# Improving Autoformalization using Type Checking

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## **Abstract**

Autoformalization, the automatic translation of unconstrained natural language into formal languages, has garnered significant attention due to its potential applications in theorem proving, formal verification, and LLM output checking. In this work, we analyze both current autoformalization methods and the processes used to evaluate them, focusing specifically on the Lean 4 theorem proving language. We demonstrate that scaling type-check filtering with self-consistency techniques on top of existing methods significantly improves performance, achieving absolute accuracy gains of up to +18.4% on ProofNet. To support reproducibility and further research, we release our code, including new symbolic equivalence for Lean formulas. We also release new benchmarks: a new research-level mathematics dataset RLM25, a corrected ProofNet, and ProofNetVerif with labeled correct and incorrect autoformalization pairs for evaluating metrics.

## 1. Introduction

Automatic verification of logical reasoning holds promise for formal verification of mathematical proofs, software verification, and artificial intelligence. Proof assistants enable users to rigorously express mathematical statements and mechanically check their proofs (Mahboubi & Tassi, 2022; Paulson, 2023; Avigad, 2024), but they require *formalization*: translating informally stated mathematical statements into a formal language. Formalization is highly nontrivial, prompting new research into methods that automate it, a task referred to as *autoformalization* (Szegedy, 2020).

Current state-of-the-art autoformalization methods rely on the few-shot formalization capabilities of large language models (Wu et al., 2022; Azerbayev et al., 2023a), distilled

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back-translation (Jiang et al., 2023; Azerbayev et al., 2023a), or retrieval augmented generation (Azerbayev et al., 2023a; Anonymous, 2024). The success rate of these techniques has been limited to this point, with reported state-of-the-art accuracy ranging from 16.1% Azerbayev et al. (2023a) to 18.9% in Anonymous (2024) on the ProofNet benchmark.

A common failure mode of these methods is their inability to generate formalizations that *type-check* in Lean 4 (Moura & Ullrich, 2021). Type checking implies respecting syntax, applying operations to arguments in their domain, and valid use of the formal libraries of math knowledge (mathlib Community, 2020). All these conditions are checked using efficient symbolic algorithms of Lean. Although type-checking a statement without a proof does not ensure that the statement is a correct translation of the informal input, it is a precursor for a correct translation. It is both deterministic and fast, enabling easy automation. We observe that the type-checking rates for these methods range from 4% to 57.2% (Jiang et al., 2023; Azerbayev et al., 2023a; Anonymous, 2024) in the literature, depending on the technique and the benchmark.

To improve final formalization performance, a line of work is studying sampling multiple formalizations from LLMs (Li et al., 2024b; Agrawal et al., 2022), showing performance improvements when sampling with up to n=10formalization attempts. In this work, we study how these improvements continue to scale as n is increased, evaluating performance across multiple models, autoformalization methods, and benchmarks. We study different ways of extracting a single prediction out of several samples, in particular through the use of type-check filtering and selfconsistency methods. Our evaluation of the produced formalizations demonstrates that, through sampling, one can substantially increase autoformalization accuracy, with a particularly notable increase on the best-performing model: the performance of GPT-40 improves from 31.0% to 45.1% accuracy on the ProofNet benchmark using 50 samples.

Automating the evaluation of autoformalization methods remains a significant challenge, which recent studies have begun to address (Anonymous, 2024; Lu et al., 2024b). To scale our experiments to  $n=1\,000$ , we develop BEq+, a relatively efficient reference-based metric inspired by BEq (Anonymous, 2024), and demonstrate it is highly corre-

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lated with human evaluations. Recognizing that reference-based metrics depend on benchmark quality, we conduct a thorough analysis of ProofNet (Azerbayev et al., 2023a), identifying and correcting formalization errors to produce ProofNet#. We additionally introduce ProofNetVerif, a dataset designed to evaluate both reference-based and reference-free metrics using manual annotations. Beyond existing benchmarks, we aim to assess autoformalization methods in more realistic research settings. To this end, we curate RLM25, a new benchmark based on real-world formalization projects.

We summarize our contributions as the following:

- We evaluate autoformalization methods on ProofNet#, our revised version of ProofNet with corrections, and on RLM25, our newly introduced benchmark based on several research-level natural language-aligned formalization projects. To the best of our knowledge, this is the first time such an evaluation is conducted on real projects without relying on synthetically generated natural inputs.
- We observe that, empirically, current LLMs struggle even to generate well-typed predictions in Lean 4. This result holds across all tested models, whether the models are general or domain-specific, and fine-tuned or not on the autoformalization task. We find that generating several candidates and filtering ill-typed statements yields large improvements. The approach is scalable as the performance still increases after n = 1000 formalization attempts, and achieves state-of-the-art results. Despite its simplicity and applicability on top of any sampling-compatible techniques, this method has been overlooked in autoformalization literature.
- Our findings indicate that correct formalizations often exist among samples generated by LLMs, but identifying them remains the key bottleneck in fully harnessing their potential. To advance research in this direction, we introduce **ProofNetVerif**, a dataset of 3 752 formalinformal pairs annotated with binary semantic equivalence labels.
- We release BEq+, a reference-based metric based on deterministic symbolic computation and running exclusively on a CPU. We demonstrate that BEq+ correlates strongly with human annotations at the benchmark level, meaning that aggregated results of this metric can be used to compare autoformalization methods.

## 2. Related Work

**LLM sampling-based methods.** Our method uses a selection step in which we employ self-consistency methods such as majority voting (Wang et al., 2023) and Self-BLEU (Zhu et al., 2018). Such methods have empirically proven to be effective across a wide range of NLP tasks (Li et al., 2024a).

In particular, Lewkowycz et al. (2022) demonstrated the effectiveness of the combination of sampling and majority voting on the MATH benchmark (Hendrycks et al., 2021). Further works in this direction improve over majority voting by using trained verifiers (Hosseini et al., 2024).

Interactive Theorem Proving. Autoformalization in mathematics depends on formal systems, such as Coq (Castéran & Bertot, 2004), Lean (Moura & Ullrich, 2021), Isabelle (Nipkow et al., 2002), and their math libraries. In this work, we focus on Lean: a powerful interactive theorem prover with a growing formal library of definitions and proven statements known as Mathlib (mathlib Community, 2020). We focus on the current version of Lean, Lean 4. de Moura et al. (2015) and Moura & Ullrich (2021) provide insights into the inner workings of Lean type-checking.

**Autoformalization.** Classical programmatic tools can be used to translate *constrained* natural language statements into formal systems (Pathak, 2024). In contrast, we are interested in translating unconstrained natural language statements. In Wu et al. (2022), the authors find LLMs to be a promising approach, capable of autoformalization through the use of in-context learning. In Azerbayev et al. (2023a) and Jiang et al. (2023), the authors demonstrate that distilled back-translation improves performance of some base models. Agrawal et al. (2022) use an advanced postprocessing step to automatically fix type errors in LLM predictions. They find that, for a given problem, keeping only well-typed predictions when generating several formalization attempts is a strong filter. In a concurrent work to ours, (Li et al., 2024b) propose a self-consistency approach specifically designed for autoformalization. Their method clusters logically equivalent formalizations using automated theorem-proving techniques. They evaluate this approach only up to n=10 samples. RAutoformalizer (Anonymous, 2024) is a recently proposed autoformalization model. Although it was not available at the time of our study, its BEq@8 results show significant gains over BEq@1, highlighting the potential of sampling-based methods to improve accuracy.

Metrics. Evaluating the accuracy of models on the statement autoformalization task is a non-trivial problem. In previous works (Wu et al., 2022; Agrawal et al., 2022; Azerbayev et al., 2023a; Jiang et al., 2023), manual evaluation is the standard practice to report statement autoformalization performance. This manual evaluation effort, though comprehensive and methodical, is an important bottleneck that limits the number of experiments that can be run. Several works suggested different methods to alleviate this issue. BLEU has been used in Wu et al. (2022); Azerbayev et al. (2023a); Ying et al. (2024b) as a proxy for accuracy. However, the use of this metric lacks theoretical and empirical evidence. In particular, in Azerbayev et al. (2023a) the authors found

that the correlation between BLEU and formalization accuracy is low. Type-check rate has been proposed as a proxy for accuracy Azerbayev et al. (2023a); Agrawal et al. (2022). As we demonstrate in subsection 6.2, this metric also has severe limitations. In recent works (Anonymous, 2024; Li et al., 2024b), promising symbolic equivalence metrics have been used to measure autoformalization performance.

**Benchmarks.** ProofNet (Azerbayev et al., 2023a) is a benchmark specifically designed for autoformalization. It consists of 371 undergraduate mathematical exercises, making it an essential benchmark for evaluating the performance of autoformalization models. In a recent work on neural theorem proving (Hu et al., 2024), the authors evaluated their automated theorem prover method on research-level formal projects. Similarly, in Anonymous (2024), the authors evaluate their statement autoformalization method on Con-NF using LLM-generated natural language statements.

## 3. Manual and Symbolic Metrics

This section presents our approach to evaluating autoformalization methods. To validate our empirical results, we rely both on manual evaluation and on newly introduced metrics backed by symbolic computation,  $BEq_L$  and BEq+. As in prior work, we define a formalization as **correct** if it is semantically equivalent to the provided natural language statement. Throughout this paper, **accuracy** refers to the proportion of statements evaluated as correct.

**Manual Evaluation.** Currently, the most reliable evaluation for autoformalization is a manual evaluation by persons with sufficient understanding of the formal proof assistant and its library. This is the approach we used for ProofNet. We evaluated each prediction twice to reduce human mistakes.

