Tu P03 10

Three Dimensional Generalized Nonhyperbolic Moveout Approximation - Application on a 3D HTI Model

Q. Hao* (Norwegian University of Science & Technology) & A. Stovas (Norwegian University of Science & Technology)

SUMMARY

Reflection moveout approximation is widely used for seismic modeling, velocity analysis, time migration etc. However, most of existed approximations are developed for 2D case. We propose a three-dimensional generalized nonhyperbolic moveout approximation (GMA). It is called generalized approximation because it follows the 2D counterpart proposed by Fomel and Stovas. In our approximation, thirteen parameters are determined from the zero-offset reference ray and two non-zero offset reference rays. By testing the accuracy of the proposed approximation in a 3D HTI model, we find that the accuracy of our 3D GMA is very good.

Introduction

Reflection moveout approximation is important for time processing including seismic modeling, velocity analysis, time migration etc. The reflection traveltime as a function of source-receiver offset is generally nonhyperbolic. For 2D case, the reflection traveltime can be accurately approximated by Fomel-Stovas' generalized nonhyperbolic moveout approximation (GMA) (Fomel and Stovas, 2010; Stovas, 2010). In their 2D GMA, five parameters are uniquely determined from zero-offset ray and a non-zero offset reference ray. We extend their approximation to 3D case. The proposed approximation includes thirteen parameters. In addition to the zero-offset reference ray, two non-zero offset reference rays with different azimuths are used to determine these parameters. The accuracy of our approximation is tested on a 3D HTI model.

3D genearlized moveout approximation

Let $t(x,y)$ represent the reflection traveltime as a function of the source-receiver offset projections x and y. We extend the 2D GMA proposed by Fomel and Stovas (2010) to 3D case,

$$t^2(x,y) = (t_0^2 + a(x,y))(1-\xi) + \xi\sqrt{t_0^4 + 2t_0^2 b(x,y) + c(x,y)} \ , \tag{1}$$

where

$$a(x,y) = a_{xx}x^2 + 2a_{xy}xy + a_{yy}y^2 \ , \tag{2}$$

$$b(x,y) = b_{xx}x^2 + 2b_{xy}xy + b_{yy}y^2 \ , \tag{3}$$

$$c(x,y) = c_{xxxx}x^4 + 4c_{xxxy}x^3y + 6c_{xxyy}x^2y^2 + 4c_{xyyy}x^2y^2 + c_{yyyy}y^4 \ . \tag{4}$$

3D GMA (1) has thirteen parameters including t_0, ξ, a_{xx}, a_{xy}, a_{yy}, b_{xx}, b_{xy}, b_{yy}, c_{xxxx}, c_{xxxy}, c_{xxyy}, c_{xyyy} and c_{yyyy}, where t_0 denotes two-way zero-offset traveltime; ξ denotes the weight.
We propose to employ zero-offset ray to determine the nine of them and two non-zero offset rays to specify the other four parameters. The Taylor expansion of equation (1) is given by

$$t^2 = t_0^2 + W_{xx}x^2 + 2W_{xy}xy + W_{yy}y^2 + \frac{1}{2t_0^2}\left(A_{xxxx}x^4 + 4A_{xxxy}x^3y + 6A_{xxyy}x^2y^2 + 4A_{xyyy}xy^3 + A_{yyyy}y^4\right) \ , \tag{5}$$

where the coefficients W_{ij} define the NMO ellipse, and coefficients A_{ijkl} are the quartic moveout coefficients. Matching the coefficients of Taylor expansion of equations (1) and equation (5) up to 4th order, we obtain eight equations,

