

# Algorithmic Fairness on Graphs

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# Acknowledgements

Many thanks to Jian Kang and Hanghang Tong for generously sharing their tutorial slides



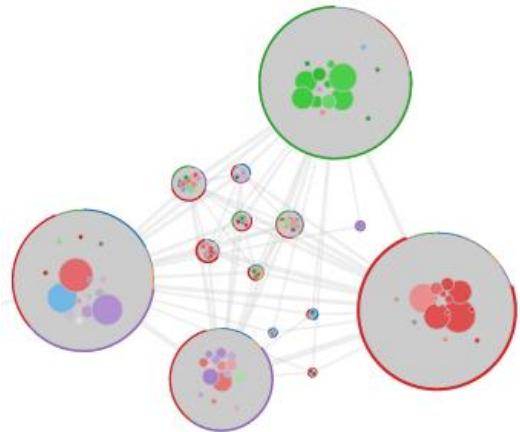
**Jian Kang**



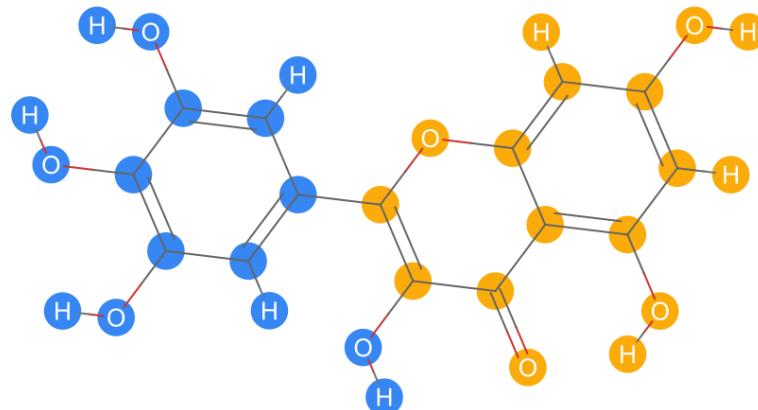
**Hanghang Tong**

**University of Illinois Urbana Champaign**

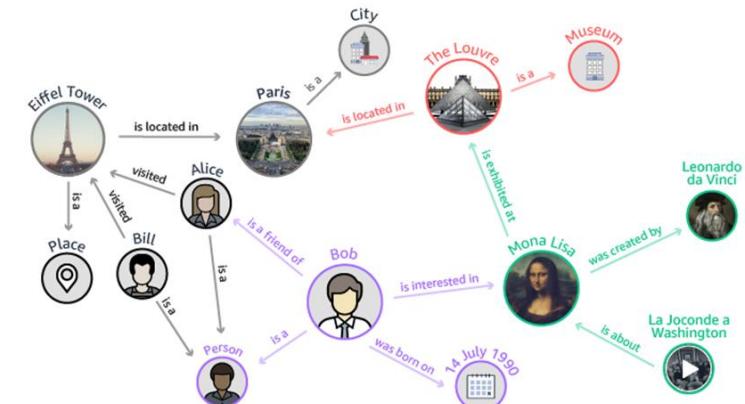
# The Ubiquity of Graphs



# Social network



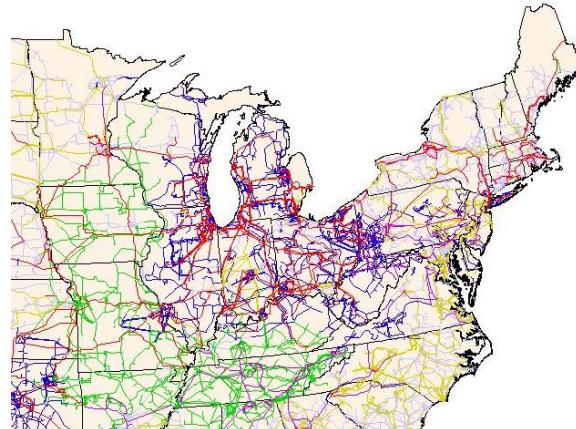
## Molecular graph



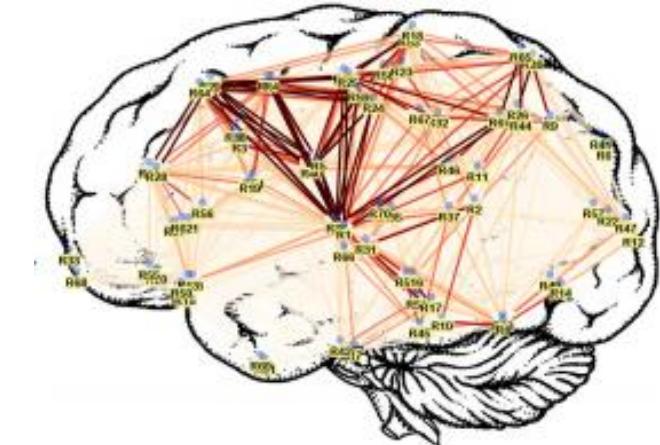