Symbolic Computation Metrics  $BEq_L$  and BEq+. To automate evaluation, we use metrics based on comparing a candidate formalization to a reference formalization by checking equivalence between two formulas using symbolic algorithms inside the proof assistant. We invoke proof scripts in Lean that try to prove each formula from the other. We used BEq in names of these metrics to acknowledge prior work (Anonymous, 2024) which, additionally, leverages a 20B LLM trained on the theorem proving task. Instead of an LLM, we employ two purely symbolic, CPU-only equivalence checks that are not only more computationally efficient and interpretable but also eliminate the need for specialized hardware, allowing us to scale our experiments effortlessly to  $n=1\,000$  samples per query.

**BEq**<sub>L</sub>. This metric is based on proving formula equivalence using the exact? tactic (tactics can be seen as proof steps). As noted by the authors (Anonymous, 2024), this tactic can prove equivalence for a wide variety of syntactic variations of equivalent formal statements. exact? can, in general,

```
Algorithm 1 BEq+ - Unidirectional
  Input: Theorem formalizations t_1 and t_2
  Output: Whether t_2 can be derived from t_1
  1. Run BEq<sub>L</sub>
  if exact? closes proof by using t_1 then
    return TRUE
  2. Leverage conclusion matching
  if apply t_1 or convert t_1 succeeds then
    Proving t_1 assumptions can be derived from t_2 ones
    if repeated applications of tauto, simp_all_arith
    !, noncomm_ring, or exact? close proof then
       return TRUE
  3. Attempt direct assumption of t_1
  if have : goal(t_1) := by apply_rules [t_1]
  succeeds then
    if repeated applications of tauto, simp_all_arith
    !, noncomm_ring, or exact? close the subproof in-
    troduced by have then
             repeated
                         applications
                                         of
       simp_all_arith!, or exact? using this
       close the main proof then
         return TRUE
  return FALSE
```

use other theorems from the libraries to prove the current formalization, resulting in a situation where unrelated true theorems are considered equivalent. We therefore restrict the exact? tactic to only use the candidate equivalent formula. Even though this metric has a relatively high false negative rate, we believe its simplicity makes it easier to get reproducible results across works.

**BEq+.** To improve recall compared to  $BEq_L$ , we introduce a new symbolic metric BEq+ implemented in Python using Lean REPL (Morrison, 2023). BEq+ explores several alternative proof strategies for each direction of the implication, as summarized in Algorithm 1 pseudocode. A description of the tactics used can be found in subsection A.1. We apply this algorithm once for each of the two implication directions.

We conduct studies on these introduced metrics in subsection 6.2, and show their strong correlation with human evaluation. Our experiments also show that BEq+ improves the recall on ProofNetVerif from 30.9% to 48.3%.

#### 4. New Benchmarks

Our paper contributes several new benchmarks. Most importantly, we introduce an entirely new benchmark RLM25 (Research-Level Mathematics 2025) based on six Lean blueprints that formalize theorems in modern mathematics (Table 1). We believe RLM25 is more representative of the intended use of autoformalization in mathematics re-

Table 1: Lean Blueprint projects used to build RLM25

#Thms	Lean	First Commit
111	4.14.0-rc2	20 Oct 2023
56	4.14.0-rc2	19 Nov 2023
84	4.7.0-rc2	22 Mar 2024
145	4.14.0-rc3	13 Nov 2023
l 99	4.14.0-rc2	9 Jan 2024
124	4.13.0-rc3	22 Feb 2024
	111 56 84 145 1 99	111 4.14.0-rc2 56 4.14.0-rc2 84 4.7.0-rc2 145 4.14.0-rc3

search. We also manually re-evaluate ProofNet and identify a large percentage of mislabeled pairs, leading us to release a corrected version, ProofNet#. From the manual evaluation, we also derive ProofNetVerif, based on the exhibited behavior of LLMs on informal mathematical statements appearing in ProofNet.

#### 4.1. RLM25: Research-Level Mathematics

To better evaluate the use of autoformalization for formalizing new mathematical results, we introduce a new benchmark, RLM25. Table 1 shows six formalization projects using the Lean blueprint framework (Massot, 2025) that we use to create the benchmark of 619 pairs of a natural language statement and its Lean formalization with context<sup>1</sup>. To alleviate data contamination concerns, Table 1 includes the first commit date of each project<sup>2</sup>. For all these projects, this commit date comes after the announced knowledge cutoff date of the models we use in this paper: October 2023 for GPT-4o, March 2023 for LLama3 8B, and August 2023 as the release date of Llemma 7B. Because these projects contain natural language-aligned formalizations of researchlevel mathematics, they are suitable for evaluating statement autoformalization methods. We explain the curation process in subsection A.3. To the best of our knowledge, we are the first to conduct such a study on real projects without relying on synthetically generated natural language inputs.

## 4.2. ProofNet#: A Corrected ProofNet for Lean 4

To compare to past autoformalization approaches (Azerbayev et al., 2023a; Jiang et al., 2023; Anonymous, 2024), we use the ProofNet benchmark (Azerbayev et al., 2023a). It originally contains 371 pairs of informal statements in undergraduate mathematics and corresponding formalizations in Lean 3. It is divided into a validation set with 185 samples and a test set with 186 samples. As we focus on Lean 4, we start from two Lean 4 ports of ProofNet (Vishwakarma, 2024; Xin et al., 2024b). Our analysis reveals that these ports are a direct translation from Lean 3 to Lean 4 with minimal changes. Our further inspection

shows that the published Lean 4 versions contain 118 entries with formalization mistakes, which is 31.8% of the total entries. When using reference-based metrics, such as BEq, this can severely bias the results. We, therefore, conducted a meticulous correction process of ProofNet, leading to a new dataset we call ProofNet# (see subsection A.2). We also make the benchmark compatible and well-typed for Lean versions 4.7.0 to the latest 4.16.0-rc2. ProofNet# remains very close to ProofNet, as only the reference formalizations are updated. Hence, the results reported in other works using reference-free metrics (e.g., human evaluation and type-check rate) remain the same.

## 4.3. ProofNetVerif for Benchmarking Metrics

Using manual annotations from this paper, we curate a new benchmark: ProofNetVerif. Achieving high performance on this benchmark serves two key purposes: (1) establishing robust evaluation metrics for autoformalization, and (2) improving selection strategies by filtering out incorrect formalizations, thereby narrowing the search space in sampling-based methods. This benchmark contains rows of the following tuples: natural language statement, reference formalization, predicted formalization, and a boolean label indicating semantic equivalence between the predicted formalization and the natural language statement. This makes it suitable to evaluate both reference-free and reference-based metrics. The benchmark contains 3 752 rows, with 1 142 entries of predicted formalizations equivalent to the natural input, and the other ones non-equivalent. We believe this benchmark to be challenging and representative of current LLMs autoformalization mistakes.

## 5. Sampling Method

We present our sampling method, illustrated in Figure 1.

#### 5.1. Base Autoformalization Methods

We consider the following autoformalization approaches as a starting point, to make sure that we have a fair baseline for comparison with our sampling method.

**In-context learning.** On ProofNet#, we use the 12-shot prompt from Azerbayev et al. (2023a) updated to Lean 4, which we share in subsection A.14. On research-level formalization projects, similar to Hu et al. (2024) for neural theorem proving, we also consider in-file context, i.e., the content preceding the official formalizations in the project files.

**Fine-tuning on MMA.** Empirically, it has been shown that LLMs are better at informalization, i.e., translating formal statements to informal mathematical statements, than autoformalization (Wu et al., 2022; Azerbayev et al., 2023a). Using this fact, Jiang et al. (2023) informalized the Lean 4

<sup>&</sup>lt;sup>1</sup>We obtained agreement from the primary authors of these projects to evaluate our models on them.

<sup>&</sup>lt;sup>2</sup>The authors of the projects confirmed that the first commit dates correspond to the first public appearance of these projects.

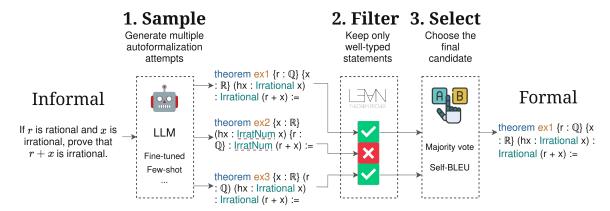


Figure 1: **Illustration of our sampling-based method. 1. Sample:** An LLM generates several candidate Lean 4 formalizations for a provided informal statement. **2. Filter:** The Lean 4 proof assistant type-checks them and filters out the statements that do not pass (the statement has hallucinated IrratNum, a type that does not exist in Mathlib4). **3. Select:** A selection heuristic, such as majority vote or Self-BLEU, is applied to the remaining candidate formalizations, and a single final formalization is returned.

Mathlib library with GPT-4 OpenAI et al. (2024a) to create a dataset, MMA, of formal-informal pairs.

**Fine-tuning on Lean Workbook.** in a recent work (Ying et al., 2024a), the authors release a synthetically generated training set for statement autoformalization. They train a model on MiniF2F and ProofNet benchmarks, and then, through active learning, they curate a train set of  $\sim 57K$  formal-informal pairs. Because of this training on the benchmarks, we only evaluate models fine-tuned on this dataset on the RLM25 benchmark.

## 5.2. Our Sampling-Based Method

Our sampling method, Figure 1, is designed as a plug-andplay improvement that can be seamlessly integrated with existing techniques to enhance their performance. Our insights come from inspecting ill-typed predictions from our baseline study. We found that the predictions are often only slightly wrong, and that a few fixes were enough to make them well-typed. In other words, tested LLMs often use correct vocabulary to formalize, but have difficulties mastering Lean type system and using correctly the Mathlib library (mathlib Community, 2020). These findings led us to the idea behind our method: exploring the local formalization distribution generated by the models using low-temperature sampling, type-check filtering to capture well-typed formalizations, and finally using a heuristic to select a candidate. For the selection method, we compare: (1) random, as a baseline, (2) majority voting (Wang et al., 2023), (3) Self-BLEU (Zhu et al., 2018), and (4) symbolic equivalence (Li et al., 2024b). A more detailed description of each component is presented in subsection A.4. In the next section, we will see that sampling with n = 50 with Self-BLEU raises accuracy of GPT 40 from 31.0% to 45.0% on ProofNet, which is substantially higher than reported so far.

