Optimization of Integrated Scheduling of Heating and Rolling in Steel-making Processes

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***Abstract*—To optimize the integrated scheduling of rolling stage of steel-making production with the collaborative process requirements of heating stage, the effect of heating time of slabs in heating stage and variation of slab specifications and completion time in rolling stage should be essentially considered. In this paper, an optimization model of integrated scheduling of heating and rolling processes in steel-making production is established, and the algorithm solving the model is designed and implemented by CPLEX optimizer. The computational experiments are conducted based on the simulated production data, a variety of comparative experiments of various problem scales are designed to verify the effectiveness of the model. Furthermore, the influence of slab parameters in heating and rolling scales with respect to solution efficiency and quality was analyzed.**

***Index Terms*—modeling and optimization, integrated scheduling, heating and rolling, steel-making processes**

I. INTRODUCTION

Hot-rolling production is an important part of integrated production in steel-making enterprises, among which the heating process is also the indispensable prerequisite of realizing the hot charging & hot transfer process. The heating furnace and rolling production scheduling is a hybrid flow-shop problem in which the slabs are divided into heating and rolling lots according to the steel types and specifications of the manufacturing order, provided keeping slabs rolled continuously and the rolling lots are taken as the planning unit to pursue the optimization of certain production criteria. Since the rolling stage is a highly complex production process, its planning plays a rather important role in production quality, cost and production efficiency. Therefore, the optimization of rolling planning has attracted the attention of many experts and scholars.

At present, the research on modeling of rolling planning is highly related to traveling salesman problem (TSP) and vehicle routing problem (VRP), and the practical solution methods are mainly based on heuristic-oriented algorithms (simulated annealing, tabu search, genetic algorithm, etc.), multi-echelon algorithm and mathematical programming (such as mixed integer programming, branch and bound, dynamic programming, etc.).

In the research fields related to heating furnace scheduling and hot-rolling planning, Li F. et al. [1] investigated the integrated slab scheduling problem of soaking pit and hot-rolling with capacity constraints at both stages, they combined heuristic algorithm with branch and bound algorithm to solve the mixed integer linear programming model. Li K. et al. [2] studied the scheduling problem of heating and hot-rolling scheduling problem based on multi-objective differential evolution. Shi et al. [3] considered the heat transfer mechanism in the hot-rolling scheduling problem and designed a large neighborhood search algorithm to solve the problem with improved space compression mechanism. A new compound mutation strategy and an adaptive parameter setting method was brought up, with respect to the reheating mass, surface oxidation loss and consumption of slab during hot-rolling process. Xia et al. [4] studied the heating scheduling of slab based on the differential evolution algorithm, and considered the heat transfer model of heating furnace and the energy consumption of heating furnace in the optimization. Zhang et al. [5] studied the hot-rolling scheduling problem with uncertain factors. In the research of hot-rolling scheduling in compact strip production, Pan et al. [6] divided the problem into two sub-problems of slab allocation and sorting of strip rolling process, and adopted a two-stage heuristic algorithm to solve the problem. With regards to the batch scheduling problem of strip production, Chen et al. [7] divided the problem into two sub-problems of strip slab grouping and slab allocation & sorting problem. With the optimization objectives of minimizing the number of virtual slabs and minimizing the change of average thickness of slabs in the rolling sequence, they also proposed an evolutionary algorithm based on artificial bee colony algorithm to solve the problem. Zhang et al. [8] proposed a mixed variable neighborhood search algorithm to solve the mathematical model with the aim of minimizing the average rolling thickness variation. In the research of multi-stage integrated production, Cowling et al. [9] introduced concept of virtual slab to describe the material form between the two stages, with respect to integrated production of continuous casting and hot-rolling, they applied mathematical programming and heuristic algorithm to solve the model. Lin et al. [10] studied the integrated production problem of continuous casting and hot-rolling, and proposed an improved evolutionary algorithm to solve the multi-objective optimization problem considering hot charging rate of slabs, operating cost and the uncertainty of time interval in production. Tan et al. [11] divided the continuous casting and hot-rolling multi-stage optimization problem into two sub-problems of scheduling on continuous casting and rolling & heating furnace, based on mixed integer programming and constraint programming, they design dedicated algorithms to solve the sub-problems. In consideration of resource conflicts and setup time constraints, the cut generation strategy based on priority relationship is designed. Suzuki et al. [12] put forward an optimization strategy for slab heating scheduling and furnace temperature control, and established a simulation model considering slab loading sequence and temperature distribution in the furnace.

