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The Offset-midpoint Traveltime Pyramid in TTI Media

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SUMMARY

Analytical representation of offset-midpoint traveltime equation is very important for pre-stack Kirchhoff migration and velocity inversion in anisotropic media. For VTI media, the offset-midpoint traveltime resembles the shape of Cheop's pyramid. In this study, we have extended the offset-midpoint traveltime pyramid to the case of TTI media. The stationary phase method is employed to derive the analytical representation of traveltime equation. The Shanks transformation is applied to improve the accuracy of horizontal slowness and vertical slowness in traveltime equation. The traveltime pyramid is derived both depth- and time-domain. Numerical example indicates that the minimum of the traveltime pyramid in TTI media is shifted due to the influence of tilted angle of TI symmetry axis. The potential application of the derived offset-midpoint traveltime equation is the prestack Kirchhoff migration and anisotropic parameter estimation in TTI media.

Introduction

Analytical traveltime equation is very important for pre-stack Kirchhoff migration and velocity inversion. For isotropic media, the analytical traveltimes as a function of offset and midpoint could be given by a simple DSR equation in homogeneous media. However, it is difficult to obtain the analytical traveltime equation for anisotropic media, since the explicit relation between group velocity and ray angle does not exist in anisotropic media. Even for transversely isotropic media with a vertical symmetry axis (VTI), the traveltime are often calculated numerically. Alkhalifah (2000b) derived the offset-midpoint traveltime equation, Cheop's pyramid equation for VTI media through stationary phase method. In this abstract, we extend the offset-midpoint traveltime equation, Cheop's pyramid equation to the case of transversely isotropic media with a tilt symmetry axis (TTI). The characteristics of traveltime pyramid for different TI media are discussed.

Theory

The pre-stack phase-shift migration operator defined in offset-midpoint domain can be given by (Yilmaz, 2001),

$$P(x,h=0,z,t=0)=\int d\omega\tilde{P}(x_0,h_0,z=0,\omega)\int dk_h\int dk_x\exp(-i\omega T)\,, \qquad (1)$$

where T is the traveltime shift,

$$T=(q_s+q_g)z+2p_x(x-x_0)-2p_h h_0\,, \qquad (2)$$

and $\tilde{P}$ and P are the seismic data before and after pre-stack depth migration, respectively, (x,z) denotes the position of image point, x_0 is the midpoint location, h_0 is the half-offset, q_s, q_g are the vertical projections of the slowness vector defined at source and receiver positions, respectively, and p_x, p_h are the horizontal slowness defined in midpoint-offset space.

The stationary phase method was introduced by Alkhalifah (2000b) to calculate the horizontal slowness component at stationary point by solving equation (2) for source slowness p_s and receiver slowness p_g instead of p_h and p_x, according to the following linear relation,

$$p_s = p_x - p_h\ ,$$

$$p_g = p_x + p_h\ .$$

Hence, equation (2) can be written in terms of slowness for the source and receiver given by,

$$T=(q_s+q_g)z+p_s y_s+p_g y_g\ , \qquad (3)$$

where y_s denotes the lateral distance between image point and source given by $y_s=(x-x_0+h_0)$ and y_g denotes the lateral distance between image point and receiver given by $y_g=(x-x_0-h_0)$.

By setting derivatives of travletime T in equation (3) with respect to p_s and p_g to zero, we obtain the equation of the type,

$$\frac{dq}{dp}=-\frac{y}{z}\ , \qquad (4)$$

where p, q and y are either p_s, q_s and y_s for source or p_g, q_g and y_g for receiver, respectively.

In order to get the horizontal slowness p from equation (4), we should consider the slowness surface equation for TTI media which describes the relation between horizontal slowness p and vertical slowness q. The 2D slowness surface for acoustic wave in VTI media has been given by Alkhalifah(1998, 2000a),

$$F_{VTI}=-2\eta\upsilon_0^2\upsilon_{nmo}^2 p_v^2 q_v^2+\upsilon_0^2 q_v^2+(1+2\eta)\upsilon_{nmo}^2 p_v^2=0\ , \qquad (5)$$

where p_v and q_v are the horizontal and vertical slowness, respectively. υ_0 and $\upsilon_{nmo} = \upsilon_0\sqrt{1+2\delta}$ are the vertical and normal moveout velocities, respectively. δ and η are the anisotropic parameters (Thomsen, 1986).
The 2D slowness surface equation for TTI media can be obtained by applying the rotation operator,

$$p_v = pcos\theta - qsin\theta \ , \tag{6}$$

$$q_v = psin\theta + qcos\theta \ , \tag{7}$$

where θ is the angle of the TI symmetry axis measured from vertical.
Inserting the expressions for p and q obtained from equations (6) and (7) into equation (4), we obtain the expression for $\dfrac{dq_v}{dp_v}$ given by

$$\frac{dq_v}{dp_v} = -\frac{y\cos\theta - z\sin\theta}{z\cos\theta + y\sin\theta} = -a \ . \tag{8}$$

