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

R: 11

x: 70

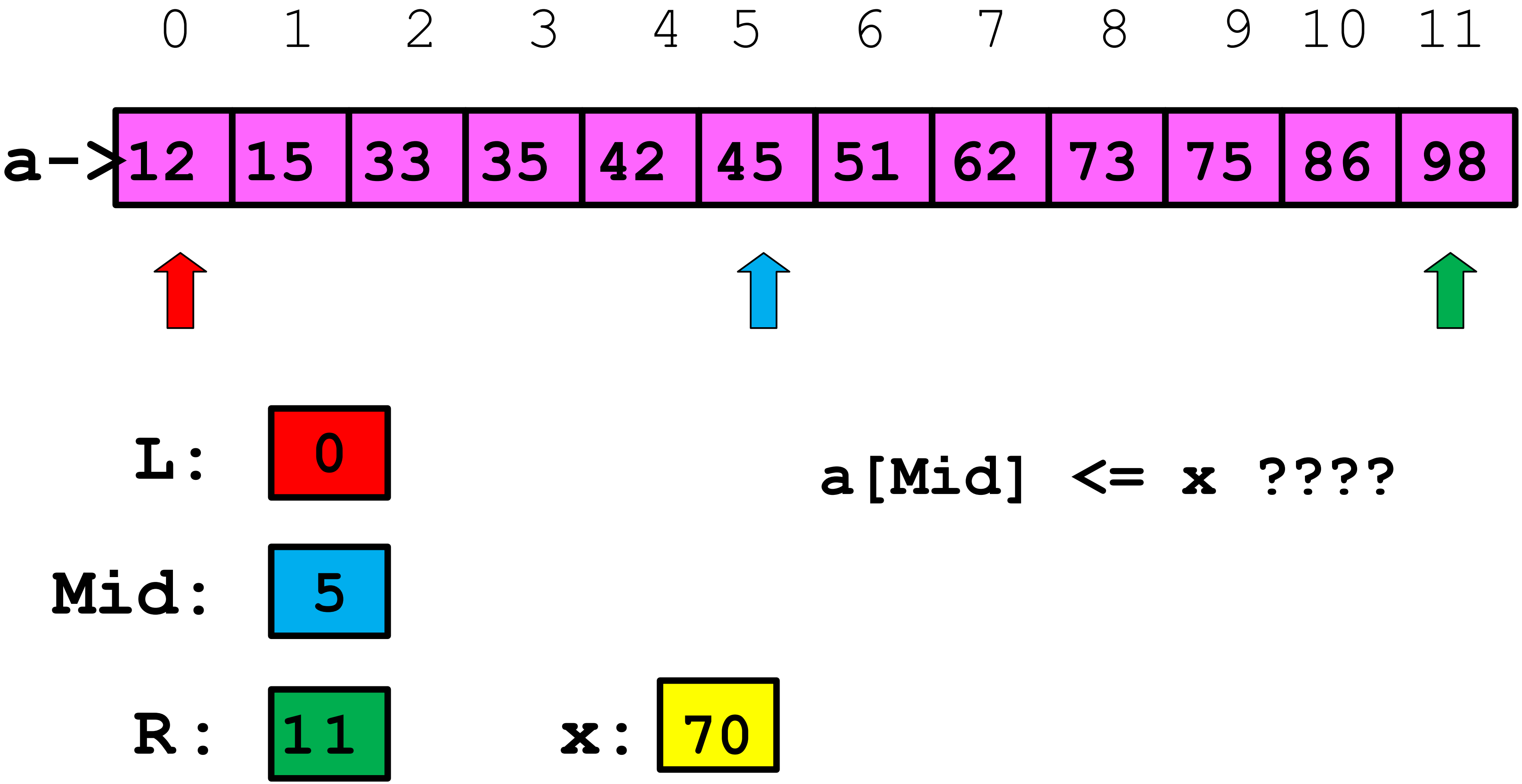
a[Mid] <= x ????