Based on the above research, it showed that the optimization objectives of the heating and hot-rolling stage mainly focus on minimizing the energy consumption of the heating furnace and minimizing the cost of the hot-rolling machine setup time and attract a lot of attention. The solutions to the production planning and hot-rolling scheduling mainly relied on applying intelligent optimization methods to solve optimization models. Since mathematical programming methods are limited practically in solving efficiency, the existing researches mostly combine mathematical optimization with intelligent optimization methods to solve the hot-rolling planning and scheduling problem.

The rest of the paper is separated into following sections: Firstly, the concerned production process of heating and hot-rolling stages is introduced and the corresponding mathematical optimization model is formulated in section II. In section III, computational experiments are carried out for heating and hot-rolling scheduling model to verify the efficiency of the model. Finally, the conclusions and future suggestions are presented in section IV.

II. PROBLEM FORMULATION

A. Heating and hot-rolling Production Process

The heating and hot-rolling stages are located at the rear of the steel production process. With respect to steel production, slabs in the continuous casting stage are transferred along the production line to the rolling-oriented stages for heating and hot-rolling processing.

Heating and hot-rolling production is a process in which slabs are processed for a series of products that meet the requirements of production orders. The process flow chart of heating and hot-rolling is shown in Fig. 1, where the processing equipments mainly consists of two types: heat holding furnaces and rolling mill.

1) Heating Stage: The slabs processed on the heat holding furnaces are mainly from sources of continuous casting machine, slabs warehouse and soaking pit. The initial temperatures of slabs may vary in different slab sources. The process in which the slabs transferred directly from continuous casting machine to the heat holding furnaces is called hot transfer and hot charging process, it has important research significance for reducing production time and energy cost saving. In this paper, the slabs in the heating process come from the continuous casting machine, which is transferred to the heat holding furnaces according to the production planning of continuous casting, i.e. a set of completion times for these slabs output

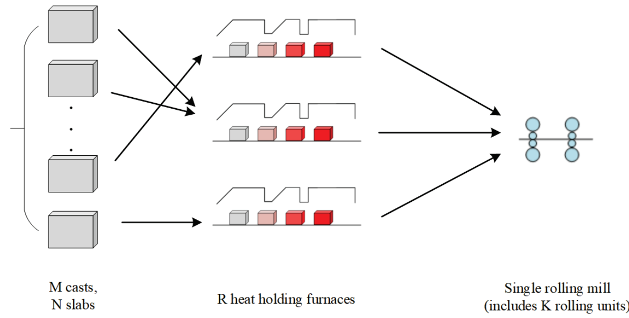

Fig. 1. Heating and rolling process.

by the continuous casting machine. The slabs in heat holding furnaces are transferred to the furnace outlet through the beam crane driven by stepping motor. To ensure that the slabs can meet the temperature requirements of hot-rolling processing, they cannot be heated until the temperature reaches the preset lowest limit.

2) Hot-Rolling Stage: Hot-rolling process as the downstream process of heat holding furnaces, after the slabs are heated properly and out of the furnace, they are seamlessly transported to the rolling mill. After the slab is heated in the furnace to the preset temperature for rolling, the strip steel products are obtained by a sequence of rolling steps from roughing to refining. The strips are cooled after finishing the last rolling step, and then rewound into coils by the coiler. Finally, the steel coils may be processed further according to the requirements of various orders, such as cutting stock process or packaging, etc.

In this paper, a typical hot-rolling production line is investigated, which consists parallel heat holding furnaces in the heating stage and single rolling mill in the hot-rolling stage. Without loss of generality, the assumptions made in this paper on the production process are as follows: 1) the continuous casting capacity matches the rolling capacity of the rolling mill, and the hot transfer and hot charging process is mainly considered in this paper; 2) the continuous casting production plan is known in advance, i.e. the rolling time of continuous casting slabs is given. 3) Parallel heat holding furnaces have different production capacity, that is, the standard heating processing time of the same slab may vary with different heat holding furnaces. 4) The furnace temperature and standard heating processing time of slab in the heating stage are known. 5) Within a rolling unit, the widths of pre-rolling material are sequentially non-decreasing, while the widths of the roll product does not increase in sequence. 6) Specifications of continuous casting slabs may vary from batch to batch.

