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# Generalized Group- and Phase-domains Moveout Approximations for Converted-wave in a Horizontal and Homogeneous VTI Layer

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## SUMMARY

For a horizontal and homogeneous transversely isotropic reflector with a vertical symmetry axis (VTI), we derive the generalized moveout approximations for P-SV converted-wave in group (t-x) domain and phase (τ-p) domain. They are called generalized approximation due to following the concept proposed by Fomel and Stovas. Both generalized moveout approximations include five parameters, separately; three parameters are determined from the zero-offset (horizontal slowness) ray and other two parameters are obtained from a non-zero offset (horizontal slowness) ray in group (phase) domain. The accuracy of our approximations is tested on a VTI model. Numerical results show that our group-domain approximation has good accuracy even for the large offset; our phase-domain approximation is equivalent to the exact solution. The potential applications of the proposed approximations include seismic modeling, traveltime inversion etc in a horizontal and homogeneous VTI layer.

## Introduction

Elastic wave incident on any elastic interface can produce mode-conversions including reflection and transmission of other types. Converted wave (C-wave) data can provide information about P- and S-wave. In this abstract, we derive the C-wave traveltime approximations in group *(t-x)* domain and phase ($\tau$-*p*) domain for a horizontal and homogeneous VTI layer. We imply C-wave is the mode for which the conversion happens only from an incident P-wave to a reflected S-wave. For a homogeneous VTI layer, C-wave corresponds to the converted P-SV wave. To derive the C-wave traveltime approximation, we refer to the generalized moveout approximations (GMAs) in group-domain (Fomel and Stovas, 2010) and in phase-domain (Stovas and Fomel, 2012a). Both generalized moveout approximations include five parameters, separately; three parameters are determined from the zero-offset (horizontal slowness) ray, and another two parameters are obtained from a non-zero offset (horizontal slowness) reference ray in group (phase) domain. Besides GMAs, some references are also published to investigate the C-wave moveout approximations in group domain (e.g. Tsvankin and Thomsen, 1994; Thomsen, 1999) and in phase domain (e.g. van der Baan and Kendall, 2003; Stovas and Fomel, 2012b).

## Traveltime parameters for C-wave

In homogeneous VTI media, P- and SV-waves can be characterized by the vertical velocities $\alpha_0$ and $\beta_0$ for P- and SV-waves and two Thomsen (1986) parameters $\varepsilon$ and $\delta$. Furthermore, to represent the reflection traveltime of C-wave, we adopt Ursin and Stovas' notation (Ursin and Stovas, 2006; Stovas, 2010) of traveltime parameters for P- and SV-waves in a horizontal VTI layer: P-wave vertical velocity $\alpha_0$; the ratio between vertical SV- and P-waves velocities $r_0 = \beta_0 / \alpha_0$; P- and SV-waves one-way vertical traveltimes $t_{P0} = z / \alpha_0$ and $t_{S0} = t_{P0} / r_0$ with $z$ being the layer thickness; P- and SV-waves NMO factors $a_0 = 2\delta$ and $b_0 = 2(\varepsilon - \delta) / r_0^2$.

For a horizontal and homogeneous VTI layer, the C-wave offset $x$ and traveltime $t$ are represented in terms of horizontal slowness $p$ as follows

$$x_c(p) = x_p(p) + x_S(p) \ , \tag{1}$$

$$t_c(p) = t_p(p) + t_S(p) \ , \tag{2}$$

where subscripts "c", "p" and "s" denotes C-, P- and SV-waves, respectively; the offset and traveltime formulations are given by

$$x_P(p) = p\alpha_0^2 t_{P0} \frac{1 + a_0 + S(p) + H(p)}{\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))}} \ , \tag{3}$$

$$t_P(p) = t_{P0} \frac{1 + p^2\alpha_0^2 H(p)}{\sqrt{1 - p^2\alpha_0^2(1 + a_0 + S(p))}} \ , \tag{4}$$