# Knowledge graph



## Road network



# Power grid



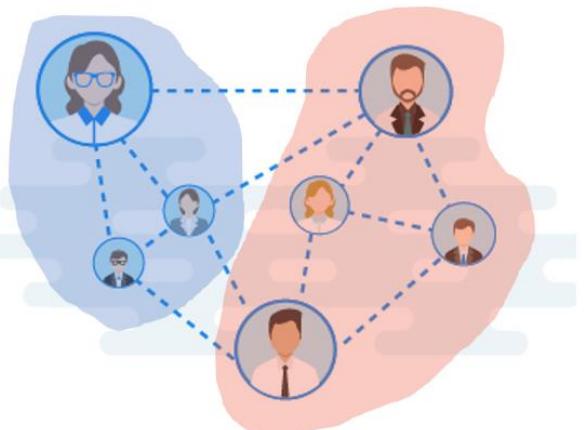
# Brain network

# This talk: Graphs = Networks

# Graph Mining: Applications



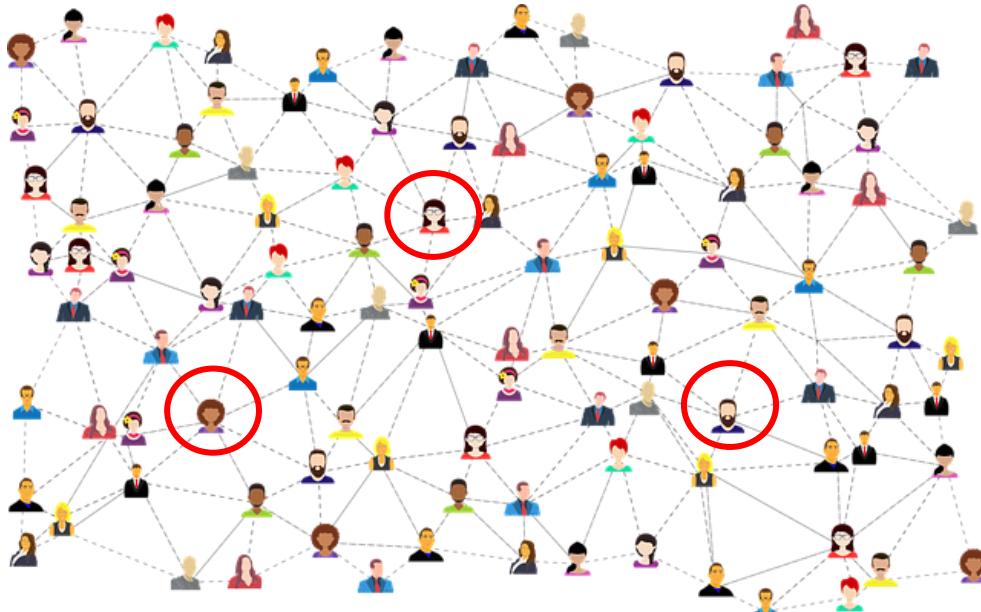
Credit scoring



User community detection



Financial fraud detection



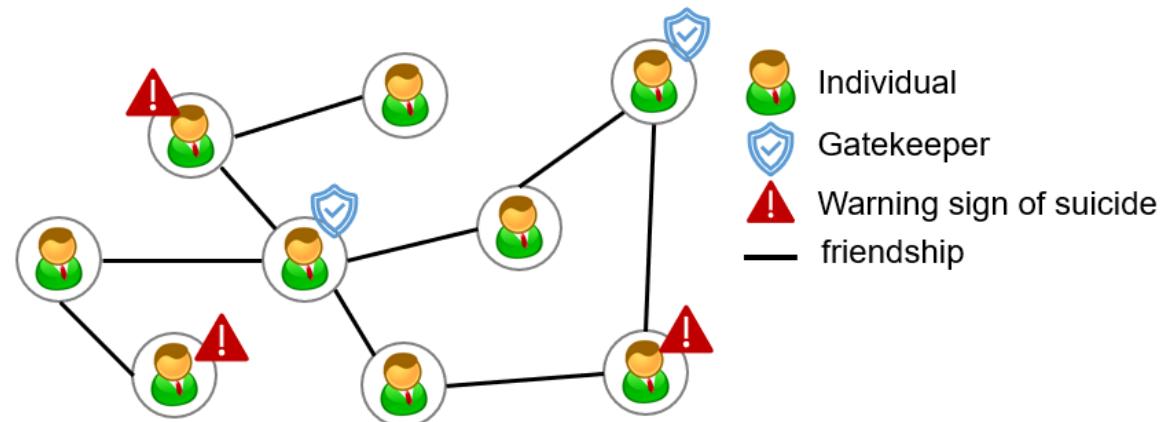
Identifying influencers

# Algorithmic Fairness on Graphs: Suicide Prevention

- Suicide is one of the leading causes of death in US

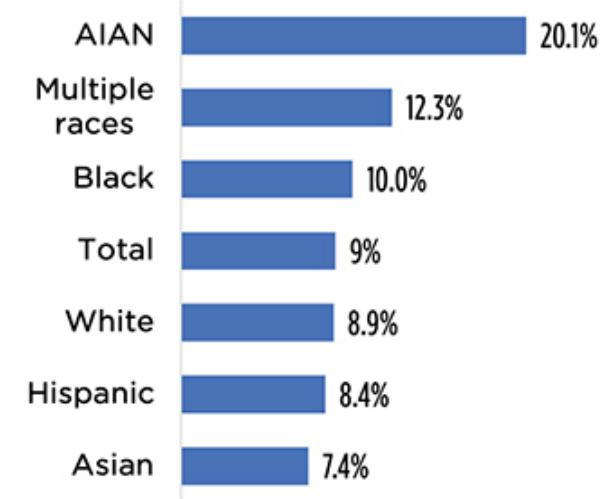


Gatekeeper training programs



Toy example of a gatekeeper training program