From equations (5) and (8), we obtain the equations for slowness p_v and q_v, independently,

$$p_v^2\upsilon_{nmo}^4 - a^2\upsilon_0^2(-1+2p_v^2\upsilon_{nmo}^2\eta)^3(-1+p_v^2\upsilon_{nmo}^2(1+2\eta)) = 0 \,, \tag{9}$$

$$a^2q_v^2\upsilon_0^4 - (1-q_v^2\upsilon_0^2)\upsilon_{nmo}^2(1+2\eta-2q_v^2\upsilon_0^2\eta)^3 = 0 \ . \tag{10}$$

Both equations (9) and (10) are quartic equations with respect variables p_v^2 and q_v^2, respectively. To derive the trial solutions, we use the perturbation of p_v and q_v with anisotropic parameter η,

$$p_v = p_{v0} + p_{v1}(2\eta) + p_{v2}(2\eta)^2 + \dots \ , \tag{11}$$

$$q_v = q_{v0} + q_{v1}(2\eta) + q_{v2}(2\eta)^2 + \dots \ , \tag{12}$$

Substituting trial solution (11) into equation (9), the coefficients p_{vi}, $i = 0,1,2$ are given by,

$$p_{v0} = \frac{a\upsilon_0}{\upsilon_{nmo}\sqrt{a^2\upsilon_0^2 + \upsilon_{nmo}^2}} \ ,$$

$$p_{v1} = -\frac{a^3\upsilon_0^3(a^2\upsilon_0^2 + 4\upsilon_{nmo}^2)}{2\upsilon_{nmo}(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{5/2}} \ ,$$

$$p_{v2} = \frac{3\upsilon_0^5(a^9\upsilon_0^4 + 4a^7\upsilon_0^2\upsilon_{nmo}^2 + 24a^5\upsilon_{nmo}^4)}{8\upsilon_{nmo}(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{9/2}} \ ,$$

In a similar way, we obtain the corresponding coefficients q_{vi}, $i = 0,1,2$

$$q_{v0} = \frac{\upsilon_{nmo}}{\upsilon_0\sqrt{a^2\upsilon_0^2 + \upsilon_{nmo}^2}} \ ,$$

$$q_{v1} = \frac{3a^4\upsilon_0^3\upsilon_{nmo}}{2(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{5/2}} \ ,$$

$$q_{v2} = \frac{3\upsilon_0^5\upsilon_{nmo}(-a^8\upsilon_0^2 + 20a^6\upsilon_{nmo}^2)}{8(a^2\upsilon_0^2 + \upsilon_{nmo}^2)^{9/2}} \ ,$$

The expression for horizontal slowness p can be obtained by the coordinate rotation (6) and (7),

$$p = p_0 + p_1(2\eta) + p_2(2\eta)^2 + \dots \ , \tag{13}$$

where Taylor expansion coefficients p_i, $i = 0,1,2$ are the linear combination of p_{vi} and q_{vi}, given by

$$p_i = p_{vi} cos\theta + q_{vi} sin\theta \ , \quad i = 0,1,2, \tag{14}$$

The Shanks transformation is employed to improve the accuracy of equation (13). The final approximation for p takes the form

$$p = p_0 + \frac{2p_1^2\eta}{p_1 - 2p_2\eta} \quad . \tag{15}$$

The horizontal slowness at source and receiver locations can be computed using this equation. Note that squaring of horizontal slowness from equation (13) and setting the tilt value to be zero results in equation (10) in from Alkhalifah (2000b) derived for VTI media. The slowness surface for a TTI medium is non-symmetric with respect to the vertical axis, and, therefore, the functional forms for $q(p)$ taken at source and receiver positions are different (Golikov and Stovas, 2012b; Stovas and Alkhalifah, 2012).

The similar approximation of the TTI slowness surface has been developed by Stovas and Alkhalifah (2012),

$$q = q_0 + \frac{2q_1^2\eta}{q_1 - 2q_2\eta} \ , \tag{16}$$

where the coefficients q_i , $i = 0,1,2$ are the first- and second-order perturbation coefficients. Equation (16) could be used to calculate vertical slowness for source and receiver. For a given horizontal slowness p, we can evaluate two vertical slownesses q corresponding to downward and upward waves from equation (16). And the correct one for us could be chosen according to equation (4).

