Question 3) a)

$$F(A,B,C,D) = (\overline{A} + B \cdot \overline{D}) \cdot (C \cdot B \cdot A + \overline{C} \cdot D)$$

A	B	C	D	A'	C'	D'	BD'	A'+BD'	CBA	C'D	CBA $+C'$D	(A'+BD')($CBA + C'$D)
0	0	0	0	1	1	1	0	1	0	0	0	0
0	0	0	1	1	1	0	0	1	0	1	1	1
0	0	1	0	1	0	1	0	1	0	0	0	0
0	0	1	1	1	0	0	0	1	0	0	0	0
0	1	0	0	1	1	1	1	1	0	0	0	0
0	1	0	1	1	1	0	0	1	0	1	1	1
0	1	1	0	1	0	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	1	1	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0	1	1	0
1	0	1	0	0	0	1	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0	0	0
1	1	0	0	0	1	1	1	1	0	0	0	0
1	1	0	1	0	1	0	0	0	0	1	1	0

1	1	1	0	0	0	1	1	1	1	0	1	1
1	1	1	1	0	0	0	0	0	1	0	1	0

Question 3 2)

$$\text{(b)}\ \ F(W,X,Y,Z) = \overline{(W+\overline{X})(Z\overline{Y}+X)}$$

W	X	Z	Y	X’	Y’	(W+X’)	ZY’	(ZY’+X)	((W+X’)*(ZY’+X))	¬[(W+X’)(ZY’+X)]
0	0	0	0	1	1	1	0	0	0	1
0	0	0	1	1	0	1	0	0	0	1
0	0	1	0	1	1	1	1	1	1	0
0	0	1	1	1	0	1	0	0	0	1
0	1	0	0	0	1	0	0	1	0	1
0	1	0	1	0	0	0	0	1	0	1
0	1	1	0	0	1	0	1	1	0	1
0	1	1	1	0	0	0	0	1	0	1
1	0	0	0	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	0	0	1

1	0	1	0	1	1	1	1	1	1	0
1	0	1	1	1	0	1	0	0	0	1
1	1	0	0	0	1	1	0	1	1	0
1	1	0	1	0	0	1	0	1	1	0
1	1	1	0	0	1	1	1	1	1	0
1	1	1	1	0	0	1	0	1	1	0

Question 3) a)

$$F(A,B,C,D) = (\bar{A} + B \cdot \bar{D}) \cdot (C \cdot B \cdot A + \bar{C} \cdot D)$$

ii) Sum of products;

from the truth table, cases where F is true are highlighted

Read off the table :

$$F = (\bar{A}\bar{B}\bar{C}D) + (\bar{A}B\bar{C}D) + (ABC\bar{D})$$

$$= \bar{A}(\bar{B}\bar{C}D + B\bar{C}D) + (ABC\bar{D})$$

$$= \bar{A}(D(\bar{B}\bar{C} + B\bar{C})) + (ABC\bar{D})$$

$$= \bar{A}(D(\bar{C}\underbrace{(\bar{B}+B)}_{1})) + (AB\bar{C}\bar{D})$$

$$= \bar{A}(D\bar{C}) + (ABC\bar{D})$$

$$= \bar{A}D\bar{C} + ABC\bar{D}$$

$\therefore$ the sum of product is $[(\bar{A}\bar{C}D) + (ABC\bar{D})]$

iii) Product of Sums :

Read off the truth table :

$$\overline{F} = (\bar{A}\bar{B}\bar{C}\bar{D}) + (\bar{A}\bar{B}C\bar{D}) + (\bar{A}\bar{B}CD) + (\bar{A}B\bar{C}\bar{D})$$
$$+ (\bar{A}BC\bar{D}) + (\bar{A}BCD) + (A\bar{B}\bar{C}\bar{D}) + (A\bar{B}\bar{C}D)$$
$$+ (A\bar{B}C\bar{D}) + (A\bar{B}CD) + (AB\bar{C}\bar{D}) + (AB\bar{C}D)$$
$$+ (ABCD)$$

iii) continues:

negate both sides:

$$\bar{\bar{F}} = \overline{(\bar{A}\bar{B}\bar{C}\bar{D})} \cdot \overline{(\bar{A}\bar{B}C\bar{D})} \cdot \overline{(\bar{A}\bar{B}CD)} \cdot \overline{(\bar{A}B\bar{C}\bar{D})}$$
$$\cdot \overline{(\bar{A}BC\bar{D})} \cdot \overline{(\bar{A}BCD)} \cdot \overline{(A\bar{B}\bar{C}\bar{D})} \cdot \overline{(A\bar{B}\bar{C}D)}$$
$$\cdot \overline{(A\bar{B}C\bar{D})} \cdot \overline{(A\bar{B}CD)} \cdot \overline{(AB\bar{C}\bar{D})} \cdot \overline{(AB\bar{C}D)}$$
$$\cdot \overline{(ABCD)}$$