# 6. Experiments

## 6.1. Models Used in Experimental Setup

We consider the following models for our experiments.

Llemma-7B & 34B (Azerbayev et al., 2023b). These open models are based on CodeLlama 7B and 34B (Rozière et al., 2024) and have been further pre-trained on the ProofPile-2 collection of mathematical data (explicitly excluding ProofNet), which was introduced along with these models. Due to their training data, these math models are particularly suited for formal-related tasks.

**Llama3-8B-Instruct** (**Grattafiori et al., 2024**). This is a state-of-the art open 8B model from the LLama3 family.

GPT-4-turbo (OpenAI et al., 2024a) and GPT-4o (OpenAI et al., 2024b). These are state-of-the-art general LLMs. We use versions gpt-4-turbo-2024-04-09 and gpt-4 o-2024-05-13 for reproducibility.

#### 6.2. Metric Study

Table 2: Binary performance, in percentage, of  $BEq_L$  and BEq+ metrics on ProofNetVerif.

Metric	$\mathbf{BEq}_L$	BEq+
Precision	100.0	98.0
Recall	30.9	48.3
F1 Score	47.2	64.7

We use symbolic equivalence metrics (Section 3) to scale up our experiments as manual evaluation is costly and timeconsuming. It is therefore important to validate whether

Table 3: Correlation between human-reported accuracy and different automated metrics on ProofNet# using data from all models evaluated in this paper.

Metric	Pearson	Kendall
Type-Check	0.655	0.560
$\mathrm{BEq}_L$	0.966	0.846
BEq+	0.974	0.872

these metrics are accurate enough to approximate human evaluation. In the related work (Anonymous, 2024), the authors conduct a study on 200 sampled formalization attempts and show that BEq is quite accurate at the instance level, but with a relatively high amount of false negatives. Similarly, as reported in Table 2, both our BEq-inspired metrics have low recall while having near-perfect precision on ProofNetVerif. This means that current BEq metrics underestimate the real accuracy of evaluated models. More details can be found in subsection A.5.

While measuring the agreement of these metrics at the instance level provides some insight into their usefulness, we argue that it fails to capture their behavior at the benchmark level, specifically, how they perform on the aggregated value computed across all entries in the benchmark. We show that BEq+ has strong correlation with accuracy. Leveraging all our manual annotations on ProofNet# conducted for this work, we measure the correlation of our BEg implementations and human evaluation at the benchmark level, amounting to 65 data points (different models and method choices, see subsection A.16), and report the results in Table 3. Correlation factors between our BEq implementations and human evaluation are fairly high, with a Kendall coefficient of 0.872 for BEq+, confirming that symbolic equivalence provides a strong automated metric for autoformalization.

We also find that the type-check rate correlation with human evaluation is relatively low, experimentally confirming that this is not robust enough to be used as a substitute to human evaluation. However, our empirical results in Figure 4 suggest that type-check rates can probably be used to approximate accuracy when comparing different experiments for a *single* model. This can be particularly useful in reference-free setups outside of benchmarks where no reference formalizations are available.

#### 6.3. Empirical Analysis of Sampling-Based Methods

In this section, we conduct an ablation study of various parameters involved in our sampling-based method. By default, we use a temperature of  $T=0.7,\,n=50$  samples, and 12-shot prompting in our experiments. All the results in this section are conducted on the **validation** split of the ProofNet# benchmark.

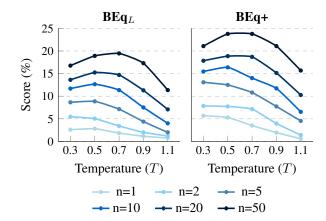


Figure 2: **Evolution of BEq+ and BEq**<sub>L</sub> **pass@n scores for different temperatures on top of type-check filtering.** We evaluate Llemma 7B on ProofNet# validation split.

**Optimal temperature** In Figure 2, we report the evolution of BEq+ pass@ $n^3$  metric using the Llemma 7B model. We find that the optimal temperature for balancing exploration and coherent outputs depends on the number of samples, and that this optimal temperature increases with the number of samples. For the rest of this study, we continue with the value of T=0.7 for our sampling-based experiments.

**Optimal selection method** In Figure 3, we report BEq+ scores of the different selection methods, along with the optimal score that can be achieved with a perfect selection method, which is represented by pass@n. First, BEq+ pass@n scores steadily increase with the number of samples, going from 4.11% at n = 1, to 40.54% at n = 1000, meaning that there is great potential in enhancing autoformalization performance through sampling. Additional studies on BEq+ pass@n can be found in subsection A.6. Second, we find that the studied self-consistency methods performance monotonically increases with the number of samples. We confirm these findings with more models and manual evaluation in Figure 4. Finally, we find that the symbolic equivalence method does not yield better empirical results for selection compared to simpler and less computeintensive methods such as Self-BLEU or majority voting. We therefore continue with Self-BLEU and majority voting in subsequent experiments.

**Type-Check Filtering** In Figure 5, we empirically study the contribution of the filtering component of our method by evaluating with and without filtering, as well as with different selection heuristics.

While majority voting (No filter + Majority voting) and Self-

<sup>&</sup>lt;sup>3</sup>pass@n measures the percentage of tasks where at least one of the n outputs is correct, as measured by BEq+ here.

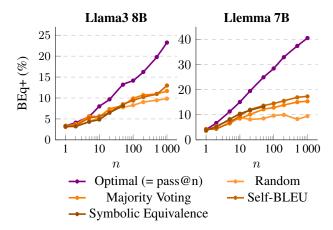


Figure 3: Evolution of BEq+ metric for different selection methods on top of type-check filtering. We evaluate Llama3 8B and Llemma 7B on ProofNet# validation split with a number of candidate samples up to  $n=1\,000$ . Given its quadratic scaling with n and high computational cost, the symbolic equivalence method is limited to  $n\leq 50$  candidate samples.

BLEU selection (*No filter* + *Self-BLEU*) generally improve the accuracy of random sampling, both struggle to increase the performance of random sampling beyond that of the greedy decoding baseline. Meanwhile, adding type-check filtering substantially outperforms the greedy decoding baseline even without any final selection heuristic (*Filter* + *Random selection*). We conclude that the type-check filter is a critical component in our method and that it should be applied before selection.

## 6.4. Empirical Analysis on RLM25

Table 4: Results, in percentage, on RLM25 for different methods and models using only the natural language statement as input (i.e. no in-file context is provided).

Model	Method	Type-Check	$\mathbf{BEq}_L$	BEq+
Llomo 2 OD	12-shot	2.80	0.20	0.40
Llama3 8B	MMA	3.18	0.00	0.54
	12-shot	5.06	2.78	2.78
Llemma 7B	MMA	8.65	0.79	1.16
	Lean Workbook	23.06	0.17	1.13
GPT-40	12-shot	10.55	0.57	1.50

We start by conducting an initial study on RLM25 using the current approach in the literature for statement autoformalization: only the natural language statement is provided as input to the methods. We report our results in Table 4. These results are very similar to the ones presented in Anonymous (2024) on their semi-synthetic Con-NF benchmark, with BEq results close to 0% for all methods.

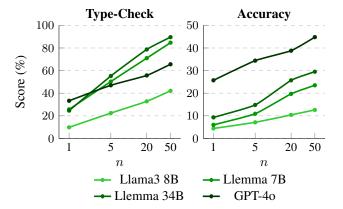


Figure 4: **Type-Check rate and Accuracy scaling trends** with respect to the number of samples on ProofNet# validation split using 12-shot prompting and our method (type-check filter + Self-BLEU). The number of samples varies from n = 1 to 50 (where Llemma 34B has top type-check rate and GPT-40 top accuracy). Numbers are in Table 15.

A manual analysis of the predictions quickly reveals a key issue: without in-file context, autoformalization methods lack access to crucial information. While some missing information from a Lean file, such as local definitions, can be tackled through current retrieval-based methods, others, such as opened namespaces and local variables are still missing. To assess the importance of different file components, we conducted an ablation study to identify which contextual elements help LLMs generate accurate formalizations. As shown in Table 5, the best performance across models is achieved when proofs are removed while retaining all other contextual elements. In contrast, removing both proofs and theorems significantly degrades performance, particularly for less capable models. We hypothesize that retaining theorems may serve as implicit few-shot examples.

Table 5: **Ablation study on prompt content** using various models on RLM25. We report BEq+ (%) performance. In the prompt column, '-' represents removal from the context.

Prompt	Llama3 8B	Llemma 7B	GPT-40
12-shot	0.40	2.78	1.50
Full file context	18.67	22.15	20.64
- theorems & proofs	6.60	13.82	17.20
- proofs	20.29	24.16	24.56

In Table 10, we find using in-file context prompting substantially improve performance across all methods and models compared to Table 4. Additionally, we find that fine-tuning on existing autoformalization datasets does not yield improvement over base models on RLM25. This suggests that context-aware formalization datasets are needed for tackling the autoformalization task on research-level projects.

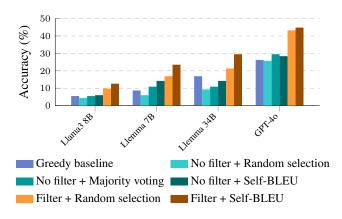


Figure 5: **Type-Check Filtering Ablation Study**. Accuracy scores are reported on ProofNet# validation split for various ablations of our method. More details and exact numbers are reported in Table 14.

