The Construction of the Graduated Handicap Tables for Target Archery.

This paper gives the final algorithms which I developed to compute the Graduated Handicap Tables for Target Archery. Most of the research into the validity of the various assumptions which had to be made was discussed in earlier papers – see the end of this note.

I define the central trajectory as that trajectory which results in the arrow arriving at the pinhole. This is the archer's goal and if each shot could be launched along the central trajectory each arrow would hit the pinhole and the score would be the maximum. But no-one is perfect and errors occur in each arrow launch. Many studies of shot distributions have shown that these errors are distributed around the central trajectory in a bi-normal distribution. Further errors may occur along the trajectory because of changes in the air and wind conditions about which the archer has no information and no control. These errors contribute to the excess dispersion which is discussed later.

The basic measure of an archer's skill which I have used is the root mean square (rms) value σ_θ of the angular deviation of the launch of shots around the central trajectory.

To connect this fundamental measure of skill to a Handicap Rating, a logarithmic scale was chosen so that each change of one Handicap step represents a constant percentage change in the archer's accuracy. The final relationship that I used is:-

$$\sigma_\theta = 1.036^{(H+12.9)} * 5 * (10^{-4})$$

where σ_θ is measured in radians and H is the Handicap Rating.

Each Handicap step therefore represents a change in the archer's skill of 3.6%.

The constant 12.9 was introduced right at the end of the development of the tables to adjust the Handicap scale so that the score for a Handicap of 0 was 1400 on the FITA Gentlemen round.

An archer with a Handicap of 0 is achieving an rms deviation of 0.789 mrad while a Handicap of 100 corresponds to an rms deviation of 27.107 mrad. Note that the limiting resolution of the unaided human eye is about 0.5 mrad.

If gravity and air resistance did not exist, the trajectory of the arrow would be a straight line and an RMS angular deviation of σ_θ would result in an RMS deviation of the hits from the pinhole given by:-

$$\sigma_r = R * \sigma_\theta \qquad \text{where R is the range to the target in metres}$$

and $$\sigma_r = 100 * R * 1.036^{(H+12.9)} * 5 * (10^{-4})$$ where σ_r is measured in cm.

Without gravity and air resistance the time that the arrow is in the air and the size of the group on the target, would increase linearly with the target range. However, because of gravity, the elevation of the shot has to be increased as the target range is increased. The time that the arrow is in the air and the dispersion of the group increases more nearly as the square of the target range rather than linearly. This effect, known as the excess dispersion, is made worse by the drag of the air on the arrow.

The major problem for the construction of Handicap Tables is that the magnitude of the excess dispersion depends critically on the arrow launch velocity. The 1980 Comprehensive Tables avoided this problem by using the same excess dispersion factor at all Handicap levels. The factor was chosen to match archers shooting to FITA Star standards who at that time were achieving arrow launch speeds of about 150 f/s. The 1980 Tables were therefore very unfair to Juniors shooting bows with low arrow speeds. During the 1980's the improvements in bow efficiency, the introduction of composite arrows and the compound bow achieved arrow speeds well over 200 f/s and the tables also became unrealistic at low Handicaps.

The solution which I adopted was to recognize the correlation between arrow speeds and handicap ratings. Thus I used an excess dispersion factor of the form :-

$F = (1 + K * R^{2)}$ where the coefficient K was a function of the Handicap rating H.

After a great deal of trial and error to make sure that the resulting tables provided a reasonable match to actual shooting results and covered the full range from seniors shooting fast compound bows to juniors in the Under 10 age group, I used the following expression for K :-

$$K = (1.429 * 10^{-6}) * 1.07^{(H+4.3)}$$

And $\quad F = (1 + (1.429 * 10^{-6}) * 1.07^{(H+4.3)} * R^2)$

Thus the final expression which I used for the RMS deviation from the pinhole is:-

$$\sigma_r = 100 * R * 1.036^{(H+12.9)} * 5 * (10^{-4}) * (1 + 1.429 * 10^{-6} * 1.07^{(H+4.3)} * R^2)$$

The probability distribution of r (the deviation from the pinhole) is the Rayleigh Distribution:-

$$p(r) = \frac{2r}{\sigma_r^2} \exp\left(\frac{-r^2}{\sigma_r^2}\right)$$

Thus the average score of each arrow is:-

$$\bar{S} = \int_0^\infty S(r).\frac{2r}{\sigma_r^2}.exp\left(\frac{-r^2}{\sigma_r^2}\right).dr$$

where *S(r)* is the staircase function representing the particular scoring system on the particular target size in use.

