COGENT: Certified Compilation for a Functional Systems Language

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Abstract

We present a self-certifying compiler for the Cogent systems language. Cogent is a restricted, polymorphic, higher-order, and purely functional language with linear types and without the need for a trusted runtime or garbage collector. It compiles to efficient C code that is designed to interoperate with existing C functions. The language is suited for layered systems code with minimal sharing such as file systems or network protocol control code.

For a well-typed Cogent program, the compiler produces C code, a high-level shallow embedding of its semantics in Isabelle/HOL, and a proof that the C code correctly implements this embedding. The aim is for proof engineers to reason about the full semantics of real-world systems code productively and equationally, while retaining the interoperability and leanness of C.

We describe the formal verification stages of the compiler, which include automated formal refinement calculi, a switch from imperative update semantics to functional value semantics formally justified by the linear type system, and a number of standard compiler phases such as type checking and monomorphisation. The compiler certificate is a series of language-level meta proofs and per-program translation validation phases, combined into one coherent top-level theorem in Isabelle/HOL.

Categories and Subject Descriptors F.3.2 [Logics and Meanings of Programs]: Semantics of Programming Languages

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1. Introduction

Imagine writing low-level systems code in a purely functional language and then reasoning about this code equationally and productively in an interactive theorem prover. Imagine doing this without the need for a trusted compiler, runtime or garbage collector and letting this code interoperate with native C parts of the system, including your own efficiently implemented and formally verified additional data types and operations.

Cogent achieves this goal by certified compilation from a high-level, pure, polymorphic, functional language with linear types, specifically designed for certain classes of systems code. For a given well-typed Cogent program, the compiler will produce a high-level shallow embedding of the program's semantics in Isabelle/HOL [Nipkow and Klein 2014], and a theorem that connects this shallow embedding to the C code that the compiler produces: any property proved of the shallow embedding is guaranteed to hold for the generated C.

The compilation target is C, because C is the language most existing systems code is written in, and because with the advent of tools like CompCert [Leroy 2006, 2009] and gcc translation valida-

tion [Sewell et al. 2013], C is now a language with well understood semantics and existing formal verification infrastructure.

If C is so great, why not verify C systems code directly? After all, there is an ever growing list of successes [Klein et al. 2009, 2014; Gu et al. 2015; Beringer et al. 2015] in this space. The reason is simple: verification of manually written C programs remains expensive. Just as high-level languages increase programmer productivity, they should also increase verification productivity. Certifying compilation of a language with verification-friendly semantics is a key step in achieving this goal for Cogent.

The state of the art for certified compilation of a full featured functional language is CakeML [Kumar et al. 2014], which covers an entire ML dialect. Cogent is targeted at a substantially different point in the design space. CakeML includes a verified runtime and garbage collector, while Cogent works hard to avoid these so it can be applicable to low-level embedded systems code. CakeML covers full turing-complete ML with complex semantics that works well for code written in theorem provers. Cogent is a restricted language of total functions with intentionally simple semantics that are easy to reason about equationally. CakeML is great for application code; Cogent is great for systems code, especially layered systems code with minimal sharing such as the control code of file systems or network protocol stacks. Cogent is not designed for systems code with closely-coupled, cross-cutting sharing, such as microkernels.

Cogent's main restrictions are the (purposeful) lack of recursion and iteration and its linear type system. The former ensures totality, which is important for both systems code correctness as well as for a simple shallow representation in higher-order logic. The latter is important for memory management and for making the transition from imperative C semantics to functional value semantics. Even in the restricted target domains of Cogent, real programs will of course contain some amount of iteration. This is where Cogent's integrated foreign function interface comes in: the engineer provides her own verified data types and iterator interfaces in C and uses them seamlessly in Cogent, including in formal reasoning.

Cogent is restricted, but it is not a toy language. We have used it to implement two efficient full-scale Linux file systems — a custom Flash file system and an implementation of standard Linux ext2. We plan to report on the experience with these implementations in separate work. The focus of this paper is what can be learned from Cogent about the formal verification of certifying compilation.

In particular, this paper discusses in detail the following contributions: a) the self-certifying Cogent compiler and language; b) the formal semantics of the Cogent language and the switch from imperative update semantics to functional value semantics formally justified by the linear type system (§3); c) the top-level compiler certificate (§4.1), which is a series of language-level meta proofs and per-program translation validation phases; d) the verifi-

cation stages that make up the correctness theorem (§4), including automated refinement calculi, formally verified type checking, Anormalisation, and monomorphisation; and e) the lessons learned in this project on functional language formalisation and compiler correctness proofs (§5).

2. Overview

Our aim in this paper is to build a self-certifying compiler from Cogent to efficient C code, such that a proof engineer can reason equationally about its semantics in Isabelle/HOL and apply the compiler theorem to derive properties about the generated C code. Formally, the certificate theorem is a refinement statement between the shallow embedding and the C code. This generated C code can be compiled by CompCert. It also falls into the subset of the gcc translation validation tool by Sewell et al. [2013], whose theorem would compose directly with our compiler certificate.¹

Shallow embeddings are nice for the human user, but they do not provide much syntactic structure for constructing the compiler theorem. Therefore, the compiler also generates a deep embedding for each Cogent program to use in the internal proof chain. There are two semantics for this deep embedding. (1) a formal functional value semantics where programs evaluate to values and (2) a formal imperative update semantics where programs manipulate references to mutable global state.

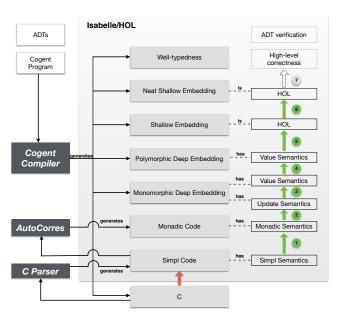


Figure 1: A detailed overview of the verification chain.

Fig. 1 shows an overview of the program representations generated by the compiler and the break-down of the automatic refinement proof that makes up the compiler certificate. The program representations are, from the bottom of Fig. 1: the C code, the semantics of the C code expressed in Isabelle/Simpl [Schirmer 2006], the same expressed as a monadic functional program [Greenaway et al. 2012, 2014], a monomorphic A-normal deep embedding of the Cogent program, a polymorphic A-normal deep embedding of the same, an A-normal shallow embedding, and finally a 'neat' shallow embedding of the Cogent program that is syntactically close to the

COGENT input of the compiler. Most of the theorems assume that the COGENT program is well-typed, which is discharged automatically in Isabelle with type inference information from the compiler.

The solid arrows on the right-hand side of the figure represent refinement proofs and the labels on these arrows correspond to the numbers in the following description. The only arrow that is not formally verified is the one crossing from C code into Isabelle/HOL at the bottom of Fig. 1 — this is the C-to-Isabelle parser [Tuch et al. 2007], which is a mature verification tool used in a number of large-scale verifications. As mentioned, it could additionally be checked by translation validation. We briefly describe each intermediate theorem, starting with the Simpl code at the bottom of the figure. For well-typed Cogent programs, we automatically prove:

- Theorem: The Simpl code produced by the C parser corresponds to a monadic representation of the C code. The proof is generated using an adjusted version of the AutoCorres tool.
- Theorem: The monadic program terminates and is a refinement of the monomorphic Cogent deep embedding under the update semantics.
- 3. Theorem: If a Cogent deep embedding evaluates in the update semantics then it evaluates to the same result in the value semantics. This is a known consequence of linear type systems [Hofmann 2000], but to our knowledge it is the first mechanised proof of such a property, esp. for a full-scale language.
- Theorem: If a monomorphic Cogent deep embedding evaluates in the value semantics then the polymorphic deep embedding evaluates equivalently in the value semantics.
- Theorem: If the polymorphic Cogent deep embedding evaluates in the value semantics then the Cogent shallow embedding evaluates to a corresponding shallow Isabelle/HOL value.
- 6. Theorem: The A-normal shallow embedding is (extensionally) equal in Isabelle/HOL to a syntactically neater shallow embedding, which is more convenient for human reasoning. This human-friendly shallow embedding corresponds to the Cogent code before the compiler's A-normalisation phase.

Arrow 7 indicates verification of user-supplied abstract data types (ADTs) implemented in C and further manual high-level proofs on top of the human-friendly shallow embedding. These are enabled by the previous steps, but are not part of this paper.

