From cocontact to multicontact

Xavier Rivas

Universidad Internacional de La Rioja

The Geometry of Field Theories

June 13–15, 2024

Outline

Outline

Cocontact manifolds

Definition

Let M be a $(2n+2)$-dimensional manifold. A **cocontact structure** on M is a pair (τ, η) of one-forms on M such that

1. $\mathrm{d}\tau = 0$,
2. $\tau \wedge \eta \wedge (\mathrm{d}\eta)^{\wedge n}$ is a volume form on M.

In this case, (M, τ, η) is a cocontact manifold.

Note that

- $\ker \tau$ is an integrable distribution giving a foliation of M with contact leaves.
- $\ker \eta$ is a non-integrable distribution.

Examples

Example

Let (P, η_0) be a contact manifold. The product $M = \mathbb{R} \times P$ is a cocontact manifold with cocontact structure $(\mathrm{d}t, \eta)$ where t denotes the canonical coordinate of $\mathbb{R}$ and η is the pull-back of η_0 to M.

Example

Let $(P, \tau, -\mathrm{d}\theta_0)$ be an exact cosymplectic manifold. The product $M = P \times \mathbb{R}$ is a cocontact manifold with cocontact structure $(\tau, \mathrm{d}s - \theta)$, where s denotes the canonical coordinate of $\mathbb{R}$ and θ is the pull-back of θ_0 to M.

Canonical cocontact manifold

Consider an n-dimensional manifold Q with coordinates (q^i) and its cotangent bundle T^*Q with induced natural coordinates (q^i, p_i).
Consider the product manifolds $\mathbb{R} \times \mathrm{T}^*Q$ with coordinates (t, q^i, p_i), $\mathrm{T}^*Q \times \mathbb{R}$ with coordinates (q^i, p_i, s) and $\mathbb{R} \times \mathrm{T}^*Q \times \mathbb{R}$ with coordinates (t, q^i, p_i, s) and the projections

$$
\begin{tikzcd}
 & \mathbb{R} \times \mathrm{T}^*Q \times \mathbb{R} \arrow[dl, "\rho_1"'] \arrow[dr, "\rho_2"] \arrow[dd, "\pi"] & \\
\mathbb{R} \times \mathrm{T}^*Q \arrow[dr, "\pi_2"'] & & \mathrm{T}^*Q \times \mathbb{R} \arrow[dl, "\pi_1"] \\
 & \mathrm{T}^*Q &
\end{tikzcd}
$$

Let $\theta_0 \in \Omega^1(\mathrm{T}^*Q)$ be the canonical 1-form of T^*Q given by $\theta_0 = p_i \mathrm{d}q^i$.
If $\theta_2 = \pi_2^*\theta$, we have that $(\mathrm{d}t, \theta_2)$ is a cosymplectic structure in $\mathbb{R} \times \mathrm{T}^*Q$.

Denoting by $\theta_1 = \pi_1^*\theta_0$, we have that $\eta_1 = \mathrm{d}s - \theta_1$ is a contact form in $\mathrm{T}^*Q \times \mathbb{R}$.

The $\flat$ isomorphism

Proposition

Let (M, τ, η) be a cocontact manifold. We have the following isomorphism of vector bundles:

$$\begin{array}{rccc} \flat = \flat_{\tau,\eta}\colon & \mathrm{T}M & \longrightarrow & \mathrm{T}^*M \\ & v & \longmapsto & (\iota_v\tau)\tau + \iota_v \mathrm{d}\eta + (\iota_v\eta)\,\eta \end{array}$$

Proof.