Apply De Morgan's Law:

$$F = (A+B+C+D) \cdot (A+B+\bar{C}+D) \cdot (A+B+\bar{C}+\bar{D}) \cdot (A+\bar{B}+C+D)$$
$$\cdot (A+\bar{B}+\bar{C}+D) \cdot (A+\bar{B}+\bar{C}+\bar{D}) \cdot (\bar{A}+B+C+D) \cdot (\bar{A}+B+C+\bar{D})$$
$$\cdot (\bar{A}+B+\bar{C}+D) \cdot (\bar{A}+B+\bar{C}+\bar{D}) \cdot (\bar{A}+\bar{B}+C+D) \cdot (\bar{A}+\bar{B}+C+\bar{D})$$
$$\cdot (\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

$\because (A+B)(A+C) = A+BC$

$$F = (A+B+D+C\bar{C}) \cdot (A+B+\bar{C}+\bar{D}) \cdot (A+\bar{B}+C+D)$$
$$\cdot (A+\bar{B}+\bar{C}+D\bar{D}) \cdot (\bar{A}+B+C+(D\bar{D}))$$
$$\cdot (\bar{A}+B+\bar{C}+D\bar{D}) \cdot (\bar{A}+\bar{B}+C+D\bar{D}) \cdot (\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

$$= (A+B+D) \cdot (A+B+\bar{C}+\bar{D}) \cdot (A+\bar{B}+C+D)$$
$$\cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C)$$
$$\cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C) \cdot (\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

distributive law

$(A \cdot B)+C = (A+C)(B+C)$

$$= [(D \cdot (\bar{C}+D)) + (A+B)] \cdot [(\bar{C} \cdot (C+D)) + (A+\bar{B})]$$
$$\cdot [(C \cdot \bar{C}) + (\bar{A}+B)] \cdot [(C \cdot (\bar{C}+\bar{D})) + (\bar{A}+\bar{B})]$$

$$= (A+B+D\bar{C}) \cdot (A+\bar{B}+D\bar{C}) \cdot (\bar{A}+B) \cdot (\bar{A}+\bar{B}+C\bar{D})$$

$$= [(A+D\bar{C}) + (B \cdot \bar{B})] \cdot [(B \cdot (\bar{B}+C\bar{D})) + \bar{A}]$$

$$= (A+D\bar{C}) \cdot (BC\bar{D}+\bar{A})$$

distributivity & associativity

$$= (A+D) \cdot (A+\bar{C}) \cdot (B+D) \cdot (B+\bar{C}) \cdot (C+D) \cdot (C+\bar{A}) \cdot (\bar{C}+\bar{D})$$

$$= (A \cdot B \cdot C+D) \cdot (AB\bar{D}+\bar{C}) \cdot (C+\bar{A})$$

$$= (A+D)\cdot(B+D)\cdot(C+D)\cdot(\overline{C}+\overline{D})\cdot(C+\overline{A})$$

$$= (ABC\overline{D})+(\overline{A}\,\overline{C}D)$$

$$= (A+\overline{C})\cdot(B+\overline{C})\cdot(\overline{A}+\overline{D})\cdot(C+D).$$

$\therefore$ The Product of sums is $\left[(A+\overline{C})\cdot(B+\overline{C})\cdot(\overline{A}+\overline{D})\cdot(C+D)\right]$

Question 3) b) is on the following page

Question 3)b)

From the truth table :

$$F = (\bar{W}\bar{X}\bar{Z}\bar{Y}) + (\bar{W}\bar{X}\bar{Z}Y) + (\bar{W}\bar{X}ZY) + (\bar{W}X\bar{Z}\bar{Y})$$
$$+ (\bar{W}X\bar{Z}Y) + (\bar{W}XZ\bar{Y}) + (\bar{W}XZY) + (W\bar{X}\bar{Z}\bar{Y}) + (W\bar{X}\bar{Z}Y) + (W\bar{X}ZY)$$