$$a_{xx} = (W_{xx} - \xi b_{xx})/(1-\xi) \ , \tag{6}$$

$$a_{xy} = (W_{xy} - \xi b_{xy})/(1-\xi) \ , \tag{7}$$

$$a_{yy} = (W_{yy} - \xi b_{yy})/(1-\xi) \ , \tag{8}$$

$$c_{xxxx} = b_{xx}^2 + A_{xxxx}/\xi \ , \tag{9}$$

$$c_{xxxy} = b_{xx}b_{xy} + A_{xxxy}/\xi \ , \tag{10}$$

$$c_{xxyy} = b_{xx}b_{yy}/3 + 2b_{xy}^2/3 + A_{xxyy}/\xi \ , \tag{11}$$

$$c_{xyyy} = b_{xy}b_{yy} + A_{xyyy}/\xi \ , \tag{12}$$

$$c_{yyyy} = b_{yy}^2 + A_{yyyy}/\xi \ , \tag{13}$$

in addition to zero-offset traveltime t_0. To determine other four parameters, we consider three different methods: for method I, we adopt the traveltime T and its y-direction derivative $\partial_y T$ for the reference ray closed to x-axis, and traveltime T and its x-direction derivative $\partial_x T$ for the other ray closed to y-axis; for method II, we use the traveltime and the sum of its partial derivatives $\partial_x T + \partial_y T$ for both rays; for method III, we employ traveltime T, partial derivatives $\partial_x T$ and $\partial_y T$ for both rays, respectively. By introducing a new function

$$F(x,y)=A(x,y)\big/(T^2(x,y)-t_0^2-W(x,y)) \ , \quad (14)$$

where

$$W(x,y)=W_{xx}x^2+2W_{xy}xy+W_{yy}y^2 \ , \quad (15)$$

$$A(x,y)=A_{xxxx}x^4+4A_{xxxy}x^3y+6A_{xxyy}x^2y^2+4A_{xyyy}x^2y^2+A_{yyyy}y^4 \ , \quad (16)$$

and considering equations (6)-(16), we rewrite equation (1) as

$$2F(x,y)b(x,y)\xi-(F^2(x,y)-2t_0^2F(x,y))\xi+A(x,y)=0 \ . \quad (17)$$

Equation (17) corresponds to the traveltime for a non-zero offset ray. By differentiating equation (17) with respect to x and y, we obtain

$$\partial_x F(x,y)b(x,y)\xi+F(x,y)\partial_x b(x,y)\xi+\partial_x F(x,y)\left(t_0^2-F(x,y)\right)\xi+\partial_x A(x,y)/2=0 \ , \quad (18)$$

$$\partial_y F(x,y)b(x,y)\xi+F(x,y)\partial_y b(x,y)\xi+\partial_y F(x,y)\left(t_0^2-F(x,y)\right)\xi+\partial_y A(x,y)/2=0 \ . \quad (19)$$

Equations (17)-(19) are three basic equations for all methods we are going to test. Substitution of equation (3) into equations (17)-(19) gives a linear system of four equations for parameters ξb_{xx}, ξb_{xy}, ξb_{yy} and ξ used in methods I and II. For method III, we obtain a linear system of six equations (equations (17)-(19) defined for each ray) for these parameters. Once parameters b_{xx}, b_{xy}, b_{yy} and ξ are determined, the eight parameters given in equations (2) and (4) can be obtained from equations (6) -(13).

Parameterization of 3D HTI media

In our study, we focus on moveout of P-wave in acoustic approximation (Alkhalifah, 1998). The parameters of an acoustic VTI medium are: the vertical velocity of P-wave υ_0, the NMO velocity $\upsilon_{nmo}=\upsilon_0\sqrt{1+2\delta}$ and anelliptical parameter $\eta=(\varepsilon-\delta)/(1+2\delta)$. For a 3D HTI medium, we have an additional parameter, the azimuth of the symmetry axis φ measured from x-axis. For a horizontal reflector in 3D HTI media, we can derive the moveout coefficients given in equation (5) analytically. In case of a homogeneous 3D HTI medium, the horizontal rays with different azimuths can be selected to compute the approximation parameters given in 3D GMA.

Numerical Examples

To test our 3D GMA (1), we use a horizontal reflector in 3D HTI media. The model parameters include $z=1km$, $\upsilon_0=2km/s$, $\delta=0.1$, $\eta=0.2$, $t_0=2z\big/\left(\upsilon_n\sqrt{1+2\eta}\right)\approx 1.543s$. The exact traveltime is calculated by two-point ray tracing method. Figure 1 compares the relative absolute error in traveltime of 3D GMA for methods I, II, and III mentioned in previous section, respectively. In this Figure, we choose two reference rays with equal radial offset ($4km$). Figures 2 show the results for the case of two reference rays with different radial offsets. From Figures 1 and 2, we can observe that the accuracy of 3D GMAs for all methods is model-dependent, and method III has the best accuracy compared to others. We find that the maximum relative error of 3D GMA for method III with opening angles of $\pi/6$ and $\pi/3$ is no more than 0.046% for equal-offset reference rays and 0.27% for unequal-offset ones. We conclude that 3D GMA with two large-offset reference rays results in better traveltime accuracy compared with the one computed from GMA with a shorter-offset reference ray. Figure 3 shows an accuracy test of 3D GMA with a hyperbolic approximation and rational approximation (Al-Dajani and Tsvankin, 1998). We can see that 3D GMA performs best. We find the maximum relative error for hyperbolic approximation, nonhyperbolic approximation and 3D GMA are about of 15%, 3% and 0.034%, respectively.

