MADRID 2015
1-4 June 2015 | IFEMA Madrid

We N114 13

The Offset-midpoint Traveltime Pyramid for P-waves in Orthorhombic Media

Q. Hao* (Norwegian University of Science & Technology) & A. Stovas (Norwegian University of Science and Technology)

SUMMARY

For homogeneous isotropic or anisotropic media, the offset-midpoint traveltime surface resembles the shape of Cheop's pyramid. We propose an analytic approximation for the P-wave offset-midpoint traveltime pyramid for orthorhombic media. In this approximation, the perturbation expansion and Shanks transform are implemented to obtain the horizontal slowness components of P-waves in orthorhombic media. Numerical examples illustrate the azimuthal variations of the proposed traveltime pyramid and its accuracy test on a horizontal orthorhombic layer. The proposed offset-midpoint traveltime equation is useful for P-wave prestack Kirchhoff time migration in orthorhombic media.

Introduction

Analytical representation of the offset-midpoint traveltime equation is useful for pre-stack time migration in anisotropic media. For a homogeneous anisotropic media, the P-wave offset-midpoint traveltime surface resembles the shape of Cheops' pyramid (Alkhalifah, 2000; Hao and Stovas, 2014a, b; 2015). In this extended abstract, the principle axes of a homogeneous acoustic orthorhombic (ORT) medium are constrained to coincide with the coordinates of the Cartesian acquisition system.

P-waves phase-shift migration operator

The single-trace response of the 3D pre-stack phase-shift migration defined in the half-offset-midpoint domain for homogenous anisotropic media reads (Hao and Stovas, 2015)

$$P(x_1,x_2,h_1=0,h_2=0,z,t=0)=\iiiint\!\!\int \tilde{P}(x_1^0,x_2^0,h_1^0,h_2^0,z=0,\omega)\exp(i\omega T)d\omega dk_{h1}dk_{h2}dk_{x1}dk_{x2} \quad , \quad (1)$$

where $\tilde{P}$ and P are the single seismic trace before pre-stack depth migration and the seismic image after pre-stack depth migration; subscripts 1 and 2 denote x and y axes, respectively; $(x_1^0,\, x_2^0)$ is the midpoint position on acquisition surface; $(x_1,\, x_2,\, z)$ is the position of the image point; $(h_1^0,\, h_2^0)$ is the source-receiver half-offset; q_s and q_g are the vertical slowness components defined at source and receiver positions; $(p_{x1},\, p_{x2})=\left(k_{x1}/(2\omega),k_{x2}/(2\omega)\right)$ and $(p_{h1},\, p_{h2})=\left(k_{h1}/(2\omega),k_{h2}/(2\omega)\right)$ are the horizontal slowness vectors defined in midpoint-half-offset space, where (k_{x1},k_{x2}) and $(k_{h1},\, k_{h2})$ are the corresponding wavenumber vectors. T is the traveltime shift given by

$$T=(q_s+q_g)z+p_{s1}y_{s1}+p_{s2}y_{s2}+p_{g1}y_{g1}+p_{g2}y_{g2} \quad , \quad (2)$$

where $(y_{s1},\, y_{s2})$ denote projections of the lateral distance between image point and source given by $(y_{s1},\, y_{s2})=(x_1^0-h_1^0-x_1,\, x_2^0-h_2^0-x_2)$ and $(y_{g1},\, y_{g2})$ denotes projections of the lateral distance between image point and receiver given by $(y_{g1},\, y_{g2})=(x_1^0+h_1^0-x_1,\, x_2^0+h_2^0-x_2)$. Equation (2) represents the traveltime of the plane wave propagating from the source to the image point and back to the receiver.

The integrals in equation (1) can be estimated by the stationary phase method approximately. The stationary point of phase function in equation (1) corresponds to the local minimum or maximum of traveltime shift (2). Hence, the stationary point is found from equations

$$\frac{\partial q_{(s,g)}}{\partial p_{i(s,g)}}=-\frac{y_{i(s,g)}}{z} \quad , \ i=1,2 \, . \quad (3)$$

where the vector $(p_1,\, p_2)$, q and $(y_1,\, y_2)$ are either $(p_{s1},\, p_{s2})$, q_s and $(y_{s1},\, y_{s2})$ for the source or $(p_{g1},\, p_{g2})$, q_g and $(y_{g1},\, y_{g2})$ for the receiver. Traveltime shift (2) at the stationary point represents the exact traveltime of seismic ray from the source to the image point then back to the receiver.