Mid = (L+R)/2

The Midpoint Computations

L	R	(L+R)/2
0	11	5
2	6	4
1	100	50

Let's Look For x in a



a->

0	1	2	3	4	5	6	7	8	9	10	11
12	15	33	35	42	45	51	62	73	75	86	98

L: 0

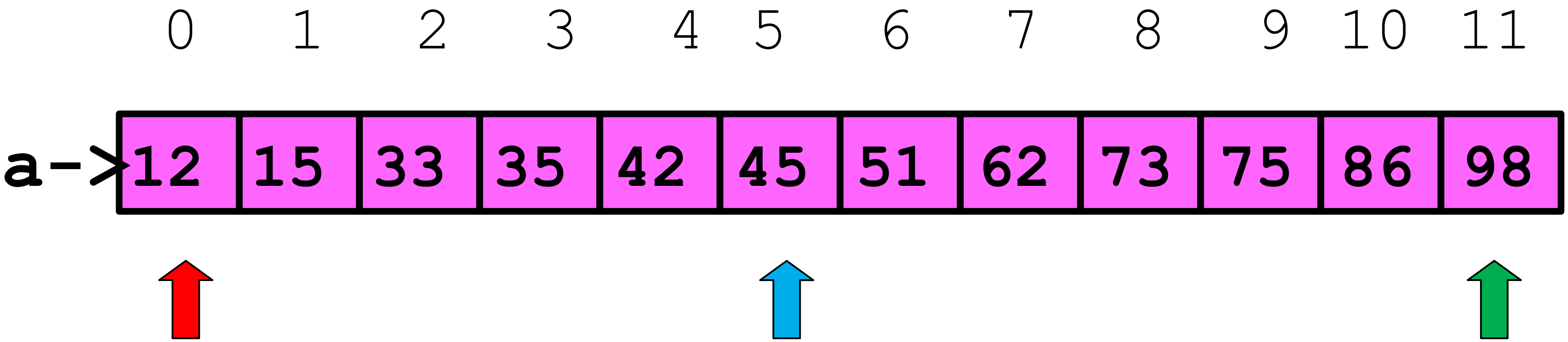
Mid: 5

R: 11

x: 70

a[Mid] <= x ????

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

`L:` 0

`Mid:` 5

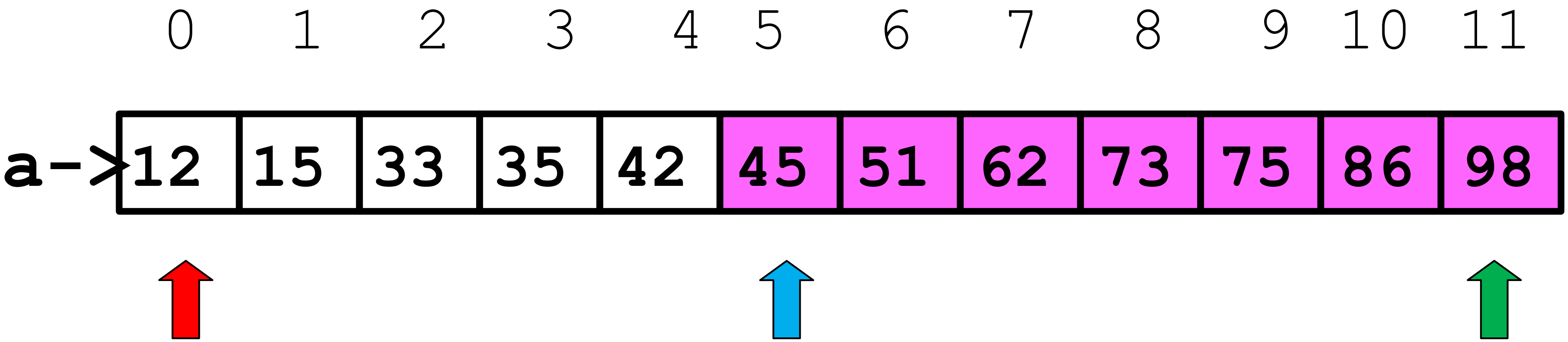
`R:` 11

`x:` 70

`a[Mid] <= x`

Yes!
So throw away
The "left half"

