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Anelliptic Approximation for P-wave Phase and Group Velocities in Orthorhombic Media

Q. Hao* (Norwegian University of Science & Technology) & A. Stovas (Norwegian University of Science and Technology)

SUMMARY

We derive two anelliptic approximations for P-wave phase and group velocities in orthorhombic media. Numerical examples illustrate the good accuracy of both approximations.

Introduction

An orthorhombic medium includes nine independent elastic parameters and three mutually orthogonal planes of mirror symmetry. In each symmetry plane, the medium exhibits transversely isotropy. For P-waves in orthorhombic media, the exact phase and group velocities can be calculated from the Christoffel equation (Tsvankin, 1997) and ray tracing equations (Cerveny, 2001). Sripanich and Fomel (2014) recently proposed an anelliptic approximation for P-wave phase and group velocities.
In our abstract, we propose an alternative method to approximate P-wave phase and group velocities in an orthorhombic medium by analogy with the generalized moveout approximation (Fomel and Stovas, 2010; Stovas, 2010; Stovas and Fomel, 2012) and the anelliptic approximation (Fomel, 2004) for VTI media. The proposed approximation includes nine independent parameters for elastic orthorhombic media (Tsvankin, 1997) and six independent parameters for acoustic orthorhombic media (Alkhalifah, 2003).

Anelliptic approximation for P-wave phase velocity

We approximate the P-wave phase velocity defined in a given vertical plane of an orthorhombic medium by the GMA-type anelliptic approximation,

$$\upsilon_p^2(\theta,\varphi)=(1-\xi)\left(a\cos^2\theta+b\sin^2\theta\right)+\xi\sqrt{a^2\cos^4\theta+2da\cos^2\theta\sin^2\theta+e^2\sin^4\theta}\ , \tag{1}$$

where θ and φ are tilt and azimuth in the phase space; $\xi=\xi(\varphi)$ as an azimuth-dependent weight is used to link the elliptic and anelliptic parts of υ_p^2; a is the vertical velocity squared; $b=b(\varphi)$, $d=d(\varphi)$, $e=e(\varphi)$ are functions of azimuth φ. Equation (1) is called the GMA-type approximation since its form is analogous to the generalized moveout approximation (Fomel and Stovas, 2010; Stovas, 2010; Stovas and Fomel, 2012). We consider the Taylor expansion of the exact P-wave velocity squared around the vertical and horizontal directions to determine all parameters in equation (1). The P-wave phase velocity squared around the z-axis is expanded up to fourth order in terms of θ for a specified azimuth,

$$\upsilon_p^2(\varphi,\theta)=f_0+f_2(\varphi)\,\theta^2+f_4(\varphi)\theta^4\ , \tag{2}$$

where f_i, $i=0,2,4$ stands for the *i*-th order Taylor coefficient. The analogous expansion around the horizontal direction is given by

$$\upsilon_p^2(\theta,\varphi)=g_0(\varphi)+g_2(\varphi)\left(\theta-\pi/2\right)^2\ . \tag{3}$$

The coefficients f_i , i=0,2,4 from equations (2) and coefficients g_i, i=0,2 from equation (3) can be analytically computed from the Christoffel equation. By matching Taylor expansions (2) and (3) with the corresponding expansions of approximation (1), we can determine all parameters in the GMA-type approximation (1).
We also propose the extended Fomel approximation as an extension of the four-parameter anelliptic approximation (Fomel, 2004) to orthorhombic media,

$$\upsilon_p^2(\theta,\varphi)=(1-s)(a\cos^2\theta+c\sin^2\theta)+s\sqrt{(a\cos^2\theta+c\sin^2\theta)^2+2\frac{f}{s}\cos^2\theta\sin^2\theta}\ , \tag{4}$$

where parameter a is explained after equation (1); parameters $c=c(\varphi)$, $f=f(\varphi)$ and $s=s(\varphi)$ can be obtained by analogy with parameters in the GMA-type approximation. However, in this case, we do not use the second-order term g_2 from equation (3).

Anelliptic approximation for P-wave group velocity

We consider the P-wave ray propagation in the direction of the tilt Θ and azimuth Φ for an homogeneous orthorhombic medium. Similar to the phase velocity approximation (1), the GMA-type group-velocity approximation is defined as

$$\frac{1}{V_p^2(\Theta,\Phi)} = (1-\Xi)\left(A\cos^2\Theta + B\sin^2\Theta\right) + \Xi\sqrt{A^2\cos^4\Theta + 2DA\cos^2\Theta\sin^2\Theta + E\sin^4\Theta} \ , \qquad (5)$$

where $\Xi = \Xi(\Phi)$ is the azimuth-dependent weight; A is the inverse of P-wave vertical velocity squared; $B = B(\Phi)$, $D = D(\Phi)$, $E = E(\Phi)$ are functions of azimuth Φ . To determine all these parameters, we propose to match the Taylor expansion of approximation (5) with the corresponding exact expansions at vertical and horizontal directions. The reciprocal of the P-wave velocity squared with a fixed propagation azimuth Φ is expanded into a series with respect to the group tilt $\Theta = 0$,

$$\frac{1}{V_p^2(\Theta,\Phi)} = F_0 + F_2(\Phi)\Theta^2 + F_4(\Phi)\Theta^4 + \ldots \ . \qquad (6)$$