It is clear that $\ker \flat = 0$, since M is a cocontact manifold. Hence, it follows that $\flat$ has to be an isomorphism. $\square$

This isomorphism can be extended to an isomorphism of $\mathscr{C}^\infty(M)$-modules:

$$\begin{array}{rccc} \flat\colon & \mathfrak{X}(M) & \longrightarrow & \Omega^1(M) \\ & X & \longmapsto & (\iota_X\tau)\tau + \iota_X \mathrm{d}\eta + (\iota_X\eta)\,\eta \end{array}$$

Reeb vector fields

Proposition

*Given a cocontact manifold (M, τ, η), there exist two unique vector fields $R_t, R_s \in \mathfrak{X}(M)$, called **Reeb vector fields**, satisfying the conditions*

$$
\begin{aligned}
\iota_{R_t}\tau &= 1\,, \qquad & \iota_{R_t}\eta &= 0\,, \qquad & \iota_{R_t}\mathrm{d}\eta &= 0\,,\\
\iota_{R_s}\tau &= 0\,, \qquad & \iota_{R_s}\eta &= 1\,, \qquad & \iota_{R_s}\mathrm{d}\eta &= 0\,.
\end{aligned}
$$

*R_t is the **time Reeb vector field** and R_s is the **contact Reeb vector field**.*

Using $\flat$, we can redefine the Reeb vector fields:

$$
R_t = \flat^{-1}(\tau)\,, \quad R_s = \flat^{-1}(\eta)\,.
$$

Georges Reeb

Darboux coordinates

Theorem (Darboux theorem for cocontact manifolds)

Let (M, τ, η) be a cocontact manifold.
Then, for every point $p \in M$, there exists a local chart $(U; t, q^i, p_i, s)$ around p such that

$$\tau|_U = \mathrm{d}t \ , \quad \eta|_U = \mathrm{d}s - p_i \mathrm{d}q^i \ .$$

These coordinates are called ***canonical*** *or* ***Darboux*** *coordinates. Moreover, in Darboux coordinates, the Reeb vector fields are*

$$R_t|_U = \frac{\partial}{\partial t} \ , \quad R_s|_U = \frac{\partial}{\partial s} \ .$$

Gaston Darboux

Note that if $(M, \tau = \mathrm{d}t, \eta)$ is a cocontact manifold, then

$$\Omega = \mathrm{d}(e^{-t}\eta) = e^{-t}(\eta \wedge \tau + \mathrm{d}\eta)$$

is an exact symplectic form on M.

Outline

Cocontact Hamilton equations for curves

Definition

A **cocontact Hamiltonian system** is a tuple (M, τ, η, H), where (M, τ, η) is a cocontact manifold and $H \in \mathscr{C}^\infty(M)$ is a Hamiltonian function.

The **cocontact Hamilton equations for a curve** $\psi : I \subset \mathbb{R} \to M$ are

$$\begin{cases} \iota_{\psi'} \mathrm{d}\eta = (\mathrm{d}H - (R_s H)\eta - (R_t H)\tau) \circ \psi \,, \\ \iota_{\psi'} \eta = -H \circ \psi \,, \\ \iota_{\psi'} \tau = 1 \,, \end{cases}$$

where $\psi' : I \subset \mathbb{R} \to \mathrm{T}M$ denotes the canonical lift of ψ to $\mathrm{T}M$.

In Darboux coordinates, these equations read

$$\dot{t} = 1 \,, \qquad \dot{q}^i = \frac{\partial H}{\partial p_i} \,, \qquad \dot{p}_i = -\frac{\partial H}{\partial q^i} - p_i \frac{\partial H}{\partial s} \,, \qquad \dot{s} = p_i \frac{\partial H}{\partial p_i} - H \,.$$

Cocontact Hamilton equations for vector fields

Definition

Let (M, τ, η, H) be a cocontact Hamiltonian system. The **cocontact Hamilton equations for a vector field** $X \in \mathfrak{X}(M)$ are

$$\iota_X \mathrm{d}\eta = \mathrm{d}H - (R_s H)\eta - (R_t H)\tau\,, \qquad \iota_X \eta = -H\,, \qquad \iota_X \tau = 1\,,$$

or equivalently $\flat(X) = \mathrm{d}H - (R_s H + H)\eta + (1 - R_t H)\tau$.