Substituting both of the slowness expressions p_s and p_g from equations (15) and (16) into equation (3) results in the depth domain offset-midpoint traveltime equation for TTI media,

$$T(x, x_0, h, z) = \left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right) z + p_s y_s + p_g y_g \ , \tag{17}$$

which is also called the Cheop's pyramid for TTI media.

In order to obtain the time-domain traveltime pyramid for TTI media, zero-offset two-way traveltime τ is obtained by setting the half offset $h = 0$ and lateral coordinate of image point equal to lateral midpoint coordinate, $x = x_0$ in equation (17),

$$\tau = T(x, x_0 = x, h = 0, z) = 2q_{z0}z \ , \tag{18}$$

where

$$q_{z0} = \frac{1}{2} \left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right) \Bigg|_{x=x_0,\ h=0}$$

denotes the vertical slowness for the zero-offset seismic ray and q_{z0} can be reduced to $1/\upsilon_0$ for VTI media.

Consequently, substituting equation (18) into equation (17), we obtain the time-domain traveltime pyramid for acoustic wave in TTI media,

$$T(x, x_0, h, \tau) = \frac{1}{2q_{z0}} \left(q_{s0} + \frac{2q_{s1}^2\eta}{q_{s1} - 2q_{s2}\eta} + q_{g0} + \frac{2q_{g1}^2\eta}{q_{g1} - 2q_{g2}\eta} \right) \tau + p_s y_s + p_g y_g \ . \tag{19}$$

Figure 1 shows traveltime calculated using equation (19) as a function of midpoint and half offset for isotropic, VTI, tilted elliptic and TTI cases. The tilted elliptic case can be obtained from the TTI case by setting $\eta = 0$ (Golikov and Stovas, 2012a). The lateral location of image point is assumed to be at x =0. Obviously, all traveltime pyramids are symmetric with respect to half-offset. The traveltime pyramids for isotropic and VTI cases are also symmetric with respect to midpoint. However, the traveltime pyramids for tilted elliptic and TTI cases are non-symmetric with respect to midpoint position. For these cases, the group velocity surface is not symmetric with respect to the vertical axis, and, the minimum traveltime is shifted away from the zero midpoint position due to the influence of tilted angle of TI symmetry axis.

Conclusions

The offset-midpoint traveltime equation for TTI media is derived using the stationary phase method. Perturbation in anisotropic parameter η and the following Shanks transformation are involved to obtain a relatively simple analytical form for this equation.

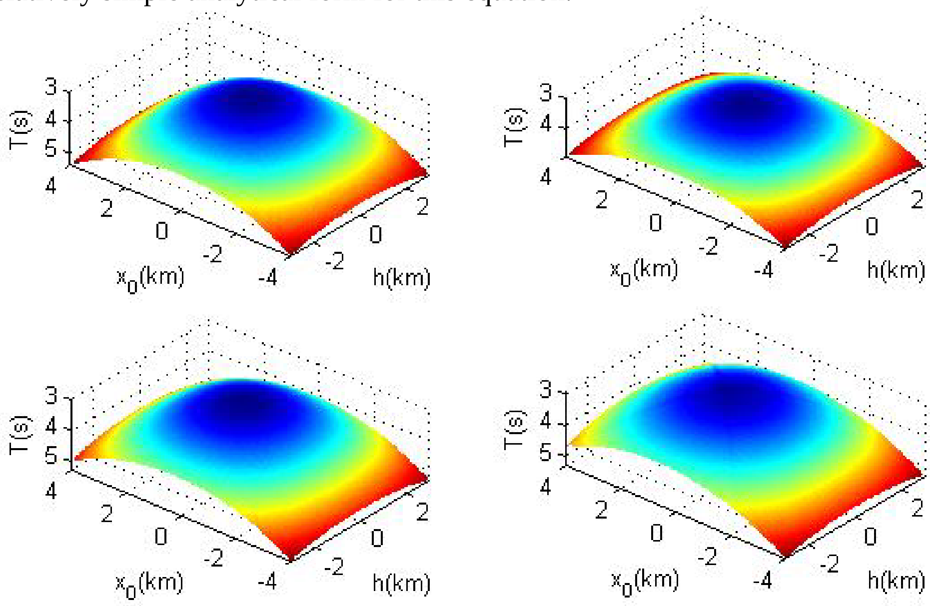

Figure 1 *Traveltime as a function of half offset h and midpoint x_0 for isotropic case (top left), VTI case with η =0.1 (top right), tilted elliptic medium with η =0 and θ =30^0 (bottom right) and TTI case with η =0.1and θ =30^0 (bottom right). Velocity υ_0 is 2km/s. The two-way vertical traveltime τ is 3s.*

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