Therefore, the problems in this paper can be described as: given a set of slabs to be processed on the rolling mill and the specification of each slab, subject to the processing constraints on the time of slabs in the furnace, rolling width & thickness, maximum length of rolling unit, etc. In the stage of processing on the heat holding furnaces, the slabs are grouped and allocated to each furnace, where the slab groups are sequenced in time, in order to optimize the slab heating times. Furthermore, in the hot-rolling stage, the slabs out of heat holding furnaces are aggregated in terms of rolling units to determine the slab rolling times and the production makespan (i.e. maximum completion time of all rolling units).

B. Symbol and Parameter Definition

1) Indices and Symbols:

N: the number of slabs.

R: the number of heat holding furnaces.

K: the number of all possible rolling units.

$i, i' \in \{1, 2, ..., N\}$: slab index.

$l \in \{1, 2, ..., R\}$: furnace index.

$k, k' \in \{1, 2, ..., K\}$: rolling units index.

T_i : completion time of slabs in continuous casting stage.

P_{il}^{std} : standard heating time of slab i in furnace l.

P_{il}^{max} : maximum heating time of slab i in furnace l.

p_i : slab rolling time.

W_i : slab rolling width.

G_i : slab rolling thickness.

L_i : slab rolling length.

G_{max} : maximum rolling thickness variation of slabs in a rolling unit.

W_{max} : maximum rolling width variation of slabs in a rolling unit.

L_{max} : maximum rolling length variation of slabs in a rolling unit.

M : large positive integer with respect to the problem instance.

a : objective weight.

2) Decision Variables:

t_{il} : heating start time of slab i in the furnace l.

e_{il} : heating completion time of slab i in the furnace l.

s_{ik} : rolling start time of slab i in the rolling unit k.

r_k : processing start time of the rolling unit k.

C_k : processing completion time of the rolling unit k.

x_{il}: 0-1 variable, equals to 1 if slab i is processed on the furnace l, otherwise 0.

y_{ik}: 0-1 variable, equals to 1 if slab i is processed on the rolling unit k, otherwise 0.

$z_{kk'}$: 0-1 variable, equals to 1 if the rolling unit k is processed before the rolling unit k', otherwise 0.

$q_{ii'l}$: 0-1 variable, equals to 1 if slab i is processed before slab i' on the furnace l, otherwise 0.

$o_{ii'k}$: 0-1 variable, equals to 1 if slab i is processed before slab i' on the rolling unit k, otherwise 0.

C. Mathematical Model

Total objective function:

$$min\ f = af_1 + (1-a)f_2 \tag{1}$$

where

$$f_1 = \frac{max\ C_k}{\sum_{i=1}^{N} p_i}$$

which is the delay rate of makespan with respect to sum of rolling processing times,

$$f_2 = \sum_{l=1}^{R}\sum_{i=1}^{N}\left(\frac{e_{il} - t_{il}}{P_{il}^{std}}\right)$$

which is the heating timeout rate with respect to sum of standard heating processing times, $i = 1, .., N,\ l \in R$.

Constraints:

$$e_{il} - t_{il} \leq\ P_{il}^{max},\quad i = 1, .., N, l = 1, .., R \tag{2}$$

$$e_{il} - t_{il} \geq\ P_{il}^{std},\quad i = 1, .., N, l = 1, .., R \tag{3}$$

$$\sum_{l=1}^{R} x_{il} = 1, i = 1, .., N \tag{4}$$

$$t_{il} - M\left(1 - x_{il}\right) \geq\ T_i,\quad i = 1, .., N, l = 1, .., R \tag{5}$$

$$e_{il} - e_{i'l} - M\left(q_{ii'l} - 1\right) \leq\ 0,\quad i = 1, .., N, l \in R \tag{6}$$

$$e_{il} - s_{ik} - M\left(x_{il} + y_{ik} - 2\right) \leq\ 0,\quad i = 1, .., N, l \in R, k = 1, .., K \tag{7}$$

$$\sum_{k=1}^{K} y_{ik} = 1, \quad i = 1, .., N \tag{8}$$

$$r_{k'} - r_k - \sum_{i=1}^{N} y_{ik}\ p_i + M\left(1 - z_{kk'}\right) \geq\ 0,\quad k, k' = 1, .., K \tag{9}$$