P-wave slowness surface in ORT media

Under the acoustic assumption, the P-wave kinematics in a homogeneous ORT medium can be normally characterized by the Alkhalifah (2003) notation including the P-wave vertical velocity υ_{p0}; the NMO velocity υ_{n2} and the anellipticity parameter $\eta_2\equiv(\varepsilon_2-\delta_2)/(1+2\delta_2)$ defined in the vertical symmetry ([x, z]) plane; the NMO velocity υ_{n1} and the anellipticity parameter $\eta_1\equiv(\varepsilon_1-\delta_1)/(1+2\delta_1)$ defined in the vertical symmetry ([y, z]) plane; as well as the anisotropy parameter δ_3 defined in the horizontal symmetry ([x, y]) plane. We slightly modify the Alkhalifah (2003) notation by introducing the anellipticity parameter $\eta_3\equiv(\varepsilon_1-\varepsilon_2-\delta_3(1+2\varepsilon_2))/((1+2\delta_3)(1+2\varepsilon_2))$ (Vasconcelos and Tsvankin, 2006) defined in the [x, y] plane instead of the anisotropy parameter δ_3. The P-wave vertical slowness component q can thus be written as a function of the horizontal slowness vector (p_1, p_2),

$$q^2 = \frac{1}{\upsilon_{p0}^2(1+2\eta_3)}\frac{f_1(p_1,p_2)}{f_2(p_1,p_2)} , \tag{4}$$

where

$$f_1(p_1,p_2) = 1-(1+2\eta_2)p_1^2\upsilon_{n2}^2 - (1+2\eta_1)p_2^2\upsilon_{n1}^2 + 2\eta_3\left(1-(1+2\eta_2)p_1^2\upsilon_{n2}^2\right)\left(1-(1+2\eta_1)p_2^2\upsilon_{n1}^2\right) , \tag{5}$$

$$f_2(p_1,p_2) = 1-2\eta_2\upsilon_{n2}^2 p_1^2 - 2\eta_1\upsilon_{n1}^2 p_2^2 - \Omega\upsilon_{n1}^2\upsilon_{n2}^2 p_1^2 p_2^2 , \tag{6}$$

$$\Omega = 2(1+\eta_1+\eta_2+\eta_3 - \sqrt{1+2\eta_1}\sqrt{1+2\eta_2}\sqrt{1+2\eta_3} - 4\eta_1\eta_2\eta_3)/(1+2\eta_3) . \tag{7}$$

Analytic approximation for P-wave horizontal slowness components

Combining equations (3) and (4) allows us to derive two algebraic equations

$$a\upsilon_{p0}^2(1+2\eta_3)AB^3 - p_1^2\upsilon_{n2}^4 C^2 = 0 , \quad b\upsilon_{p0}^2(1+2\eta_3)AB^3 - p_2^2\upsilon_{n1}^4 D^2 = 0 . \tag{8}$$

where $a = y_1/z$ and $b = y_2/z$ stand for the offset-depth ratios; and

$$A = (1+2\eta_3)(1-p_1^2\upsilon_{n2}^2(1+2\eta_2) - p_2^2\upsilon_{n1}^2(1+2\eta_1)) + 2p_1^2p_2^2\upsilon_{n1}^2\upsilon_{n2}^2\eta_3(1+2\eta_1)(1+2\eta_2) , \tag{9}$$

$$B = 1-2p_1^2\upsilon_{n2}^2\eta_2 - 2p_2^2\upsilon_{n1}^2\eta_1 - \Omega p_1^2p_2^2\upsilon_{n1}^2\upsilon_{n2}^2 , \tag{10}$$

$$\begin{aligned} C = {} & 1+2\eta_3 + p_2^4\upsilon_{n1}^4(1+2\eta_1)(4\eta_1\eta_3(1+2\eta_2)+\Omega(1+2\eta_3)) \\ & - p_2^2\upsilon_{n1}^2(2(\eta_1-\eta_2)+2\eta_3(1+4\eta_1+4\eta_1\eta_2)+\Omega(1+2\eta_3)) \end{aligned} , \tag{11}$$

$$\begin{aligned} D = {} & 1+2\eta_3 + p_1^4\upsilon_{n2}^4(1+2\eta_2)(4\eta_2\eta_3(1+2\eta_1)+\Omega(1+2\eta_3)) \\ & - p_1^2\upsilon_{n2}^2(2(\eta_2-\eta_1)+2\eta_3(1+4\eta_2+4\eta_1\eta_2)+\Omega(1+2\eta_3)) \end{aligned} . \tag{12}$$