In §4 we define in more detail the relations that formally link the values (and states, when applicable) that these programs evaluate to. Steps (3) and (4) are general properties about the language and we therefore prove them manually once and for all. Steps (1), (2), (5), and (6) are generated by the compiler for every program. The proof for step (1) is generated by AutoCorres. For steps (2) and (5) we define compositional refinement calculi that ease the automation of these proofs. Step (6), the correctness of A-normalisation, is straightforward to prove via rewriting because at this stage we can already use equational reasoning.

3. Language

In this section we formally define Cogent, including its linear type system, its two dynamic semantics — update and value — mentioned earlier in §2, and the refinement theorem between them. We begin the section by walking through an example Cogent programs.

3.1 Example

2

Fig. 2 shows an excerpt of our Cogent ext2 implementation. The example uses not all, but many features of the language.

The first line in Fig. 2 shows the Cogent side of the foreign function interface. It declares an abstract Cogent data type ExSt, implemented in C. Line 2 shows a parametric abstract type, and line 9

¹ At the time of writing, Cogent's occasionally larger stack frames lead to gcc emitting memcpy() calls that, while conceptually straightforward to handle, the translation validator does not yet cover.

```
1 type ExSt
2 type UArray a
3 type Opt a = <None () | Some a>
4 type Node = #{mbuf:Opt Buf, ptr:U32, fr:U32, to:U32}
5 type Acc = (ExSt, FsSt, VfsInode)
6 type Cnt = (UArray Node,
    (U32, Node, Acc, U32, UArray Node) -> (Node, Acc))
9 uarray_create: all (a :< E). (ExSt, U32)</pre>
    -> <Success (ExSt, UArray a) | Err ExSt>
10
12 ext2_free_branch: (U32, Node, Acc, U32)
13 -> (Node, Acc, <Expd Cnt | Iter ()>
14 ext2_free_branch (depth,nd,(ex,fs,inode),mdep) =
15
    if depth + 1 < mdep
16
       then
         uarray_create[Node] (ex,nd.to-nd.fr) !nd
         | Success (ex, children) =>
18
           let nd_t { mbuf } = nd
and (children, (ex, inode, _, mbuf)) =
19
20
21
              uarray_map_no_break #{
                arr = children,
                     = ext2_free_branch_entry,
23
                acc = (ex, inode, node_t.fr, mbuf),
24
25
                ... } !nd_t
26
           and nd = nd_t { mbuf }
           in (nd, (ex, fs, inode),
27
28
             Expd (children, ext2_free_branch_cleanup))
29
         | Err ex -> (nd, (ex,fs,inode), Iter ())
30
      else
```

Figure 2: Cogent example

shows a corresponding abstract function uarray_create(), also implemented in C. Note that this abstract function is polymorphic, with a kind constraint E (see §3.2) on type argument a.

The integration of such foreign functions is seamless on the Cogent side, but naturally has requirements on the corresponding C code. The C side must respect the Cogent type system, and, for example, keep all shared state internal to the abstract type to comply with linearity constraints. It must also be terminating and implement the user-supplied semantics that appear in the corresponding shallow embedding of the Cogent program in Isabelle/HOL — ideally the user should provide a formal proof to discharge the corresponding assumption of the compiler certificate theorem.

Abstract functions can be higher-order and provide the iteration constructs that are intentionally left out from core Cogent. E.g. line 21, uarray_map_no_break() implements a map iterator for arrays. In our file system applications we have found it sufficient to provide a small library of iterators for types such as arrays. We also interfaced to an existing mature red-black tree implementation.

Returning to the example in Fig. 2, lines 3–7 show basic type constructors and declarations of variants, records and tuples using type variables and the primitive type U32. For instance, type Cnt is defined as a pair of UArray Node and a function type. Types in Cogent are structural [Pierce 2002], i.e. two types with the same structure but different names are intensionally equal.

Moreover, line 17 calls the abstract polymorphic function uarray_create(), instantiated with type argument Node. The !nd notation temporarily turns a linear object of type Node into a read-only one (see §3.3.1). The two basic, non-linear fields to and fr in type Node can directly be accessed read-only using projection functions. Line 18 and 29 are pattern matches on the result of the function invocation. Line 19 shows surface syntax for Cogent's linear take construct (see §3.3.3), accessing and binding the mbuf field of nd to the name mbuf (punning as in Haskell), as well as binding the rest of the record to the name nd_t.

The linear type system tracks that the field mbuf is logically absent in nd_t. It also tracks that nd on line 19 has been used,

```
U8 | U16 | U32 | U64 | Bool
prim. types
                                                     ::=
                                                                \alpha \mid \alpha! \mid ()
types
                                       \tau, \rho
                                                                t \mid \mathtt{T}\,\overline{\tau}\,m \mid \tau \to \rho
                                                                \langle \overline{\mathsf{C} \, \tau} \rangle \mid \{ \overline{\mathsf{f} \, :: \, \tau^?} \} \, m
                                       	au^{?}
field types
                                                                \tau \mid \tau
                                                     ::=
                                       P
permissions
                                                                {D, S, E}
                                                      =
kinds
                                                      \subseteq
                                                                P
                                                                \forall (\overline{\alpha} ::_{K} \overline{\kappa}). \tau
polytypes
                                                    ::=
                                       \pi
                                                                Read-only | Writable | Unboxed
modes
                                                     ::=
type variables
                                                      ∋
                                                                \alpha, \beta
abs. type names
                                                      \ni
                                                                T, U
                                                                \overline{\alpha:_{\mathrm{K}}\kappa}
kind context
                                       Δ
                                                     ::=
type context
                                       Г
                                                     ::=
                                                                \overline{x}:\tau
   \Delta \vdash \Gamma_1 \stackrel{\text{weak}}{\leadsto} \Gamma_2
                                                                      \Delta \vdash \overline{x_i : \tau_i}, \Gamma \stackrel{\text{weak}}{\leadsto} \Gamma
                                                                  for each i: \Delta \vdash \tau_i :_K \{S\}
\Delta \vdash \Gamma_1 \leadsto \Gamma_2 \boxplus \Gamma_3
                                               \overline{\Delta} \vdash \overline{x_i : \tau_i}, \Gamma_1, \Gamma_2 \leadsto \overline{x_i : \tau_i}, \Gamma_1 \boxplus \overline{x_i : \tau_i}, \Gamma_2
                        (overbar indicates lists, i.e. zero or more)
```

Figure 3: Type Structure of Cogent & structural context operations

so cannot be accessed again. Thus the programmer is safe to bind a new object to the same name nd (on line 26) without worrying about name shadowing. Line 26 shows surface syntax for **put**, the dual to **take**, which re-establishes the mbuf fields in the example.

3.2 Types and Kinding

3

Wadler [1990] first noted that linear types can be used as a way to safely model mutable state and similar effects while maintaining a purely functional semantics. Hofmann [2000] later proved Wadler's intuition by showing that, for a linear language, imperative C code can implement a simple set-theoretic semantics. We use linear types for two reasons: to ensure safe handling of heap-allocated objects, without the need for runtime support, and to allow us to assign to Cogent programs a simple, equational, purely functional semantics implemented via mutable state and imperative effects.

The type structure and associated syntax of Cogent is presented in Fig. 3. Our type system is loosely based on the polymorphic λ_{URAL} of Ahmed et al. [2005]. We restrict this polymorphism to be rank-1 and predicative, in the style of ML, to permit easy implementation by specialisation with minimal performance penalty.

To ease implementation, and to eliminate any direct dependency on a heap allocator, we require that all functions be defined on the top-level. This eliminates the need for linear function types: any top-level function can be shared freely because they cannot capture *any* local variables, let alone linear ones.

We include a set of primitive integer types (U8, U16 etc.). Records $\{\overline{f} :: \tau^2\}$ m comprise (1) a sequence of fields $f :: \tau^2$, where τ is the type on an inaccessible field, and (2) a mode m (see §3.3.3 and §3.2.1 for a more detailed description). We also have polymorphic variants $\langle \overline{C} \tau \rangle$, a generalised sum type in the style of OCaml, the mechanics of which are briefly described in §3.3.2. Abstract types T $\overline{\tau}$ m are also parametrised by modes. We omit product types from this presentation; they are desugared into unboxed records.