Thus it is clear that for every Hamiltonian function $H \in \mathscr{C}^\infty(M)$ there exists a unique $X_H \in \mathfrak{X}(M)$, called the **cocontact Hamiltonian vector field**, satisfying the Hamilton equations for the function H. In Darboux coordinates, it reads

$$X_H = \frac{\partial}{\partial t} + \frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i} - \left(\frac{\partial H}{\partial q^i} + p_i \frac{\partial H}{\partial s}\right)\frac{\partial}{\partial p_i} + \left(p_i \frac{\partial H}{\partial p_i} - H\right)\frac{\partial}{\partial s}\,.$$

The integral curves of $X \in \mathfrak{X}(M)$ satisfy the cocontact Hamilton equations for curves iff X satisfies the cocontact Hamilton equations for vector fields.

Outline

Euler–Lagrange equations

The Hamiltonian formulation has a Lagrangian counterpart. It is based in **Herglotz variational principle**. Given a manifold Q, consider a Lagrangian function $L : \mathbb{R} \times \mathrm{T}Q \times \mathbb{R} \to \mathbb{R}$. We can modify Hamilton's variational principle to only consider curves on $\mathbb{R} \times \mathrm{T}Q \times \mathbb{R}$ of the form $\sigma(t) = (t, q^i(t), \dot{q}^i(t), s(t))$ such that

$$\dot{s}(t) = L(t, q^i(t), \dot{q}^i(t), s(t))\,.$$

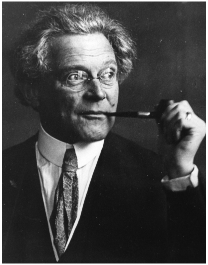
We obtain
the so-called **Herglotz–Euler–Lagrange equations**:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial v^i}\right) - \frac{\partial L}{\partial q^i} = \frac{\partial L}{\partial s}\frac{\partial L}{\partial v^i}\,, \qquad \dot{s} = L\,.$$

These Lagrangians are called *action-dependent*.

There exists a Poincaré–Cartan-like formulation of these equations so that they can be written as

$$\begin{cases} \iota_{X_L}\mathrm{d}\eta_L = \mathrm{d}E_L - (R_t^L E_L)\mathrm{d}t - (R_s^L E_L)\eta_L\,, \\ \iota_{X_L}\eta_L = -E_L\,, \\ \iota_{X_L}\mathrm{d}t = 1\,. \end{cases}$$



Gustav Herglotz

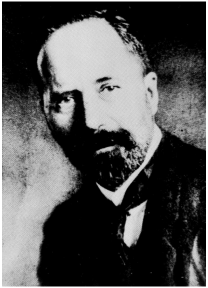
Example: Duffing's equation (I)

The Duffing equation is a non-linear second-order differential equation which can be used to model certain damped and forced oscillators. The Duffing equation is

$$\ddot{x} + \delta \dot{x} + \alpha x + \beta x^3 = \gamma \cos \omega t \,,$$

where $\alpha, \beta, \gamma, \delta, \omega$ are constant parameters. Note that

- if $\gamma = 0$, i.e. the system does not depend on time, we are in the case of contact mechanics.
- if $\delta = 0$, namely there is no damping, we have a cosymplectic system.
- if $\beta = \delta = \gamma = 0$, we obtain the equation of a simple harmonic oscillator.



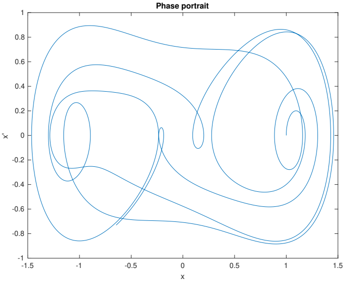
Georg Duffing

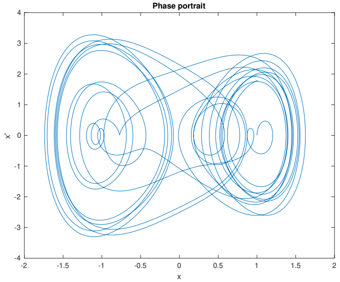
Duffing's equation models a damped forced oscillator with a stiffness different from the one obtained by Hooke's law.