$$s_{ik} + p_i - s_{i'k} - M\left(o_{ii'k} - 1\right) \leq\ 0,\quad i, i' = 1, .., N, k = 1, .., K \tag{10}$$

$$C_k - r_k - \sum_{i=1}^{N} y_{ik}\ p_i \geq\ 0,\quad k = 1, .., K \tag{11}$$

$$s_{ik} - r_k + M\left(1 - y_{ik}\right) \geq\ 0,\quad i = 1, .., N, k = 1, .., K \tag{12}$$

$$C_{k'} - C_k - \sum_{i=1}^{N} y_{ik}\ p_i + M\left(1 - z_{kk'}\right) \geq\ 0,\quad k, k' = 1, .., K \tag{13}$$

$$\sum_{i=1}^{N} y_{ik}\ L_i \leq\ L_{max}, \quad i, i' = 1, .., N, k = 1, .., K \tag{14}$$

$$o_{ii'k}\left|W_{i'} - W_i\right| \leq\ W_{max},\quad i, i' = 1, .., N, k = 1, .., K \tag{15}$$

$$o_{ii'k}\left|G_{i'} - G_i\right| \leq\ G_{max},\quad i, i' = 1, .., N, k = 1, .., K \tag{16}$$

Constraint (2), (3) ensures the heating time of each slab in the furnace. Constraint (4) indicates that a slab is assigned to only one heat holding furnace for heating. Constraint (5) ensures heating start time not earlier than compleition time of continuous casting and coupled with furnace assignment variable x_{il}. Constraint (6) denotes the heating completion time of slabs should concide with their sequence into the same furnace. Constraint (7) means that slabs' heating completion time not less than its rolling start time and coupled with rolling assignment variable y_{ik} of each slab. Constraint (8) indicates that a slab is assigned to only one rolling unit. Constraint (9) ensures the validity of sequential start time of the rolling units. Constraint (10) restricts the processing start time of adjacent slabs within a rolling unit. Constraint (11) imposes the relation of start time, processing & completion time of a rolling unit, also coupled with with rolling assignment variable y_{ik} of each slab. Constraint (12) implies that the rolling start time of each slab in a rolling unit is not earlier than the start time of the rolling unit. Constraint (13) denotes the relation of completion time of two adjacent rolling units. Constraint (14) indicates the maximum rolling length limitation of any rolling unit. Constraint (15) and (16) are the variation restrictions on adjacent slabs in a rolling unit, respectively on widths and thickness.

III. COMPUTATIONAL EXPERIMENTS

In order to verify the effectiveness of the rolling-oriented multi-stage production optimization model, which is a mixed integer programming (MIP), a dedicated algorithm is designed and implemented with CPLEX optimizer, based on branch and cut method and associated search mechanisms.

In this paper, with regard to the problem instances for the experiments, multiple problem scales are considered. Based on the availability of practical production data, the data of each problem instance is generated with random number generator. To make experimental comparison, with respect to heating process, the parameters about the problem scale of heat holding furnaces scheduling are considered, including slab batch scales, quantity of heat holding furnaces; with respect to rolling process, the scale factors affecting hot-rolling scheduling problem are also considered, including rolling unit scale, variation of slab specifications and so on. The numbers of slabs, heat holding furnaces and rolling units are set up in each problem instance, whose data structure also includes as slab id, completion time of slabs in continuous casting stage, slab specification (width and thickness), rolling specification (thickness and width), standard heating time, standard rolling time, etc. To simply show the effectiveness of the mathematical model, a small-scale problem instance is firstly presented, where the production process parameters of heating and rolling constraints are set as: $G_{max} = 3, W_{max} = 30, L_{max} = 4000, a = 0.36$.

In order to investigate the effect of parameters variations (in casts scales, number of heat holding furnaces and rolling units) on the solution quality and time consuming, the experiments are set up each with fixed number of slabs (i.e. 50 slabs) for comparison, and other parameters varies in the number of batches, number of heat holding furnaces and rolling units, respectively.