The most obvious similarity to λ_{URAL} is our use of *kinds* to determine if a type may be freely shared or discarded, as opposed to earlier linear type systems, such as that of Wadler [1990], where

$$\frac{\Delta \vdash \tau :_{K} \kappa}{\Delta \vdash () :_{K} \kappa} KUNIT} \frac{\Delta \vdash \tau :_{K} \kappa}{\Delta \vdash t :_{K} \kappa} KPRIM} \frac{\Delta \vdash \tau \to \rho :_{K} \kappa}{\Delta \vdash \tau \to \rho :_{K} \kappa} KFUN}{\frac{(\alpha :_{K} \kappa') \in \Delta \quad \kappa \subseteq k'}{\Delta \vdash \alpha :_{K} \kappa}} KVAR} \frac{(\alpha :_{K} \kappa') \in \Delta \quad \kappa \subseteq bang(\kappa')}{\Delta \vdash \alpha !_{K} \kappa} KVAR!} KVAR!} \frac{(\alpha :_{K} \kappa') \in \Delta \quad \kappa \subseteq bang(\kappa')}{\Delta \vdash \alpha !_{K} \kappa} KVAR!} KVAR!ANT} \frac{m :_{K} \kappa' \quad \kappa \subseteq \kappa'}{\Delta \vdash \tau_{i} :_{K} \kappa} KVARIANT} KABS} \frac{m :_{K} \kappa' \quad \kappa \subseteq \kappa'}{\Delta \vdash \tau_{i} :_{K} \kappa} KABS} \frac{m :_{K} \kappa' \quad \kappa \subseteq \kappa'}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC} \frac{m :_{K} \kappa}{\Delta \vdash \{f_{i} ::: \tau_{i}^{2}\} m :_{K} \kappa} KREC} KREC}$$

Figure 4: Kinding rules for Cogent types and the $bang(\cdot)$ operator

a type's linearity is encoded directly into its syntactic structure. Kinds in Cogent are sets of *permissions*, denoting whether a variable of that type may be discarded without being used (D), shared freely and used multiple times (S), or safely bound in a **let**! expression (E). A *linear* type, values of which must be used exactly once, has a kind that excludes D and S, and so forbids it being discarded or shared. We discuss **let**! expressions in §3.2.2.

Another similarity to λ_{URAL} is that we explicitly represent the context operations of weakening and contraction, normally relegated to structural rules, as explicit judgements: $\Delta \vdash \Gamma \stackrel{\text{weak}}{\leadsto} \Gamma'$ for weakening (discarding assumptions) and $\Delta \vdash \Gamma \leadsto \Gamma_1 \boxplus \Gamma_2$ for contraction (duplicating them). The rules for these judgements are presented in Fig. 3. For a typing assumption to be discarded (respectively duplicated), the type must have kind {D} (resp. {S}).

The full kinding rules for the types of Cogent are given in Fig. 4. Basic types such as () or U8, as well as functions, are simply passed by value and do not contain any heap references, so they may be given any kind. Kinding for structures and abstract functions is discussed shortly in §3.2.1.

A type may have multiple kinds, as a nonlinear type assumption may be used linearly, never being shared and being used exactly once. Therefore, a type with a permissive kind, such as {D,S},

would be an acceptable instantiation of a type variable of kind \emptyset , as we are free to *waive* permissions that are included in a kind. We can prove formally by straightforward rule induction:

Lemma 1 (Waiving rights). *If* $\Delta \vdash \tau :_K \kappa$ *and* $\kappa' \subseteq \kappa$, *then* $\Delta \vdash \tau :_K \kappa'$.

This result allows for a simple kind-checking algorithm, not immediately apparent from the rules. For example, the maximal kind of an unboxed structure with two fields of type τ_1 and τ_2 respectively can be computed by taking the intersection of the computed maximal kinds of τ_1 and τ_2 . This result ensures that this intersection is also a valid kind for τ_1 and τ_2 .

3.2.1 Kinding for Records and Abstract Types

Recall that Cogent may be extended with *abstract types*, implemented in C, which we write as T $\overline{\tau_i}$ m in our formalisation. We allow abstract types to take any number of *type parameters* τ_i , where each specific instance corresponds to a distinct C type. For example, a List abstract type, parameterised by its element type, would correspond to a family of C List types, each one specialised to a particular concrete element type. Because the implementations of these types are user supplied, the user is free to specialise implementations based on these type parameters, for example representing an array of boolean values as a bitstring, so long as they can show that every different operation implementation is a refinement of the same user-supplied CDSL semantics for that operation.

Values of abstract types may be represented by references to heap data structures. Specifically, an abstract type or structure is stored on the heap when its associated *storage mode m* is not "Unboxed". For boxed records and abstract types, the storage mode distinguishes between those that are "Writable" vs. "Read-only". The same is true for record types, written $\{\overline{\mathbf{f}} :: \tau^2\}$ m, which are discussed in more detail in §3.3.3.

The storage mode m affects the maximal kind that can be assigned to the type. For example, an unboxed structure with two components of type U8 is freely shareable, but if the structure is instead stored on the heap, then a writable reference to that structure must be linear. Thus, the type given to such references has the "Writable" mode, whose kind is $\{E\}$, thereby preventing such a reference from being assigned a nonlinear kind such as $\{D, S\}$.

3.2.2 Kinding and bang

Like Wadler [1990], we allow linear values to be shared read-only in a limited scope. This is useful for practical programming in a language with linear types, as it makes our types more informative. For example, to write a function to determine the size of a (linear) buffer object, a naive approach would be to write a function:

$$\texttt{size}: \texttt{Buf} \to \texttt{U32} \times \texttt{Buf}$$

This function has a cumbersome additional return value just so that the linear argument is not discarded. Further, the type above does not express the fact that the input buffer and output buffer are identical — this would need to be established by additional proof. To address this problem, we include a type operator $\mathbf{bang}(\cdot)$, in the style of Wadler's! operator, which changes all writable modes in a type to read-only ones. The full definition of $\mathbf{bang}(\cdot)$ is in Fig. 4. We can therefore write the type of our function as:

$$size: bang(Buf) \rightarrow U32$$

For any valid type τ , the kind of $\mathbf{bang}(\tau)$ will be nonlinear, which means that our size function no longer needs to be encumbered by the extra return value. This kinding result is formally stated as:

Lemma 2 (Kinding for **bang**(·)). For any type τ , if $\Delta \vdash \tau :_K \kappa$ then $\Delta \vdash bang(\tau) :_K bang(\kappa)$.

```
{+, *, /, <=, ==, | |, <<, ...}
primops
                                            {123, True, 'a',...}
literals
                             \ell
                                     \in
expressions
                                            x \mid () \mid f[\overline{\tau}] \mid o(\overline{e}) \mid e_1 e_2
                                            \mathbf{let}\ x = e_1\ \mathbf{in}\ e_2
                                            \mathbf{let}!(\overline{y}) \ x = e_1 \ \mathbf{in} \ e_2
                                            if e_1 then e_2 else e_3
                                             \ell \mid \mathbf{cast} \ t \ e \mid \mathbf{promote} \ \langle \overline{\mathsf{C}} \ \tau \rangle \ e
                                            case e_1 of C x \rightarrow e_2 else y \rightarrow e_3
                                            \{f = e\} \mid e.f \mid \mathbf{put} \ e_1.f \coloneqq e_2
                                            take x \{f = y\} = e_1 \text{ in } e_2
function def.
                                            \langle f :: \pi, f | x = e \rangle \mid \langle f :: \pi, \blacksquare \rangle
programs
function names
                                     €
                                            f, g
variables
                                     ∋
                                            x, y
constructors
                                             A, B, C
                                     \ni
record fields
 primopType(\cdot)
                              : o \to \bar{t} \times t
                                                                        (primop types)
 funDef(\cdot)
                                                        (definition environment)
                                   f \rightarrow d
                                   t \to \mathbb{N}
                                                                  (maximum value)
```

Figure 5: Syntax of Cogent programs (after desugaring)

To integrate this type operator with parametric polymorphism, we borrow a trick from Odersky's Observer types [Odersky 1992], and tag type variables that have been made read only, using the syntax α !. Whenever a variable α is instantiated to some concrete type τ , we also replace α ! with **bang**(τ). The lemma above ensures that our kinding rule for such tagged variables is sound, and enables us to prove the following:

Lemma 3 (Type instantiation preserves kinds). *For any type* τ , $\overline{\alpha_i} :_{\mathbf{K}} \kappa_i \vdash \tau :_{\mathbf{K}} \kappa \text{ implies } \Delta \vdash \tau[\overline{\rho_i}/\overline{\alpha_i}] :_{\mathbf{K}} \kappa \text{ when, for each } i, \Delta \vdash \rho_i :_{\mathbf{K}} \kappa_i$.

