

A MORE DETAILS OF OUR KD-TREE-BASED INDEX SOLUTION

• **KDT-tree construction.** Given a list of BC-points L , we can construct the KDT-Index level by level in a top-down manner: we first choose the dimension D and select the BC-point with medium coordinate in the dimension for the root, and then go through L to get the A, S for the root. Afterward, the unselected BC-points in L will be sent to the children of the root, and we continue to construct the KDT-index in the children nodes.

• **KDT-tree-based binary δ -temporal triangle counting.** Given a time window $[t_s, t_e]$ and a duration δ , the counting result equals the summarized R of nodes in the KDT-index whose A is contained by the cube $[t_s, t_e] \times [t_s, t_e] \times [0, \delta]$. We apply a recursive method starting from the root to find the counting result, as shown in Algorithm 6. Initially, we let $node$ be the root of KDT-Index (line 1). Then, we use a function $KDsum$ to sum the R values of nodes whose A is contained by the cube (line 2). If the cube A of the current node is contained by the query cube, we return the R of the node (line 3). If the intersection of the cube and A is empty, we return 0 (line 5); otherwise, we continue the recursion in the children nodes (line 6).

Algorithm 6: Sum the R values of nodes in a cube

Input: KDT-Index, a query cube $[t_s, t_e] \times [t_s, t_e] \times [0, \delta]$
Output: Total R value of nodes whose A is contained by the cube

```

1  $node \leftarrow$  root of KDT-Index;
2 Function  $KDsum(node)$ :
3   if  $node.A \subseteq [t_s, t_e] \times [t_s, t_e] \times [0, \delta]$  then return  $node.S$ ;
4   else
5     if  $node.A \cap [t_s, t_e] \times [t_s, t_e] \times [0, \delta] = \emptyset$  then return 0;
6     else return  $KDsum(node.lChild) + KDsum(node.rChild)$ ;
```

Based on analysis of [34], we can conclude that Algorithm 6 completes in $O(\Delta^{\frac{2}{3}})$ time.

• **KDT-tree maintenance.** For the dynamic temporal graph, when a new BC-point is generated, we update the KDT-Index starting from the root node in a top-down manner. Specifically, we first update the cube A and sum R in the current node, and then compare the coordinates of the new BC-point and the BC-point of the current node in the selected dimension. If the new BC-point has a larger value in the selected dimension, it will be in the right child to continue the update; otherwise, it will be in the left child.