AP 1 : Vector and Induced Matrix Norms through Visualization

I compared two different vector norms and their corresponding induced matrix norms. Specifically, the Euclidean norm (L_2) and the Infinity norm (L_∞) were used. Those were used to compute distances and to visualize as unit balls with this constraint:

$$||x - x_r|| \leq 1.$$

Two random 4-dimensional vectors were generated using NumPy's `randn()` function. Each vector was then reshaped into a 2×2 matrix.

For the **vector norms**, the following definitions were implemented:

$$||v||_2 = \sqrt{\sum v_i^2}, \quad ||v||_\infty = \max(|v_i|).$$

For the **induced matrix norms**, the corresponding definitions were used:

$$||A||_2 = \sigma_{\max}(A), \quad ||A||_\infty = \max_i \sum_j |a_{ij}|,$$

where $\sigma_{\max}(A)$ is the largest singular value of A.

Distances between pairs (v_1, v_2) and (A_1, A_2) were computed using these norms to quantify their separation in the respective spaces.

To interpret the geometry of these norms, sections of the 4D unit balls were visualized. Since direct 4D visualization is not possible, two dimensions were fixed and the first two were plotted as a 2D slice.

- For the L_2 norm, the unit ball appears as a **circle**, representing isotropic distance (equal scaling in all directions).
- For the L_∞ norm, the unit ball forms a **square**, showing that this norm measures distance by the largest single component deviation.

Each norm emphasizes distinct geometric properties: L_2 reflects overall magnitude, while L_∞ focuses on the largest component difference.