for incident P-wave, and

$$x_S(p) = p\alpha_0^2 r_0 t_{P0} \frac{1 + b_0 - S(p) - H(p)}{\sqrt{1 - p^2\alpha_0^2 r_0^2(1 + b_0 - S(p))}} \ , \tag{5}$$

$$t_S(p) = t_{P0} \frac{1 - p^2\alpha_0^2 r_0^2 H(p)}{r_0\sqrt{1 - p^2\alpha_0^2 r_0^2(1 + b_0 - S(p))}} \ , \tag{6}$$

for reflected SV-wave. Here, anisotropy functions are

$$H(p) = \frac{S(p)}{\sqrt{Q(p)}} \ , \tag{7}$$

$$S(p) = \frac{2b_0 p^2\alpha_0^2 r_0^2 (1 - r_0^2 + a_0)}{(1 - r_0^2)\left(1 + \dfrac{(a_0 - b_0)}{(1 - r_0^2)} p^2\alpha_0^2 r_0^2 + \sqrt{Q(p)}\right)} \ , \tag{8}$$

$$Q(p) = 1 + \frac{2(a_0 - b_0)}{(1 - r_0^2)} p^2 \alpha_0^2 r_0^2 + \frac{(4(1 - r_0^2) b_0 + (a_0 + b_0)^2)}{(1 - r_0^2)^2} p^4 \alpha_0^4 r_0^4 \ . \tag{9}$$

**Generalized traveltime approximations in group domain**

We use the GMA proposed by Fomel and Stovas (2010) to derive the analytical traveltime approximation for C-wave in group domain. Let C-wave traveltime $t_c$ as a function of offset $x$, we obtain its GMA formula given by

$$t_c^2 = t_{c0}^2 + \frac{x^2}{\upsilon_{cn}^2} + \frac{Ax^4}{\upsilon_c^4 \left( t_{c0}^2 + B_1 \frac{x^2}{\upsilon_{cn}^2} + \sqrt{t_{c0}^4 + 2B_1 t_{c0}^2 \frac{x^2}{\upsilon_{cn}^2} + C_1 \frac{x^4}{\upsilon_{cn}^4}} \right)} \ , \tag{10}$$

where $t_{c0}$ denotes the zero-offset two-way traveltime; Parameters $\upsilon_{cn}$ and $A$ denote the NMO velocity and the quartic moveout coefficient for C-wave. The Taylor expansion of approximation (10) at the zero offset is written as

$$t_c^2 = t_{c0}^2 + \frac{x^2}{\upsilon_c^2} + \frac{Ax^4}{2\upsilon_c^4 t_{c0}^2} + \ldots \ . \tag{11}$$

By matching the corresponding coefficients from equation (11) and the Taylor expansion of exact traveltime squared at zero-offset ray, we derive the explicit expressions for $t_{c0}$, $\upsilon_{cn}$ and $A$ given by (Tsvankin and Thomsen, 1994; Thomsen, 1999)

$$t_{c0} = t_{p0} + t_{s0} = t_{p0} \left( 1 + \frac{1}{r_0} \right) , \tag{12}$$

$$\upsilon_c = \alpha_0 \sqrt{\frac{r_0 (1 + a_0 + (1 + b_0) r_0)}{1 + r_0}} \ , \tag{13}$$

$$A = -\frac{(1 + a_0 + (-1 + b_0) r_0^2)^2}{2 r_0 (1 + a_0 + (1 + b_0) r_0)^2} \ . \tag{14}$$