Distributivity

$$= \bar{W}(\bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z}Y + \bar{X}ZY + X\bar{Z}\bar{Y} + X\bar{Z}Y + XZ\bar{Y} + XZY) +$$
$$W(\bar{X}\bar{Z}\bar{Y} + \bar{X}\bar{Z}Y + \bar{X}ZY)$$

$$= \bar{W}(\bar{X}(\bar{Z}\bar{Y} + \bar{Z}Y + ZY) + X(\bar{Z}\bar{Y} + \bar{Z}Y + Z\bar{Y} + ZY)) +$$
$$W(\bar{X}(\bar{Z}\bar{Y} + \bar{Z}Y + ZY))$$

$$= \bar{W}(\bar{X}(Y(Z+\bar{Z}) + \bar{Y}\bar{Z}) + X(Y(\bar{Z}+Z) + \bar{Y}(\bar{Z}+Z))) +$$
$$W(\bar{X}(Y(\bar{Z}+Z) + \bar{Y}\bar{Z}))$$

$$= \bar{W}(\underbrace{\bar{X}(Y+\bar{Y}\bar{Z})}_{A} + X) + W(\underbrace{\bar{X}(Y+\bar{Y}\bar{Z})}_{A})$$

$$= \bar{W}(A + X) + W(A)$$
$$= \bar{W}A + \bar{W}X + WA$$
$$= A(\bar{W} + W) + \bar{W}X$$
$$= \bar{X}(Y + \bar{Y}\bar{Z}) + \bar{W}X$$
$$= X\bar{W} + \bar{X}Y + \bar{X}\bar{Z}$$

$\therefore$ The sum of products form: $(X\bar{W} + \bar{X}Y + \bar{X}\bar{Z})$

Question 3) b) continues :

iii) Product of sums :

read off the truth table :

$$\overline{F} = (\overline{W}\,\overline{X}\,Z\,\overline{Y}) + (W\,\overline{X}\,Z\,\overline{Y}) + (W\,X\,\overline{Z}\,\overline{Y}) + (W\,X\,\overline{Z}\,Y)$$
$$+ (W\,X\,Z\,\overline{Y}) + (W\,X\,Z\,Y)$$

negate both sides

$$\overline{\overline{F}} = \overline{(\overline{W}\,\overline{X}\,Z\,\overline{Y})} \cdot \overline{(W\,\overline{X}\,Z\,\overline{Y})} \cdot \overline{(W\,X\,\overline{Z}\,\overline{Y})} \cdot \overline{(W\,X\,\overline{Z}\,Y)}$$
$$\cdot \overline{(W\,X\,Z\,\overline{Y})} \cdot \overline{(W\,X\,Z\,Y)}$$

$$= (W + X + \overline{Z} + Y) \cdot (\overline{W} + X + \overline{Z} + Y) \cdot (\overline{W} + \overline{X} + Z + Y)$$
$$\cdot (\overline{W} + \overline{X} + Z + \overline{Y}) \cdot (\overline{W} + \overline{X} + \overline{Z} + Y) \cdot (\overline{W} + \overline{X} + \overline{Z} + \overline{Y})$$

Distributive Law

$$= [(X + \overline{Z} + Y) + (W \cdot \overline{W})] \cdot [(\overline{W} + \overline{X} + Z) + (\overline{Y} \cdot Y)]$$
$$\cdot [(\overline{W} + \overline{X} + \overline{Z}) + (Y \cdot \overline{Y})]$$

$$= (X + \overline{Z} + Y) \cdot (\overline{W} + \overline{X} + Z) \cdot (\overline{W} + \overline{X} + \overline{Z})$$

$$= (X + \overline{Z} + Y) \cdot [(\overline{W} + \overline{X}) + (Z + \overline{Z})]$$

$$= (X + \overline{Z} + Y) \cdot (\overline{W} + \overline{X})$$

$$= (X\overline{W} + \cancel{X\overline{X}} + \overline{Z}\,\overline{W} + \overline{Z}\,\overline{X} + Y\overline{W} + Y\overline{X})$$

$$= X\overline{W} + \overline{X}(\overline{Z} + Y) + \overline{W}(\overline{Z} + Y)$$

$$= X\overline{W} + Y\overline{X} + \overline{X}\,\overline{Z}$$

∴ The product of sums is $(X\overline{W} + Y\overline{X} + \overline{X}\,\overline{Z})$

This is the end of Question 3. Thank you so much for your time!