## 6.5. Final Results

We conducted a detailed baseline study along with human evaluation of the different autoformalization methods on ProofNet#, which we report in the appendix in Table 8. Confirming results from prior works (Jiang et al., 2023; Azerbayev et al., 2023a), we note the large proportion of errors for all methods due to type-check failures. In Table 6, we report side-by-side final performance results between classical greedy decoding and our sampling-based method using Self-BLEU on both ProofNet# and RLM25. These results confirm the consistent performance improvement of sampling for all tested models on the two benchmarks, ProofNet# and RLM25. We also report the effect of sampling on MMA and Lean Workbook fine-tuned models in Table 9. However, we find that applying our sampling strategy on base models with few-shot learning achieves better absolute accuracy. We report additional results on other benchmarks such as PDA (Lu et al., 2024a) and MiniF2F (Zheng et al., 2022) in subsection A.10.

## 7. Discussion

**Limitations.** Our method requires more computational resources than the greedy decoding baseline. However, by using the same prompt for the sampling phase, our technique benefits from parallel sampling where different optimizations, such as paged attention (Kwon et al., 2023), exist. The inference times remain acceptable even on consumer hardware: on RTX4090 with 24 GB VRAM, running sampling with Llemma 7B and n=50 on all 371 sentences from ProofNet takes  $\sim 660$  seconds.

Our type-check filtering approach shows promise but is inherently constrained by the autoformalization capabilities

Table 6: Performance comparison between greedy decoding and our sampling-based method on ProofNet# and the new RLM25 benchmark. 12-shot is used for ProofNet#, and in-file context with proofs removed is used for RLM25. We sample using T=0.7 and n=50.

Model	Method	ProofN	RLM25	
111041		Accuracy	BEq+	BEq+
I lama 2 OD	Greedy	3.3	3.3	20.3
Llama3 8B	Filter + Self-BLEU	12.0	<u>9.2</u>	<u>23.9</u>
Llemma 7B	Greedy	10.9	6.5	24.2
Lieiiiiia /B	Filter + Self-BLEU	<u>29.3</u>	17.9	28.8
GPT-40	Greedy	31.0	18.5	24.6
Ur 1-40	Filter + Self-BLEU	45.1	23.4	31.6

of the underlying method. While Figure 3 highlights encouraging results, achieving perfect accuracy through sampling would require both a highly effective selection strategy and a substantial number of samples. However, as illustrated in Figure 4, more powerful base models, such as GPT-40, exhibit stronger initial performance, significantly reducing the number of samples needed to reach a given accuracy level.

**Data contamination.** Our in-depth study in subsection A.15 suggests that data contamination is unlikely among the models we evaluated.

#### 8. Conclusion

We introduced a sampling-based method for autoformalization that leverages type-check filtering and simple self-consistency techniques, achieving up to +18.4% absolute accuracy gains on ProofNet. By scaling up to  $n=1\,000$  samples, we found that performance continues to improve, highlighting the potential of larger-scale sampling and better selection strategies.

To enhance benchmarking and evaluation, we released **RLM25**, a research-level autoformalization benchmark leveraging data from modern formalization projects, **ProofNet#**, a corrected version of ProofNet, and **ProofNetVerif**, a dataset for evaluating formalization metrics. We also developed **BEq+**, a symbolic, CPU-only equivalence-checking metric that strongly correlates with human evaluation.

Our findings suggest that large-scale sampling is a promising approach for autoformalization. Additionally, we show that incorporating broader contextual information, beyond just definitions, is essential for better training datasets and enhancing the performance of statement autoformalization methods. We hope our contributions will inspire further progress toward more accurate formalization systems.

## **Impact Statement**

This paper presents work whose goal is to advance the field of Machine Learning. There are many potential societal consequences of our work, none of which we feel must be specifically highlighted here.

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# A. Appendix

## A.1. Tactics used in BEq+

To develop BEq+, we checked existing tactics in Lean and Mathlib using the list provided at https://github.com/haruhisa-enomoto/mathlib4-all-tactics/blob/main/all-tactics.md. We provide a brief description of the tactics we ended up using in BEq+ using the official Mathlib documentation (mathlib Community, 2020):

- exact?: Searches environment for definitions or theorems that can solve the goal using exact with conditions resolved by solve\_by\_elim. While we are not directly interested in searching the library, this tactic is also capable of handling a few transformations on the conclusion, but also variable/hypothesis assignment. For both BEq<sub>L</sub> and BEq+, if exact? succeeds we check that it is using the other formalization to close the goal. Otherwise it could lead to false positives.
- apply e tries to match the current goal against the conclusion of e's type. If it succeeds, then the tactic returns as many subgoals as the number of premises that have not been fixed by type inference or type class resolution.
- convert: The exact e and refine e tactics require a term e whose type is definitionally equal to the goal. convert e is similar to refine e, but the type of e is not required to exactly match the goal. Instead, new goals are created for differences between the type of e and the goal using the same strategies as the congr! tactic. We use this tactic to try partial matching with the conclusion of the other theorem. In particular we use the convert using n variation, where n determines the matching depth. We vary n between 0 and 5.
- tauto breaks down assumptions of the form \_ ∧ \_, \_ ∨ \_, \_ ↔ \_ and ∃ \_, \_ and splits a goal of the form \_ ∧ \_, \_ ↔ \_ or ∃ \_, \_ until it can be discharged using reflexivity or solve\_by\_elim.
- simp\_all\_arith!: simplifies multiple times target and all (propositional) hypotheses using the other hypotheses. Additionally, it uses normalization by linear arithmetic.
- noncomm\_ring: A tactic for simplifying identities in not-necessarily-commutative rings. It is pretty general and works on all types having a ring structure.
- have : t := ... adds the hypothesis this : t to the current goal.
- apply\_rules [1, 12, ...] tries to solve the main goal by iteratively applying the list of lemmas [1, 12, ...] or by applying a local hypothesis. If apply generates new goals, apply\_rules iteratively tries to solve those goals. apply\_rules will also use rfl, trivial, congrFun and congrArg.

## A.2. Methodology used to correct ProofNet

We iteratively ran 3 times the following steps on the Lean 4 port of ProofNet to detect and fix formalization errors:

- Manual pass through all the statements.
- Running DeepSeek-Prover-V1.5 (Xin et al., 2024b) to find proofs on the current iteration of ProofNet. We then manually analyze these proofs to check if flaws in the formalizations not detected by the manual pass have been exploited.
- Finding counter-examples using plausible / slim\_check and finding contradicting hypotheses using a method similar to the one in Xin et al. (2024a).

In total this lead to the discovery of mistakes in 118 entries out of the 371 of the benchmark.

#### A.3. Curating RLM25

To obtain benchmarks out of Lean blueprint projects, we use plasTeX (plasTeX Development Team, 2024) to extract natural language statements from blueprint latex files along with the Lean labels. We then use LeanDojo (Yang et al., 2023) to extract formal statements along with their context from the Lean files. Finally, we align the natural language statements with their formal counterparts using the Lean labels in the latex files.

#### A.4. Our Sampling-Based Method

Our method is composed of three steps: (1) sampling, (2) type-check filtering, and then (3) selecting. We present them in this section.

#### A.4.1. SAMPLING

In our experiments, unless otherwise stated, we employ temperature sampling with T=0.7 and generate n=50 autoformalization attempts per informal statement. Depending on the models, we use the vLLM library (Kwon et al., 2023)

or the OpenAI API to generate predictions.

Cleaning: Certain models often try to provide proofs after generating formal statements. Furthermore, we find that generated names for theorems sometimes clash with names in the Mathlib library. To avoid being considered invalid by the Lean type-checker, we trim proofs, substitute theorem names for dummy identifiers, and normalize whitespace when parsing the generated theorems. Additionally, the Lean proof assistant requires theorems to be accompanied by proofs. To address this, we append a safe dummy sorry proof to each theorem (which indicates to Lean that the proof will be provided later).

## A.4.2. FILTERING

We use Lean REPL (Morrison, 2023) to implement our filtering step. For any formal statement, if the statement is valid, REPL will return declaration uses 'sorry', which means that the statement is well-typed and that we should provide an actual proof instead of sorry. Otherwise, the tool will return error messages explaining why the formal statement is ill-formed, which we use as an indicator to filter out such statements. de Moura et al. (2015) and Moura & Ullrich (2021) provide detailed insights into the inner workings of the Lean type system and type-checking.

#### A.4.3. SELECTION

In our selection process, we employ and compare four distinct heuristics to refine and choose the best outputs generated by the models: random selection, majority voting (Wang et al., 2023), Self-BLEU (Zhu et al., 2018), and the symbolic equivalence method presented in Li et al. (2024b).

**Random:.** As a baseline strategy, we randomly choose an output from the set of generated candidates.

**Majority voting.** (Wang et al., 2023): We aggregate multiple outputs and select the most frequently occurring candidate as the final choice, relying on consensus to mitigate the impact of any single erroneous output. Our cleaning process after the sampling step normalize the generated outputs, increasing the chance of exact string match between the predictions.

**Self-BLEU.** (Zhu et al., 2018): We evaluate the similarity of the generated outputs by calculating the BLEU score between all pairs of candidates. We then select the generated candidate with the highest aggregated BLEU score.

**Symbolic Equivalence.** (Li et al., 2024b): the core idea of this method is to compute equivalence classes of the generated predictions, using logical equivalence. A prediction from the largest equivalence class is then selected as final prediction. As the original work has been conducted in the Isabelle formal language (Nipkow et al., 2002), no implementation of this method is available in Lean. We therefore implemented a Lean version relying on our BEq+ method to compute the equivalence between statements.

#### A.5. BEq<sub>L</sub> & BEq+ Performance on ProofNetVerif

Table 7: Binary performance, in percentage, of  $BEq_L$  and BEq+ metrics on ProofNetVerif. The evaluation is performed on 3 splits based on the reference formalizations length. These splits contain roughly 1 250 entries each.