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

R: 11

x: 70

`a[Mid] <= x`

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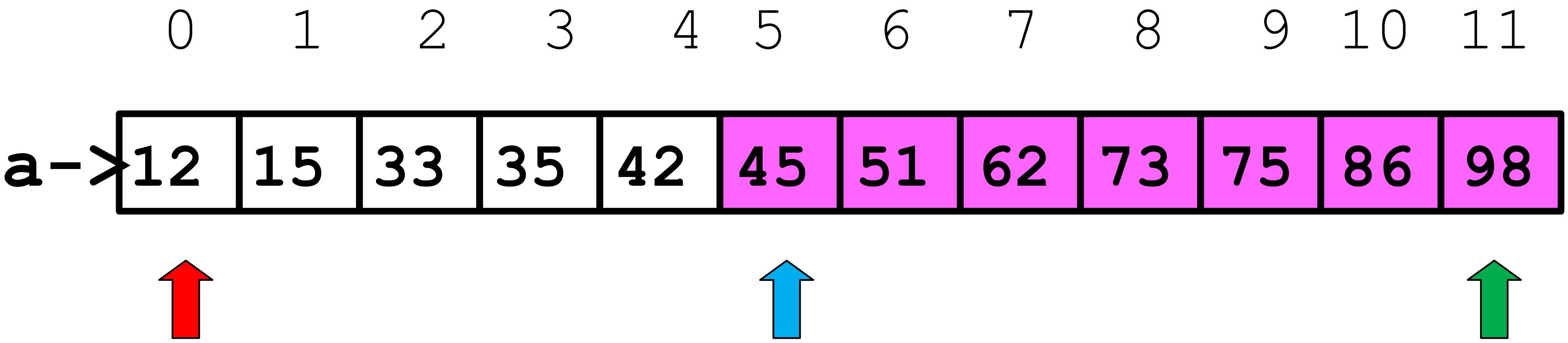
Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 0