The perturbation solution of equations (8) is defined as

$$p_1^2 = c^{(0)} + \sum_i^3 c_i^{(1)}\eta_i + \sum_{i,j=1,i\le j}^3 c_{ij}^{(2)}\eta_i\eta_j , \quad p_2^2 = d^{(0)} + \sum_i^3 d_i^{(1)}\eta_i + \sum_{i,j=1,i\le j}^3 d_{ij}^{(2)}\eta_i\eta_j . \tag{13}$$

where $c^{(0)}$, $c_i^{(1)}$, $c_{ij}^{(2)}$, $d^{(0)}$, $d_i^{(1)}$ and $d_{ij}^{(2)}$ are the perturbation coefficients. They can be analytically determined by substituting equations (13) into equations (8) then equating the coefficients of the polynomial with respect to anellipticity parameters η_i. The zero-order coefficients $c^{(0)}$ and $d^{(0)}$ corresponding to elliptical ORT media ($\eta_i = 0$, $i = 1,2,3$) are given by

$$c^{(0)} = \frac{a\upsilon_{n1}^2\upsilon_{p0}^2}{\upsilon_{n2}^2(\upsilon_{p0}^2(a\upsilon_{n1}^2+b\upsilon_{n2}^2)+\upsilon_{n1}^2\upsilon_{n2}^2)} , \quad d^{(0)} = \frac{b\upsilon_{n2}^2\upsilon_{p0}^2}{\upsilon_{n1}^2(\upsilon_{p0}^2(a\upsilon_{n1}^2+b\upsilon_{n2}^2)+\upsilon_{n1}^2\upsilon_{n2}^2)} . \tag{14}$$

The magnitude of the horizontal slowness squared p^2 is equal to the sum of equations (13),

$$p^2 = (c^{(0)}+d^{(0)}) + \sum_i^3 (c_i^{(1)}+d_i^{(1)})\eta_i + \sum_{i,j=1,i\le j}^3 (c_{ij}^{(2)}+d_{ij}^{(2)})\eta_i\eta_j . \tag{15}$$

Shanks transformation (e.g. Hao and Stovas, 2014b) is employed to improve the accuracy of approximation (15). Consequently, we find

$$p^2 = E_0 - \frac{(E_1-E_0)^2}{(E_2+E_0-2E_1)} , \tag{16}$$

where

$$E_0 = c^{(0)}+d^{(0)} , \quad E_1 = \sum_i^3 (c_i^{(1)}+d_i^{(1)})\eta_i , \quad E_2 = \sum_{i,j=1,i\le j}^3 (c_{ij}^{(2)}+d_{ij}^{(2)})\eta_i\eta_j . \tag{17}$$

To further calculate the values of p_1 and p_2, we define the non-physical azimuth α as $\alpha = \arctan(p_1^2/p_2^2)$, which represents the azimuth of vector (p_1^2, p_2^2) measured from *x*-axis. From equations (13), we find the value of azimuth α from the equation

$$\tan\alpha = \frac{c^{(0)} + \sum_{i}^{3} c_i^{(2)}\eta_i + \sum_{i,j=1,i\le j}^{3} c_{ij}^{(2)}\eta_i\eta_j}{d^{(0)} + \sum_{i}^{3} d_i^{(2)}\eta_i + \sum_{i,j=1,i\le j}^{3} d_{ij}^{(2)}\eta_i\eta_j} \;, \; \alpha \in [0, \pi/2] \; . \tag{18}$$

From these operations, it follows that the horizontal slowness components are given by

$$p_1 = sgn(y_1)\, p\sqrt{\cos\alpha} \;\;,\;\; p_2 = sgn(y_2)\, p\sqrt{\sin\alpha} \;\; . \tag{19}$$

In equations (19), the operator $sgn(y_i)$, $i = 1,2$ makes sure that the horizontal slowness component p_i, $i = 1,2$ and the horizontal projection of propagation distance y_i always have the same sign. These conditions preserve the horizontal projections of the slowness vector and the ray velocity vector being in the same quadrant of the [*x*, *y*] plane.

