0.1 Investment Decisions

Time Value of Money - Time and Uncertainty (risk). Dollar today does not equal dollar tomorrow.

Always Draw A Timeline To Visualize The Timing of Cashflows

- 1. Identify timing and amount of the cash flows. Visualize them in a timeline
- 2. Compute relevant discount factors for different points in time: $(1+r)^{-t}$
- 3. Discount the single cash flows to time 0 using the corresponding discount factors
- 4. Aggregate the cash flows at time 0

$$c0 - - - c1 - - - c2 - - - c3$$

To move money forward:

$$FV_t(c) = C \cdot (1+r)^t$$

To move money backward:

$$PV_t(C) = \frac{C}{(1+r)^t}$$

Compounding:

$$FV = \sum_{t} C_t \cdot (1+r)^{T-t}$$

Discounting:

$$PV = \sum_{t} \frac{C_t}{(1+r)^{T-t}}$$

Perpetuity:

$$PV_{perpetuity} = \frac{C}{r}$$

Important: The first cash flow arrives one period from today Non-zero-start perpetuities:

$$V_{future} = \frac{\left(\frac{C}{r}\right)}{(1+r)_{future}^t}$$

or

$$V_0 = C_{immediate} + \frac{C}{r}$$

Annuity - a finite stream of cash flows of identical magnitude and equal spacing in time:

$$PV_{annuity} = \frac{C}{r} (1 - \frac{1}{(1+r)^T})$$

Growing perpetuity - when r > g:

$$PV_{gp} = \frac{C}{r - g}$$

Growing annuity:

$$PV_{ga} = \frac{C}{r - g} \cdot (1 - (\frac{1 + r}{1 + g})^{-T})$$

Risky cash flows:

 $ExpectedValue(CF) = p_1(C_1) + p_2(C_2)$

If you are risk averse, discount even higher rate:

$$\frac{C}{(1+r) + RP}$$

Terminal value: steady state performance - FCF grow at constant rate

NPV = PV(benefits - costs)

$$NPV = \sum_{t=0}^{T} E(FCF_t)/(1+k)^t$$

$$IRR = \sum_{t=0}^{T} E(FCF_t)/(1 + IRR)^t == 0$$

where RP is risk premium

In other words, risk needs compensation:

$$PV = \sum_{t} \frac{E(C_t)}{(1+r+RP)^t}$$

Accepting projects:

- 1. NPV > 0
- 2. IRR > 0 though IRR has problems with multiple roots
- $3. \ PP < PredefinedCutoffPeriod$
- 4. $PI = \frac{NPV}{InitialCashOutlay} > 0$

0.2 Financing Decisions

Bonds (debt) and Stocks (equity): remember take the bootstrapping method for individual spot rates of a bond Interest rate risk: interest rate and bond price are inversely related: no interest rate risk if you hold till maturity, and risk is lower for bonds with shorter maturities and larger coupons

The discount rate that equates the present value of a zero-coupon bond with maturity T to its current market price P

is called the spot rate.

$$sr_T = \left(\frac{F}{P_t}\right)^{-1/T} - 1$$

For coupon-bearing bond with varying annual rates:

$$P_T = \frac{CP}{1+y} + \frac{CP_2}{(1+y)^2} + \ldots + \frac{CP+F}{(1+y)^T}$$

where y is Yield-to-Market - the unique rate that makes PV of all future payments equal to price Invested capital:

$$IC = PPE + WC$$

and

$$ROIC = \frac{NOPLAT}{IC}$$
 and $EBIT = OPREV - OPEXP - DEPR$

$$NOPLAT = EBIT \cdot (1 - \tau)$$

where

NWC = OperatingCash + Inventory +Receivables + PrepaidExpences -

Payables-Deferred Revenue-Accrued Expenses

$$FCF = NOPLAT + DEPR - CAPEX - \Delta NWC$$

Credit risk is

$$c_{debt} = risk_{free-rate} + creditspread \\$$

 $\begin{array}{ll} \textbf{Stocks} & \text{Returns: } R = \frac{dividend + P_{t+1} - P_t}{P_t} \\ \text{Covariance: } Cov[R_A, R_B] = \sigma_{AB} == \rho_{AB}\sigma_A\sigma_B \\ \text{Correlation: } \rho_{AB} = \frac{\sigma_{AB}}{\sigma_A\sigma_B} \end{array}$

The portfolio of two stocks, A and B:

$$\mathbf{E}(R_n) = w_a E[R_a] + w_b E[R_b]$$

$$\mathbf{V}(R_p) = w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2w_a w_b \sigma_{ab}$$

Risk plane: $\{x = VOL, y = RATE\}$

CAPM Two-Fund Separation Theorem: the optimal portfolio needs only two funds: the risk-free asset and the Tangent Portfolio

$$Sharpe: SR_p = \frac{\mathbf{E}[R_p] - R_f}{\sigma[R_p]}$$

 $CAPM|SML : \mathbf{E}[R_i] = R_f + \beta_f \cdot (\mathbf{E}[R_m] - R_f)$

For CML vs SML: both explain expected returns and feature the risk-free rate viz the market portfolio Differences: volatility vs beta, sharpe vs market premium, and efficient Portfolios vs all Stocks

0.3Valuations

Capital Structure No taxes $(\tau = 0)$

1. MM1: V is independent of cap structure $D + E_L = V_L$

2. MM2: $R_{E,L} = R_A + (R_A - R_D) \frac{D}{E_L}$, where $R_A = R_{E,U}$

3. MM3: $WACC_L = \frac{E_L}{D+E_L}R_{E,L} + \frac{D}{D+E_L}R_D$

With taxes $(\tau > 0)$

1. MM1: $V + PV(ITS) = V_L$

2. MM2: $R_{E,L} = R_A + (R_A - R_D)(1 - \tau) \frac{D}{E_L}$

3. MM3: $WACC_L = \frac{E_L}{D + E_T} R_{E,L} + \frac{D}{D + E_T} R_D (1 - \tau)$

DCF, APV, and Relative Valuation using Multiples

$$\begin{split} DCF &= EV_0 = \frac{FCF_1}{(1 + WACC)} + \ldots + \frac{FCF_T \cdot TV_T}{(1 + WACC)^T} \\ APV : V_L &= V_U + PV(ITS) \end{split}$$

$$PV(ITS) = \frac{ITS_1}{(1+R_D)} + \dots + \frac{ITS_T + TV_T}{(1+R_D)^T}$$

where

$$WACC = \frac{E}{D+E}R_E + \frac{D}{D+E} \cdot (1-\tau)R_D$$

$$TV_T = \frac{FCF_T \cdot (1+g)}{WACC - g}$$

$$B_L = B_U \cdot (1 + \frac{D}{E}), with \ \beta_U = \beta_A$$

For comparison:

1. Unlever the β_L of comparable firms to get their β_U

2. Compute the average of β_U

3. Relever the β_U with the firm's $\frac{D}{E}$ ratio

Using Multiples:

1. multiple consistently defined

2. multiple uniformly estimated

3. problems: difficult to define comparable firms, challenge quantifying differences in different multiples, backward-looking, you may overpay due to lack of fun-

4. example: $V_n = AvgComparable[\frac{EV}{EBIT}] \cdot EBIT_n$