Percentage of high schoolers reporting a suicide attempt in the past 12 months, by race/ethnicity



Suicide attempts by race/ethnicity

- **Observation:** existing suicide prevention efforts disproportionately affect individuals of different demographics

[1] <https://www.cdc.gov/nchs/data/vsrr/vsrr024.pdf>

[2] <https://988lifeline.org/>

[3] <https://www.childtrends.org/publications/addressing-discrimination-supports-youth-suicide-prevention-efforts>

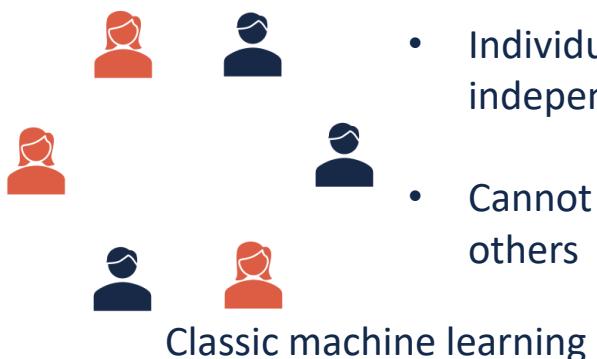
# Challenge

- Assumption

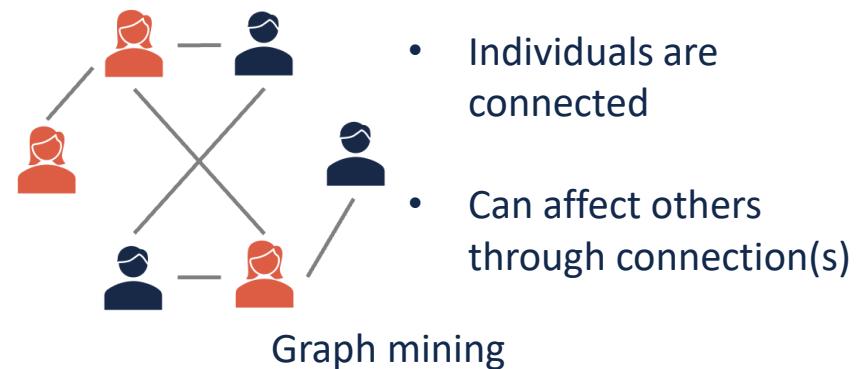
	Classic machine learning	Graph mining
Data	IID samples	Non-IID graph