The other two parameters $B_1$ and $C_1$ given in equation (10) are related to traveltime approximation at large offset, which can be determined from the traveltime and the horizontal slowness for a non-zero offset reference ray. For the horizontal reference ray, its offset becomes infinite. For this case, the corresponding expressions for $B_1$ and $C_1$ are derived by Fomel and Stovas (2010),

$$B_1 = \frac{t_{c0}^2 (1 - \upsilon_c^2 P_\infty^2)}{t_{c0}^2 - T_\infty^2} - \frac{A}{1 - \upsilon_c^2 P_\infty^2} \ , \tag{15}$$

$$C_1 = \frac{t_{C0}^4 (1 - \upsilon_C^2 P_\infty^2)^2}{(t_{C0}^2 - T_\infty^2)^2} \ , \tag{16}$$

where asymptotic parameters $P_\infty$ and $T_\infty$ are determined from

$$P_\infty = \lim_{x \to \infty} \left( \frac{dt_c}{dx} \right) , \tag{17}$$

$$T_\infty^2 = \lim_{x \to \infty} \left( t_c^2(x) - \frac{dt_c(x)}{dx} t_c(x) x \right) \ . \tag{18}$$

Substitution of equations (1) and (2) into equations (17) and (18) gives the explicit expressions for $P_\infty$ and $T_\infty$. Consequently, the expressions for $B_1$ and $C_1$ given in equations (15) and (16) become

$$B_1 = \frac{(1+r_0)(1+a_0+(-1+b_0)r_0^2)^2(1+a_0+b_0r_0^2)}{2r_0(1+a_0+(1+b_0)r_0)^2(1+a_0+(-1+b_0r_0)r_0^2)} \quad , \tag{19}$$

$$C_1 = 0 \ . \tag{20}$$

**Generalized traveltime approximations in phase domain**

The phase-domain C-wave traveltime in a horizontal and homogeneous VTI layer is

$$\tau_c(p) = t_c(p) - px_c(p) \ , \tag{21}$$

where $\tau_c$ denotes the traveltime intercept; functions $x_c(p)$ and $t_c(p)$ are given in equations (1) and (2), respectively.
The C-wave traveltime in phase-domain becomes nonelliptic even for a homogeneous isotropic layer. The generalized nonelliptic moveout approximation in the *τ-p* domain reads (Stovas and Fomel, 2012a)

$$\tau_c^2 = \tau_{c0}^2\left(1 + p^2\upsilon_c^2 + \frac{Ap^4\upsilon_c^4}{1 - B_2p^2\upsilon_c^2 + \sqrt{1 - 2B_2p^2\upsilon_c^2 + C_2p^4\upsilon_c^4}}\right) \ , \tag{22}$$

where $\tau_{c0}$ is the two-way zero-horizontal-slowness traveltime equal to $t_{c0}$ given in equation (10); $\upsilon_{cn}$ and $A$ have the same meanings shown in previous subsection; Parameters $B_2$ and $C_2$ control the accuracy of traveltime approximation (22) at large horizontal slowness $p$. By expanding equation (22) into a series with respect to horizontal slowness $p$, we obtain

$$\tau_c^2 = \tau_{c0}^2(1 - p^2\upsilon_c^2 + \frac{A}{2}p^4\upsilon_c^4 + \ldots) \ . \tag{23}$$

As an alternative approach, parameters $\tau_{c0}$, $\upsilon_{cn}$ and $A$ can also be specified by matching the corresponding coefficients of equations (23) and the phase-domain exact traveltime expansion at $p = 0$. Parameters $B_2$ and $C_2$ given in equation (22) can be determined from a reference ray with a non-zero valued horizontal slowness $P \le p_h$. Let $\tau(P) = \hat{\tau}$ and $\frac{d\tau}{dp}(P) = \hat{\tau}'$, the expressions for $B_2$ and $C_2$ are derived from equation (22) (Stovas and Fomel, 2012),

$$B_2 = \frac{p\upsilon_c^2\tau_0^2 + \hat{\tau}\hat{\tau}'}{p\upsilon_c^2\left(\tau_{c0}^2 - \hat{\tau}^2 + p\hat{\tau}\hat{\tau}'\right)} + \frac{A\tau_{c0}^2p^2\upsilon_c^2}{\tau_{c0}^2(1 - p^2\upsilon_c^2) - \hat{\tau}^2} \ , \tag{24}$$

$$C_2 = \frac{(p\upsilon_c^2\tau_{c0}^2 + \hat{\tau}\hat{\tau}')^2}{p\upsilon_c^2(\tau_{c0}^2 - \hat{\tau}^2 + p\hat{\tau}\hat{\tau}')^2} + \frac{2A\tau_{c0}^2}{\tau_{c0}^2(1 - p^2\upsilon_c^2) - \hat{\tau}^2} \ . \tag{25}$$