It can be derived from the Lagrangian function

$$L(t, x, v, s) = \frac{1}{2} v^2 - \frac{1}{2} \alpha x^2 - \frac{1}{4} \beta x^4 - \delta s + \gamma x \cos \omega t \,.$$

Example: Duffing's equation (II)





(a) $\alpha = -1$, $\beta = 1$, $\gamma = 0.44$, $\delta = 0.3$, $\omega = 1.2$, $x(0) = 1$, $\dot{x}(0) = 0$

(b) $\alpha = 1$, $\beta = 5$, $\gamma = 8$, $\delta = 0.02$, $\omega = 0.5$, $x(0) = 1$, $\dot{x}(0) = 0$

Outline

Idea

We want to develop a geometric framework to describe non-conservative field theories generalizing the notion of **cocontact manifold** and compatible with the k-**contact** and k-**cocontact** formalisms.

This new geometric framework has to lead to the **Herglotz–Euler–Lagrange equations**:

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial y^i_\mu}\right) = \frac{\partial L}{\partial y^i} + \frac{\partial L}{\partial s^\mu}\frac{\partial L}{\partial y^i_\mu}\,.$$

In order to find this new structure, we first consider the fiber bundle $J^1\pi$ of $\pi : E \to M$.

We also consider $\Lambda^{m-1}(\mathrm{T}^* M)$ which, based on the Herglotz's variational principle for fields, is the natural structure to define the new variables s^μ that represent the dependence of the Lagrangian on the action.

Idea

In these fiber bundles we have several natural forms: the **Poincaré–Cartan m-form** associated with a Lagrangian function L in $J^1\pi$, the **tautological form** associated to $\Lambda^{m-1}(\mathrm{T}^*M)$, and a **volume form** on M.

We want to obtain a new form defined in an appropriate extension of the jet bundle, whose coordinate expression reads

$$\Theta_{\mathcal{L}} = -\frac{\partial L}{\partial y^i_\mu}\mathrm{d}y^i \wedge \mathrm{d}^{m-1}x_\mu + \left(\frac{\partial L}{\partial y^i_\mu}y^i_\mu - L\right)\mathrm{d}^m x + \mathrm{d}s^\mu \wedge \mathrm{d}^{m-1}x_\mu\,,$$

for a Lagrangian function L defined in that jet bundle extension. The new variables s^μ must give account for the “non-conservation”.

This form will be used to characterize the field equations so that we reach the **Herglotz–Euler–Lagrange equations**.

Geometric elements

Let P be a manifold with $\dim P = m + N$ and $N \geq m \geq 1$, and two forms $\Theta, \omega \in \Omega^m(P)$ with constant rank. These forms play different roles: one of them, ω, is a "reference form", while the other, Θ, is the one that gives the structure that we want to introduce, properly said.

First, given a regular distribution $\mathcal{D} \subset \mathrm{T}P$, consider $\Gamma(\mathcal{D})$, the set of sections of $\mathcal{D}$. For every $k \in \mathbb{N}$, define

$$\mathcal{A}^k(\mathcal{D}) := \{\alpha \in \Omega^k(P) \mid \iota_Z \alpha = 0 \ , \ \forall Z \in \Gamma(\mathcal{D})\} \, ;$$

that is, the set of differential k-forms on P vanishing by the vector fields of $\Gamma(\mathcal{D})$.

At a point $\mathrm{p} \in P$, the point-wise version is

$$\mathcal{A}^k_{\mathrm{p}}(\mathcal{D}) := \left\{\alpha \in \Lambda^k \mathrm{T}^*_{\mathrm{p}} P \mid \iota_v \alpha = 0 \ , \ \forall v \in \mathcal{D}_{\mathrm{p}}\right\} .$$

Geometric elements

Lemma

If $\mathcal{D}$ is an involutive distribution and $\alpha \in \mathcal{A}^k(\mathcal{D})$, we have

$$\iota_X \iota_Y \mathrm{d}\alpha = 0\,,$$

for every $X, Y \in \Gamma(\mathcal{D})$.

