Question 4.2)

One bit set to 1: output: 001
Two bits set to 1: output 010
Three bits set to 1: output: 011
Four bits set to 1: output: 100

A	B	C	D	F2	F1	F0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	1	1
1	0	0	0	0	0	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	1	1

1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	0

Question 4.2

ii) from the truth table:

$$F_0 = (\bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}C\bar{D}) + (\bar{A}B\bar{C}\bar{D}) + (\bar{A}BCD)$$
$$+ (A\bar{B}\bar{C}\bar{D}) + (A\bar{B}CD) + (AB\bar{C}D) + (ABC\bar{D})$$

$$F_1 = (\bar{A}\bar{B}CD) + (\bar{A}B\bar{C}D) + (\bar{A}BC\bar{D}) + (\bar{A}BCD)$$
$$+ (A\bar{B}\bar{C}D) + (A\bar{B}C\bar{D}) + (A\bar{B}CD) + (AB\bar{C}\bar{D})$$
$$+ (AB\bar{C}D) + (ABC\bar{D})$$

$$F_2 = (A+B+C+D)$$

derive minimized sum of products form:

$$F_0 = \bar{A}(\bar{B}\bar{C}D + \bar{B}C\bar{D} + B\bar{C}\bar{D} + BCD) +$$
$$A(\bar{B}\bar{C}\bar{D} + \bar{B}CD + B\bar{C}D + BC\bar{D})$$

$$= \bar{A}(\bar{B}\overbrace{(\bar{C}D + C\bar{D})}^{Q} + B\overbrace{(\bar{C}\bar{D} + CD)}^{P}) +$$
$$A(\bar{B}\underbrace{(\bar{C}\bar{D} + CD)}_{P} + B\underbrace{(\bar{C}D + C\bar{D})}_{Q})$$

$$= \bar{A}(\bar{B}Q + BP) + A(\bar{B}P + BQ)$$

$$= \bar{A}\bar{B}Q + \bar{A}BP + A\bar{B}P + ABQ$$

Seems that it can not be further simplified

Check its K-map to make sure each element is unique

K-Map:

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	1
$\bar{A}B$	1	0	1	0
AB	0	1	0	1
$A\bar{B}$	1	0	1	0

Can't group things togther...

$$F_0 = (\bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}C\bar{D}) + (\bar{A}B\bar{C}\bar{D}) + (\bar{A}BCD)$$
$$+ (A\bar{B}\bar{C}\bar{D}) + (A\bar{B}CD) + (AB\bar{C}D) + (ABC\bar{D})$$

F_1 continues on the following page.

From the truth table:

$$F_1 = (\bar{A}\bar{B}CD) + (\bar{A}B\bar{C}D) + (\bar{A}BC\bar{D}) + (\bar{A}BCD)$$
$$+ (A\bar{B}\bar{C}D) + (A\bar{B}C\bar{D}) + (A\bar{B}CD) + (AB\bar{C}\bar{D})$$
$$+ (AB\bar{C}D) + (ABC\bar{D})$$

$$= \bar{A}(\bar{B}CD + B\bar{C}D + BC\bar{D} + BCD) +$$
$$A(\bar{B}\bar{C}D + \bar{B}C\bar{D} + \bar{B}CD + B\bar{C}\bar{D} + B\bar{C}D + BC\bar{D})$$

$$= \bar{A}(B(\overbrace{\boxed{\bar{C}D + C\bar{D} + CD}}^{(C+D)}) + \bar{B}CD) +$$
$$A(\bar{B}(\underbrace{\boxed{\bar{C}D + C\bar{D} + CD}}_{(C+D)}) + B(\underbrace{\boxed{\bar{C}\bar{D} + \bar{C}D + C\bar{D}}}_{= \bar{C} + C\bar{D} = (\bar{C}+\bar{D})}))$$

$$= \bar{A}(B(C+D) + \bar{B}CD) +$$

By Distributivity $\quad \bar{A}(\bar{B}(C+D) + B(\bar{C}+\bar{D}))$

$$)$$

$A+BC$ $\quad = \bar{A}(BC + BD + \bar{B}CD) + A(\bar{B}C + \bar{B}D + B\bar{C} + B\bar{D})$

$=(A+B)(A+C)$ $\quad = \bar{A}(BC + D(\underline{B + \bar{B}C})) + A(\bar{B}C + \bar{B}D + B\bar{C} + B\bar{D})$

$$= \bar{A}(BC + D(\underline{B\bar{B} + BC})) + A(\bar{B}C + \bar{B}D + B\bar{C} + B\bar{D})$$

$$= \bar{A}(BC + \underline{BD + CD}) + A(\bar{B}C + \bar{B}D + B\bar{C} + B\bar{D})$$

$$= \bar{A}(BC + BD + CD) + A(\bar{B}(C+D) + B(\bar{C}+\bar{D}))$$

$$= \bar{A}(BC + BD + CD) + A((\bar{B}(C+D) + B) \cdot (\bar{B}(C+D) + (\bar{C}+\bar{D})))$$

$$= \bar{A}(BC + BD + CD) + A(\underbrace{(B + \bar{B})}_{1} \cdot (B + (C+D)) \cdot ((\bar{C}+\bar{D}) + \bar{B}) \cdot \underbrace{(\bar{C}+\bar{D}+C+D)}_{1})$$

$$B + C + D \cdot (\bar{B} + \bar{C} + \bar{D})$$

$$= \bar{A}(BC+BD+CD) + A((B+C+D)\cdot(\bar{B}+\bar{C}+\bar{D}))$$

$$= \bar{A}(BC+BD+CD) + A(B\bar{C}+B\bar{D}+C\bar{B}+D\bar{C})$$

$$= \bar{A}BC + \bar{A}BD + \bar{A}CD + AB\bar{C} + AB\bar{D} + AC\bar{B} + AD\bar{C}$$

Rearrange the terms.

$$= AB\bar{C} + AC\bar{D} + AD\bar{B} + AD\bar{C} + BC\bar{D} + BD\bar{C} + CD\bar{A}$$

$$= A(B\bar{C}+C\bar{D}+D\bar{B}+D\bar{C}) + BC\bar{D} + BD\bar{C} + CD\bar{A}$$

$$= AB\bar{D} + A\bar{B}C + A\bar{C}D + BC\bar{D} + BD\bar{C} + CD\bar{A}$$

∴ The sum of product form is

$$F_1 = (AB\bar{D} + A\bar{B}C + A\bar{C}D + BC\bar{D} + BD\bar{C} + CD\bar{A})$$

$$\Updownarrow$$

$$(\bar{A}CD + \bar{A}BD + BC\bar{D} + A\bar{B}D + A\bar{B}C + AB\bar{C})$$

$$F_2 = (A+B+C+D)$$

Can not be further simplified.