Reference formalization length	Binary Metric	$\mathbf{BEq}_L$	BEq+
	Precision	100.0	97.1
Less than 115 characters	Recall	39.9	62.5
	F1 Score	57.0	<b>76.1</b>
	Precision	100.0	99.2
Between 115 and 165 characters	Recall	28.2	41.0
	F1 Score	44.0	58.0
	Precision	100.0	100.0
More than 165 characters	Recall	17.2	29.6
	F1 Score	29.3	45.6
	Precision	100.0	98.0
All	Recall	30.9	48.3
	F1 Score	47.2	64.7

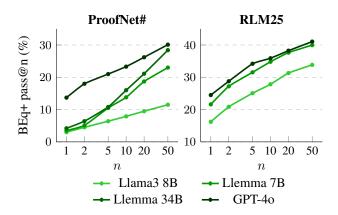


Figure 6: **BEq+ pass@n scaling trends with respect to the number of samples** on ProofNet# validation split and RLM25 using in-context learning prompting. We vary the number of candidate samples from n = 1 to 50.

In Table 7, we report the performance of our metrics at the instance level using binary metrics. While BEq+ outperforms  $BEq_L$ , we find that overall both struggle with a low recall. Empirically, we find that both BEq implementations show better recall on short statements than on long statements. Intuitively, long statements involve more logical clauses, generally making equivalence proving harder.

These results are not directly comparable to the ones presented in Anonymous (2024) as we do not use the same samples to evaluate. For instance, regarding their results without LLM use, which correspond to  $BEq_L$ , they get a recall of 67.14% on their 200 sampled predictions, vs 30.9% on ProofNetVerif in our case. However, given the large improvement of BEq+ over  $BEq_L$  showcased in Table 7, it is very likely that BEq+ outperforms the original LLM-based BEq implementation. In fact, in their most compute-intensive setup, BEq achieves a recall of 72.86% on their samples, which is only slightly better than the 67.14% baseline. On ProofNetVerif, BEq+ outperforms  $BEq_L$  with a larger relative improvement for the recall: 48.3% vs. 30.9%.

A typical example of two non-semantically equivalent statements that are considered equivalent by BEq+:

```
theorem ground_truth (a b : \mathbb{Z}) : (ofInt a : GaussianInt) | ofInt b \rightarrow a | b := sorry theorem prediction (a b : \mathbb{Z}) (ha : a | b) : a | (b : \mathbb{Z}) := sorry
```

The main issue here lies from the fact that it is trivial to prove one formalization assuming the other. In BEq+, this is caused by our use of the simp tactic after the conclusion matching.

## A.6. BEq+ pass@n Additional Results

In Figure 6, we conduct a study on BEq+ pass@n with 4 models: Llama3 8B, Llemma 7B, Llemma 34B, and GPT-40 on both ProofNet# and RLM25. For ProofNet#, we use 12-shot prompting. For RLM25, we prompt models with in-file context with proofs removed as described in subsection 6.4. Llemma 34B model has not been run on the RLM25 benchmark.

## A.7. ProofNet#: Baseline study

We report a baseline performance in Table 8. We evaluated the models described in subsection 6.1 using greedy decoding, coupled with either 12-shot learning or a fine-tuning on either MMA (Jiang et al., 2023) or Lean Workbook (Ying et al., 2024a).

#### A.8. Detailed Results of Our Method on ProofNet#

We present detailed results about the use of our methods on different models and autoformalization methods in Table 9.

In Figure 7, we report results on the ProofNet# test dataset by supplementing tested models using our self-consistency method described in subsection 5.2. Overall, we observe a consistent and significant improvement over the greedy baseline

Table 8: **Baseline performance on ProofNet# using greedy decoding**. Except for Codex, which has been evaluated on Lean 3 in Azerbayev et al. (2023a) (indicated with an asterisk \*, only the results on the test split are available), all models are evaluated on Lean 4.

Method	Model	Validation			Test				
11201100	1,1000	Type-Check	Accuracy <sup>↑</sup>	$\mathbf{BEq}_L \uparrow$	BEq+↑	Type-Check	Accuracy <sup>↑</sup>	$\mathbf{BEq}_L \uparrow$	BEq+↑
12-shot	Codex	-	-	-	-	23.7*	13.4*	-	-
Prompt retrieval	Codex	-	-	-	-	45.2*	16.1*	-	-
MMA	Llama3-8B	12.6	4.9	2.2	3.3	4.3	-	0.5	0.5
MMA	Llemma-7B	14.2	6.0	2.7	4.4	9.7	-	0.0	1.1
Lean Workbook <sup>1</sup>	Llemma-7B	39.5	-	5.4	7.0	39.3	-	5.4	6.5
12-shot	Llama3-8B	13.7	5.5	3.3	4.4	13.0	3.3	1.6	3.3
12-shot	Llemma-7B	26.8	8.7	3.3	6.0	29.9	10.9	5.4	6.5
12-shot	Llemma-34B	33.3	16.9	5.5	9.3	29.9	12.5	5.4	7.1
12-shot	GPT-4-turbo	24.6	19.7	8.7	12.6	27.7	22.8	13.0	16.8
12-shot	GPT-4o	33.3	26.2	10.9	16.4	42.9	31.0	13.6	18.5

# Accuracy on ProofNet# test split

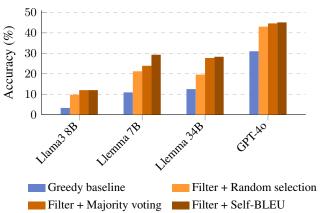


Figure 7: **Autoformalization accuracy on the ProofNet# test set.** All methods use 12-shot prompting in this figure. Detailed results are reported in Table 8 and Table 9.

across all selection methods and all models evaluated. Interestingly, even using random selection over filtered generated statements is enough to outperform the greedy decoding baseline substantially. This demonstrates the practical efficacy of extensively leveraging the type-checker in the context of statement autoformalization. We find that the overall best strategy is to use type-check filtering and Self-BLEU for the selection step. One can notice that the absolute improvement in accuracy achieved by this method ranges from +8.7% to +18.4% over greedy decoding, with relative improvements between 1.4x and 3x. We also report results of our method on other classical benchmarks, PDA and MiniF2F, in subsection A.10.

#### A.9. Detailed Results on RLM25

We present detailed results about the use of our methods on different models and autoformalization methods in Table 10.

#### A.10. Other Benchmarks

While we focused our work on the ProofNet# and RLM25 benchmarks, our method is not specific to this benchmark, and we believe it should yield improvements out of the box on other statement autoformalization benchmarks.

In this section, we report results on two other benchmarks: Process-Driven Autoformalization (PDA) (Lu et al., 2024a), and MiniF2F (Zheng et al., 2022). The PDA benchmark from Lu et al. (2024a) has been designed to evaluate statement and proof autoformalization. Given the size of this benchmark, 3 test splits of 1000 theorems, and the fact that not all of

Table 9: Evaluation results (in percentage) of our method on ProofNet#. For all these results, for each informal statement in the benchmark, we sampled 50 formalization attempts per model and filtered type-checking ones before applying a selection method. We observe some performance differences between the two splits which are caused by the small size of the ProofNet# benchmark (2x 185 statements).

Model	Selection	Validation			Test				
Model	method	Type-Check	<b>Accuracy</b> \( \)	$\mathbf{BEq}_L \uparrow$	BEq+↑	Type-Check	<b>Accuracy</b> ↑	$\mathbf{BEq}_L \uparrow$	BEq+↑
MMA fine-tu	ne								
	Random	33.3	9.3	3.3	4.9	29.0	-	2.2	4.8
Llama3-8B	Majority	33.3	8.7	3.3	4.4	29.0	-	1.1	4.8
	Self-BLEU	33.3	8.7	3.3	4.4	29.0	-	1.1	4.3
	Random	61.2	9.3	4.4	5.5	52.2	-	3.2	5.9
Llemma-7B	Majority	61.2	10.9	3.8	5.5	52.2	-	3.8	7.0
	Self-BLEU	61.2	13.7	4.9	6.6	52.2	-	5.4	8.1
Lean Workbo	ok fine-tune								
	Random	86.0	_	7.6	9.2	86.6	-	7.0	8.6
Llemma-7B	Majority	86.0	-	9.2	10.8	86.6	-	5.9	8.6
	Self-BLEU	86.0	-	9.7	12.4	86.6	-	6.5	10.2
12-shot									
	Random	42.1	9.8	4.4	6.6	45.7	13.6	4.9	10.3
Llama3-8B	Majority	42.1	12.0	4.9	7.1	45.7	14.7	4.9	10.9
	Self-BLEU	42.1	12.6	6.0	8.2	45.7	12.0	4.9	9.2
	Random	84.7	16.9	4.9	7.7	88.6	21.2	9.8	11.4
Llemma-7B	Majority	84.7	23.0	8.2	9.8	88.6	23.9	10.3	12.0
	Self-BLEU	84.7	23.5	7.7	11.5	88.6	29.3	11.4	17.9
	Random	89.6	21.3	4.9	9.8	84.2	19.6	5.4	11.4
Llemma-34B	Majority	89.6	27.3	<b>8.7</b>	12.6	84.2	27.7	10.9	14.1
	Self-BLEU	89.6	29.5	<b>8.7</b>	13.1	84.2	28.3	9.8	14.7
	Random	65.6	43.2	16.4	23.5	70.1	42.9	15.8	22.8
GPT-4o	Majority	65.6	45.4	15.8	21.3	70.1	44.6	15.8	22.8
	Self-BLEU	65.6	44.8	15.3	21.3	70.1	45.1	16.3	23.4

these subsets contain reference formalizations, we sample 50 random problems for each test split and conduct a manual evaluation. We report our results in Table 11 .

