An acoustic eikonal equation for attenuating VTI media

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Summary

We present an acoustic eikonal equation governing the complex-valued travel time of P-waves in attenuating, transversely isotropic media with a vertical symmetry axis (VTI). This equation is based on the assumption that the P-wave complex-valued travel time is independent of the S-wave velocity parameter υ_{S0} in Thomsen's notation and the attenuation coefficient A_{S0} in the Thomsen-type notation for attenuating VTI media. We combine perturbation theory and Shanks transform to develop practical approximations to the attenuating acoustic eikonal equation, capable of admitting analytical description of the attenuation in homogeneous media. For a horizontal, attenuating VTI layer, we also derive non-hyperbolic approximations for the real and imaginary parts of the complex-valued reflection travel time.

Introduction

Modeling the anelastic attenuating nature of earth is becoming important as our analysis of recorded data includes a closer look at amplitude for inversion purposes. Since waves tend to exhibit anisotropic behavior as a result of the natural thin layering of the earth, we would expect the same waves to experience an anisotropic attenuation for the same reason. Describing such behavior in an efficient matter via complex traveltimes and accordingly acquiring insights of its influence on seismic data are important for proper wave propagation description and inversion.

For generally attenuating anisotropic media, the stiffness coefficients may be expressed in Voigt notation. Actually, Vavrycuk (2009) and Rasolofosaon (2010) proposed two different weak anisotropy-attenuation notations to describe the attenuation of waves in generally attenuating anisotropic media. For attenuating, transversely isotropic media with a vertical symmetry axis (VTI) and orthorhombic media, Zhu and Tsvankin (2006, 2007) proposed Thomsen-type notations to describe the attenuation coefficients of plane waves.

In this expanded abstract, the complex-valued travel time of waves in an attenuating medium is expressed by $\tau = \tau_R + i\tau_I$, where i denotes the imaginary unit; the real part τ_R corresponds to the phase of waves; the imaginary part τ_I describes the decay of displacement amplitude of waves due to energy absorption. The complex-valued travel time satisfies the complex eikonal equation (e.g. Cerveny and Psencik, 2009), which may be solved by complex-valued ray tracing methods (e.g. Zhu and Chun, 1994; Chapman et al., 1999; Kravtsov et al., 1999). However, the complex ray tracing requires the analytic continuation of the model parameters defined in real space to complex space and this continuation may be realized only for simple attenuating models (e.g. Kravtsov et al., 1999; Cerveny and Psencik, 2009; Vavrycuk, 2008). The perturbation methods are developed to approximately calculate the complex-valued travel time in attenuating media (e.g. Gajewski and Psencik, 1992; Cerveny and Psencik, 2009; Klimes and Klimes, 2011). Vavrycuk (2008, 2012) proposed a real ray tracing method to solve the complex eikonal equation.

An acoustic attenuating eikonal equation

We use Thomsen's (1986) notation and the Thomsen-type notation (Zhu and Tsvankin, 2006) to fully describe an attenuating VTI medium, where Thomsen's (1986) notation is used to describe the non-attenuating reference of the medium, and the Thomsen-type notation is used to describe the attenuation coefficients in the medium. The S-wave velocity parameter υ_{S0} in Thomsen's (1986) notation barely affects the P-wave velocity in non-attenuating VTI media (Alkhalifah, 1998, 2000), and the attenuation of plane P-waves is approximately independent of the S-wave attenuation coefficient A_{S0} (Zhu and Tsvankin, 2006). Therefore, setting $\upsilon_{S0}=0$ and $A_{S0}=0$ should have little influence on P-wave travel time and attenuation in VTI media in which these parameters do not equal zero. In this case, we use Alkhalifah's (1998) notation including the P-wave vertical velocity υ_{P0}, the P-wave normal moveout (NMO) velocity υ_n, the anellipticity parameter η to describe an acoustic non-attenuating VTI medium, which corresponds to the real parts of the stiffness coefficients of the acoustic, attenuating VTI medium. We also use the P-wave attenuation coefficient A_{P0}, the attenuation-anisotropy parameters ε_Q and δ_Q in the Thomsen-type notation (Zhu and Tsvankin, 2006) to describe the attenuation of P-waves.