Conclusions

We propose a 3D GMA that includes thirteen parameters. Only three reference rays are required to determine these parameters, one zero-offset ray and two nonzero-offset rays. We also propose three methods to determine corresponding parameters in 3D GMA. A 3D HTI model test illustrates that the accuracy of 3D GMA is very good. Numerical examples show that the azimuth of the HTI symmetry axis, the azimuth and the offset of reference ray significantly affect the accuracy of our approximation.

Acknowledgements

We would like to acknowledge the ROSE project for financial support.

References

Al-Dajani, A. and Tsvankin, I. [1998] Nonhyperbolic reflection moveout for horizontal transversely isotropy. *Geophysics,* **63**, 1738-1753.
Alkhalifah, T. [1998] Acoustic approximations for seismic processing in transversely isotropic media. *Geophysics,* **63**, 623–631.
Fomel, S. and Stovas, A. [2010] Generalized nonhyperbolic moveout apprimation. *Geophysics,* **75**, U9-U18.
Stovas, A. [2010] Generalized moveout approximation for qP- and qSV waves in a homogeneous transversely isotropic medium. *Geophysics,* **75**, D79-D84.

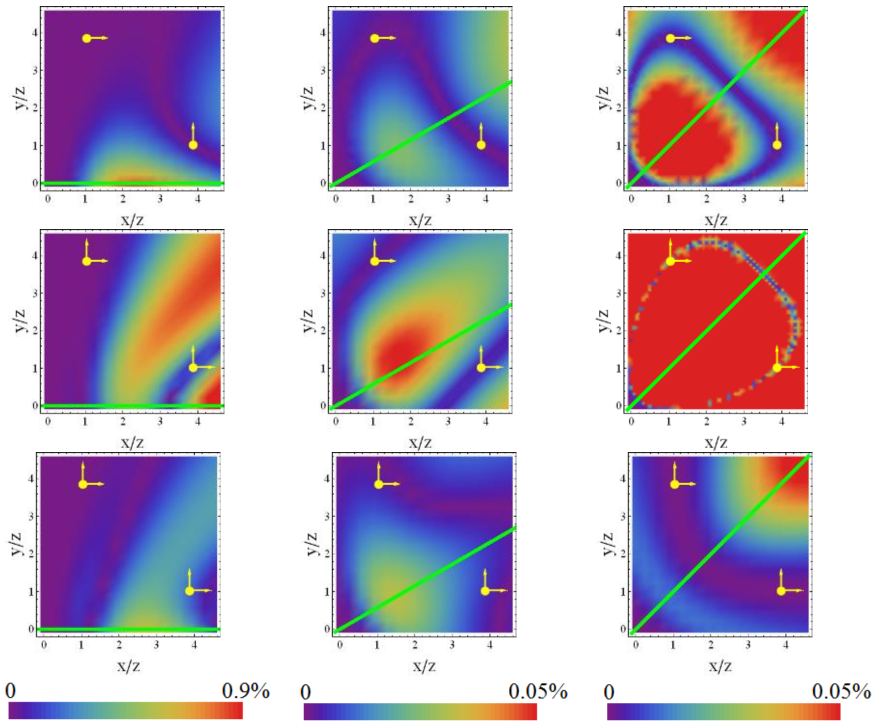

Figure 1 *Relative absolute error of 3D GMA equation* (1) *as a function of offset-depth ratio for methods I (top row), II (middle row) and III (bottom row). The yellow point corresponds to the offsets of the reference ~~rays~~, and both rays have same radial offset of* $4km$. *The green line denotes the azimuth of HTI symmetry axis. The yellow arrow shows the direction of traveltime spatial derivatives. In each plot, the azimuths of two reference rays are* $\pi/12$ *and* $5\pi/12$. *The opening angle formed by two reference rays equal to* $\pi/3$ *in all plots. From left to right, the plots in each row correspond to the azimuths of symmetry axis equal to* 0, $\pi/6$ *and* $\pi/4$, *respectively. From left to right, the scale bars correspond to methods I, II and III, respectively.*