For the horizontal reference ray, $\hat{\tau}$ and $\hat{\tau}'$ becomes zero and infinite, respectively. The limit of variable $\hat{\tau}\hat{\tau}'$ can be determined from traveltime equation (21) and parametric equations (1) and (2). Consequently, we derive the analytical expressions for $B_2$ and $C_2$ given by

$$B_2 = \frac{(1+r_0)(1+a_0+(1+b_0)r_0^2)}{2r_0(1+a_0+(1+b_0)r_0)} \quad , \tag{26}$$

$$C_2 = \frac{(1+r_0)^2(1+a_0+b_0r_0^2)}{(1+a_0+(1+b_0)r_0)^2} \quad . \tag{27}$$

**Numerical Examples**

To test C-wave traveltime approximations (10) and (22), we adopt the horizontal and homogeneous Greenhorn shale model in Stovas (2010). The model parameters are: $\alpha_0 = 3.094 km/s$, $\beta_0 = 1.51 km/s$, $\varepsilon = 0.256$, and $\delta = -0.0505$. The thickness of this horizontal model is $1 km$. The group-domain exact traveltime (2) is calculated by inverting horizontal slowness $p$ from equations (1), (3) and (5). The phase-domain exact traveltime is directly calculated from equations (1)-(6) and (21). Figure 1 compares the group-domain approximation (10) with Tsvankin and Thomsen (1994) rational approximation. We can see that our approximation has good accuracy (maximum relative error is about 0.44%) even for large offset. Figure 2 shows the relative absolute error of the phase-domain traveltime approximation (22). It illustrates that our phase-domain traveltime approximation is practically equivalent to the exact solution.

## Conclusions

We derive the C-wave generalized traveltime approximations in group-domain and phase-domain. Each approximation needs five independent parameters, in which three are determined from zero-offset ray and the other two are obtained from a non-zero offset reference ray. Numerical results show that both approximations have good accuracy, and the phase-domain approximation is equivalent to the exact solution.

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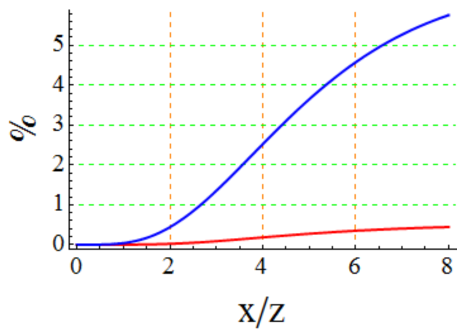


***Figure 1** Relative absolute error of traveltime versus offset-depth ratio. Red and blue lines correspond to our GMA (10) and Tsvankin and Thomsen (1994) approximation, respectively.*

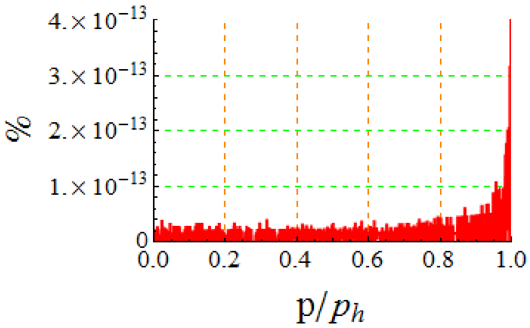


***Figure 2** Relative absolute error of traveltime intercept as a function of normalized horizontal slowness. Parameter $p_h$ denotes the maximum horizontal slowness for C-wave.*