From the Christoffel equation (Cerveny and Psencik, 2005, 2009), we derive the 2D acoustic attenuating VTI eikonal equation,

$$A\left(\frac{\partial \tau}{\partial x}\right)^2 + B\left(\frac{\partial \tau}{\partial z}\right)^2 - C\left(\frac{\partial \tau}{\partial x}\right)^2\left(\frac{\partial \tau}{\partial z}\right)^2 = 1 , \qquad (1)$$

where the coefficients A, B and C are given by

$$A = \upsilon_n^2(1+2\eta) - i\frac{2\upsilon_n^2 A_{P0}(1+\varepsilon_Q)(1+2\eta)}{1-A_{P0}^2} , \qquad (2)$$

$$B = \frac{\upsilon_{P0}^2(1-iA_{P0})^2}{1-A_{P0}^2} , \qquad (3)$$

$$C = \upsilon_{P0}^2\left(2\upsilon_n^2\eta + \frac{(2-iA_{P0})b_1 - b_0}{\upsilon_n^2(1-A_{P0}^2)} + \frac{d}{\upsilon_n^2(1-A_{P0}^2)^2}\right), \quad (4)$$

with

$$b_0 = \upsilon_{P0}^4\delta_Q^2 + 8\upsilon_n^4\eta , \quad (5)$$

$$b_1 = 8\upsilon_n^4\eta - 2\upsilon_n^2\upsilon_{P0}^2\delta_Q + 2\upsilon_n^4\varepsilon_Q(1+2\eta) , \quad (6)$$

$$d = 4\upsilon_n^2\upsilon_{P0}^2\delta_Q + \upsilon_{P0}^4\delta_Q^2 - 4\upsilon_n^4(\varepsilon_Q + 2(1+\varepsilon_Q)\eta) . \quad (7)$$

Note that the imaginary part disappears when $A_{P0} = 0$, implying no attenuation, and the travel time has only a real part. In this case, the eikonal equation reduces to the one for acoustic, non-attenuating VTI media (Alkhalifah, 1998, 2000).

An approximate solution to the eikonal equation

We define the vector $\ell = (\eta, \varepsilon_Q, \delta_Q)^T$ to represent the trial solution to equation 1,

$$\tau = \tau_0 + \sum_{i=1}^{3}\tau_i\ell_i + \sum_{i,j=1;i\le j}^{3}\tau_{ij}\ell_i\ell_j . \quad (8)$$

Substitution of equation 8 into equation 1 gives a second-order expansion of the eikonal equation with respect to the anellipticity parameter and the Thomsen-type attenuation anisotropy parameters. Since all zero-, first- and second-order coefficients in the expansion must equal zero to satisfy the eikonal equation, we derive the equations for the travel time coefficients as follows:

- The zero-order coefficient τ_0 satisfying

$$\upsilon_n^2\left(\frac{\partial\tau_0}{\partial x}\right)^2 + \upsilon_{P0}^2\left(\frac{\partial\tau_0}{\partial z}\right)^2 = \frac{1-A_{P0}^2}{(1-iA_{P0})^2} . \quad (9)$$

- The first-order coefficients τ_i (i=1,2,3) satisfying

$$\upsilon_n^2\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_i}{\partial x} + \upsilon_{P0}^2\frac{\partial\tau_0}{\partial z}\frac{\partial\tau_i}{\partial z} = f_i(\tau_0) . \quad (10)$$

- The second-order coefficients τ_{ij} (i,j=1,2,3, and $i \le j$) satisfying

$$\upsilon_n^2\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_{ij}}{\partial x} + \upsilon_{P0}^2\frac{\partial\tau_0}{\partial z}\frac{\partial\tau_{ij}}{\partial z} = f_{ij}(\tau_0,\tau_1,\tau_2,\tau_3) . \quad (11)$$