Mid: 5

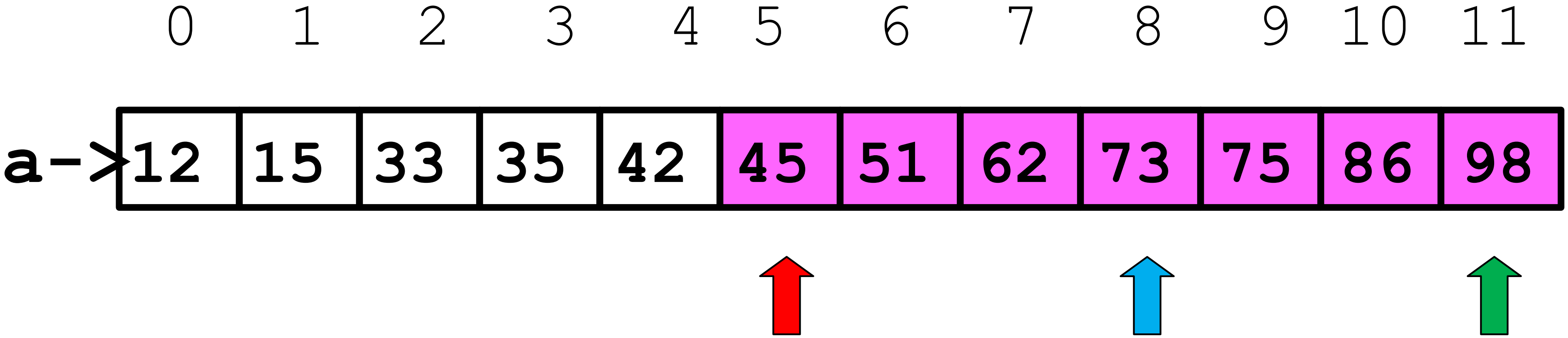
R: 11

x: 70

`a[Mid] <= x`

Revise L and Mid

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

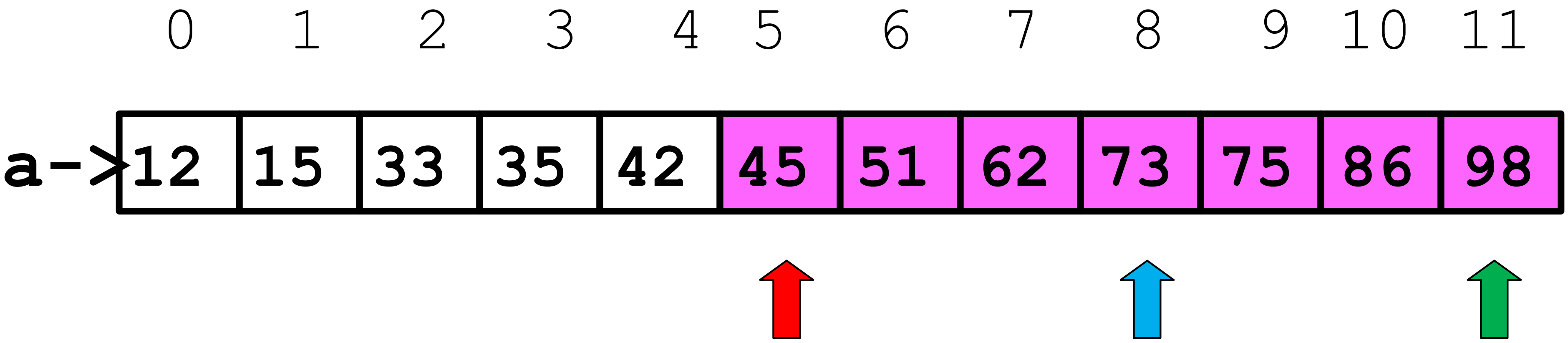
Mid: 8

R: 11

a[Mid] <= x ???

x: 70

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 8

R: 11

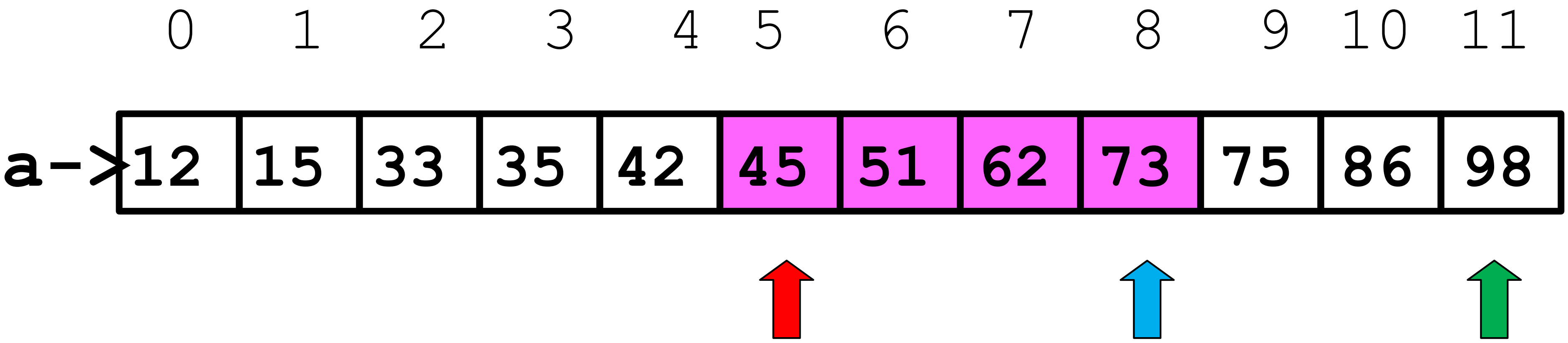
x: 70

`a[Mid] <= x`

No

So throw away the
“right half”