On all test splits, we observe that our method largely improves over greedy decoding. We have found the real test split to be challenging as problems are sampled from Arithmo dataset (Jindal, 2023) without solutions, therefore requiring first solving the problem before providing an accurate formalization.

The authors of the PDA benchmark reported compilation results for statement and proof autoformalization at once. However, they do not report results for statement autoformalization alone and do not report accuracy results either. This lack of data in Lu et al. (2024a) means that we cannot compare directly to their results.

We report results on the MiniF2F benchmark in Table 12. We find the Llemma 7B model fine-tuned on Lean Workbook (Ying et al., 2024a) to perform particularly well on it. However, since the Lean Workbook dataset has been synthetically generated by a model finetuned on the MiniF2F and ProofNet benchmarks, some data leakage might have happened.

#### A.11. Low-correction Effort Formalizations

One goal of autoformalization is the development of AI-assisted tools for formalization. In this setting, producing close-to-correct formal statements can already help users by providing hints and potential directions. Using the same setup as in the previous section, we report our results on the ProofNet# test split in Figure 7. Note that in this setup, contrary to Jiang et al. (2023), we are considering only well-typed statements.

Table 10: Results on RLM25 for different methods and models. In this setup, all models are prompted with in-file context and proofs are removed. Greedy decoding is used for generation.

Model	Method	Type-Check	$\mathbf{BEq}_L$	BEq+
Llama3 8B	No fine-tuning	37.19	18.90	20.29
Liailia5 6B	MMA	41.73	17.55	19.24
	No fine-tuning	55.96	22.49	24.16
Llemma 7B	MMA	51.63	21.37	22.63
	Lean Workbook	53.89	21.84	23.55
GPT-4o	No fine-tuning	51.37	21.18	24.56

Table 11: Evaluation results (in percentage) of our method on the **Process-Driven Autoformalization** benchmark. We evaluate on 50 samples on each of the "Basic", "Random", and "Real" splits of this benchmark. Greedy decoding is used for methods without sampling, i.e. when filtering is not mentioned. For sampling-based methods, we sample n=50 predictions with temperature T=0.7. We separate greedy decoding methods from sampling-based methods by a gray line for each model.

Model	Method	Basic		Random		Real	
1,10001		Type-Check	<b>Accuracy</b> ↑	Type-Check	Accuracy <sup>↑</sup>	Type-Check	<b>Accuracy</b> ↑
Llama3-8B	12-shot	18.0	16.0	20.0	16.0	20.0	4.0
Liailia5-oD	12-shot + Filter + Self-BLEU	48.0	28.0	46.0	32.0	72.0	8.0
Llemma-7B	12-shot	20.0	14.0	30.0	26.0	62.0	0.0
Liemma-/B	12-shot + Filter + Self-BLEU	76.0	58.0	72.0	54.0	100.0	8.0
GPT-4o	12-shot	30.0	28.0	42.0	38.0	4.0	0.0
	12-shot + Filter + Self-BLEU	64.0	54.0	66.0	56.0	26.0	12.0

We find that, by using our method, open-source models Llemma-7B and Llemma-34B can autoformalize  $\sim 50\%$  of the mathematical statements from the ProofNet# test benchmark in a *close-to-correct* way<sup>4</sup>. This makes these open models good fit for local autoformalization assistant, especially Llemma-7B for its relatively small size.

In this section, we present several examples of autoformalizations on ProofNet# validation split that are evaluated as incorrect yet fixable with low effort. Evaluation results on ProofNet# test split are presented in Table 13.

## **Low-correction effort examples**

RUDIN EXERCISE\_1\_1A

**Informal statement:** If r is rational  $(r \neq 0)$  and x is irrational, prove that r + x is irrational.

**Generated formalization:** 

theorem exercise\_1\_1a (hr :  $r \neq 0$ ) (hi : Irrational x) : Irrational (r + x) :=

**Issue:** r has not been declared as a rational number.

HERSTEIN EXERCISE\_4\_3\_25

**Informal statement:** Let R be the ring of  $2 \times 2$  matrices over the real numbers; suppose that I is an ideal of R. Show that I = (0) or I = R.

**Generated formalization:** 

```
theorem exercise_4_3_25 {R : Type*} [CommRing R] (I : Ideal (Matrix (Fin 2) (Fin 2) \mathbb{R}) : I = \bot \lor I = \top :=
```

**Issue:** Superfluous declaration of CommRing R.

<sup>&</sup>lt;sup>4</sup>We define *close-to-correct* formalizations as those with one slightly diverging hypothesis or conclusion, typically fixable in a matter of seconds.

Table 12: Performance of different methods on **MiniF2F**. Greedy decoding is used for methods without sampling, i.e. when filtering is not mentioned. For sampling-based methods, we sample n=50 predictions with temperature T=0.7. We separate greedy decoding methods from sampling-based methods by a gray line for each model. <sup>1</sup>Lean Workbook dataset has been curated with a model trained on the MiniF2F benchmark, thus data leakage concerns apply.

Model	Method	Type-Check	$\mathbf{BEq}_L\uparrow$	BEq+↑
	12-shot	45.9	6.8	14.6
Llama3 8B	MMA	36.7	3.1	8.8
	12-shot + Filter + Self-BLEU	93.0	10.3	21.3
	12-shot	67.0	7.6	16.0
	MMA	32.2	2.1	5.3
Llemma 7B	Lean Workbook <sup>1</sup>	89.6	14.3	28.5
	12-shot + Filter + Self-BLEU	99.8	10.3	21.3
	Lean Workbook + Filter + Self-BLEU <sup>1</sup>	99.0	14.1	28.1
GPT-4o	12-shot	24.4	8.0	13.5

# Accuracy - Low correction effort 60 40 20 Utantia 18 Utentina 18 Utentina 18 Filter + Random selection Filter + Majority voting Filter + Self-BLEU

Figure 8: Proportion of formalized statements evaluated as correct or as fixable with a low amount of effort (i.e., *close-to-correct*) on the ProofNet# test set. All models are prompted with 12-shot examples. Detailed results are reported in Table 13.

# A.12. Ablation Study: Detailed Results

We present detailed results of our ablation study on the type-check filtering step in Table 14.

## A.13. Sampling Scaling

Detailed results about our sampling study can be found in Table 15.

## A.14. 12-shot Prompt

**Note:** We translated the 12-shot prompt from ProofNet# to Lean 4, with as minimal changes as possible, for the accuracy comparison with previous results to be as fair as possible. In particular, we did not remove/change the statements leaked from the benchmark and did not correct potential formalization mistakes in this prompt to make our results comparable with the results in Azerbayev et al. (2023a).

Table 13: Models performance (in percentage) on ProofNet# test split when accounting for formalizations that can be corrected with a low amount of efforts.

Model	Method	Accuracy <sup>†</sup>
	Greedy	9.1
Llama3-8B	Filter + Random	23.7
Liailia3-8D	Filter + Majority	25.8
	Filter + Self-BLEU	25.3
	Greedy	16.7
Llemma-7B	Filter + Random	37.1
Lieiiiiia-/b	Filter + Majority	40.9
	Filter + Self-BLEU	48.9
	Greedy	19.9
Llemma-34B	Filter + Random	35.5
Lieiiiiia-34D	Filter + Majority	40.3
	Filter + Self-BLEU	<b>51.</b> 1
	Greedy	40.3
GPT-40	Filter + Random	60.7
Of 1-40	Filter + Majority	60.2
	Filter + Self-BLEU	61.3

## 12-shot examples

#### Natural language version:

Let P be a p-subgroup of G. Then P is contained in a Sylow p-subgroup of G.

Translate the natural language version to a Lean 4 version:

```
theorem exists_le_sylow [Group G] {P : Subgroup G} (hP : IsPGroup p P) : \exists Q : Sylow p G, P \leq Q :=
```

## Natural language version:

Let E and F be complex normed spaces and let  $f: E \to F$ . If f is differentiable and bounded, then f is constant Translate the natural language version to a Lean 4 version:

```
theorem exists_eq_const_of_bounded {E : Type u} [NormedAddCommGroup E] [NormedSpace \mathbb{C} E] {F : Type v} [NormedAddCommGroup F] [NormedSpace \mathbb{C} F] {f : E \rightarrow F} (hf : Differentiable \mathbb{C} f) (hb : IsBounded (range f)) : \exists c, f = const E c :=
```

## Natural language version:

Let X be a topological space; let A be a subset of X. Suppose that for each  $x \in A$  there is an open set U containing x such that  $U \subset A$ . Then A is open in X.

Translate the natural language version to a Lean 4 version:

```
theorem subset_of_open_subset_is_open (X : Type*) [TopologicalSpace X] (A : Set X) (hA : \forall x \in A, \exists U : Set X, IsOpen U \land x \in U \land U \subseteq A): IsOpen A :=
```

## Natural language version:

Two multiplicative functions  $f,g:\mathbb{N}\to R$  are equal if and only if  $f(p^i)=f(g^i)$  for all primes p.

Translate the natural language version to a Lean 4 version:

```
theorem eq_iff_eq_on_prime_powers [CommMonoidWithZero R] (f : ArithmeticFunction R) (hf : f.IsMultiplicative) (g : ArithmeticFunction R) (hg : g.IsMultiplicative) : f = g \leftrightarrow \forall \ p \ i : \mathbb{N}, Nat.Prime p \to f (p ^ i) = g (p ^ i) :=
```

Table 14: Models performance (in percentage) on ProofNet# validation split removing different aspects of our method. We also report Greedy baseline and the Filter + Self-BLEU results as reference.