- IID: independent and identically distributed

- Example



- Individuals are independent
- Cannot affect others



- Individuals are connected
- Can affect others through connection(s)

- Challenges: implication of non-IID nature on

- Measuring bias
  - Dyadic fairness, degree-related fairness
- Mitigating unfairness
  - Enforce fairness by graph structure imputation

# Roadmap

- Network Centrality Fairness
- Spectral Clustering Fairness
- Fair Graph Embeddings
- Graph Neural Network Fairness

# Roadmap

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# The Pagerank Algorithm

- The best-known algorithm for measuring the centrality/importance of nodes in a graph, introduced by Google
- **Assumption:** important webpage → linked by many others
- Pagerank performs a **random walk with restarts**:
- At each step of the random walk:
  - With probability  $c$  perform a transition according to the **transition probability matrix  $A$**
  - With probability  $1 - c$  restart to a randomly selected node according to **teleportation (jump) vector  $e$**
- The Pagerank vector is the **stationary distribution  $r$**  of this random walk

# Preliminary: PageRank

- **Formulation**

- Iterative method for the following linear system

$$\mathbf{r} = c\mathbf{A}^T \mathbf{r} + (1 - c)\mathbf{e}$$

- $\mathbf{A}$ : transition matrix
    - $\mathbf{r}$ : PageRank vector
    - $c$ : damping factor
    - $\mathbf{e}$ : teleportation vector

- Closed-form solution

$$\mathbf{r} = (1 - c)(\mathbf{I} - c\mathbf{A}^T)^{-1}\mathbf{e}$$

- **Variants**

- Personalized PageRank (PPR)

[1] Page, L., Brin, S., Motwani, R., & Winograd, T. (1999). The PageRank Citation Ranking: Bringing Order to the Web. Stanford InfoLab 1999.

[2] Haveliwala, T. H. (2003). Topic-sensitive PageRank: A Context-Sensitive Ranking Algorithm for Web Search. TKDE 2003.

[3] Tong, H., Faloutsos, C., & Pan, J. Y. (2006). Fast Random Walk with Restart and Its Applications. ICDM 2006.

# Unfairness in PageRank

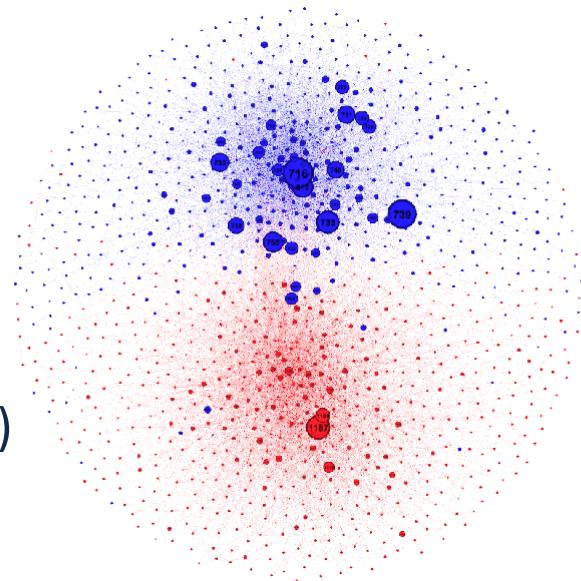
- Pagerank distributes the importance values to the nodes in the network
  - But is it **fair**?
- **Example**
  - **Network:** 1222 nodes of political blogs
  - **Groups:** red (left-leaning) and blue (right-leaning)



## Unfair ranking

Similar number of red nodes vs. blue nodes (**48% red** vs. **52% blue**)

Much less PageRank mass of red nodes (**33% red** vs. **67% blue**)