Let's Look For x = 70



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 8

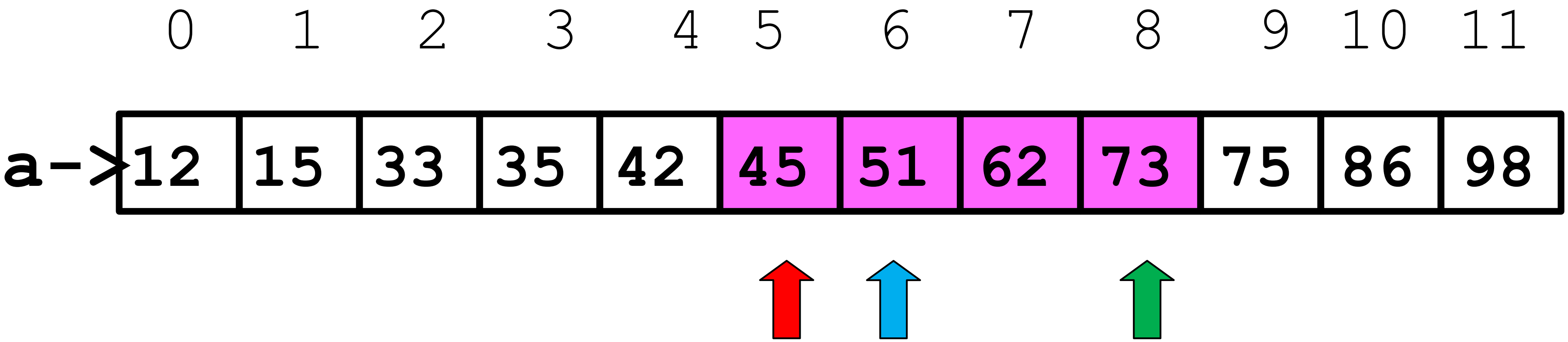
R: 11

x: 70

`a[Mid] <= x`

Revise R and Mid

Let's Look For x = 70



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 6

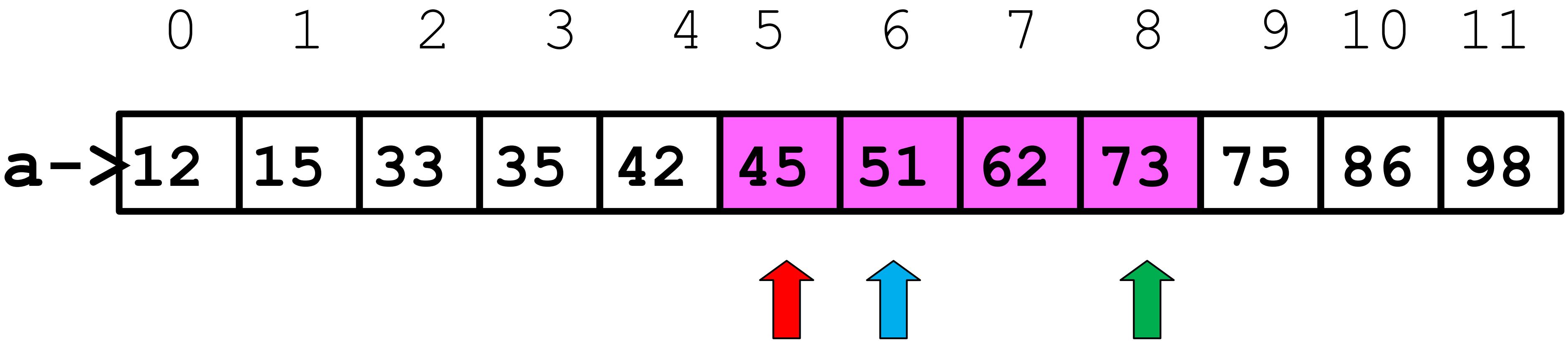
R: 8

x: 70

`a[Mid] <= x`

Revise R and Mid

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

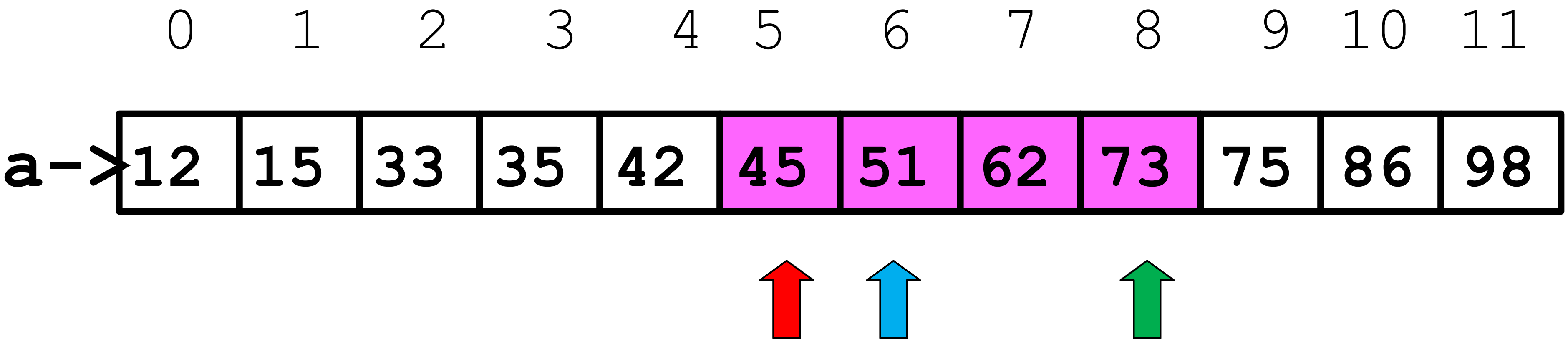
a[Mid] <= x ????