Model	Method	Type-Check	Accuracy↑	$\mathrm{BEq}_L \uparrow$	BEq+↑
	Greedy	13.7	5.5	3.3	4.4
Llama3-8B	No filter + Random	9.8	4.4	2.7	2.7
	No filter + Majority	13.1	5.5	3.3	3.8
	No filter + Self-BLEU	14.2	6.0	2.7	3.8
	Filter + Random	42.1	9.8	4.4	6.6
	Filter + Self-BLEU	42.1	12.6	6.0	8.2
	Greedy	26.8	8.7	3.3	6.0
	No filter + Random	25.7	6.0	3.3	3.8
Llemma-7B	No filter + Majority	25.1	10.9	4.4	4.9
Lieiiiiia-/D	No filter + Self-BLEU	32.2	14.2	6.6	9.3
	Filter + Random	84.7	16.9	4.9	7.7
	Filter + Self-BLEU	84.7	23.5	7.7	11.5
	Greedy	33.3	16.9	5.5	9.3
	No filter + Random	24.6	9.3	2.7	4.4
Llemma-34B	No filter + Majority	24.6	10.9	4.9	7.1
Licillia-34D	No filter + Self-BLEU	32.8	14.2	3.8	6.0
	Filter + Random	89.6	21.3	4.9	9.8
	Filter + Self-BLEU	89.6	29.5	<b>8.7</b>	13.1
	Greedy	33.3	26.2	10.9	16.4
GPT-40	No filter + Random	33.3	25.7	9.8	14.8
	No filter + Majority	35.5	29.5	12.0	16.4
	No filter + Self-BLEU	36.1	28.4	12.0	16.9
	Filter + Random	65.6	43.2	16.4	23.5
	Filter + Self-BLEU	65.6	44.8	15.3	21.3

#### Natural language version:

```
If z_1, ..., z_n are complex, then |z_1 + z_2 + ... + z_n| \le |z_1| + |z_2| + ... + |z_n|.
```

Translate the natural language version to a Lean 4 version:

```
theorem abs_sum_leq_sum_abs (n : \mathbb{N}) (f : \mathbb{N} \to \mathbb{C}) : abs (\Sigma i in Finset.range n, f i) \leq \Sigma i in Finset.range n, abs (f i) :=
```

## Natural language version:

If x and y are in  $\mathbb{R}^n$ , then  $|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$ .

Translate the natural language version to a Lean 4 version:

```
theorem sum_add_square_sub_square_eq_sum_square (n : \mathbb{N}) (x y : EuclideanSpace \mathbb{R} (Fin n )) : \|x + y\|^2 + \|x - y\|^2 = 2*\|x\|^2 + 2*\|y\|^2 :=
```

## Natural language version:

If x is an element of infinite order in G, prove that the elements  $x^n$ ,  $n \in \mathbb{Z}$  are all distinct.

Translate the natural language version to a Lean 4 version:

```
theorem distinct_powers_of_infinite_order_element (G : Type*) [Group G] (x : G) (hx_inf : \forall n : \mathbb{N}, x ^ n \neq 1) : \forall m n : \mathbb{Z}, m \neq n \rightarrow x ^ m \neq x ^ n :=
```

Natural language version:

Table 15: Evaluation results (in percentage) of our method on ProofNet# validation split for different numbers of formalizations sampled during the sampling phase of our method (represented by the number n in this table). We used a 12-shot prompt with the filter+Self-BLEU variant of our method and a temperature of 0.7.

Model	Type-Check			<b>Accuracy</b> ↑			$\mathbf{BEq}_L \!\!\uparrow$				BEq+↑					
	n=1	n=5	n=20	n=50	n=1	n=5	n=20	n=50	n=1	n=5	n=20	n=50	n=1	n=5	n=20	n=50
Llama3-8B	9.8	22.4	32.8	42.1	4.4	7.1	10.4	12.6	2.7	3.3	4.4	6.0	2.7	4.9	6.0	8.2
Llemma-7B	25.7	50.3	71.0	84.7	6.0	10.9	19.7	23.5	3.3	3.3	7.1	7.7	3.8	6.0	10.4	11.5
Llemma-34B	24.6	55.2	78.7	89.6	9.3	14.8	25.7	29.5	2.7	5.5	8.2	8.7	4.4	8.7	12.0	13.1
GPT-40	33.3	47.0	55.7	65.6	25.7	34.4	38.8	44.8	9.8	14.2	13.7	15.3	14.8	20.2	19.7	21.3

A set of vectors  $\{v_i\}_{i\in I}$  orthogonal with respect to some bilinear form  $B: V \times V \to K$  is linearly independent if for all  $i \in I, B(v_i, v_i) \neq 0$ .

Translate the natural language version to a Lean 4 version:

```
theorem linear_independent_of_is_Ortho {V K : Type*} [Field K] [AddCommGroup V] [Module K V] {n : Type*} {B : BilinForm K V} {v : n \rightarrow V} (hv<sub>1</sub> : B.iIsOrtho v) (hv<sub>2</sub> : \forall (i : n), \negB.IsOrtho (v i) (v i)) : LinearIndependent K v :=
```

## Natural language version:

Suppose that V is an n-dimensional vector space. Then for some set of vectors  $\{v_i\}_{i=1}^k$ , if k > n then there exist scalars  $f_1, \ldots, f_k$  such that  $\sum_{i=1}^k f_k v_k = 0$ .

Translate the natural language version to a Lean 4 version:

```
theorem exists_nontrivial_relation_sum_zero_of_dim_succ_lt_card {K V : Type*} [DivisionRing K] [AddCommGroup V] [Module K V] [FiniteDimensional K V] {t : Finset V} (h : FiniteDimensional.finrank K V + 1 < t.card) : \exists (f : V \rightarrow K), t.sum (\lambda (e : V) => f e · e) = 0 \land t.sum (\lambda (e : V) => f e) = 0 \land \exists (x : V) (H : x \in t), f x \neq 0 :=
```

#### Natural language version:

A group is commutative if the quotient by the center is cyclic.

Translate the natural language version to a Lean 4 version:

```
theorem comm_group_of_cycle_center_quotient {G H : Type*} [Group G] [Group H] [IsCyclic H] (f : G \rightarrow* H) (hf : f.ker \leq (center G : Subgroup G)): CommGroup G :=
```

## Natural language version:

If H is a p-subgroup of G, then the index of H inside its normalizer is congruent modulo p to the index of H. Translate the natural language version to a Lean 4 version:

```
theorem card_quotient_normalizer_modEq_card_quotient {G : Type*} [Group G] [Fintype G] {p : N} {n : N} [hp : Fact p.Prime] {H : Subgroup G} (hH : Fintype.card H = p ^ n) : Fintype.card (normalizer H / Subgroup.comap ((normalizer H).subtype : normalizer H \rightarrow* G) H) \equiv Fintype.card (G / H) [MOD p] :=
```

## Natural language version:

Suppose X,Y,Z are metric spaces, and Y is compact. Let f map X into Y, let g be a continuous one-to-one mapping of Y into Z, and put h(x)=g(f(x)) for  $x\in X$ . Prove that f is uniformly continuous if h is uniformly continuous. Translate the natural language version to a Lean 4 version:

```
theorem uniform_continuous_of_continuous_injective_uniform_continuous_comp {X Y Z : Type*} [MetricSpace X] [MetricSpace Y] [MetricSpace Z] (hY : CompactSpace Y) (f : X \rightarrow Y) (g : Y \rightarrow Z) (hgc : Continuous g) (hgi : Function.Injective g) (h : UniformContinuous (g \circ f)) : UniformContinuous f :=
```

#### A.15. Data Contamination

Data contamination is a serious issue in today's LLM benchmarks. In fact, large language models are trained on large-scale training data, thus, despite the filtering efforts, data leakage might happen. For the new dataset RLM25 we introduce, as stated in subsection 4.1, all projects selected for evaluation were made available after the knowledge cutoff dates of the evaluated models. In particular, Llemma 7B, performing almost on par with GPT-40 on this benchmark, is open-weight and has been released in August 2023, thus before the first commit of any of these projects.

ProofNet 3 was released in February 2023, and an unofficial port to Lean 4 has been publicly available since March 2024. Since the cutoff training dates for all models used in these experiments are before March 2024, Lean 4 data contamination due to training is not possible. However, it remains theoretically possible that some models were trained on the Lean 3 version and weakly generalized to Lean 4. Such data leakage for the Llemma models family (Azerbayev et al., 2023b) seems unlikely as the authors claim they have specifically excluded ProofNet from their training data.

For our data contamination study, we use an unofficial Lean 4 port (Vishwakarma, 2024) of the ProofNet benchmark made by an independent research team. This port shows minimal differences from the original Lean 3 ProofNet benchmark, preserving the order of hypotheses and terms. Upon analyzing the raw predictions of all models, we did not find any exact matches with the Lean 4 ground truths. This is primarily because the theorems in the benchmark follow an exercise\_number naming scheme, which the models do not produce. Consequently, we employed fuzzy matching for our data contamination checks. This involved normalizing whitespaces and removing comments and theorem names. We found a maximum of 2.2% matches (4 statements out of 185/186) for each model independently on the validation split, including the 2 statements leaked by the prompt. Given that the space of correct formal statements is heavily constrained, this hit rate is quite reasonable. Below, we provide a list of all unique hits found across all models and experiments. Most of these hits are very short and almost unavoidable. Considering these results, it seems unlikely that significant data leakage occurred during the training of these models.