The zero-, first- and second-order travel time coefficients can be successively calculated from equations 9-11. Equation 9 denotes the acoustic eikonal equation for attenuating, elliptically isotropic media, which may be numerically solved by a modification of the finite-difference method (Vidale, 1988, 1990) developed for non-attenuating isotropic media. Once the zero-order travel time is calculated from equation 9, equations 10 and 11 may be successively solved by using the method proposed by Alkhalifah (2011, 2013). After obtaining the zero-, first- and second-order travel time coefficients, we obtain the complex-valued travel time from equation 8. The complex-valued travel time may be further improved using Shanks transform (Bender and Orszag, 1978),

$$\tau = T_0 + \frac{T_1^2}{T_1 - T_2} , \quad (12)$$

with

$$T_0 = \tau_0 , \ T_1 = \sum_{i=1}^{3}\tau_i\ell_i , \ T_2 = \sum_{i,j=1;i\le j}^{3}\tau_{ij}\ell_i\ell_j . \quad (13)$$

A special case: homogenous, attenuating VTI media

Let us consider the special case of a homogeneous, attenuating VTI medium to derive the analytic solutions to equations 9-11. Assuming that the source is located at the origin of the [x, z] plane, we obtain

$$\tau_0 = \frac{\sqrt{(1-A_{P0}^2)(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)}}{(1-iA_{P0})\upsilon_n\upsilon_{P0}} , \quad (14)$$

$$\tau_1 = -\frac{x^4\upsilon_{P0}^3\sqrt{1-A_{P0}^2}}{\upsilon_n(1-iA_{P0})(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{3/2}} , \quad (15)$$

$$\tau_2 = \frac{iA_{p0}\sqrt{1-A_{P0}^2}\upsilon_{P0}^3x^4}{(1-iA_{p0})^3\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{3/2}} , \quad (16)$$

$$\tau_3 = \frac{iA_{p0}\sqrt{1-A_{P0}^2}\upsilon_{P0}^3x^2z^2}{(1-iA_{p0})^3\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{3/2}} , \quad (17)$$

$$\tau_{11} = \frac{3\sqrt{1-A_{P0}^2}\upsilon_{P0}^5(\upsilon_{P0}^2x^2+4\upsilon_n^2z^2)x^6}{2(1-iA_{p0})\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} , \quad (18)$$

$$\tau_{22} = -\frac{3A_{P0}^2\sqrt{1-A_{P0}^2}\upsilon_{P0}^5x^6(\upsilon_{P0}^2x^2+4\upsilon_n^2z^2)}{2(1-iA_{p0})^5\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} , \quad (19)$$

$$\tau_{33} = -\frac{3A_{P0}^2\sqrt{1-A_{P0}^2}\upsilon_{P0}^5x^2z^2(\upsilon_{P0}^4x^4-\upsilon_{P0}^2\upsilon_n^2x^2z^2+\upsilon_n^4z^4)}{2(1-iA_{p0})^5\upsilon_n^3(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} , \quad (20)$$

$$\tau_{12} = -\frac{iA_{P0}\sqrt{1-A_{P0}^2}\upsilon_{P0}^3x^4(\upsilon_{P0}^4x^4+8\upsilon_{P0}^2\upsilon_n^2x^2z^2-2\upsilon_n^4z^4)}{(1-iA_{P0})^3\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} , \quad (21)$$

$$\tau_{13} = \frac{3iA_{P0}\sqrt{1-A_{P0}^2}\upsilon_{P0}^5x^4z^2(\upsilon_{P0}^2x^2-2\upsilon_n^2z^2)}{(1-iA_{P0})^3\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} , \quad (22)$$

$$\tau_{23} = \frac{3A_{P0}^2\sqrt{1-A_{P0}^2}\upsilon_{P0}^5x^4z^2(\upsilon_{P0}^2x^2-2\upsilon_n^2z^2)}{(1-iA_{P0})^5\upsilon_n(\upsilon_{P0}^2x^2+\upsilon_n^2z^2)^{7/2}} . \quad (23)$$