Mid: 6

R: 8

x: 70

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 5

Mid: 6

R: 8

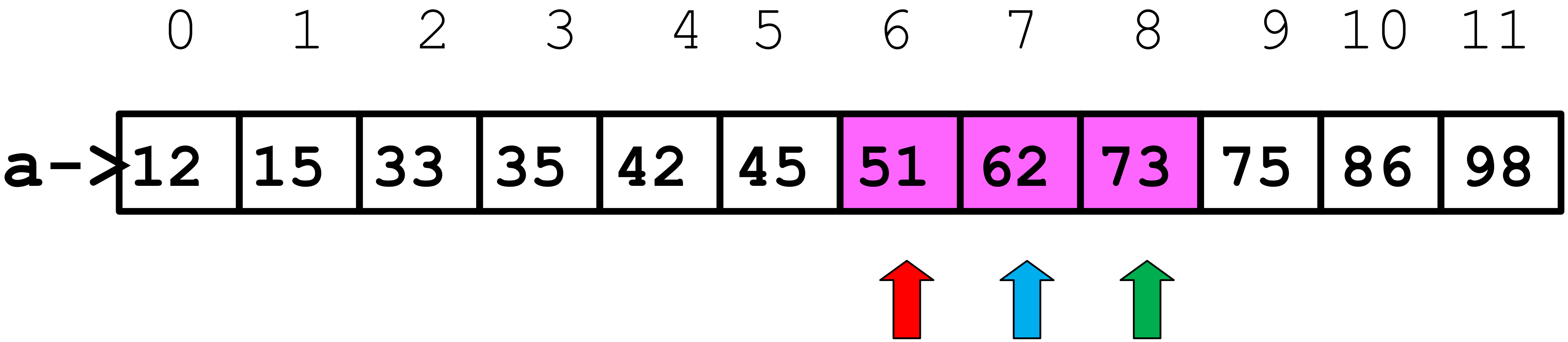
x: 70

`a[Mid] <= x`

Yes

Throw away the
Left half

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

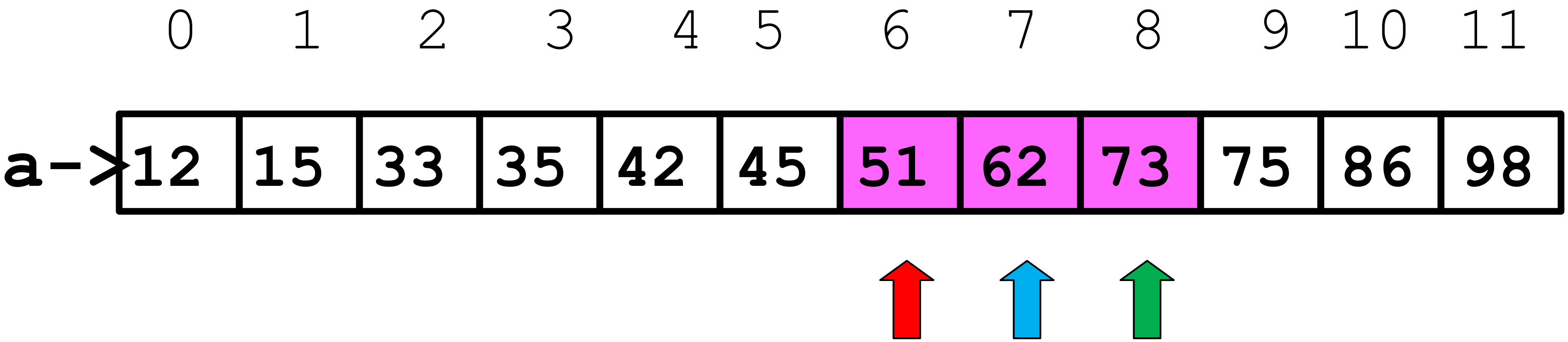
R: 8

x: 70

a[Mid] <= x

Yes

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

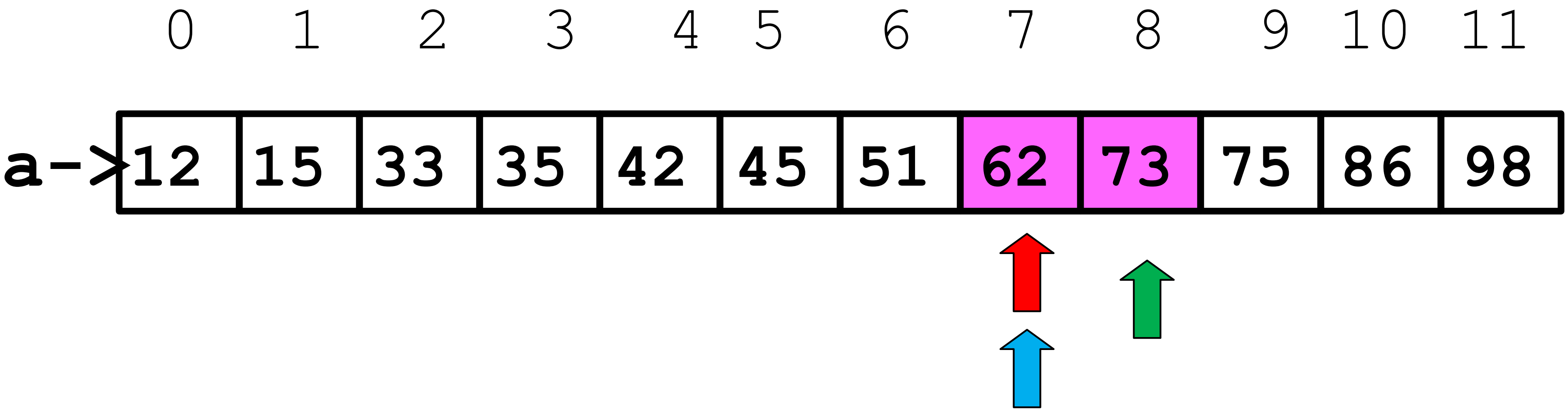
R: 8

x: 70

`a[Mid] <= x`

Throw away the left half

Let's Look For x in a



	0	1	2	3	4	5	6	7	8	9	10	11
a->	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 7

R: 8

x: 70

Done! At this point we just compare **x** with **a[L]** and **a[L+1]**.

