Reply to the reviews of manuscript

"TGRS-2022-00377 A Generalized Seismic Attenuation Compensation Operator Optimized By 2D Mathematical Morphology Filtering "

We thank the reviewers for their valuable comments and suggestions. Please find the revised version of our manuscript which takes account of the various comments and suggestions of the two reviewers. Our responses to each and every question of Reviewers 1 and 2 are given in bold text below, immediately following each comments.

1 Response to Reviewer 1

1. Some minor typos, e.g., P2, L24, "two-dimensiona" and P2, L44 "as follows.". Please check the grammar one more time.

Reply: Thank you for your careful review~~ed~~ of the paper. We have checked the manuscript and corrected any mistake~~s~~ or point~~s~~ of confusion, as well as ~~improve~~ the language.

2. For eq12, how the constant in this equation are obtained? Is there any reference or the authors obtain them?

Reply: This equation is given by Wang's method, the details of which can be found in references 8 and 9. This equation is derived for an empirical relationship between the stabilization factor and the specified gain limit to control explicitly the amplitude magnification in inverse Q-filtering.

3. What is the input of Figure 5? The authors said it is the results of different filter types, but what is the input, I did not get it?

Reply: We are sorry ~~to cause any~~ confusion. Figure 5a is the input and others diagrams in the figure are the filtered results by 2D MMF, 2D Wiener filtering and 2D median filtering. We have corrected this in the revised manuscript.

4. For Figures 7 and 11, it is better to show the clean records and the removed noise corresponding to the different methods.

Reply: Thank you for your advice. We have added a figure of the clean data in Figure 7. The 2D MMF based coefficient H in equation 32 is aimed at estimating the high SNR segments of the data and then ~~multiply~~ with the GACO for compensation. This processing shows the compensated results but not the denoised data. We also added a comparison of the H coefficients with 2D MMF, 2D Wiener and 2D median filtering to illustrate the superiority of our approach.

But to address your point of view, we additionally calculated the denoised results directly using the H coefficients with the difference method, as shown below. The result based on the 2D morphological coefficient H is smooth and retains the components with high SNR, whereas for the data parts overwhelmed by noise, it is more suppressed compared to the other methods. In the following compensation, we agree with the results of coefficient H based on 2D MMF. For low SNR regions, no compensation can be made, but only those high SNR regions are compensated. However, this function also suppresses the signal if it is evaluated as a denoising capability. Such denoising results are not the purpose of attenuation compensation in this paper. The proposed method aims to provide robust compensation whilst maintaining high SNR. Therefore, we only show the comparison here (below), not

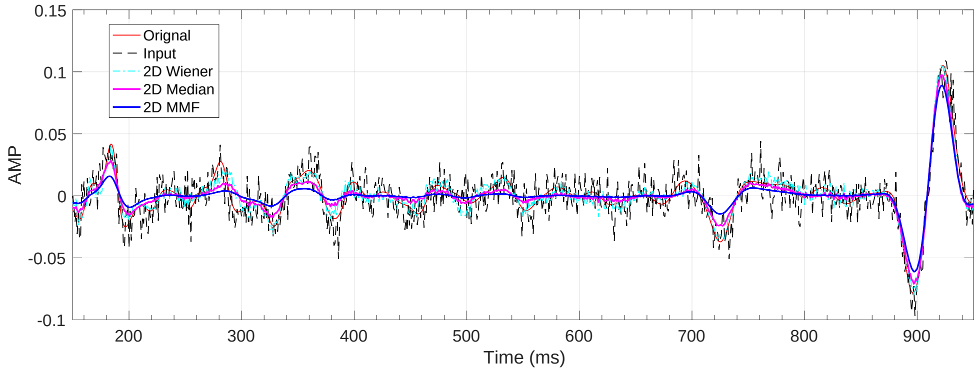

Figure 1: The denoised results directly using the H coefficients ~~with the difference method~~.

in the actual manuscript.

5. *State the SNR values for the synthetic examples corresponding to different methods.*

Reply: Thank you for your suggestion. The 2D MMF based coefficient H is the percentage of energy compensation in estimated regions of high SNR. The purpose of this function is to highlight the higher SNR areas that should be compensated to suppress low SNR areas but it does not calculate the true SNR value by 2D MMF. We did not and could not calculate or estimate the precise SNR value. Instead, we estimate the area where the effective signal energy is concentrated. This coefficient should not be considered the SNR values. When the noise energy in a certain region ~~is strong~~, the H value will become smaller, and the compensation will also become smaller, that is, the noise will neither be amplified nor suppressed.

6. *Enlarge Figure 8. It is too small to compare different methods.*

Reply: Thank you for your suggestion. We have modified Figure 8 accordingly to improve the contrast.

7. *Can you please illustrate Figure 14 with more details? I can see some artifacts in Figure 14(d).*

Reply: Thank you for your carefully scrutiny. There are really some artifacts in Figure 14d in the time interval $400-1000$ ms. There are several reasons for this, as explained below:
A. The SNR of the whole noisy records is 15, but when the amplitude of the local reflection recording is weak, the SNR will be low~~er~~, even lower than 1, as in the regions where the artifacts occur.
B. Comparing Figure 14b and c, the resolution of Figure 14c is higher than Figure 14b. This is caused by different compensation. In Figure 14b, ~~the~~ $G_{lim} = 25$, whereas that based on GACO compensation is more, thus causing the noise to be amplified. Figure 14d applies the same GACO parameters, which have improved the SNR due to the incorporation of 2D MMF-based H coefficients, but there are still some residuals.
C. These artifacts can be reduced by changing the length of n in 2D MMF.