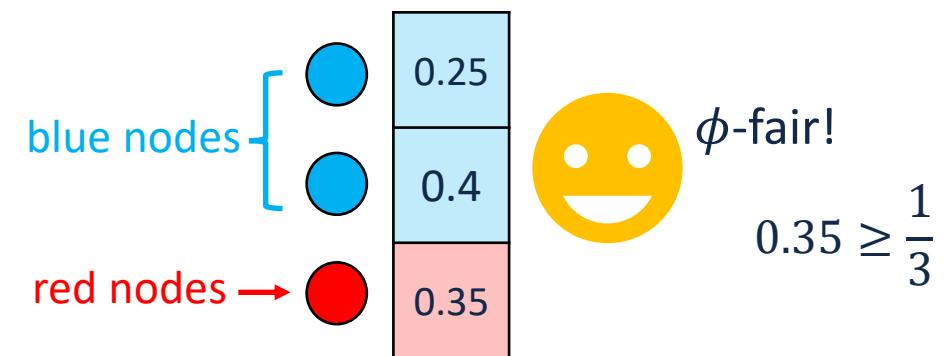
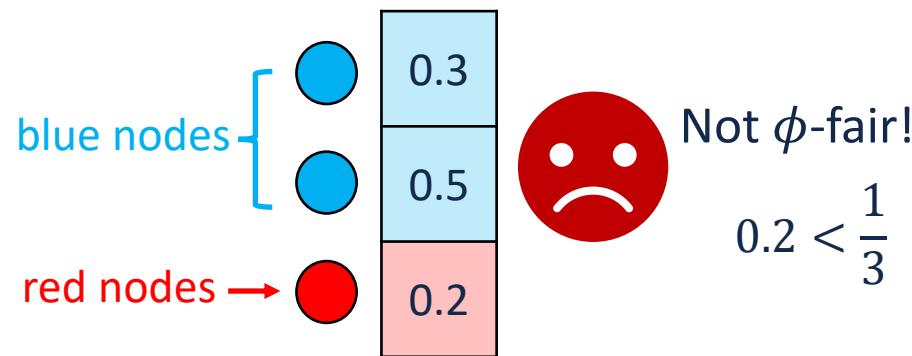
- How can we define Pagerank fairness?
- How do we make Pagerank fair?

[1] Tsoutsouliklis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

[2] Tsoutsouliklis, S., Pitoura, E., Semertzidis, K., & Tsaparas, P. (2022). Link Recommendations for PageRank Fairness. WWW 2022.

# Fairness Measure: $\phi$ -Fairness

- **Given:** (1) a graph  $G$ ; (2) a parameter  $\phi$
- **Definition:** a PageRank vector is  $\phi$ -fair if at least  $\phi$  fraction of total PageRank mass is allocated to the protected group
- **Variants and generalizations**
  - Statistical parity  $\rightarrow \phi = \text{fraction of protected group}$
  - Affirmative action  $\rightarrow \phi = \text{a desired ratio (e.g., 20%)}$
- **Example**
  - Protected group = red nodes
  - $\phi = 1/3$



[1] Tsoutsouliklis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

[2] Tsoutsouliklis, S., Pitoura, E., Semertzidis, K., & Tsaparas, P. (2022). Link Recommendations for PageRank Fairness. WWW 2022.

# Problem Definition: Fair PageRank

- **Given**
  - A graph with transition matrix  $A$
  - Partitions of nodes
    - Red nodes ( $\mathcal{R}$ ): protected group
    - Blue nodes ( $\mathcal{B}$ ): unprotected group
- **Produce:** a fair PageRank vector  $\tilde{\mathbf{r}}$  that is
  - $\phi$ -fair
  - Close to the original PageRank vector  $\mathbf{r}$  (minimizes the utility loss)

[1] Tsoukatos, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

[2] Tsoukatos, S., Pitoura, E., Semertzidis, K., & Tsaparas, P. (2022). Link Recommendations for PageRank Fairness. WWW 2022.

# Fair PageRank: Solutions

- **Recap:** closed-form solution for PageRank

$$\mathbf{r} = (1 - c)(\mathbf{I} - c\mathbf{A}^T)^{-1}\mathbf{e}$$

- **Parameters in PageRank**

- **Damping factor  $c$**  avoids sinks in the random walk (i.e., nodes without outgoing links)
- **Teleportation vector  $e$**  controls the starting node where a random walker restarts
  - Can we control where the walker teleports to? ← Solution #1: fairness-sensitive PageRank
- **Transition matrix  $A$**  controls the next step where the walker goes to
  - Can we modify the transition probabilities?
  - Can we modify the graph structure?