Setting A_{P0} to zero, only τ_0, τ_1, τ_{11} are not zero and the eikonal solution 12 reduces to the travel time approximation presented in Alkhalifah (2011), which is highly accurate. The addition of the attenuation anisotropy parameters ε_Q and δ_Q allows us to solve for an anisotropic attenuation provided by the imaginary part of the travel time.

A horizontal, attenuating VTI layer

Let us now consider a horizontal, attenuating VTI layer. We denote the source-receiver offset by x , and the real and imaginary parts of the reflection travel time by t_R and t_I . We denote the zero-offset two-way travel time for the non-attenuating reference of an attenuating VTI layer by $t_0 = 2z / \upsilon_{P0}$, where z stands for the thickness of the layer. Considering the weak attenuation assumption $(A_{P0} \ll 1)$, from the one-way travel time equation 12, we derive the fourth-order expansions of the squared real and imaginary parts of the two-way reflection travel time at the zero-offset $x = 0$,

$$t_R^2 = t_0^2 + \frac{x^2}{\upsilon_n^2} - \frac{2\eta x^4}{t_{R0}^2 \upsilon_Q^4} \ , \tag{24}$$

$$t_I^2 = A_{P0}^2 \left(t_0^2 + \frac{x^2}{\upsilon_Q^2} - \frac{2\eta_Q x^4}{t_{R0}^2 \upsilon_Q^2} \right) \ . \tag{25}$$

Equation 24 is totally the same as for acoustic non-attenuating VTI media. In equation 25, the parameters υ_Q and η_Q are given by

$$\upsilon_Q = \frac{\upsilon_{P0}(1+2\delta)}{\sqrt{1+2\delta+2\delta_Q}} = \frac{\upsilon_n \sqrt{1+2\delta}}{\sqrt{1+2\delta+2\delta_Q}} \ , \tag{26}$$

$$\eta_Q = -\frac{\delta_Q^2 - 2(1+2\delta)\delta_Q(1+6\eta) + 2(1+2\delta)^2(\varepsilon_Q - \eta + 2\varepsilon_Q \eta)}{2(1+2\delta+2\delta_Q)^2} \ , \tag{27}$$

where δ is the Thomsen (1986) parameter for the non-attenuating reference of an attenuating VTI medium.
For VTI media with the isotropic attenuation coefficient $(\varepsilon_Q = \delta_Q = 0)$, υ_Q and η_Q reduce to υ_n and η .
Furthermore, we also obtain the existing fraction approximation (Tsvankin and Thomsen, 1994; Alkhalifah, 1997) for the real part of the complex-valued travel time,

$$t_R^2 = t_0^2 + \frac{x^2}{\upsilon_n^2} - \frac{2\eta x^4}{t_0^2 \upsilon_n^2 (1+\xi x^2)} \ , \tag{28}$$

with

$$\xi = \frac{2\eta}{t_0^2 \upsilon_n^4 \left(\frac{1}{\upsilon_n^2} - \frac{1}{\upsilon_h^2} \right)} \ , \tag{29}$$

where $\upsilon_h = \upsilon_n \sqrt{1+2\eta}$ denotes the velocity of the horizontally propagating P-wave in a non-attenuating VTI medium.