Time-domains traveltime pyramid

Considering the relationship between the depth z and the minimum two-way zero-offset traveltime τ and equation (4), equation (2) becomes the time-domain traveltime pyramid in ORT media,

$$T(x_1,x_2,x_1^0,x_2^0,h_1^0,h_2^0,\tau) = \frac{\tau}{2\sqrt{1+2\eta_3}}\left(\frac{\sqrt{f_1(p_{s1},p_{s2})}}{\sqrt{f_2(p_{s1},p_{s2})}} + \frac{\sqrt{f_1(p_{g1},p_{g2})}}{\sqrt{f_2(p_{g1},p_{g2})}}\right) \;\; . \tag{20}$$
$$+\, p_{s1}y_{s1} + p_{s2}y_{s2} + p_{g1}y_{g1} + p_{g2}y_{g2}$$

For a single diffraction point $(x_1, x_2, z = \upsilon_{p0}\tau/2)$ in a homogeneous ORT medium, the diffraction traveltime corresponding to the midpoint (x_1^0, x_2^0) and the offset (h_1^0, h_2^0) is calculated by equation (20), where the horizontal slowness components (p_{s1}, p_{s2}) for source and (p_{s1}, p_{s2}) for receiver are calculated from equations (16)-(19); the vector (y_{s1}, y_{s2}) from source to midpoint and the vector (y_{g1}, y_{g2}) from receiver to midpoint are defined after equation (2).

Numerical examples

Figure 1 shows azimuth-dependent traveltime pyramids for P-waves in ORT media. The traveltime pyramid (20) can be implemented to calculate the P-wave reflection traveltime in ORT media. The accuracy of the proposed approximation (20) is compared with the Vasconcelos and Tsvankin (2006) approximation in Figure 2.

Conclusions

We propose an analytic approximation for P-waves traveltime pyramid in ORT media. It can be extended to the case of azimuthal ORT media by the slowness surface rotation in the horizontal plane.

Acknowledgements

We thank the ROSE project for its financial support.

References

Alkhalifah, T. [2000] The offset-midpoint traveltime pyramid in transversely isotropic media. Geophysics, **65**, 1316-1325.
Alkhalifah, T. [2003] An acoustic wave equation for orthorhombic anisotropy. Geophysics, **68**, 1169-1172.

Hao, Q. and Stovas, A. [2014a] P-wave diffraction and reflection traveltimes for a homogeneous 3D TTI medium. Journal of seismic exploration, **23**, 405-429.
Hao, Q. and Stovas, A. [2014b] The offset-midpoint traveltime pyramid in 2D transversely isotropic media with a tilted symmetry axis. Geophysical Prospecting, Early View.
Hao, Q. and Stovas, A. [2015] The offset-midpoint traveltime pyramid in 3D transversely isotropic media with a horizontal symmetry axis. Geophysics, **80**, T51-T62.
Vasconcelos, I. and Tsvankin, I. [2006] Non-hyperbolic moveout inversion of wide-azimuth P-wave data for orthorhombic media. Geophysical Prospecting, **54**, 535-552.