[1] Tsoutsouliklis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

[2] Tsoutsouliklis, S., Pitoura, E., Semertzidis, K., & Tsaparas, P. (2022). Link Recommendations for PageRank Fairness. WWW 2022. <sup>14</sup>

# Solution #1: Fairness-sensitive PageRank

- **Intuition**

- Find a teleportation vector  $\mathbf{e}$  to make PageRank vector  $\phi$ -fair

$$\mathbf{r} = \mathbf{Q}^T \mathbf{e}, \quad \mathbf{Q}^T = (1 - c)(\mathbf{I} - c\mathbf{A}^T)^{-1}$$

- Keep transition matrix  $\mathbf{A}$  and  $\mathbf{Q}^T$  fixed

- **Observation:** mass of PageRank  $\mathbf{r}$  w.r.t. red nodes  $\mathcal{R}$

$$\mathbf{r}(\mathcal{R}) = \mathbf{Q}^T[\mathcal{R}, :] \mathbf{e}$$

- $\mathbf{Q}^T[\mathcal{R}, :]$ : rows of  $\mathbf{Q}^T$  w.r.t. nodes in set  $\mathcal{R}$

- **(Convex) optimization problem**

$$\begin{array}{ll}\min_{\mathbf{e}} & \|\mathbf{Q}^T \mathbf{e} - \mathbf{r}\|^2 \\ \text{s.t.} & \mathbf{e}[i] \in [0, 1], \forall i \\ & \|\mathbf{e}\|_1 = 1 \\ & \|\mathbf{Q}^T[\mathcal{R}, :] \mathbf{e}\|_1 = \phi\end{array}$$

The fair PageRank  $\mathbf{Q}^T \mathbf{e}$  is as close as possible to the original PageRank  $\mathbf{r}$

The teleportation vector  $\mathbf{e}$  is a probability distribution

The fair PageRank  $\mathbf{Q}^T \mathbf{e}$  is  $\phi$ -fair

- Can be solved by any convex optimization solvers

# Fairness-sensitive PageRank: Example

- Settings:  $\phi = 1/3$  and protected node = red node
- Original PageRank

$$\begin{array}{c} \text{rows w.r.t. blue nodes} \\ \left[ \begin{array}{c} \text{blue circle} \\ \text{blue circle} \end{array} \right] \end{array} \quad Q^T = \begin{array}{|c|c|c|} \hline 0.55 & 0.3 & 0.2 \\ \hline 0.4 & 0.6 & 0.2 \\ \hline 0.05 & 0.1 & 0.6 \\ \hline \end{array} \quad e = \begin{array}{|c|} \hline 1/3 \\ \hline 1/3 \\ \hline 1/3 \\ \hline \end{array} \quad r = Q^T e = \begin{array}{|c|} \hline 0.35 \\ \hline 0.4 \\ \hline 0.25 \\ \hline \end{array}$$