TABLE I
COMPARISON EXPERIMENTS OF 50 SLABS

Instances (casts-furnaces-units)	**Iteration**	**CPU(s)**	**Best Solution**	**Average Solution**
5-2-3	82910	10.91	69.738	69.829
5-2-5	1336532	63.86	69.69	69.764
5-4-3	60992	12.83	137.791	137.832
5-4-5	101383	13.09	137.744	137.816
10-2-3	103470	7.65	70.168	70.175
10-2-5	27790	4.35	70.121	70.140
10-4-3	2562779	3635	329.894	332.28
10-4-5	2267907	3606	253.052	262.286

The statistical analysis of the data in Table I shows that the increase in the number of heat holding furnaces has a negative impact on the solving efficiency. As the numbers of heat holding furnaces and rolling units increase, the iterations increases and the solving time increases as well. By comparing the examples 10-2-3 (i.e. 50 slabs, 10 casts, 2 heat holding furnaces, 3 rolling units), 10-2-5, which are differ in rolling unit scales.It shows that in similar solving time, the instance with more rolling units can obtain the best feasible solution with fewer iterations. In terms of solution quality, the number increase of heat holding furnaces may not improve the optimal solution, and the objective function solution may benefit to some extent from the number increase of rolling units.

Fig. 2-7 show the integer solution curves of problem instances differ in casts scales and heat holding furnaces scales of 50 slabs. Fig. 2 & 6 show that the slope of the data curve for smaller casts is larger, that is, the convergence of integer solutions is faster. By comparative analysis of Fig. 2 & 3, the number increase of rolling units slows down the solving speed and increases the number of feasible solutions. Compared with the results of instances 5-2-3 and 5-4-3 (Fig. 2 & 4), the objective function shows significant difference with respect to number of heat holding furnaces. As for slabs specification, heating processing time of a slab varies with different furnaces, hence the appropriate furnaces scale is sensitive to the optimization results of production time cost.

Based on the comparatively experimental results of 50 slabs, it is analyzed that the number increase of heat holding furnaces may not have a positive effect on the scheduling results. The optimal scheduling results can be obtained by selecting the appropriate number of heat holding furnaces combined with the slabs number. With the same number of heat holding furnaces, as the number of rolling units increases, the iterations and solving time increase, the solution's objective value may be better than that of smaller number of rolling units. As shown by the last two groups of data, the solution efficiency decreases significantly as number of rolling units increase, and the gap between the best solution and the optimal solution increases, therefore, the optimal solution cannot be obtained within a satisfactory time.

The parameters related to the decision variables are given in

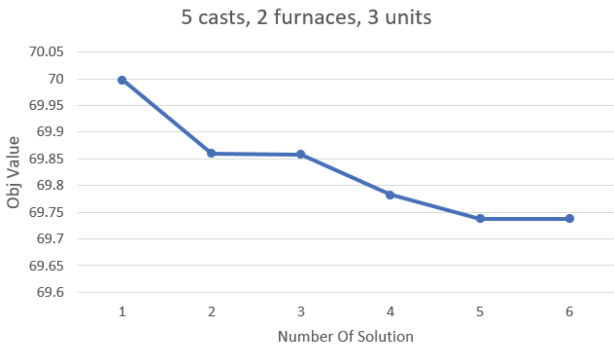

Fig. 2. 5-2-3 solution curve.

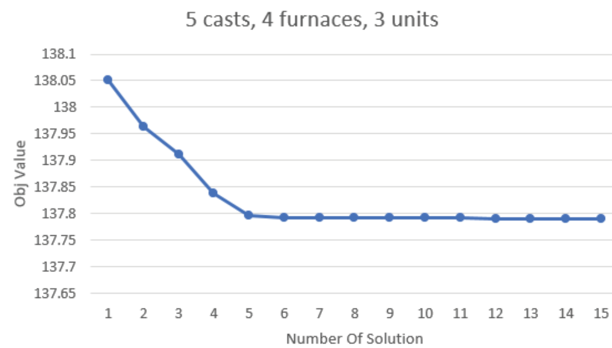

Fig. 4. 5-4-3 solution curve.

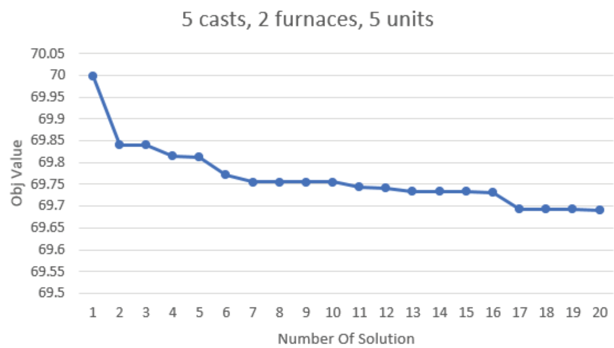

Fig. 3. 5-2-5 solution curve.