3.3 Expressions and Typing

While Cogent features a rich surface syntax, due to space constraints, we only document the (full) core language in Fig. 5 to which the surface syntax is desugared.

Fig. 6 shows the typing rules for Cogent expressions. Many of these are standard for any linear type system. We will discuss here the rules for let!, where we have taken a slightly different approach to established literature, and the rules for the extensions we have made to the type system, such as variants and record types.

3.3.1 Typing for let!

On the expression level, the programmer can use **let**! expressions, in the style of Wadler [1990], to temporarily convert variables of linear types to their read-only equivalents, allowing them to be freely shared. In this example, we wish to copy a buffer b_2 onto a buffer b_1 only when b_2 will fit inside b_1 .

```
let!(b_1, b_2) ok = (size(b_2) < size(b_1)) in if ok then copy(b_1, b_2) else ...
```

Note that even though b_1 and b_2 are used multiple times, they are only used once in a linear context. Inside the **let**! binding, they have been made temporarily nonlinear. Our kind system ensures these read-only, shareable references inside **let**! bindings cannot "escape" into the outside context. For example, the expression

let!(b) b' = b **in** copy(b, b') would violate the invariants of the linear type system, and ruin the purely functional abstraction that linear types allow, as both b and b' would refer to the same object, and a destructive update to b would change the shareable b'.

We are able to use the existing kind system to handle these safety checks with the inclusion of the E permission, for Escapable, which indicates that the type may be safely returned from within a let!. We ensure, via the typing rules of Fig. 6, that the left hand side of the binding (ok in the example) has the E permission, which excludes temporarily nonlinear references via $bang(\cdot)$ (see Fig. 4). Our solution is as powerful as Odersky's, but we encode the restrictions in the kind system directly, not as side-condition constraints that recursively descend into the structure of the binding's type.

3.3.2 Typing for Variants

A variant type $\langle \overline{C_i \ \tau_i} \rangle$ is a generalised sum type, where each alternative is distinguished by a unique *data constructor* C_i . The order in which the constructors appear in the type is not important. One can create a variant type with a single alternative simply by invoking a constructor, e.g. Some 255 might be given the type \langle Some U8 \rangle . The original value of 255 can be retrieved using the **esac** construct. The set of alternatives is enlarged by using **promote** expressions that are automatically inserted by the type-checker of the surface language, which uses subtyping to infer the type of a given variant. A similar trick is used for numeric literals and **cast**.

In order to pattern match on a variant, we provide a **case** construct that attempts to match against one constructor. If the constructor does not match, it is *removed* from the type and the reduced type is provided to the **else** branch. In this way, a traditional multi-way pattern match can be desugared by nesting:

$$\begin{array}{lll} \mathbf{case} \ x \ \mathbf{of} & \mathbf{case} \ x \ \mathbf{of} \\ \mathbf{A} \ a \rightarrow e_a & \\ \mathbf{B} \ b \rightarrow e_b & \mathbf{becomes} \\ \mathbf{C} \ c \rightarrow e_c & \mathbf{else} \ x' \rightarrow \mathbf{case} \ x' \ \mathbf{of} \\ \mathbf{B} \ b \rightarrow e_b & \\ \mathbf{else} \ x'' \rightarrow \mathbf{let} \ c = \mathbf{esac} \ x'' \ \mathbf{in} \ e_c \end{array}$$

Note that because the typing rule for **esac** only applies when only one alternative remains, our pattern matching is necessarily total.

3.3.3 Typing for Records

Some care is needed to reconcile record types and linear types. Assume that Object is a type synonym for an (unboxed) record type containing an integer and two (linear) buffers.

$$\texttt{Object} = \{size :: \texttt{U32}, b_1 :: \texttt{Buf}, b_2 :: \texttt{Buf}\} \ Unboxed$$

Let us say we want to extract the field b_1 from an Object. If we extract just a single Buf, we have implicitly discarded the other buffer b_2 . But, we can't return the entire Object along with the Buf, as this would introduce aliasing. Our solution is to return along with the Buf an Object where the field b_1 cannot be extracted again, and reflect this in the field's type, written as b_1 :: Buf. This field extractor, whose general form is take $x \{ f = y \} = e_1$ in e_2 , operates as follows: given a record e_1 , it binds the field f of e_1 to the variable f0, and the new record to the variable f1 in f2. Unless the type of the field f2 has kind f3, that field will be marked as unavailable, or taken, in the type of the new record f3.

Conversely, we also introduce a **put** operation, which, given a record with a taken field, allows a new value to be supplied in its place. The expression **put** e_1 .f := e_2 returns the record in e_1 where the field f has been replaced with the result of e_2 . Unless the type of the field f has kind {D}, that field must already be taken, to avoid accidentally destroying our only reference to a linear resource.

Unboxed records can be created using a simple struct literal $\{\overline{f_i} = e_i\}$. We also allow records to be stored on the heap to minimise unnecessary copying, as unboxed records are passed by

$$\frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash x : \tau} \bigvee_{\text{VAR}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash x : \tau} \bigvee_{\text{VAR}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash (1 : 0)} \bigvee_{\text{UNT}} \frac{\ell < |I|}{\Delta; \Gamma \vdash \ell : t} \bigvee_{\text{LTERAL}} \frac{\Delta; \Gamma \vdash \overline{e_i} : * \overline{l_i}}{\Delta; \Gamma \vdash (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : (i, e)} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash e : \tau} \bigvee_{\text{UND}} \frac{\Delta; \Gamma \vdash e : \tau}$$

Figure 6: Typing rules for Cogent

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value. These boxed records are created by invoking an externally-defined C allocator function. For these allocation functions, it is often convenient to allocate a record with all fields already taken, to indicate that they are uninitialised. Thus a function for allocating Object-like records might return values of type: {size :: U32, b_1 :: Buf, b_2 :: Buf} Writable.

For any nonlinear record (that is, (1) read-only boxed records, which cannot have linear fields, as well as (2) unboxed records without linear fields) we also allow traditional member syntax *e*.f for field access. The typing rules for all of these expressions are given in Fig. 6.

3.3.4 Type Specialisation

As mentioned earlier, we implement parametric polymorphism by specialising code to avoid paying the performance penalties of other approaches such as boxing. This means that polymorphism in our language is restricted to predicative rank-1 quantifiers.

This allows us to specify dynamic objects, such as our value typing relations (see §3.4.1) and our dynamic semantics (see §3.4), in terms of simple monomorphic types, without type variables. Thus, in order to evaluate a polymorphic program, each type variable must first be instantiated to a monomorphic type. We show that typing of the instantiated program follows from the typing of the polymorphic program, if the type instantiation used matches the kinds of the type variables.

Lemma 4 (Type specialisation). $\overline{\alpha_i :_{\kappa} \kappa_i}$; $\Gamma \vdash e : \tau$ implies Δ ; $\Gamma[\overline{\rho_i}/\overline{\alpha_i}] \vdash e[\overline{\rho_i}/\overline{\alpha_i}] : \tau[\overline{\rho_i}/\overline{\alpha_i}]$ when, for each i, $\Delta \vdash \rho_i :_{\kappa} \kappa_i$.

The above lemma is sufficient to show the monomorphic instantiation case, by setting $\Delta = \varepsilon$ (the empty context). This lemma is a key ingredient for the refinement link between polymorphic and monomorphic deep embeddings (See §4.5).

3.4 Dynamic Semantics

Fig. 8 defines the big-step evaluation rules for the *value* semantics of Cogent. The relation $V \vdash e \Downarrow_v v$ states that under environment V, the expression e evaluates to a resultant value v. These values are documented in Fig. 7. In many ways, the semantics is entirely typical of a purely functional language, albeit with some care to handle abstract function calls appropriately. This is intentional, since our goal is to automatically produce a purely functional shallow embedding from this semantics.

As functions must be defined on the top level, our function values $\langle\langle \lambda x.~e\rangle\rangle$ consist only of an unevaluated expression, which is evaluated when the function is applied. Abstract function values, written $\langle\langle abs.f|\bar{\tau}\rangle\rangle$, are instead passed more indirectly, as a pair of the function name and a list of the types used to instantiate any type variables. When an abstract function value $\langle\langle abs.f|\bar{\tau}\rangle\rangle$ is applied, the user-supplied semantics $\langle\langle f\rangle\rangle\rangle$, are invoked, which is simply a function from input value to output value.