Not  $\phi$ -fair!  
 $0.25 < \frac{1}{3}$



- Fairness-sensitive PageRank

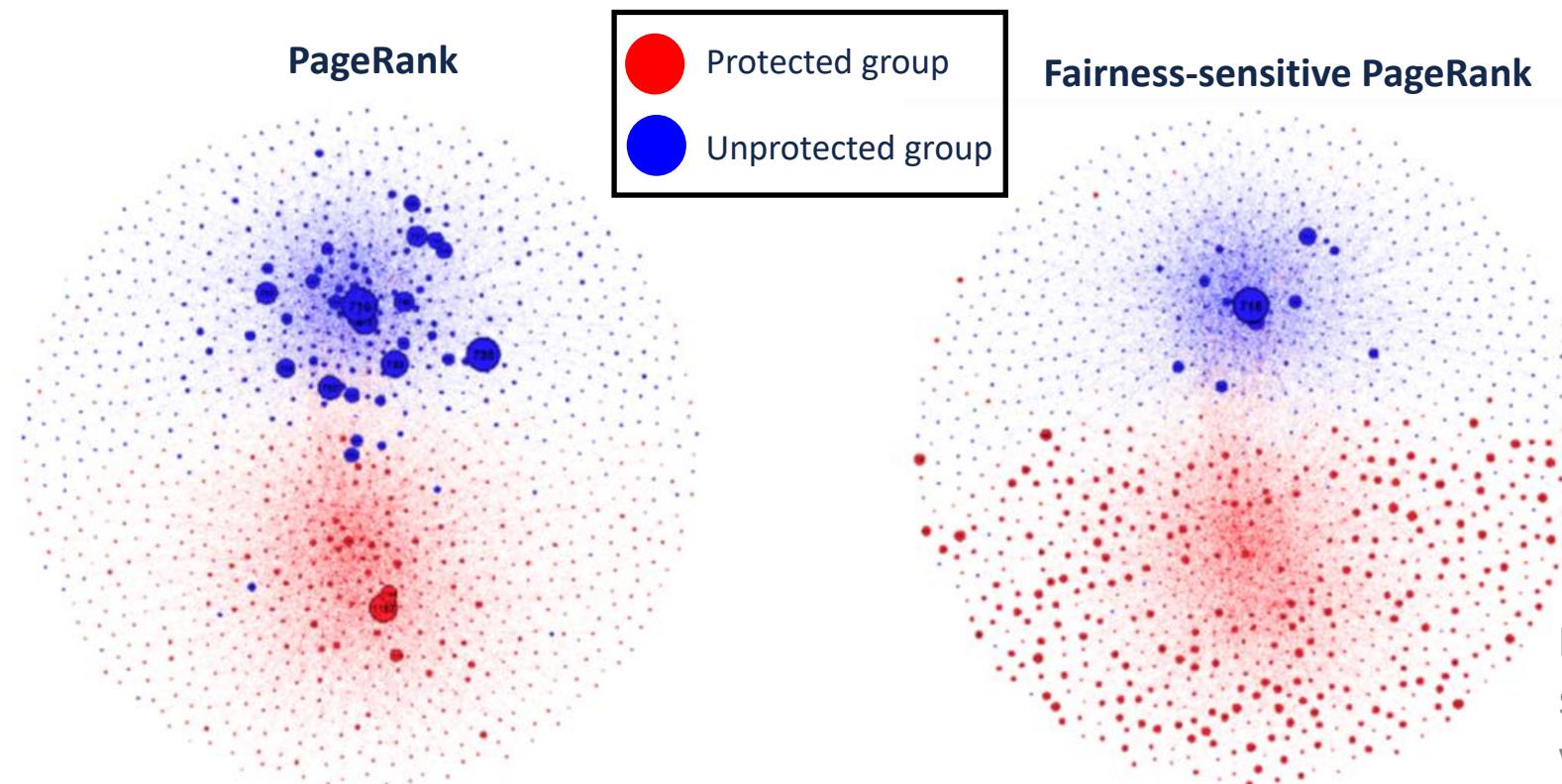
$$\begin{array}{c} \text{rows w.r.t. blue nodes} \\ \left[ \begin{array}{c} \text{blue circle} \\ \text{blue circle} \end{array} \right] \end{array} \quad Q^T = \begin{array}{|c|c|c|} \hline 0.55 & 0.3 & 0.2 \\ \hline 0.4 & 0.6 & 0.2 \\ \hline 0.05 & 0.1 & 0.6 \\ \hline \end{array} \quad \tilde{e} = \begin{array}{|c|} \hline 1/4 \\ \hline 1/4 \\ \hline 1/2 \\ \hline \end{array} \quad \tilde{r} = Q^T \tilde{e} = \begin{array}{|c|} \hline 0.31 \\ \hline 0.35 \\ \hline 0.34 \\ \hline \end{array}$$

$\phi$ -fair!  
 $0.34 \geq \frac{1}{3}$



# Fairness-sensitive PageRank: Experiment

- **Observation:** the teleportation vector allocates more weight to the red nodes, especially nodes at the periphery of the network
  - More likely to (1) restart at red nodes and (2) walk to other red nodes more often