8. *P9-L59, How did the authors obtain the SNR for the field data?*

Reply: For the field data, we did not and cannot calculate the SNR value, which is also one of the most challenging issues in field data processing. We used 2D MMF on the time-frequency spectra and then calculated the coefficient H to control the

intensity of compensation by GACO. The key parameter H could not be considered as an SNR value.

9. The same comment for the field data, the removed noise should be plotted for different noise.

Reply: Thank you for your suggestion once again. We used different 2D filtering comparisons to verify the superiority of the coefficient H based on 2D MMF. This comparison for a signal trace from the field data has been added in the revised manuscript.

10. I am interested to see how machine learning and deep learning can fit with such a problem. The authors can add one or two paragraphs to discuss such a hot topic as future work.

Reply: Thank you for the suggestion. We used 2D MMF for calculating the coefficient H and this cost a lot of computational effort. The amount of field data processing is huge, which will undoubtedly limit the application of this method. Therefore, it can be applied or combined with some artificial intelligence algorithms to accelerate the process. We have added ~~this~~ short discussion at the end of the section entitled 'Discussion'.

2 Response to Reviewer 1

1. For Figure 1, the noisy data is with different attenuation. Why the strong attenuated trace after compensation shows high SNRs from Figure b and c?

Reply: The SNR of each trace is 15. This indicates that the amplitudes of added noise with a large Q factor will be greater than for a strongly attenuated trace (small Q value) which could be found in Figure 1a. This will cause noise amplification, especially for $Q = 150$, 80. But for $Q = 30$ and 15, although the noise is not amplified, there is no significant compensation either, since its $G_{lim} = 10$. For the noisy recordings, adequate compensation necessarily amplifies the noise.

2. How to choose the parameters m and n to obtain a suitable compensation result? Does this need lots of test?

Reply: Thank you for your questions. In normal condition, $m - n = 1$. The parameters m and n have been transformed to the peak frequency f_s and the amplitude compensation coefficient (ACC). The former one is the desired compensated PF which must be larger than the input PF. The ACC is the amplification factor relative to exponential curve β^{-1} at the PF. At the same time, it also determines the steepness of GACO. The larger the value, the more suppressed the compensation of high frequency components will be. These two parameters can be obtained from the time-frequency spectra.

3. For the 2D MMF, the parameters of SE are important for the result. Please explain how to get the suitable parameters of amplitude and length?

Reply: There are three parameters of SE in this research. They are the length of the time window, the frequency window and the amplitude of SE, which are shown in Figure 4. The amplitude of SE is defaulted as the maximum amplitude of the time frequency spectra. The length of the frequency window is dependent on the domain frequency and the frequency sampling interval df. It is given by $1/f_0/df$. The length of the time window is associated with the domain frequency and the time sampling interval dt. It is about $\frac{1}{8} \sim \frac{1}{10}$ of $1/f_0/dt$. This also depends on the degree of sparseness of the signal, but generally for dense reflection seismic records,

the degree of sparseness can be ignored.

One needs to analyze the time-frequency spectra first, then choose the largest amplitude and the dominant frequency. Since the H coefficient is used, the identification process of high SNR is simplified. There is no need to test different pairs of parameters and lots of time intervals when compared with other 2D filters. We have also added a short paragraph on this point in the Discussion section.

4. Figure 5 shows the comparisons between 2D MMF, 2D Wiener and 2D median filtering. Is it possible to use one pair of parameters to make a better result for 2D wiener or median filtering? If possible, how to prove the superiority of 2D MMF?

Reply: Thank you for your questions. We also believe that this would be achieved to obtain acceptable and good results by using other 2D filtering methods. But these may require lots of testing for the optimal parameter selections. Relatively speaking, the parameter selection of 2D MMF is simpler and more straightforward. And it is based on topology theory and set theory, which is obviously different and superior to conventional mathematical methods, which ~~have~~ been proved in previous studies (References 38-44). Although from the comparison in Figure 5, the advantages of 2D MMF cannot be clearly reflected. We also utilize the H coefficient to make the regions with high SNR features more obvious, as shown in Figure 6. This has helped to establish that the H-function based on the 2D MMF method is superior.

5. In practical applications, single-parameter control is more concise, and it is convenient for operators to select parameters. In theory, two-parameter control does have better adaptability and superiority. But in practical applications, will the parameter selection be more complicated, and it is not easy to get good results?

Reply: Thank you for your comments. We also agree with you. At present, the single-parameter compensation based on Wang's method is also commonly used in field data processing, which has the characteristics of simple operation. But it needs to test different G_{lim} values to get the best compensation result. Since this step is carried out in the time-frequency domain, the amount of computation is large. The method in this paper can directly select appropriate parameters according to the results of time-frequency analysis, and its physical meaning is more obvious and easier to operate, so it will not make the compensation more complicated.

Suggestion: The 2D MMF based coefficient could be tried to be as a stand-alone noise suppression for future study.

Reply: Thank you for the suggestion. We will try this in future research.