The effect of the line cutter rule (line cutters count high) is included by increasing the radius of each scoring zone by an amount equal to the radius of the arrow. In all of the calculations I have used 0.357 cm (the radius of an 1864 arrow).

Thus if D is the diameter of the target measured in cm, the results of the integration to give $\bar{S}$ are:-

For Imperial rounds:-

$$\bar{S} = 9 - 2\sum_{n=1}^{4} \exp\left(\frac{-\left(\frac{nD}{10} + 0.357\right)^2}{\sigma_r^2}\right) - \exp\left(\frac{-\left(\frac{D}{2} + 0.357\right)^2}{\sigma_r^2}\right)$$

Note that for Imperial scoring all target ranges must be converted to metres for these computations.

For normal metric scoring:-

$$\bar{S} = 10 - \sum_{n=1}^{10} \exp\left(\frac{-\left(\frac{nD}{20} + 0.357\right)^2}{\sigma_r^2}\right)$$

For Compound 'Inner ten' metric scoring:-

$$\bar{S} = 10 - \exp\left(\frac{-\left(\frac{D}{40} + 0.357\right)^2}{\sigma_r^2}\right) - \sum_{n=2}^{10} \exp\left(\frac{-\left(\frac{nD}{20} + 0.357\right)^2}{\sigma_r^2}\right)$$

For 'Triple', 'Vegas' or FITA 5 zone faces:-

$$\bar{S} = 10 - \sum_{n=1}^{4} \exp\left(\frac{-\left(\frac{nD}{20} + 0.357\right)^2}{\sigma_r^2}\right) - 6.\exp\left(\frac{-\left(\frac{5D}{20} + 0.357\right)^2}{\sigma_r^2}\right)$$

For Vegas, Triple, or FITA 5 zone faces using inner ten scoring:-

$$\bar{S} = 10 - \exp\left(\frac{-\left(\frac{D}{40} + 0.357\right)^2}{\sigma_r^2}\right) - \sum_{n=2}^{4} \exp\left(\frac{-\left(\frac{nD}{20} + 0.357\right)^2}{\sigma_r^2}\right) - 6.\exp\left(\frac{-\left(\frac{5D}{20} + 0.357\right)^2}{\sigma_r^2}\right)$$

For the Worcester face:-

$$\bar{S} = 5 - \sum_{n=1}^{5} \exp\left(\frac{-\left(\frac{nD}{10} + 0.357\right)^2}{\sigma_r^2}\right)$$

For FITA 6 zone faces:-

$$\bar{S} = 10 - \sum_{n=1}^{5} \exp\left(\frac{-\left(\frac{nD}{20} + 0.357\right)^2}{\sigma_r^2}\right) - 5.\exp\left(\frac{-\left(\frac{6D}{20} + 0.357\right)^2}{\sigma_r^2}\right)$$

Finally, the score for a round is given by:

$$S = \sum_{1}^{m} N_m * \bar{S}(R_m, D_m)$$

where m is the number of ranges in the round, N_m is the number of arrows shot at Range m and $\bar{S}(R_m, D_m)$ is the average score per arrow shot at range R_m and target D_m using the appropriate staircase function for the type of target. S is then rounded to the nearest whole number.

At the top and bottom ends of the tables the same score can occur for adjacent Handicap Ratings. In the ‘Score for Round’ tables, the score is only included for the highest Handicap Rating which produces each score.

I have to point out that S is the average score. Many people have queried that the Tables contain ‘impossible’ scores (1295, 1293, 1291, etc) for the imperial rounds. My reply has always been ‘the National Statistics show that the average family includes 2.4 children’.

David Lane

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Earlier Papers:

The Construction of Handicap Tables for Archers - paper sent to GNAS March 1978

Handicap Tables – Toxophilus Vol II No 1 Apr/May 1979

The Variation of Accuracy with Range - Toxophilus Vol II No 3 Aug/Sep 1979