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
  # a[L]<=x<=a[R]
  Mid = (L+R)/2
  if x <= a[mid]:
      R = Mid
  else:
      L = Mid
```

A Loop Invariant

Note that a[L]<=x<=a[R] remains True throughout the loop

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
    # a[L]<=x<=a[R]
    Mid = (L+R)/2
    if x <= a[mid]:
        R = Mid
    else:
        L = Mid
```

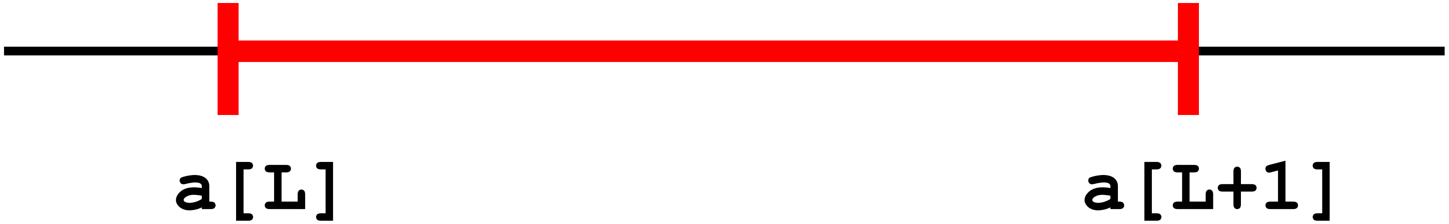
What is the situation when the loop terminates?

What We Just Did

```
L = 0
R = len(a)-1
while R-L > 1:
    # a[L]<=x<=a[R]
    Mid = (L+R)/2
    if x <= a[mid]:
        R = Mid
    else:
        L = Mid
```