Nonetheless, during our data contamination study, we found that 4 examples from the 12-shot prompt in Azerbayev et al. (2023a), which we intended to compare to, were also present in the benchmark (2 in the validation set and 2 in the test set). Fortunately, this affects the results only negligibly (at most  $\sim 1.1\%$ ). We report all our results with these statements removed.

```
List of all the hits found (using fuzzy matching) across all our experiments on the ProofNet validation split

Munkreslexercise_29_1:

theorem exercise_29_1: ¬ LocallyCompactSpace ℚ :=

Dummit-Footelexercise_1_1_22a:

theorem exercise_1_1_22a {G : Type*} [Group G] (x g : G) :
    orderOf x = orderOf (g<sup>-1</sup> * x * g) :=

Hersteinlexercise_2_1_27:

theorem exercise_2_1_27 {G : Type*} [Group G]
    [Fintype G] : ∃ (m : N), ∀ (a : G), a ^ m = 1 :=

Munkreslexercise_17_4:

theorem exercise_17_4 {X : Type*} [TopologicalSpace X]
```

```
(U A : Set X) (hU : IsOpen U) (hA : IsClosed A) :
IsOpen (U \ A) ∧ IsClosed (A \ U) :=

Hersteinlexercise_5_5_2:
theorem exercise_5_5_2 : Irreducible (X^3 - 3*X - 1 : Polynomial ℚ) :=

Munkreslexercise_32_3:
theorem exercise_32_3 {X : Type*} [TopologicalSpace X]
   (hX : LocallyCompactSpace X) (hX' : T2Space X) :
   RegularSpace X :=

Hersteinlexercise_4_3_25:
theorem exercise_4_3_25 (I : Ideal (Matrix (Fin 2) (Fin 2) ℝ)) :
   I = ⊥ ∨ I = T :=
```

```
Munkreslexercise_13_1
theorem subset_of_open_subset_is_open (X : Type*) [TopologicalSpace X]
  (A : Set X) (hA : ∀ x ∈ A, ∃ U : Set X, IsOpen U ∧ x ∈ U ∧ U ⊆ A):
        IsOpen A :=

Dummit-Footlexercise_1_1_34
theorem distinct_powers_of_infinite_order_element (G : Type*) [Group G] (x : G)
        (hx_inf : ∀ n : N, x ^ n ≠ 1) :
        ∀ m n : Z, m ≠ n → x ^ m ≠ x ^ n :=
```

#### A.16. All results on ProofNet#

We report in the table below metric results for all autoformalization methods and models on which we conducted manual evaluation. Such manual evaluation has been conducted on both ProofNet# validation and test splits.

Model	Strategy	Split	Type Check	Accuracy	$BEq_L$	BEq+	Hyp. Rej.
ensemble-12shot	50 samples + Type check filter + Majority voting	test	96.2	44.6	17.4	23.4	3.8
ensemble-12shot	50 samples + Type check filter + Random	test	96.2	33.2	9.8	14.7	2.2
ensemble-12shot	50 samples + Type check filter + Self-BLEU	test	96.2	48.4	17.9	26.6	3.3
gpt-4-turbo-2024-04-09	Greedy decoding	test	27.7	22.8	13.0	16.8	1.1
gpt-4-turbo-2024-04-09	Greedy decoding	valid	24.6	19.7	8.7	12.6	0.0
gpt-4o-2024-05-13	Greedy decoding	test	42.9	31.0	13.6	18.5	1.6
gpt-4o-2024-05-13	Greedy decoding	valid	33.3	26.2	10.9	16.4	0.0
gpt-4o-2024-05-13	50 samples + Type check filter + Majority voting	test	70.1	44.6	15.8	22.8	2.2
gpt-4o-2024-05-13	50 samples + Type check filter + Random	test	70.1	42.9	15.8	22.8	1.1
gpt-4o-2024-05-13	50 samples + Type check filter + Self-BLEU	test	70.1	45.1	16.3	23.4	2.2
gpt-4o-2024-05-13	50 samples + Majority voting	valid	35.5	29.5	12.0	16.4	0.0
gpt-4o-2024-05-13	50 samples + Random	valid	33.3	25.7	9.8	14.8	0.0
gpt-4o-2024-05-13	50 samples + Self-BLEU	valid	36.1	28.4	12.0	16.9	0.0
gpt-4o-2024-05-13	50 samples + Type check filter + Majority voting	valid	65.6	45.4	15.8	21.3	0.0
gpt-4o-2024-05-13	50 samples + Type check filter + Random	valid	65.6	43.2	16.4	23.5	0.0
gpt-4o-2024-05-13	50 samples + Type check filter + Self-BLEU	valid	65.6	44.8	15.3	21.3	0.0
gpt-4o-2024-05-13	20 samples + Type check filter + Self BLEU	valid	55.7	38.8	13.7	19.7	0.5
gpt-4o-2024-05-13	5 samples + Type check filter + Self-BLEU	valid	47.0	34.4	14.2	20.2	0.0
llama3-8b-mma	Greedy decoding	valid	12.6	4.9	2.2	3.3	2.2

## Improving Autoformalization using Type Checking

llama3-8b-mma	50 samples + Type check filter + Majority voting		33.3	8.7	3.3	4.4	2.2
llama3-8b-mma	50 samples + Type check filter + Random	valid	33.3	9.3	3.3	4.9	2.7
llama3-8b-mma	50 samples + Type check filter + Self-BLEU	valid	33.3	8.7	3.3	4.4	2.7
llama3-8b	Greedy decoding	test	13.0	3.3	1.6	3.3	1.1
llama3-8b	Greedy decoding	valid	13.7	5.5	3.3	4.4	0.5
llama3-8b	50 samples + Type check filter + Majority voting	test	45.7	14.7	4.9	10.9	2.2
llama3-8b	50 samples + Type check filter + Random	test	45.7	13.6	4.9	10.3	1.1
llama3-8b	50 samples + Type check filter + Self-BLEU	test	45.7	12.0	4.9	9.2	1.6
llama3-8b	50 samples + Majority voting	valid	13.1	5.5	3.3	3.8	0.5
llama3-8b	50 samples + Random	valid	9.8	4.4	2.7	2.7	0.5
llama3-8b	50 samples + Self-BLEU	valid	14.2	6.0	2.7	3.8	0.5
llama3-8b	50 samples + Type check filter + Majority voting	valid	42.1	12.0	4.9	7.1	1.1
llama3-8b	50 samples + Type check filter + Random	valid	42.1	9.8	4.4	6.6	1.1
llama3-8b	50 samples + Type check filter + Self-BLEU	valid	42.1	12.6	6.0	8.2	1.1
llama3-8b	20 samples + Type check filter + Self-BLEU	valid	32.8	10.4	4.4	6.0	0.5
llama3-8b	5 samples + Type check filter + Self-BLEU	valid	22.4	7.1	3.3	4.9	1.1
llemma-34b	Greedy decoding	test	29.9	12.5	5.4	7.1	1.1
llemma-34b	Greedy decoding	valid	33.3	16.9	5.5	9.3	0.0
llemma-34b	50 samples + Type check filter + Majority voting	test	84.2	27.7	10.9	14.1	3.3
llemma-34b	50 samples + Type check filter + Random	test	84.2	19.6	5.4	11.4	2.2
llemma-34b	50 samples + Type check filter + Self-BLEU	test	84.2	28.3	9.8	14.7	2.2
llemma-34b	50 samples + Majority voting	valid	24.6	10.9	4.9	7.1	0.0
llemma-34b	50 samples + Random	valid	24.6	9.3	2.7	4.4	0.5
llemma-34b	50 samples + Self-BLEU	valid	32.8	14.2	3.8	6.0	0.0
llemma-34b	50 samples + Type check filter + Majority voting	valid	89.6	27.3	8.7	12.6	1.1
llemma-34b	50 samples + Type check filter + Random	valid	89.6	21.3	4.9	9.8	2.2
llemma-34b	50 samples + Type check filter + Self-BLEU	valid	89.6	29.5	8.7	13.1	1.6
llemma-34b	20 samples + Type check filter + Self-BLEU	valid	78.7	25.7	8.2	12.0	1.1
llemma-34b	5 samples + Type check filter + Self-BLEU	valid	55.2	14.8	5.5	8.7	2.2
llemma-7b-mma	Greedy decoding	valid	14.2	6.0	2.7	4.4	0.0
llemma-7b-mma	50 samples + Type check filter + Majority voting	valid	61.2	10.9	3.8	5.5	1.6
llemma-7b-mma	50 samples + Type check filter + Random	valid	61.2	9.3	4.4	5.5	2.7
llemma-7b-mma	50 samples + Type check filter + Self-BLEU	valid	61.2	13.7	4.9	6.6	2.7
llemma-7b	Greedy decoding	test	29.9	10.9	5.4	6.5	2.2
llemma-7b	Greedy decoding	valid	26.8	8.7	3.3	6.0	0.0
llemma-7b	50 samples + Type check filter + Majority voting	test	88.6	23.9	10.3	12.0	2.7
llemma-7b	50 samples + Type check filter + Random	test	88.6	21.2	9.8	11.4	4.9
llemma-7b	50 samples + Type check filter + Self-BLEU	test	88.6	29.3	11.4	17.9	4.3
llemma-7b	50 samples + Majority voting	valid	25.1	10.9	4.4	4.9	0.5
llemma-7b	50 samples + Random	valid	25.7	6.0	3.3	3.8	1.1
llemma-7b	50 samples + Self-BLEU	valid	32.2	14.2	6.6	9.3	0.5
llemma-7b	50 samples + Type check filter + Majority voting		84.7	23.0	8.2	9.8	2.2
llemma-7b	50 samples + Type check filter + Random	valid	84.7	16.9	4.9	7.7	2.7
llemma-7b	50 samples + Type check filter + Self-BLEU	valid	84.7	23.5	7.7	11.5	1.6
llemma-7b	20 samples + Type check filter + Self-BLEU	valid	71.0	19.7	7.1	10.4	1.6
llemma-7b	5 samples + Type check filter + Self-BLEU	valid	50.3	10.9	3.3	6.0	1.6
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Table 16: Table reporting all experiments conducted on ProofNet# with accuracy manually evaluated.