By analogy with the fraction approximation 28, we derive the fraction approximation for the imaginary part of the complex-valued travel time,

$$t_I^2 = A_{P0}^2 \left(t_0^2 + \frac{x^2}{\upsilon_Q^2} - \frac{2\eta_Q x^4}{t_0^2 \upsilon_Q^2 (1+\xi_Q x^2)} \right) \ , \tag{30}$$

with

$$\xi_Q = \frac{2\eta_Q}{t_0^2 \upsilon_Q^4 \left(\frac{1}{\upsilon_Q^2} - \frac{1}{\upsilon_{hQ}^2} \right)} \ , \tag{31}$$

where υ_{hQ} denotes the inverse of the slope of the curve t_I / A_{P0} versus x , as $x \to \infty$,

$$\upsilon_{hQ} = \frac{\upsilon_n \sqrt{1+2\eta}}{1+\varepsilon_Q} \ . \tag{32}$$

If ε_Q becomes zero, υ_{hQ} will be identical to the velocity of the horizontally propagating P-wave in a non-attenuating VTI medium.

Numerical examples

First, we investigate the influence of the S-wave parameters υ_{S0} and A_{S0} on the P-wave ray velocity V_{ray} and ray attenuation A_{ray} (Vavrycuk, 2007) for a homogeneous, attenuating VTI medium. Figure 1 implies that neglecting the parameters υ_{S0} and A_{S0} in the eikonal equation for attenuating VTI media almost does not affect the P-wave complex-valued travel time.

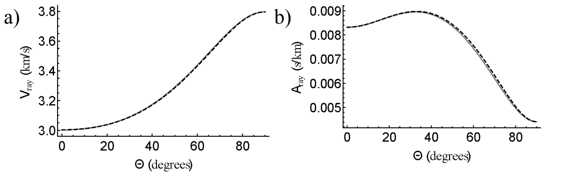

Figure 1: (a) Ray velocity V_{ray} and (b) ray attenuation A_{ray} as functions of the ray angle Θ , where the ray angle is measured from the vertical axis to the direction of the homogeneous ray-velocity vector. The gray solid lines correspond to the attenuating VTI model with the parameters $\upsilon_{P0} = 3km/s$, $\upsilon_{S0} = 1.5km/s$, $\varepsilon = 0.3km/s$, $\delta = 0.1$, $A_{P0} = 0.02498$ (corresponding to $Q_{33} = 20$, where Q_{33} denotes the quality factor of P-waves propagating along the vertical axis), $A_{S0} = 0.03330$ (corresponding to $Q_{55} = 15$, where Q_{55} denotes the quality factor of S-waves propagating along the vertical axis), $\varepsilon_Q = -0.33$, and $\delta_Q = 0.98$. The black dashed lines correspond to the attenuating VTI model with the same parameters except for $\upsilon_{S0} = 0$ and $A_{S0} = 0$. The values of parameters A_{P0} , A_{S0} , ε_Q and δ_Q are extracted from Zhu and Tsvankin (2006).

Second, we test the accuracy of the proposed approximation 8 and 12 for a homogeneous, acoustic attenuating VTI model. Figure 2 shows the real and imaginary parts of the exact complex-valued travel time. The attenuation anisotropy (corresponding to the anisotropy of the imaginary part of the complex-valued traveltime) is far stronger than the wavefront anisotropy (corresponding to the anisotropy of the real part of the complex-valued traveltime). The comparison between Figures 3 and 4 indicates that Shanks transform significantly improves the accuracy of the approximation for the complex-valued travel time.

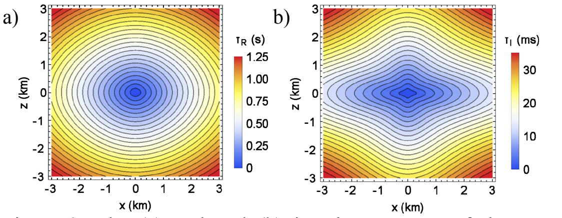

Figure 2. The (a) real and (b) imaginary parts of the exact complex-valued travel time for a homogeneous, attenuating VTI model. The model parameters are $\upsilon_{P0} = 3km/s$, $\upsilon_n = 3.286km/s$, $\eta = 0.167$, $A_{P0} = 0.02498$ (corresponding to $Q_{33} = 20$, where Q_{33} is explained in Figure 1), $\varepsilon_Q = -0.33$, and $\delta_Q = 0.98$. The values of the model parameters are from Zhu and Tsvankin (2006).

