#### k-contact reduction

B.M. Zawora (joint work with J. de Lucas, X. Rivas, and S. Vilariño) in progress

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#### Outline

- k-symplectic manifolds
- k-symplectic reduction
  - k-contact manifolds
- k-contact reduction of the geometry
- k-contact reduction of the dynamics
- an example of k-contact reduction

Let M be an n-dimensional manifold and let TM be its tangent bundle.

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#### Definition

A k-vector field on a manifold M is a section  $X: M \to \bigoplus^k TM$  of the projection  $\operatorname{pr}_M$ . Then,  $\mathfrak{X}^k(M)$  denotes the set of all k-vector fields on M.

$$TM \stackrel{\operatorname{pr}_{\alpha}}{\longleftarrow} \bigoplus^{k} TM$$

$$X_{\alpha} \downarrow \qquad \qquad X$$

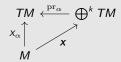
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Taking into account the diagram above, a k-vector field  $\mathbf{X} \in \mathfrak{X}^k(M)$  amounts to some vector fields  $X_1, \ldots, X_k \in \mathfrak{X}(M)$  given by  $X_\alpha = \operatorname{pr}^\alpha \circ \mathbf{X}$  with  $\alpha = 1, \ldots, k$ .

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Let  $h \in C^{\infty}(M)$  be a Hamiltonian function. A k-contact Hamiltonian k-vector field is  $\mathbf{X} = (X_1, \dots, X_k) \in \mathfrak{X}^k(M)$  satisfying

$$\begin{cases} \iota_{X_{\alpha}} d\eta^{\alpha} = dh - (R_{\alpha}h)\eta^{\alpha}, \\ \iota_{X_{\alpha}} \eta^{\alpha} = -h. \end{cases}$$

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Let  $\mathbf{X}^h = (X_1^h, \dots, X_k^h)$  be a k-contact Hamiltonian k-vector field associated with a G-invariant function  $h \in C^{\infty}(M)$  relative to the Lie group action  $\Phi$ .

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Consider a manifold  $M = \bigoplus^2 T^* \mathbb{R} \times \mathbb{R}^2$  with coordinates  $(q^1, q^2, p_1^t, p_2^t, p_1^x, p_2^x, s^t, s^x)$ .

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$$\eta = \eta^t \otimes e_1 + \eta^x \otimes e_2 = (ds^t - p_1^t dq^1 - p_2^t dq^2) \otimes e_1 + (ds^x - p_1^x dq^1 - p_2^x dq^2) \otimes e_2.$$

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$$\begin{aligned} \Phi : \mathbb{R}^2 \times M \ni \left(\lambda_1, \lambda_2; q^1, q^2, p_1^t, p_2^t, p_1^x, p_2^x, s^t, s^x\right) \\ & \mapsto \left(q^1 + \lambda_1, q^2 + \lambda_1, p_1^t, p_2^t, p_1^x, p_2^x, s^t, s^x + \lambda_2\right) \in M. \end{aligned}$$

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The fundamental vector fields associated with  $\Phi: \mathbb{R}^2 \times M \to M$  read

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B.M. Zawora

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$$\mathbf{J}: M\ni x\longmapsto \boldsymbol{\mu}=\boldsymbol{\mu}^1\otimes e_1+\boldsymbol{\mu}^2\otimes e_2=(0,1)\otimes e_1-(\boldsymbol{p}_1^t+\boldsymbol{p}_2^t,\boldsymbol{p}_1^x+\boldsymbol{p}_2^x)\otimes e_2\in (\mathbb{R}^2)^{*2}.$$

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Let us fix  $\mu=\mu_1\otimes e_1$  and recall that

$$T_x \mathbf{J}^{-1}(\mathbb{R}^{\times} \boldsymbol{\mu}) = \{ v_x \in T_x M : T_x \mathbf{J}(v_x) = \lambda \boldsymbol{\mu}, \quad \lambda \in \mathbb{R}^{\times} \},$$

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B.M. Zawora k-contact reduction

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where  $C(q^1 - q^2)$  is a coupling function between the two strings.

B.M. Zawora k-contact reduction 6

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$$\frac{\partial^2 q}{\partial t^2} - \frac{\partial^2 q}{\partial x^2} = \gamma \frac{\partial q}{\partial t} - 2 \frac{\partial C}{\partial q},$$

which is the equation of a single damped string with an external force acting on it.

# Thank you for your attention

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#### Literature:

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