For a form $\alpha \in \Omega^k(P)$, with $k > 1$, the '1-ker of α' will be simply denoted as $\ker \alpha$; that is, $\ker \alpha = \{Z \in \mathfrak{X}(P) \mid \iota_Z \alpha = 0\}$. With this in mind, the above definition of $\mathcal{A}^k(\mathcal{D})$ can be written as

$$\mathcal{A}^k(\mathcal{D}) = \left\{\alpha \in \Omega^k(P) \mid \Gamma(\mathcal{D}) \subset \ker \alpha\right\}.$$

The Reeb distribution

For a pair (Θ, ω) we define:

Definition

The **Reeb distribution** associated to the pair (Θ, ω) is the distribution $\mathcal{D}^{\mathfrak{R}} \subset \mathrm{T}P$ defined, at every point $\mathrm{p} \in P$, as

$$\mathcal{D}^{\mathfrak{R}}_{\mathrm{p}} = \left\{ v \in (\ker \omega)|_{\mathrm{p}} \;\mid\; \iota_v \mathrm{d}\Theta_{\mathrm{p}} \in \mathcal{A}^m_{\mathrm{p}}(\ker \omega) \right\},$$

and $\mathcal{D}^{\mathfrak{R}} = \bigcup_{\mathrm{p} \in P} \mathcal{D}^{\mathfrak{R}}_{\mathrm{p}}$. The set of sections of the Reeb distribution is denoted by $\mathfrak{R} := \Gamma(\mathcal{D}^{\mathfrak{R}})$, and its elements $R \in \mathfrak{R}$ are called **Reeb vector fields**. Then, if $\ker \omega$ has constant rank,

$$\mathfrak{R} = \left\{ R \in \Gamma(\ker \omega) \mid \iota_R \mathrm{d}\Theta \in \mathcal{A}^m(\ker \omega) \right\}.$$

Note that $\ker \omega \cap \ker \mathrm{d}\Theta \subset \mathfrak{R}$.

Lemma

If ω is a closed form and has constant rank, then $\mathfrak{R}$ is involutive.

Multicontact structures

Definition

The pair (Θ, ω) is a **premulticontact structure** if ω is a closed form and, for $0 \leq k \leq N - m$, we have that:

(1) $\operatorname{rank} \ker \omega = N$.

(2) $\operatorname{rank} \mathcal{D}^{\mathfrak{R}} = m + k$.

(3) $\operatorname{rank} (\ker \omega \cap \ker \Theta \cap \ker \mathrm{d}\Theta) = k$.

(4) $\mathcal{A}^{m-1}(\ker \omega) = \{\iota_R \Theta \mid R \in \mathfrak{R}\}$,

Then, the triple (P, Θ, ω) is said to be a **premulticontact manifold** and Θ is called a **premulticontact form** on P. The distribution $\mathcal{C} \equiv \ker \omega \cap \ker \Theta \cap \ker \mathrm{d}\Theta$ is called the **characteristic distribution** of (P, Θ, ω).

If $k = 0$, the pair (Θ, ω) is a **multicontact structure**, (P, Θ, ω) is a **multicontact manifold** and, in this situation, Θ is said to be a **multicontact form** on P.

The dissipation form σ_Θ

Lemma

The characteristic distribution of a (pre)multicontact manifold (P, Θ, ω) *is involutive and*

$$\ker \omega \cap \ker \Theta \cap \ker \mathrm{d}\Theta = \mathcal{D}^{\mathfrak{R}} \cap \ker \Theta \,.$$

Associated to a (pre)multicontact structure, we have the following one-form:

Proposition

Given a (pre)multicontact manifold (P, Θ, ω)*, there exists a unique* 1*-form* $\sigma_\Theta \in \Omega^1(P)$ *verifying that*

$$\sigma_\Theta \wedge \iota_R \Theta = \iota_R \mathrm{d}\Theta \,, \text{ for every } R \in \mathfrak{R} \,.$$

Definition

The 1-form σ_Θ is called the **dissipation form**.