Another expansion around the horizontal direction gives

$$\frac{1}{V_p^2(\Phi,\Theta)} = G_0(\Phi) + G_2(\Phi)(\Theta - \pi/2)^2 + \ldots \ . \qquad (7)$$

The Taylor coefficients F_i from equation (6) can be easily determined, while it is very difficult to derive exact expressions for $G_0(\Phi)$ and $G_2(\Phi)$ from equation (7). Considering $G_0(\Phi)$ is the inverse of squared group velocity defined in the horizontal plane of an orthorhombic medium, the analytic approximation for G_0 can be obtained by transforming the GMA for P-wave in TI media (Stovas, 2010) to the corresponding group velocity. The analytic approximation for $G_2(\Phi)$ can be derived by the perturbation method in which the acoustic orthorhombic media (Alkhalifah, 2003) are taken as the reference and the shear wave velocity V_{S0} in Tsvankin (1997) notation as the perturbation parameter. Thus, the parameters given in the group-velocity GMA-type approximation (5) can be determined by analogy with the parameter determination for the phase-velocity GMA-type approximation discussed in the previous section.
We also propose the extended Fomel approximation for P-wave group velocity,

$$\frac{1}{V_p^2(\Theta,\Phi)} = (1-S)(A\cos^2\Theta + C\sin^2\Theta) + S\sqrt{(A\cos^2\Theta + C\sin^2\Theta)^2 + 2\frac{F}{S}\cos^2\Theta\sin^2\Theta} \ , \qquad (8)$$

where A, $S = S(\Phi)$, $C = C(\Phi)$, $F = F(\Phi)$ are determined by analogy with the approach for the GMA-type approximation (5) but with no coefficient $G_2(\Phi)$ from equation (7).

Numerical Examples

To test the proposed approximations, we adopt the orthorhombic model used in Sripanich and Fomel (2014). The density-normalized elastic parameters include $c_{11} = 9.0$, $c_{12} = 3.6$, $c_{13} = 2.25$, $c_{22} = 9.84$, $c_{33} = 5.9375$, $c_{44} = 2.0$, $c_{55} = 1.6$, $c_{66} = 2.182$, where c_{ij} have the units of $(km/s)^2$. Figures 1 and 2 show the accuracy comparison of our approximations with other existing approximations for phase- and group-velocity, respectively. In Table 1, we show a few orthorhombic models used in comparison. Tables 2 and 3 list the maximum relative error of different approximations for P-wave phase and group velocities, respectively. From these comparisons, we can see that the extended Fomel approximation and the GMA-type approximation are very stable and provide accurate results.

Conclusions

We derive stable and accurate approximations for P-wave phase and group velocities in orthorhombic media.

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Model	c_{11}	c_{22}	c_{33}	c_{44}	c_{55}	c_{66}	c_{12}	c_{13}	c_{23}
1	15.9	15.5	11.1	3.4	3.0	3.8	7.0	6.8	6.9
2	8.70	13.25	12.25	2.89	2.34	2.28	4.68	5.07	5.13
3	13.75	18.49	21.39	8.55	7.57	7.38	2.30	2.77	2.02
4	6.30	6.871	5.411	1.00	0.80	1.50	2.70	2.25	2.393

Table 1 *Density-normalized stiffness parameters (unit:* km^2/s^2 *) for orthorhombic models. Models 1-4 are taken from Mah and Schmitt (2003), Mahmoudian et al. (2014), Sano et al. (1992) and Miller and Spencer (1994), respectively.*

Model	Tsvankin	Linearized	S & F	Extended	GMA-type
1	0.721	0.544	0.078	0.059	3.0×10^{-3}
2	0.507	0.995	0.052	0.069	2.1×10^{-4}
3	4.198	1.090	0.207	0.209	5.3×10^{-3}
4	1.282	0.932	0.724	0.186	7.0×10^{-4}

Table 2 *Maximum relative error of phase velocity approximations for models listed in Table 1. The abbreviations "S & F", "Extended" and "GMA-type" stand for the Sripanich and Fomel (2014), our extended Fomel and GMA-type approximations. "Tsvankin" and "Linearized" stand for Tsvankin (1997) approximation and the linearized approximation (Daley and Krebe, 2004).*

Model	V & T	Linearized	S & F	Extended	GMA-type
1	0.857	1.218	0.037	0.060	0.368
2	1.049	3.846	0.126	0.057	0.107
3	1.231	1.295	0.079	0.218	0.109
4	4.554	7.767	0.572	0.156	0.167