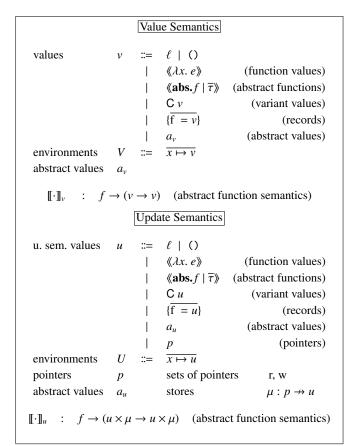


Figure 7: Definitions for Value and Update Semantics

The *update* semantics, by contrast, is much more imperative. The semantic rules can also be found in Fig. 8, with associated definitions in Fig. 7. This semantics is also an evaluation semantics, written $U \vdash e \mid \mu \Downarrow_u u \mid \mu'$ in the style of Pierce [2002]. Values in the update semantics may now be *pointers*, written p, to values in a mutable store or *heap* μ . This mutable store is modelled as a partial function from a pointer to an update semantics value.

Most of the rules in Fig. 8 only differ from the value semantics in that they thread the store μ through the evaluation of the program. However, the key differences arise in the treatment of records and of abstract types, which may now be represented as *boxed* structures, stored on the heap. In particular, note that the rule UPUT2 destructively updates the heap, instead of creating a new record value, and the semantics of abstract functions $[\cdot]_{u}$ may also modify the heap.

3.4.1 Update-Value Refinement and Type Preservation

In order to show that the update semantics is a refinement of the value semantics, we must exploit the information given to us by Cogent's linear type system. A typical refinement approach to relate the two semantics would be to define a correspondence relation between update semantics states and value semantics values, and show that an update semantics evaluation implies a corresponding value semantics evaluation. However, such a statement is not true if aliasing exists, as a destructive update (from, say, **put**) would result in multiple values being changed in the update semantics but not necessarily in the value semantics. As our type system forbids aliasing of writable references, we must include this information in our correspondence relation. Written as $u \mid \mu : v : \tau$ [**ro:** r **rw:** w], this relation states that the update semantics value u with store μ corresponds to the value semantics value v, which both have the

type τ . The sets r and w contain all pointers accessible from the value u that are read-only and writable respectively. We use this to encode the uniqueness property ensured by linear types as explicit non-aliasing constraints in the rules for the correspondence relation, which are given in Fig. 9. Read-only pointers may alias other read-only pointers, but writable pointers do not alias any other pointer, whether read-only or writable.

Because our correspondence relation includes types, it naturally implies a value typing relation for both value semantics (written $v:\tau$) and update semantics (written $u \mid \mu:\tau$ [ro: r rw: w]). In fact, the rules for both relations can be derived from the rules in Fig. 9 simply by erasing either the value semantics parts (highlighted like this) or the update semantics parts (highlighted like this). As we ultimately prove preservation for this correspondence relation across evaluation, this same erasure strategy can be applied to our proofs to produce a type preservation proof for either semantics.

Formalising uniqueness With this correspondence relation, we can prove our intuitions about linear types. For example, the following lemma, which shows that we do not discard any unique writable reference via weakening, makes use of the fact that a value is only given a discardable type when it contains no writable pointers.

Lemma 5 (Weakening respects environment typing).

```
If U \mid \mu : V : \Gamma [ro: r rw: w] and \vdash \Gamma \stackrel{weak}{\leadsto} \Gamma' then there exists r' \subseteq r such that U \mid \mu : V : \Gamma' [ro: r' rw: w].
```

We also prove a similar lemma about our context splitting judgement, which uses the fact that a value is only given a shareable type when it contains no writable pointers to conclude that the two output contexts give access to non-aliasing sets of writable pointers.

Lemma 6 (Splitting respects environment typing).

If $U \mid \mu : V : \Gamma$ [ro: r rw: w] and $\vdash \Gamma \leadsto \Gamma_1 \boxplus \Gamma_2$ then there exists r_1, r_2 and w_1, w_2 where $r = r_1 \cup r_2$ and $w = w_1 \cup w_2$, such that $U \mid \mu : V : \Gamma_1$ [ro: r_1 rw: w_1] and $U \mid \mu : V : \Gamma_2$ [ro: r_2 rw: w_2] and $w_1 \cap w_2 = \emptyset$.

In addition, we prove our main intuition about $bang(\cdot)$, necessary for showing refinement for let! expressions.

```
Lemma 7 (bang(·) makes writable read-only). If u \mid \mu : v : \tau [ro: r rw: w] then u \mid \mu : v : bang(\tau) [ro: r \cup w rw: 0]
```

Dealing with mutable state We define a *framing* relation which specifies exactly how evaluation may affect the mutable store μ . Given an input set of writable pointers w_i , an input store μ_i , an output set of pointers w_o and an output store μ_o , the relation, written $w_i \mid \mu_i$ **frame** $w_o \mid \mu_o$, ensures three properties for any pointer p:

```
Inertia If p \notin w_i \cup w_o, then \mu_i(p) = \mu_o(p).
```

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```
Leak freedom If p \in w_i and p \notin w_o, then \mu_o(p) = \bot.
```

```
Fresh allocation If p \notin w_i and p \in w_o, then \mu_i(p) = \bot.
```

Framing implies that our correspondence relation, for both values and environments, is unaffected by unrelated store updates:

Lemma 8 (Unrelated updates). Assume two unrelated pointer sets $w \cap w_1 = \emptyset$ and that $w_1 \mid \mu$ frame $w_2 \mid \mu'$, then