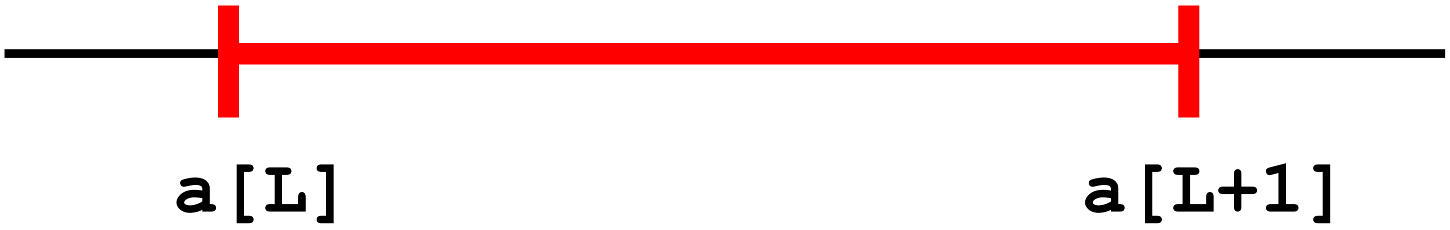
R-L<=1 implies R = L+1

After the Loop Ends



This is True: `a[L]<=x<=a[L+1]`

After the Loop Ends



```
if x==a[L]:
    return L
elif x==a[L+1]:
    return L+1
else:
    return -1
```

Measuring Execution Time

We now have two ways to search a list:

```
LinSearch(x,a)
BinSearch(x,a)
```

Intuition: BinSearch much faster.

Can we quantify this with a “stop watch”?

The timeit Module

This module can be used to time how long it takes to execute a chunk of code.

Typical chunk = some function of interest.

This is called benchmarking.

Benchmarking

Let's benchmark **`LinSearch(x,a)`** and **`BinSearch(x,a)`**.

Compare how long it takes when **`len(a)`** equals 1000, 10000, 100000, and 1000000.

Our intuition tells us that as **`len(a)`** increases, **`BinSearch`** will be dramatically faster.

BinSearch vs LinSearch

n	tBin	tLin	tLinW
1000	0.0007	0.0064	0.0119
10000	0.0009	0.0668	0.1203
100000	0.0011	0.8296	1.2082
1000000	0.0015	17.7388	13.9341

```
tBin  = time for BinSearch
tLin  = time for LinSearch (for loop version)
tLinW = time for LinSearch (while-loop version)
```

BinSearch vs LinSearch

n	tLin/tBin
1000	9
10000	74
100000	754
1000000	7095

Reporting ratios is more illuminating since we do not really care about the time units in this informal comparison

Using the `timeit` Module

We show how this module was used to get the results on the previous slides.

Our LinSearch vs BinSearch example is very typical: is one function faster than another?

A Benchmarking Framework

```
from timeit import *
S = “““
   Set-up code
”””
B = “““
   Code to Benchmark
”””
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

Yes, these are doc strings.

The Set-Up and Bench Codes

```
from random import randint as randi
from ShowSearch import BinSearch
n = 10000
s = [randi(0,10*n) for i in range(n)]
s.sort()
x = s[n/2]
```

```
k=BinSearch(x,s)
```

The set-up code is run once.

It is not timed.

It just sets up the code to be timed.

A Benchmarking Framework

```
from timeit import *
S = “““
    Set-up code
”””
B = “““
    Code to Benchmark
”””
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

An “experiment” consists of running the blue code m times.

The stopwatch will time how long it takes to do one experiment

Larger values necessary if the blue code executes very quickly

A Benchmarking Framework

```
from timeit import *
S = “““
    Set-up code
”””
B = “““
    Code to Benchmark
”””
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

Timer returns a length-p list. Each element is the stopwatch time for 1 experiment

This helps control for other stuff that may be running on your computer.

A Benchmarking Framework

```
from timeit import *
S = """
    Set-up code
”””
B = """
    Code to Benchmark
”””
p = 10; m = 100
t = min(Timer(B,setup=S).repeat(p,m))
```

In general, it is best to take the mininum as the most reliable. The benchmark time is assigned to t

This helps control for other stuff that may be running on your computer.

Why Benchmarking is Important

Confirms/refutes what our intuition might say about efficiency.

Makes us sensitive to the various issues that affect efficiency.

Steers us away from simplistic comparisons of different methods that can be used on the same problem.

Demonstration!

- Download and run the python script ShowBenchmarking.py
- TODO:
  - Run the above script for the following values of “n” (line 55)
    - 1000, 10000, 100000, 500000, 1000000, 10000000
  - Of the three search methods—binary search, linear search with for loop, linear search with while—which seems the fastest?
  - Which is faster of the following two—linear search with for loop, linear search with while loop?