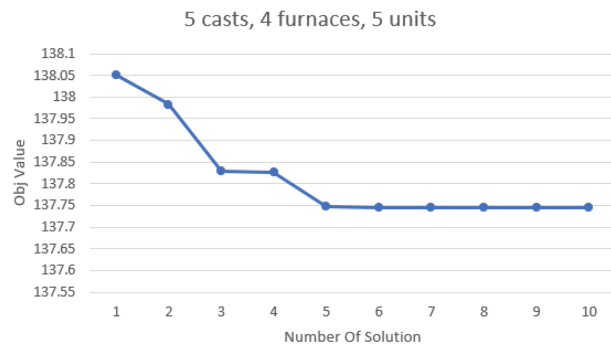

Fig. 5. 5-4-5 solution curve.

TABLE II
HEATING AND HOT-ROLLING PARAMETER STATISTICS

Instances (casts-furnaces-units)	**heating time delay**	**makespan**
5-2-3	90	449
5-2-5	90	277
5-4-3	90	450
5-4-5	90	281
10-2-3	195	513
10-2-5	195	322

TABLE III
SLAB SCALE COMPARISON EXPERIMENT

Instances (slabs-casts-furnaces-units)	**Iteration**	**CPU(s)**
50-10-2-3	958	7.61
80-10-2-3	1862	6.49
100-10-2-3	2383	10.37
120-10-2-3	2487	13.37
140-10-2-3	3152	17.66
160-10-2-3	2737	19.74
180-10-2-3	3195	25.65
200-10-2-3	5456	35.30

Table II, which involve the heating time of each heat holding furnace and the maximum completion time of all rolling units. With the 50 slabs, the number of heat holding furnacess has not affected remarkably the heating time results, but strongly correlates to the optimization results of the maximum completion time of rolling units, i.e. the rolling units plays a critical role on optimizing the makespan.

In order to verify the solving efficiency of the model with respect to different slabs number, four groups of comparative experiments with different total numbers of slabs(50 to 200) are set up with the data shown in Table III. In the same heat holding furnaces and rolling unit scales(10 casts, 2 furnaces, 3 rolling units), the different setting on the slabs number has a significant impact on the solving efficiency, among which smaller number of slabs may approximate the optimal integer solution in a shorter time.

The comprehensively analyse of solutions with the model indicates that, the constraints related to slabs assignments to either heat holding furnaces or rolling unit and related to variation of slab specifications form the boundary of the feasible domain, they tend to be binding or tighter to some sense. The time sequence dependent constraints becomes loose constraint in the process of solving.

IV. CONCLUSION AND SUGGESTIONS

In this paper, based on the actual production situation of steel-making process, the integrated scheduling of heating and hot-rolling stage in steel production is studied. By analyzing the characteristics of two-stage production process, the time

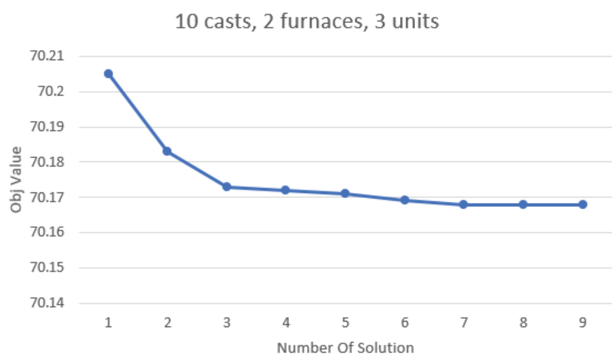

Fig. 6. 10-2-3 solution curve.