```
• If u \mid \mu : v : \tau [ro: r rw: w] then u \mid \mu' : v : \tau [ro: r rw: w] and w \cap w_2 = \emptyset.
```

• If
$$U \mid \mu : V : \Gamma$$
 [ro: r rw: w] then $U \mid \mu' : V : \Gamma$ [ro: r rw: w] and $w \cap w_2 = \emptyset$.

Refinement and preservation With the above lemmas and definitions, we are able to prove refinement between the value and the update semantics. This of course requires us to assume the same for the semantics given to abstract functions, $[\![\cdot]\!]_v$ and $[\![\cdot]\!]_u$.

$$\frac{|\nabla v_{-} \nabla v_{-}$$

Figure 8: Cogent Value and Update Semantics (some straightforward rules omitted for brevity)

Assumption 1. Let f be an abstract function with type signature $f:: \forall (\overline{\alpha_i}::_K \kappa_i). \tau \to \tau'$, and $\overline{\rho_i}$ be an instantiation of the type variables $\overline{\alpha_i}$ such that for each $i, \vdash \rho_i:_K \kappa_i$. Let u and v be update- and value-semantics values such that $u \mid \mu: v: \tau[\overline{\rho_i}/\overline{\alpha_i}]$ [ro: r rw: w]. The user-supplied meaning of f in each semantics gives $[\![f]\!]_v v = v'$ and $[\![f]\!]_u (u, \mu) = (u', \mu')$. Then, there exists $r' \subseteq r$ and w' such that $u' \mid \mu': v': \tau'[\overline{\rho_i}/\overline{\alpha_i}]$ [ro: r rw: w] and $w \mid \mu$ frame $w' \mid \mu'$.

We first prove that the correspondence relation is preserved when both semantics evaluate from corresponding environments. By erasing one semantics, this becomes a type preservation theorem for the other. Due to space constraints, we omit the details of the proof in this paper, but the full proof is available in our Isabelle/HOL formalisation.

Theorem 1 (Preservation of types and correspondence). *If* ε ; $\Gamma \vdash e : \tau$ *and* $U \mid \mu : V : \Gamma$ **[ro:** r r w: w] *and* $V \vdash e \not \downarrow_{v} v$ *and* $U \vdash e \mid \mu \not \downarrow_{u} u \mid \mu'$, *then there exists* $r' \subseteq r$ *and* w' *such that* $u \mid \mu' : v : \tau$ **[ro:** r' r w: w'] *and* $w \mid \mu$ *frame* $w' \mid \mu'$.

In order to prove refinement, we must show that every evaluation on the concrete update semantics has a corresponding evaluation in the abstract value semantics. While Theorem 1 already gets us most of the way there, we still need to prove that the value semantics can evaluate whenever the update semantics does.

Lemma 9 (Upward-propagation of evaluation). *If* ε ; $\Gamma \vdash e : \tau$ *and* $U \mid \mu : V : \Gamma$ [*ro: r rw: w*] *and* $U \vdash e \mid \mu \Downarrow_u u \mid \mu'$, *then there exists a v such that* $V \vdash e \Downarrow_v v$

Composing this lemma and Theorem 1, we can now easily prove our desired refinement statement.

Theorem 2 (Value \Rightarrow Update refinement). If ε ; $\Gamma \vdash e : \tau$ and $U \mid \mu : V : \Gamma$ [ro: r rw: w] and $U \vdash e \mid \mu \Downarrow_u u \mid \mu'$, then there exists a value v and pointer sets $r' \subseteq r$ and w' such that $V \vdash e \Downarrow_v v$, and $u \mid \mu' : v : \tau$ [ro: r' rw: w'] and $w \mid \mu$ frame $w' \mid \mu'$.

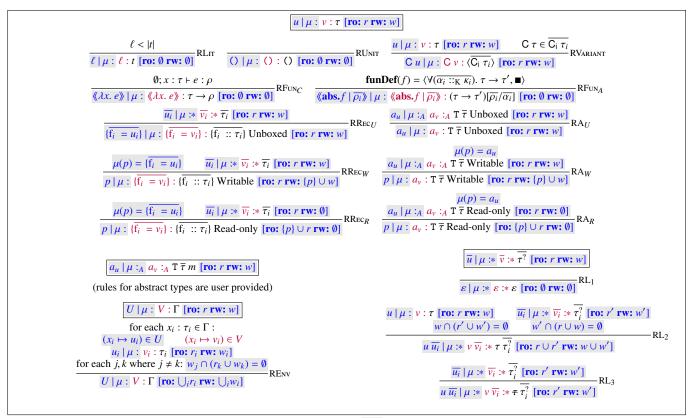


Figure 9: Value Typing and Refinement. For value typing rules, erase this text for value semantics, and this text for update semantics.

4. Verification

With the formal semantics of COGENT available, this section describes each of the proof steps that make up the compiler certificate, depicted in Fig. 1 in §2.

4.1 Top-Level Theorem

We start by describing the top-level theorem that forms the program certificate, emitted by the compiler. Recall that for a well-typed Cogent program, the compiler produces C code, a shallow embedding in Isabelle/HOL, and a refinement proof between them.

We say a C program correctly implements its Cogent shallow embedding if the following holds: (i) the C program terminates with defined execution; and (ii) if the initial C state and Cogent store are related, and the input values of the programs are related, then their output values are related.

This means, the compiler correctness theorem states that a *value relation* is preserved. This relation is concrete and can be inspected. In §3.4.1, we introduced a value typing relation between update semantics and value semantics. At each other refinement stage in the following sections, we will introduce a further relation between values of the two respective programs. By composing these value relations, we get the value relation $\mathcal V$ between the result v_m of the C program p_m and the shallow embedding s by going through the intermediate update semantics value u and value semantics result v. Note that the relation in §3.4.1 also depends on a Cogent store μ . The C state and Cogent store are related using the *state relation* $\mathcal R$, defined in detail in §4.3.

Let λe . \mathcal{M}_e r e and λv . \mathcal{M}_v r v (defined in §4.5) be two functions that monomorphise expressions and (function) values, respectively, using a rename function r provided by the compiler. Further, let R be a state relation, s a shallow embedding, e a monomorphic deep

embedding, p_m a C program, μ a Cogent store and σ a C state. Then we define **correspondence** as follows:

If $(\exists r \ w. \ U \mid \mu : V : \Gamma [\mathbf{ro:} \ r \ \mathbf{rw:} \ w])$ and $(\mu, \sigma) \in R$, then p_m successfully terminates starting at σ ; and after executing p_m , for any resulting value v_m and state σ' , there exist μ' , u, and v such that:

$$(\mu',\sigma')\in\mathcal{R}\wedge U\vdash e\mid\mu\Downarrow_uu\mid\mu'\wedge V\vdash e\Downarrow_v\mathcal{M}_vrv\wedge\mathcal{V}r\mu'v_muvs$$

Theorem 3. Given a Cogent function f that takes x of type τ as input, let p_m be its generated C code, s its shallow embedding, and e its deep embedding. Let v_m be an argument of p_m , and u and v be the update and value semantics arguments, of appropriate type, for f. If r is injective, then

$$\forall \mu \ \sigma. \ V \ r \ \mu \ v_m \ u \ v \ s \longrightarrow$$
correspondence $r \ \mathcal{R} \ (s \ v_s) \ (\mathcal{M}_e \ r \ e) \ (p_m v_m) \ U \ V \ \Gamma \ \mu \ \sigma$
where $U = (x \mapsto u), \ V = (x \mapsto v), \ and \ \Gamma = (x \mapsto \tau).$

This top-level refinement theorem additionally assumes that abstract functions in the program adhere to their specification and that their behaviour remains the same when they are monomorphised.

Intuitively, this theorem states that for related input values, all programs in the refinement chain evaluate to related output values. This can of course be used to deduce that there exist intermediate programs through which the C code and its shallow embedding are directly related. The proof engineer does not need to care what those intermediate programs are.

4.2 Well-typedness

Before we present each refinement step, we briefly describe the well-typing theorems that are used in these steps.

The Cogent compiler proves, via an automated Isabelle/HOL tactic, that the monomorphic deep embedding of the input program is well-typed. Specifically, the compiler defines $\mathbf{funDef}(\cdot)$ in Isabelle/HOL and proves that each Cogent function $f \ x = e$ is well-typed in accordance with its type as given by $\mathbf{funDef}(\cdot)$. Polymorphic well-typing is derived generically in the monomorphisation proof in §4.5.

Theorem 4 (Typing). Let f be a (monomorphic) Cogent function, where $funDef(f) = \langle f :: \tau \to \tau', f | x = e \rangle$. Then $\varepsilon; x :: \tau \vdash e :: \tau'$.

Because, as we will see in §4.3, proving refinement requires access to the typing judgements for program sub-expressions and not just for the top level, the Cogent compiler also instructs Isabelle to store all of the intermediate typing judgements established during type checking. These theorems are stored in a tree structure, isomorphic to the type derivation tree for the Cogent program. Each node is a typing theorem for a sub-expression of the program.

4.3 From C to Cogent Monomorphic Deep Embedding

This section describes the first three transformations from Fig. 1 in §2. In the first step, the C code is converted to Simpl [Schirmer 2006] by the C-to-Isabelle parser [Tuch et al. 2007], used in the seL4 project [Klein et al. 2009]. This step is kept as simple as possible and makes no effort to abstract from the details of C.

The second step in Fig. 1, which is the first link in the formal refinement chain, applies a modified version of the AutoCorres tool to produce a *monadic* shallow embedding of the C code semantics, and additionally proves that the Simpl C semantics is a refinement of the monadic shallow embedding. We modify AutoCorres to make its output more predictable by switching off its control-flow simplification and forcing it to always output the shallow embedding in the *nondeterministic state monad* of Cock et al. [2008]. In this monad, computation is represented by functions of type $state \Rightarrow (\alpha \times state) set \times bool$. Here state is the global state of the C program, including global variables, while α is the return-type of the computation. A computation takes as input the global state and returns a set, results, of pairs with new state and result value. Additionally the computation returns a boolean, failed, indicating whether it failed (e.g. whether there was undefined behaviour).

While AutoCorres was designed to facilitate manual reasoning about C code, here we use it as the foundation for automatically proving correspondence to the Cogent input program. One of the main benefits AutoCorres gives us is a *typed* memory model. Specifically, the *state* of the AutoCorres monadic representation contains a set of *typed heaps*, each of type 32 word $\Rightarrow \alpha$, one for each type α used on the heap in the C input program.

Proving that the AutoCorres-generated monadic embedding never fails implies that the C code is type- and memory-safe, and is free of undefined behaviour [Greenaway et al. 2014]. We prove non-failure as a side-condition of the refinement statement from the AutoCorres shallow embedding to the Cogent monomorphic deep embedding in its update semantics, essentially using Cogent's type system to guarantee C memory safety during execution.

This refinement proof is the third step in Fig. 1. To phrase the refinement statement we first define how deeply-embedded Cogent values and types relate to their corresponding monadic shallowly-embedded C values. The value-mapping is captured by the *value relation val-rel*, defined in Isabelle/HOL automatically by the Cogent compiler using ad hoc overloading. *val-rel* is defined separately for each Cogent program because the types used in the shallow C embedding depend on those used in the C program as, e.g., C structs are represented directly as Isabelle/HOL records.

The type relation *type-rel* is used to determine, for a Cogent value v of type τ , which typed heap in the state of the monadic

shallow embedding *v* should appear in. As with *val-rel* it is defined automatically for each Cogent program.

Given val-rel and type-rel for a particular Cogent program, the state relation $\mathcal R$ defines the correspondence between the store μ over which the Cogent update semantics operates, and the state σ of the monadic shallow embedding.

Definition 1 (Monad-to-Update State Relation). $(\mu, \sigma) \in \mathcal{R}$ *if and only if: for all pointers p in the domain of* μ *, there exists a value* ν *in the appropriate heap of* σ *(as defined by type-rel) at location p, such that val-rel* $\mu(p)$ ν *holds.*

With \mathcal{R} and val-rel, we define refinement generically between a monadic computation p_m and a Cogent expression e, evaluated under the update semantics. We denote the refinement predicate **corres**. Because \mathcal{R} changes for each Cogent program, we parameterise **corres** by an arbitrary state relation R. It is parameterised also by the typing context Γ and the environment U, as well as by the initial update semantics store μ and monadic shallow embedding state σ .

Definition 2. Monad-to-Update Correspondence