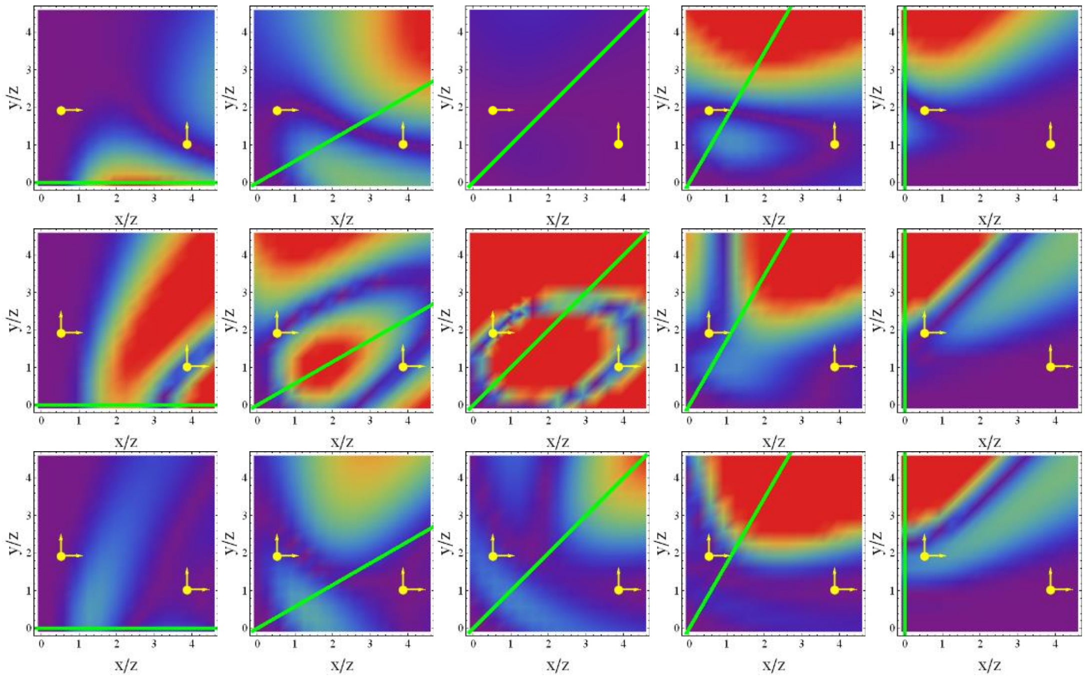

Figure 2 *Similar to Figure 1 but with a shorter radial offset (2km) for one reference ray with azimuth of* $5\pi/12$. *From left to right, the plots in each row correspond to the azimuths of symmetry axis equal to* 0, $\pi/6$, $\pi/4$, $\pi/3$ *and* $\pi/2$, *respectively. The scale bars are the same as Figure 1.*

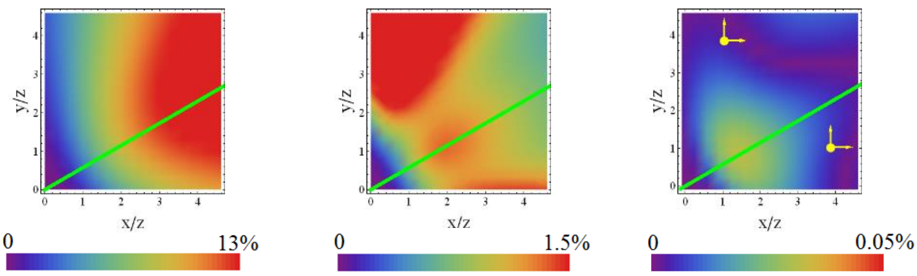

Figure 3 *Comparison of relative absolute error in reflection traveltime for hyperbolic approximation (left), rational approximation (middle), and 3D GMA (right). Green line denotes the HTI symmetry axis with azimuth of* $\pi/6$. *For the right plot, the yellow points correspond to the offsets of reference rays with azimuths of* $\pi/12$ *and* $5\pi/12$, *respectively. The yellow arrows denote the directions of traveltime spatial derivatives.*