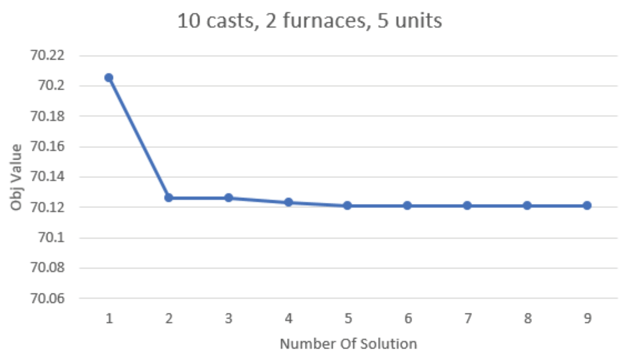

Fig. 7. 10-2-5 solution curve.

cost in each stage is essential optimization factor, and a set of processe constraints are taken into account, such as the variation of slab specification in the hot-rolling stage, capacity of heat holding furnacess, etc. The mathematical model of optimization of integrated scheduling of heating and rolling in steel-making processes is proposed, which aims to optimize the total processing time and the unnecessary delay time. A dedicated algorithm is coded to solved the MIP featured model by CPLEX optimizer. Various experiments based on the practical production environment are designed to testify the effectiveness of the model and algorithm. The experimental results and comparative analysis show that the proposed mathematical model may effectively validate the processes and time sequence constraints in the hot-rolling oriented two-stage production. However, the algorithm has some limitations in problem scales, even though it is efficient and not sensitive to the numbers of heat holding furnaces and rolling units for smaller-scale problems; as the slabs number increases, the solving efficiency decreases obviously and only compromised solution can be obtained. It is necessary to further investigate the model and the algorithm to reduce the solving time for larger-scale problems, e.g. with respect to the model's coefficient matrix, the model can be divided into smaller tractable subproblems controlled by the associated restrict master problem,so as to reduce the iteration scale and improve the solving efficiency.

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REFERENCES

[1] F. Li, Y. Zhang, H. Wei, and X. Lai, "Integrated problem of soaking pit heating and hot rolling scheduling in steel plants," *Computers & Operations Research*, vol. 108, no. AUG., pp. 238–246, 2019.

[2] K. Li and H. Tian, "Integrated scheduling of reheating furnace and hot rolling based on improved multiobjective differential evolution," *Complexity*, vol. 2018, pp. 1–19, 2018.

[3] Y. Shi and S. Liu, "Very large-scale neighborhood search for steel hot rolling scheduling problem with slab stack shuffling considerations," *IEEE Access*, vol. 9, pp. 47 856–47 863, 2021.

[4] Q. Xia, X. Wang, and L. Tang, "Operation optimization of slab reheating process based on differential evolution," *Isij International*, vol. 56, no. 11, pp. 2006–2015, 2016.

[5] R. Zhang, S. Song, and C. Wu, "The hot strip mill scheduling problem with uncertainty: Robust optimization models and solution approaches," *IEEE Transactions on Cybernetics*, pp. 1–15, 2022.

[6] Chen, Qing-da, Pan, Quan-Ke, Gao, Liang, Wang, Bing, Meng, and Tao, "A mathematical model and two-stage heuristic for hot rolling scheduling in compact strip production," *Applied mathematical modelling*, vol. 48, pp. 516–533, 2017.

[7] Q. Chen, Q. Pan, B. Zhang, J. Ding, and J. Li, "Effective hot rolling batch scheduling algorithms in compact strip production," *IEEE Transactions on Automation Science and Engineering*, vol. 16, no. 4, pp. 1933–1951, 2019.

[8] B. Zhang, Q. K. Pan, L. Gao, X. L. Zhang, and Q. D. Chen, "A hybrid variable neighborhood search algorithm for the hot rolling batch scheduling problem in compact strip production," *Computers & Industrial Engineering*, vol. 116, no. FEB., pp. 22–36, 2017.

[9] P. Cowling and W. Rezig, "Integration of continuous caster and hot strip mill planning for steel production," *Journal of Scheduling*, vol. 3, no. 4, pp. 185–208, 2015.

[10] J. Lin, M. Liu, J. Hao, and S. Jiang, "A multi-objective optimization approach for integrated production planning under interval uncertainties in the steel industry," *Computers & Operations Research*, vol. 72, no. aug., pp. 189–203, 2016.

[11] Y. Tan, M. C. Zhou, Y. Wang, X. Guo, and L. Qi, "A hybrid mip-cp approach to multistage scheduling problem in continuous casting and hot-rolling processes," *IEEE Transactions on Automation Science and Engineering*, vol. PP, no. 99, pp. 1–10, 2019.

[12] M. Suzuki, K. Katsuki, J. I. Imur, J. I. Nakagawa, T. Kurokawa, and K. Aihara, "Simultaneous optimization of slab permutation scheduling and heat controlling for a reheating furnace," *Journal of Process Control*, no. 24-1, 2014.