```
corres R \ e \ p_m \ U \ \Gamma \ \mu \ \sigma = (\exists r \ w. \ U \ | \ \mu : \Gamma \ [ro: \ r \ rw: \ w]) \longrightarrow (\mu, \sigma) \in R \longrightarrow (\neg failed \ (p_m \ \sigma) \ \land (\forall v_m \ \sigma'. \ (v_m, \sigma') \in results \ (p_m \ \sigma) \longrightarrow (\exists \mu' \ u. \ U \vdash e \ | \ \mu \ \downarrow_u \ u \ | \ \mu' \land (\mu', \sigma') \in R \land val\text{-}rel \ u \ v_m)))
```

The definition states that if the state relation R holds initially, then the monadic computation p_m cannot fail and, moreover, for all executions of p_m there must exist a corresponding execution under the update semantics of the expression e such that the final states are related by R and val-rel holds between their results. AutoCorres proves automatically that: \neg failed $(p_m \sigma) \longrightarrow results (p_m \sigma) \neq \emptyset$.

Refinement Proof The refinement proof is automatic in Isabelle, driven by a set of syntax-directed **corres** rules, one for each Cogent construct. The proof procedure makes use of the fact that the Cogent term is in A-normal form to reduce the number of cases that need to be considered and to simplify the higher-order unification problems that some of the proof rules pose to Isabelle.

This refinement theorem does not need an explicit formal assumption of well-typedness of the COGENT program. The proof tactic will simply fail for programs that are not well-typed.

Fig. 10 depicts two **corres** rules, one for expressions x that are variables and the other for **let** x = a **in** b. These correspond respectively to the two basic monadic operations **return**, which yields values, and >>=, for sequencing computations.

Observe that the rule Corres-Let is *compositional*: to prove that **let** x = a **in** b corresponds to a' >>= b' the rule involves proving that (1) a corresponds to a' and (2) that b corresponds to b' when each are executed over corresponding results v_u and v_m (e.g. as yielded by a and a' respectively). This compositionality significantly simplifies the automation of the correspondence proof. The typing assumptions of Corres-Let are discharged by appealing to the type theorem tree generated by the compiler (see §4.2).

The rules for some of the other constructs, such as **take**, **put**, and **case**, contain non-trivial assumptions about \mathcal{R} and about the types used in the program. Once a program and its \mathcal{R} are fixed, a set of simpler rules is automatically generated by *specialising* the generic **corres** rules for each of these constructs to the particular \mathcal{R} and types used in the input program. This in effect discharges the non-trivial assumptions of these rules once-and-for-all, allowing the automated proof of correspondence to proceed efficiently.

Conceptually, the refinement proof proceeds bottom-up, starting with the leaf functions of the program and ending with the top-level