The operator $\overline{\mathrm{d}}$

Using this dissipation form we can define the following operator, which will be used later to set the field equations in a (pre)multicontact manifold.

Definition

Let $\sigma_\Theta \in \Omega^1(P)$ be the dissipation form. We define the operator

$$
\begin{aligned}
\overline{\mathrm{d}} : \Omega^k(P) &\longrightarrow \Omega^{k+1}(P) \\
\beta &\longmapsto \overline{\mathrm{d}}\beta = \mathrm{d}\beta + \sigma_\Theta \wedge \beta \,.
\end{aligned}
$$

We have that $\overline{\mathrm{d}}^2 = 0$ if, and only if, $\mathrm{d}\sigma_\Theta = 0$. In this case, it induces a *Lichnerowicz–Jacobi cohomology*. One consequence is that, locally, there exists a function such that $\sigma_\Theta = \mathrm{d}f$ and $\overline{\mathrm{d}}\beta = e^{-f}\mathrm{d}(e^f\beta)$. In this case, we say that the pair (Θ, ω) is a **closed multicontact structure**. This is also the condition required in order to consider variational higher-order contact Lagrangian field theories.

The field equations

Given a variational (pre)multicontact bundle (P, Θ, ω), the **(pre)multicontact field equations for τ-transverse, locally decomposable multivector fields $\mathbf{X} \in \mathfrak{X}^m(P)$** are

$$\iota_{\mathbf{X}}\Theta = 0\,, \qquad \iota_{\mathbf{X}}\overline{\mathrm{d}}\Theta = 0\,,$$

where the condition of τ-transversality is $\iota_{\mathbf{X}}\omega = 1$.

With this setting, one can develop Lagrangian and Hamiltonian formalisms, whose field equations read, respectively,

$$\frac{\partial s^\mu}{\partial x^\mu} = L \circ \boldsymbol{\psi}\,, \qquad \frac{\partial}{\partial x^\mu}\left(\frac{\partial L}{\partial y^i_\mu} \circ \boldsymbol{\psi}\right) = \left(\frac{\partial L}{\partial y^i} + \frac{\partial L}{\partial s^\mu}\frac{\partial L}{\partial y^i_\mu}\right) \circ \boldsymbol{\psi}\,,$$

and

$$\frac{\partial s^\mu}{\partial x^\mu} = \left(p_i^\mu\,\frac{\partial H}{\partial p_i^\mu} - H\right) \circ \boldsymbol{\psi}\,, \qquad \frac{\partial y^i}{\partial x^\mu} = \frac{\partial H}{\partial p_i^\mu} \circ \boldsymbol{\psi}\,,$$

$$\frac{\partial p_i^\mu}{\partial x^\mu} = -\left(\frac{\partial H}{\partial y^i} + p_i^\mu\,\frac{\partial H}{\partial s^\mu}\right) \circ \boldsymbol{\psi}\,.$$

Outline

Cocontact structures

Cocontact Hamiltonian systems

Cocontact Lagrangian systems

Multicontact field theories

From cocontact to multicontact

From cocontact to multicontact I

Every cocontact structure (τ, η) on a manifold M along with a Hamiltonian function H allow us to define a multicontact 1-form $\Theta \in \Omega^1(M)$ given by

$$\Theta = H\tau + \eta\,.$$

In fact, we can take the 1-form $\omega = \tau$. In this case, we have

$$\ker\omega = \ker\tau = \left\langle \frac{\partial}{\partial q^i}, \frac{\partial}{\partial p_i}, \frac{\partial}{\partial s} \right\rangle, \qquad \mathfrak{R} = \left\langle \frac{\partial}{\partial s} \right\rangle.$$