Table 3 *Similar to Table 2 but for group velocity. The abbreviation "V & T" stands for the Vasconcelos and Tsvankin (2006) approximation.*

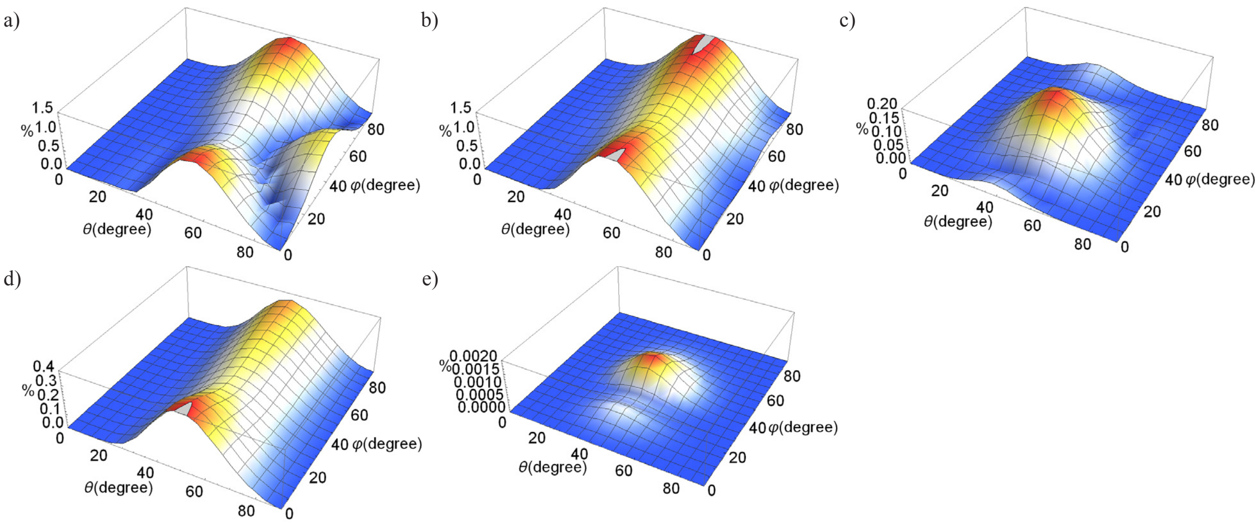

Figure 1 *Relative error in P-wave phase velocity for Tsvankin (1997) (a), Linearized (Daley and Krebes, 2004) (b), Sripanich and Fomel (2014) (c), the extended Fomel (d) and the GMA-type (e) approximations.*

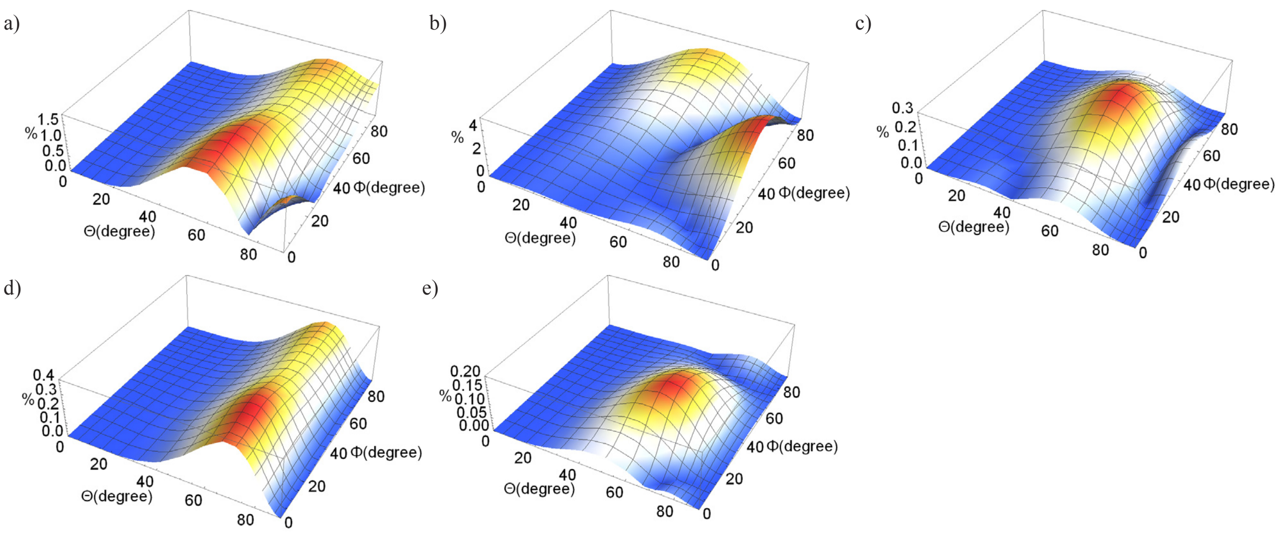

Figure 2 *Similar to Figure 1 but for P-wave group velocity.*