```
\frac{(x \mapsto u) \in U \quad val\text{-}rel \ u \ v_m}{\operatorname{\mathbf{corres}} R \ x \ (\mathbf{return} \ v_m) \ U \ \Gamma \ \mu \ \sigma} \quad \text{Corres-Var}
\vdash \Gamma \leadsto \Gamma_1 \boxplus \Gamma_2 \quad \varepsilon; \Gamma_1 \vdash a : \tau \quad \mathbf{corres} \ R \ a \ a' \ U \ \Gamma_1 \ \mu \ \sigma \quad (\forall v_u \ v_m \ \mu' \ \sigma'. \ val\text{-}rel \ v_u v_m \ \longrightarrow} \quad \underbrace{(\forall v_u \ v_m \ \mu' \ \sigma'. \ val\text{-}rel \ v_u v_m \ \longrightarrow}_{\mathbf{corres} \ R \ b \ (b' \ v_m) \ (x \mapsto v_u, U) \ (x : \tau, \Gamma_2) \ \mu' \sigma')}_{\mathbf{corres} \ R \ (\mathbf{let} \ x = a \ \mathbf{in} \ b) \ (a' >>= b') \ U \ \Gamma \ \mu \ \sigma \quad (\mathsf{Corres-Let})}^{\mathsf{Corres-Let}}
```

Figure 10: Two example corres rules

entry points; **corres** results proved earlier are used to discharge **corres** assumptions for callees. The **corres** proof tactic thus follows the call-graph of the input program. Currently, the tactic is limited to computing call graphs correctly only for programs containing up to second-order functions. We did not need higher orders in our applications yet, but the tactic can certainly be extended if needed.

The resulting refinement theorem at this stage assumes that **corres** holds for all the abstract functions used in the program.

Theorem 5. Let f be a (monomorphic) Cogent function, such that $funDef(f) = \langle f :: \tau \to \tau', f | x = e \rangle$. Let p_m be its monadic shallow embedding, as derived from its generated C code. Let u and v_m be arguments of appropriate type for f and p_m respectively. Then:

$$\forall \mu \ \sigma. \ val\text{-rel } u \ v_m \longrightarrow \mathbf{corres} \ \mathcal{R} \ e \ (p_m \ v_m) \ (x \mapsto u) \ (x : \tau) \ \mu \ \sigma$$

4.4 From Update to Value Semantics

To complete this step, the compiler simply applies Theorem 2.

4.5 From Monomorphic to Polymorphic Deep Embedding

Having made the transition to the value semantics, the proof now establishes the correctness of the compiler's monomorphisation pass, moving upwards in Fig. 1 from a monomorphic to a polymorphic deep embedding of the input program.

In this pass, the compiler generates an injective renaming function r that, for a polymorphic function name f_p and types $\overline{\tau}$, yields the specialised monomorphic function name f_m , mapping names downwards, from the polymorphic to the monomorphic level. Just as we assume abstract functions are correctly implemented in C, we also assume that their behaviour remains consistent under r.

To establish correctness of monomorphisation, we essentially have an Isabelle function that repeats the monomorphisation process on behalf of the Cogent compiler, and prove that (1) the monomorphised program it produced is identical to that produced by the compiler, and (2) that the monomorphised program is a correct refinement of the polymorphic one. We define two Isabelle/HOL functions, both parameterised by r: one for monomorphising expressions, called \mathcal{M}_{ν} , and the other for monomorphising (function) values, called \mathcal{M}_{ν} . The functions specialise function calls and use r to monomorphise all function calls in expressions and values, respectively. The functions are defined compositionally for all other Cogent constructs.

Step (1) is proved by straightforward rewriting, and is automated on a per-program basis. Step (2) is embodied in the following refinement theorem, which we prove, once and for all, by rule induction over the value semantics. The specialisation Lemma 4 of §3.3.4, is a key ingredient of this proof.

Theorem 6 (Monomorphisation). Let f be a (polymorphic) Co-GENT function whose definition given by **funDef**(·) is f x = e. Let v be an appropriately-typed argument for f. Let r be an injective renaming function. Then:

$$\forall v'. (x \mapsto \mathcal{M}_v r v) \vdash \mathcal{M}_e r e \downarrow_v \mathcal{M}_v r v' \longrightarrow (x \mapsto v) \vdash e \downarrow_v v'$$

Note that on the left-hand-side of the implication, the computation runs under the value semantics where the renaming is applied across the **funDef**(·) of the right-hand side.

The compiler generates a well-typedness proof for the monomorphic deeply embedded program ($\S4.2$). We use the top-level theorem's injectivity assumption on r to infer well-typedness of the polymorphic deeply embedded program.

4.6 From Deep to Shallow Embedding

In this section, the proof makes the transition from deep to shallow embedding, where the shallow embedding is a pure function in Isabelle/HOL. This shallow embedding is still in A-normal form and is produced by the compiler as a separate Isabelle/HOL theory file. There is a second, neater shallow embedding, explained in the following section, that is closer to the Cogent input program.

For each Cogent type, the compiler generates a corresponding Isabelle/HOL type definition, and for each Cogent function, a corresponding Isabelle/HOL constant definition. We can drop the linear types at this stage and remain in Isabelle's simple types, because we have already made use of them: we are in the value semantics.

In addition to these definitions, the compiler produces a theorem that the deeply embedded polymorphic Cogent term under the value semantics correctly refines this Isabelle/HOL function. Refinement is formally defined here by the predicate **scorres** that defines when a shallowly embedded expression s is refined by a deeply embedded one e when evaluated under the environment V.

Definition 3 (Deep to Shallow Correspondence).

scorres
$$s \in V \equiv \forall r. \ V \vdash e \downarrow_v r \longrightarrow \text{valRel } s \ r$$

That is, s corresponds to e under variable bindings V if whenever e evaluates to an r under V, then s and v are in the value relation valRel. Similarly to the proof from monadic C to update semantics, the value relation here is one polymorphic constant in Isabelle/HOL, defined incrementally via ad-hoc overloading.

The program-specific refinement theorem produced is:

Theorem 7 (Deep to Shallow Refinement). Let f be an A-normal Cogent function such that $funDef(f) = \langle f :: \pi, f \mid x = e \rangle$, and let s be f's shallow embedding. Then

$$\forall v_s \ v. \ \text{valRel} \ v_s \ v \longrightarrow \mathbf{scorres} \ (s \ v_s) \ e \ (x \mapsto v)$$

Note that valRel v_s v ensures that v_s and v are of matching type, and that the shallow expression s v_s ensures in Isabelle's type system that it is the appropriate one. Like the C refinement proof in §4.3, this proof is automatic and driven by a set of syntax-directed **scorres** rules, specialised to Cogent A-normal form.

4.7 From Shallow Embedding to Neat Shallow Embedding

Fig. 11 depicts the final top-level shallow embedding, only mildly polished for presentation, for the Cogent example of Fig. 2. As Fig. 11 shows, the Isabelle definitions use the same names as the Cogent input program and they have the same structure as the input program. In this example, it remains visible that the compiler replaces tuples from the surface syntax with records in the core language, e.g. Cnt.mk is the Isabelle record constructor for the type Cnt, and instead of tuple pattern matching, the compiler generates

a sequence of take expressions. In practice, these disappear by rewriting when reasoning about the function. Tuple syntax could be reconstructed in an additional small proof pass if so desired.

The correctness statement for this phase is simple: it is pure Isabelle/HOL equality between the A-normal and neat shallow embedding for each function. For instance:

Shallow.ext2_free_branch = Neat.ext2_free_branch

The proof is simple as well. Since we can now use equational reasoning with Isabelle's powerful rewriter, we just unfold both

Figure 11: Shallow embedding for the example from §2.

sides, apply extensionality and the proof is automatic given the right congruence rules and equality theorems for functions lower in the call graph. This proof stage was the easiest and fastest of the stages to construct; it took about 1 person day.

This is a strong indication that this representation of the program is well suited for further reasoning on top.

5. Discussion and Lessons Learned

Language Restrictions: Totality The current version of Cogent purposefully omits primitive constructs for iteration and recursion, because we wanted to ensure that the language was total for a neat shallow embedding in HOL (which is total). However, since our language meta-level proofs do not require totality, we only require that each program is terminating. We are therefore contemplating to relax this restriction and allow Cogent iterator constructs where termination is obvious enough for Isabelle to prove automatically.

Formal Language Semantics The Cogent semantics in Isabelle departs slightly from that presented in §3. In particular, we enriched the update semantics to carry enough value type information to infer their corresponding C types, and adjusted the typing rules accordingly. While not needed for any of the proofs of §3, this information is used in the automatic C-correspondence proof. In addition, we found ourselves repeating parts of the (linear) type preservation proof in rule inductions on the semantics that make use of typing assumptions. This means, while type erasure is an important property for languages to enjoy (and is enjoyed by Cogent), dynamic semantics with type information are helpful for mechanised reasoning. Ideally, there should be an erased and a typed dynamic semantics, with type safety implying their equivalence.

Optimisation The current Cogent compiler performs little optimisation when generating C code and leaves low-level optimisation to gcc or CompCert. Clever optimisations in the Cogent-to-C stage would complicate our current syntax-directed correspondence approach. Cogent-to-Cogent optimisations, however, are different. The ease by which we prove the correctness of the A-normalisation over the shallow embedding via rewriting, suggests fruitful ground for optimisation. We leave exploring this idea for future work.

Effort and Size Cogent has been under development for over 2 years and has continually evolved as we have scaled the language to ever larger applications. All up, the combined language development and certifying compiler took ≈ 5 person-years. Engineering the Cogent compiler, excluding ≈ 33.5 person-months spent on proof automation and proof framework development, consumed ≈ 10 person-months. The remaining ≈ 18 person-months was for the design, formalisation and proof of Cogent and its properties (e.g. the theorems of §3), a small amount of which was also spent on early compiler development. The total size of the development

in the Isabelle theorem prover is $\approx 17,000$ lines of code (including comments and whitespace), which includes the once-and-for-all language proofs plus automated proof tactics to perform the translation validation steps, given appropriate hints from the Cogent compiler. The Cogent compiler, written in Haskell, is $\approx 9,500$ source lines of code (excluding comments and whitespace). For 6,454 lines of etx2 Cogent code we generate 76,759 lines of Isabelle/HOL proofs and embeddings.

6. Related Work

Like us, the High-Assurance Systems Programming HASP project [2010] project seeks to improve systems software by combining formal methods and programming language research. Like Cogent, HASP's systems language, Habit, is a domain specific functional language. McCreight et al. [2010] show the correctness of a garbage collector in this project; however, to the best of our knowledge, there exist no full formal language semantics yet.

Ivory [Pike et al. 2014] is a domain specific language embedded in Haskell also for implementing correct systems software. It generates well-defined, memory safe C code; however, unlike Cogent it does not *prove* correctness of the generated code.

Linear types have been used in several general purpose imperative languages to ensure memory safety without depending on a runtime, such as in Vault [Fahndrich and DeLine 2002] and Rust. PacLang [Ennals et al. 2004] is an imperative domain-specific language which uses linear types to guide optimisation of packet processing applications on network processors. Similar substructural type systems, namely uniqueness types, have been integrated into functional programming languages such as Clean [Barendsen and Smetsers 1993]. However, the type system there is only used as a way to provide a purely functional abstraction over effects, and thus Clean still depends on a run-time garbage collector.

To the best of our knowledge, Hofmann [2000] is the only work which proves the equivalence of the functional and imperative interpretation of a language with a linear type system. The proof is by pen and paper, from a first order functional language with linear types to its translation in C. Cogent in comparison is higher order and its compiler produces a machine checked proof linking a purely functional shallow embedding to its C implementation.

Examples for verified compilers for high-level languages are CakeML [Kumar et al. 2014], discussed in more detail in §1, which compiles a full ML dialect, including verified runtime and garbage collection. In contrast, [Neis et al. 2015] focuses on a compositional approach to compiler verification for a relatively simple functional language, Pilsner, to an idealised assembly language.

Charguéraud [2010, 2011] also generate a shallow embedding representation of a program to facilitate proofs about properties via a proof assistant, as we do. However, they do not address the verification of the code generated by the compiler.

7. Conclusions

We have presented the Cogent language, its self-certifying compiler, their formal definitions and top-level compiler certificate theorem, and the correctness theorems for each compiler stage. The language targets systems code where data sharing is minimal or can be abstracted, performance and small memory footprint are requirements, and formal verification is the aim.

COGENT is a pure, total functional language to enable productive equational reasoning in an interactive theorem prover. It is higher-order and polymorphic to increase conciseness. It uses linear types to make memory management bugs compile time errors, and to enable efficient destructive in-place update. It avoids garbage collection and a trusted runtime to reduce footprint. It supports a formally

modelled foreign-function interface to interoperate with C code and to implement additional data types, iterators and operations.

It does all of these with full formal proof of compilation correctness and type-safety in Isabelle/HOL.

Cogent sets a new benchmark for trustworthy systems languages, and demonstrates, through the careful application of language design with verified compilation in mind, that writing systems code that supports purely functional equational reasoning is possible.

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