The conditions stated in the definition of multicontact structure hold obviously for this structure taking into account that $k = 0$, $m = 1$, and $N = 2n + 1$, and that $\mathcal{A}^0(\ker\omega) = \mathscr{C}^\infty(M)$. In coordinates, we have

$$\begin{aligned}
\sigma_\Theta &= \frac{\partial H}{\partial s}\mathrm{d}t\,,\\
\mathrm{d}\Theta &= \frac{\partial H}{\partial q^i}\mathrm{d}q^i \wedge \mathrm{d}t + \frac{\partial H}{\partial p_i}\mathrm{d}p_i \wedge \mathrm{d}t + \frac{\partial H}{\partial s}\mathrm{d}s \wedge \mathrm{d}t + \mathrm{d}q^i \wedge \mathrm{d}p_i\,,\\
\overline{\mathrm{d}}\Theta &= \left(\frac{\partial H}{\partial q^i} + p_i\frac{\partial H}{\partial s}\right)\mathrm{d}q^i \wedge \mathrm{d}t + \frac{\partial H}{\partial p_i}\mathrm{d}p_i \wedge \mathrm{d}t + \mathrm{d}q^i \wedge \mathrm{d}p_i\,.
\end{aligned}$$

From cocontact to multicontact II

Consider now a 1-multivector field $X \in \mathfrak{X}^1(M)$ with local expression

$$X = f\frac{\partial}{\partial t} + F^i\frac{\partial}{\partial q^i} + G_i\frac{\partial}{\partial p_i} + g\frac{\partial}{\partial s}\,.$$

Imposing the multicontact field equations, we obtain

$$\begin{aligned} f &= 1\,,\\ g &= p_iF^i - H\,,\\ G_i &= -\left(\frac{\partial H}{\partial q^i} + p_i\frac{\partial H}{\partial s}\right)\,,\\ F^i &= \frac{\partial H}{\partial p_i}\,,\\ 0 &= \left(\frac{\partial H}{\partial q^i} + p_i\frac{\partial H}{\partial s}\right)F^i + G_i\frac{\partial H}{\partial p_i}\,, \end{aligned}$$

where the last equation holds identically when the above ones are taken into account.

From cocontact to multicontact III

Thus, the local expression of the vector field X is

$$X = \frac{\partial}{\partial t} + \frac{\partial H}{\partial p_i}\frac{\partial}{\partial q^i} - \left(\frac{\partial H}{\partial q^i} + p_i\frac{\partial H}{\partial s}\right)\frac{\partial}{\partial p_i} + \left(p_i\frac{\partial H}{\partial p_i} - H\right)\frac{\partial}{\partial s}\,.$$

This local expression coincides with the local expression given above of the cocontact Hamiltonian vector field X_H associated to the Hamiltonian function H.

Thus, we have checked that time-dependent contact mechanics is a particular case of the multicontact setting.

Conversely, given a multicontact structure (Θ, ω) with $m = 1$, consider the 1-forms $\tau = \omega$ and $\eta = \Theta - H\omega$. Since ω is closed, $\tau \wedge \eta \wedge (\mathrm{d}\eta)^n$ is a volume form on M and thus (τ, η) define a cocontact structure on M.

Thanks for your attention!

- M. de León, J. Gaset, X. Gràcia, M. C. Muñoz-Lecanda and X. Rivas. Time-dependent contact mechanics. *Monatsh. Math.* **201**:1149-1183, 2023. doi: 10.1007/s00605-022-01767-1

- M. de León, J. Gaset, M. C. Muñoz-Lecanda, X. Rivas and N. Román-Roy, Multicontact formulation for non-conservative field theories. *J. Phys. A: Math. Theor.* **56**(2):025201, 2023. doi: 10.1088/1751-8121/acb575

- M. de León, J. Gaset, M. C. Muñoz-Lecanda, X. Rivas and N. Román-Roy. A contact formulation of action-dependent field theories. Applications to physics. *In preparation*