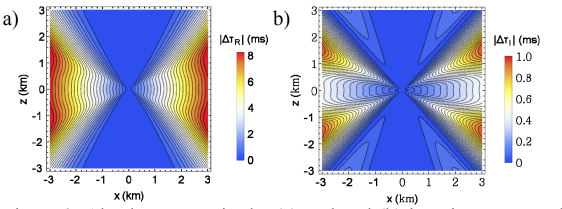

Figure 3. Absolute errors in the (a) real and (b) imaginary parts of the complex-valued travel time from the approximation 8 for a homogeneous, attenuating VTI medium. The model parameters are the same as in Figure 2.

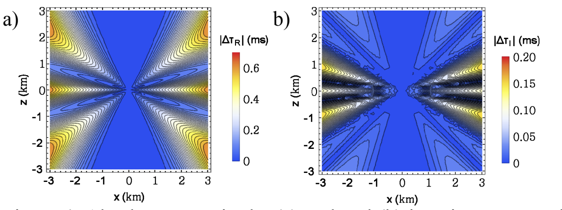

Figure 4. Absolute errors in the (a) real and (b) imaginary parts of the complex-valued travel time from the approximation 12 for a homogeneous, attenuating VTI medium. The model parameters are the same as in Figure 2.

Third, we investigate the reflection travel time for a homogeneous, attenuating VTI layer. The plot (a) in Figure 5 shows that the fraction approximation 28 matches very well with the exact solution for small to large offset-depth ratios, which implies that we may still use the existing methods such as velocity analysis (Alkhalifah, 1997) to invert for the NMO velocity υ_n and the anellipticity parameter η. The plot (b) in Figure 5 shows that the fraction approximation 30 is very accurate for the offset-depth ratios less than 1.7, and the curve of the imaginary part of the complex-valued travel time from this approximation has the same tendency with the increase in the offset-depth ratio as has the curve of the imaginary part of the exact complex-valued travel time.

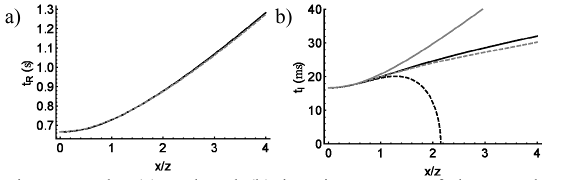

Figure 5. The (a) real and (b) imaginary parts of the complex-valued travel time as a function of the ratio between the source-receiver offset and the depth of the reflector. Except the depth of the reflector $z = 1km$, the medium parameters are the same as in Figure 2. For the plot (a), the black solid line corresponds to the exact solution; the gray dashed line corresponds to the fraction approximation 28. For the plot (b), the black solid line corresponds to the exact solution; the gray line corresponds to the second-order approximation described by the first two terms in the right hand side of equation 25; the black dashed line corresponds to the fourth-order approximation 25; the gray dashed line corresponds to the fraction approximation 30.

Conclusions

We derive an acoustic eikonal equation for attenuating VTI media under the assumption that the influence of the S-wave velocity parameter υ_{S0} in Thomsen's notation and the S-wave attenuation coefficient A_{S0} in the Thomsen-type notation on the complex-valued travel time of P-waves in attenuating VTI media is negligible. Combining perturbation theory and Shanks transform leads to an accurate solution to the acoustic eikonal equation. For a horizontally, homogeneous, attenuating VTI layer, the influence of the parameters A_{P0}, ε_Q and δ_Q in the Thomsen-type notation on the real part of the complex-valued travel time is so weak that we may use the existing non-hyperbolic approximations such as the fraction approximation to describe the real part of the complex-valued travel time; the imaginary part of the complex-valued travel time also has a non-hyperbolic shape, which may be expressed by a fraction approximation.

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EDITED REFERENCES

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