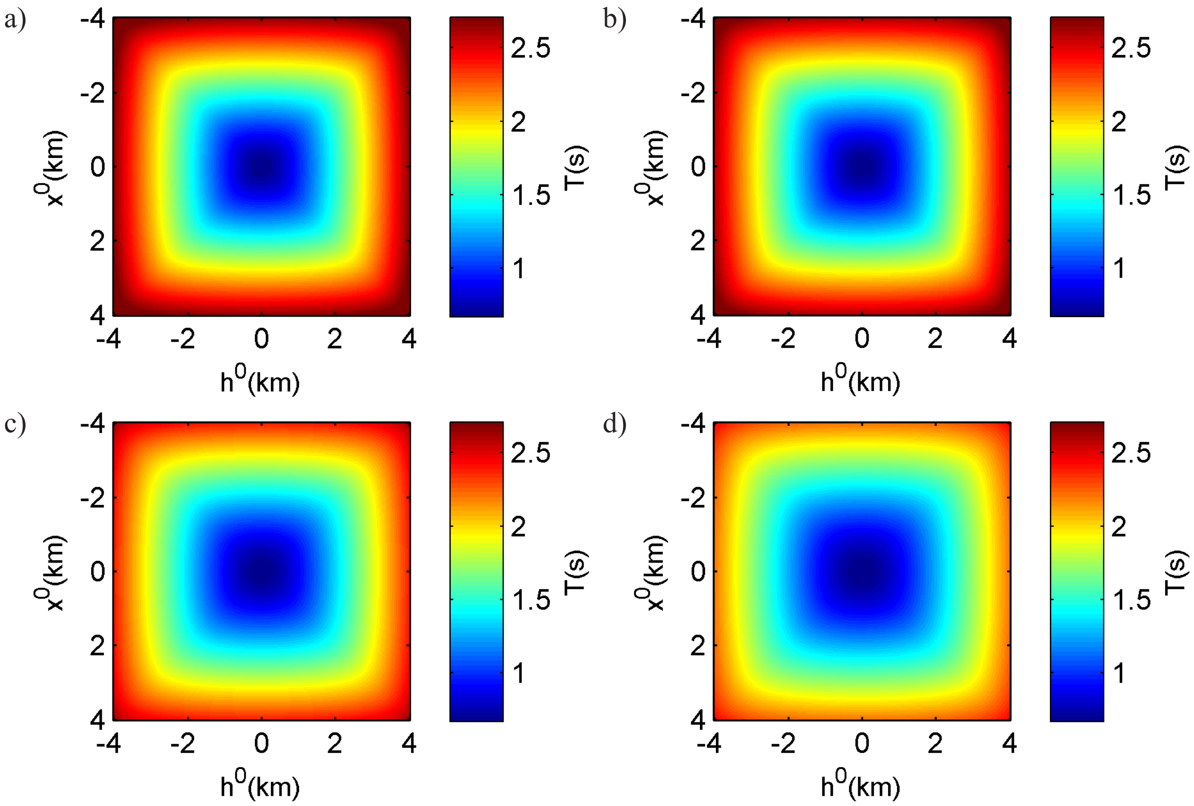

Figure 1 *The offset-midpoint traveltime pyramids along the acquisition azimuths 0 (a), $\pi/6$ (b), $\pi/3$ (c) and $\pi/2$ (d) in an ORT medium. x^0 and h^0 denote the midpoint and offset along an acquisition azimuth. The medium parameters include the P-wave vertical velocity $\upsilon_{p0}=3km/s$; NMO velocities $\upsilon_{n1}=3.5km/s$ and $\upsilon_{n2}=2.5km/s$; anellipticity parameters $\eta_1=0.1$, $\eta_2=0.3$ and $\eta_3=0.2$. The single diffraction point is located behind the coordinate origin. The minimum zero-offset two-way traveltime is $\tau=0.67s$; All plots are illustrated in the same color scale.*

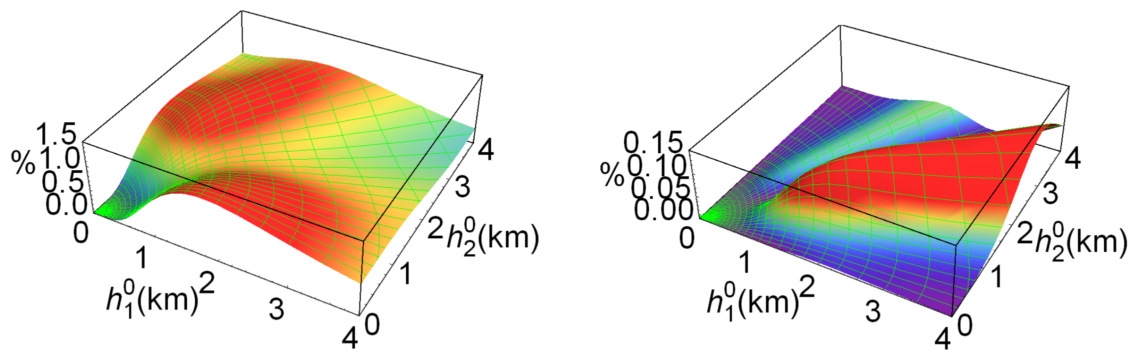

Figure 2 *The relative error comparison between Vasconcelos and Tsvankin (2006) approximation (left) and the proposed approximation* (20) *(right) for the P-wave traveltime in a horizontal reflector in a 3D ORT medium. The medium parameters from Figure 1 are adopted. The zero-offset two-way traveltime is $\tau=0.67s$.*