**NOTE:** size is proportional to score in the teleportation vector

# Fair PageRank: Solutions

- **Recap:** closed-form solution for PageRank

$$\mathbf{r} = (1 - c)(\mathbf{I} - c\mathbf{A}^T)^{-1} \mathbf{e}$$

- **Parameters in PageRank**

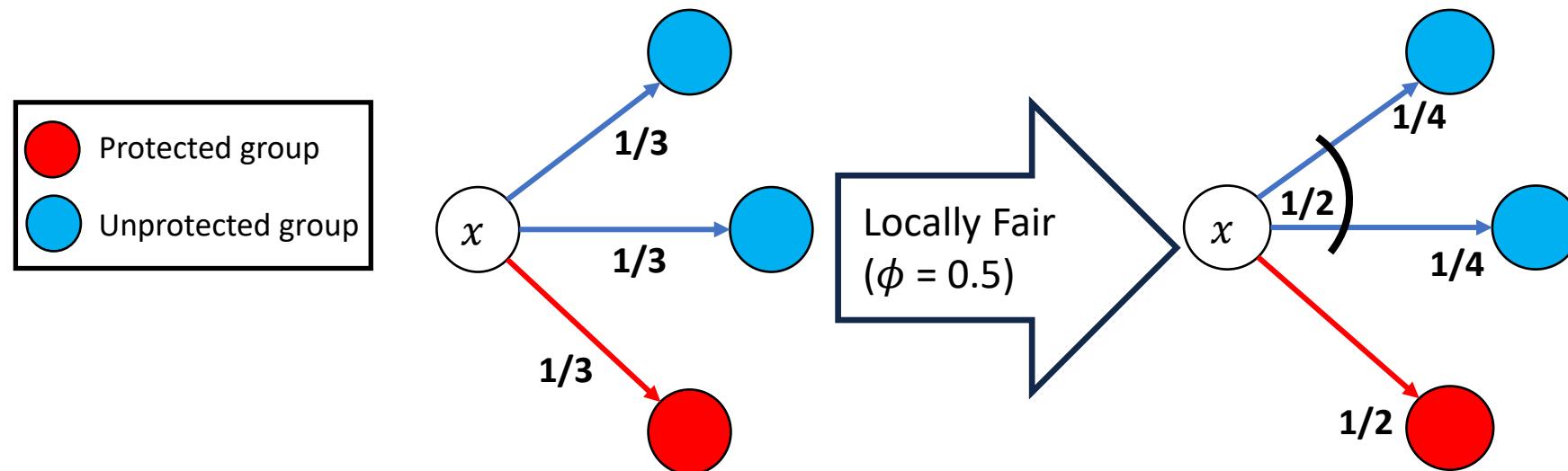
- **Damping factor  $c$**  avoids sinks in the random walk (i.e., nodes without outgoing links)
- **Teleportation vector  $e$**  controls the starting node where a random walker restarts
  - Can we control where the walker teleports to?
- **Transition matrix  $A$**  controls the next step where the walker goes to
  - Can we modify the transition probabilities? ← Solution #2: locally fair PageRank
  - Can we modify the graph structure?

[1] Tsoutsouliklis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

[2] Tsoutsouliklis, S., Pitoura, E., Semertzidis, K., & Tsaparas, P. (2022). Link Recommendations for PageRank Fairness. WWW 2022.

# Solution #2: Locally Fair PageRank

- **Intuition:** adjust the transition matrix A to obtain a fair random walk
- **Neighborhood locally fair PageRank**
  - **Key idea:** jump with probability  $\phi$  to red nodes and  $(1- \phi)$  to blue nodes
  - **Example**



[1] Tsoutsoulikis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

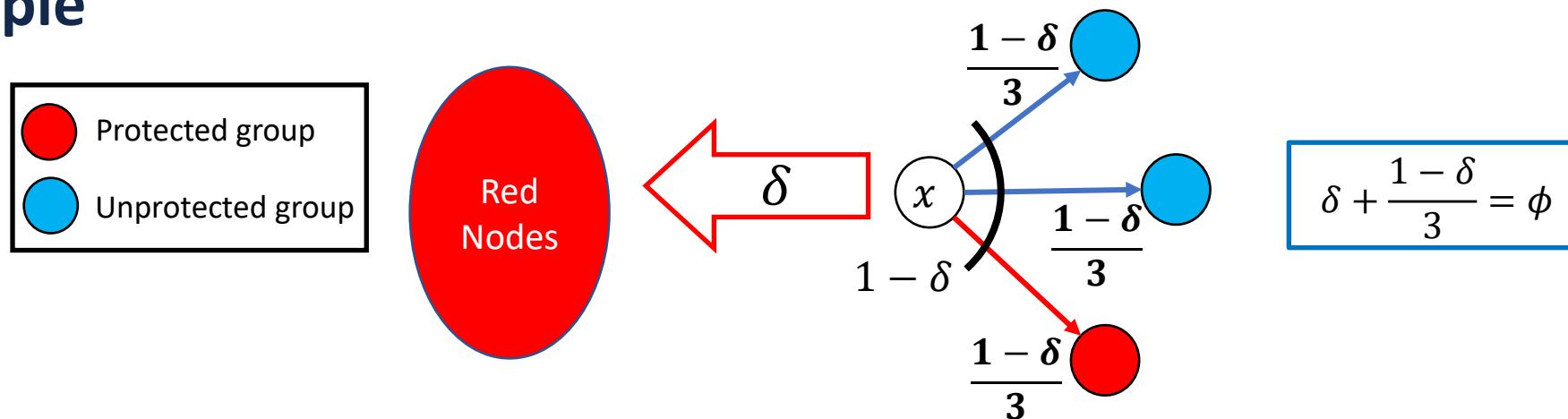
# Solution #2: Locally Fair PageRank

- **Residual locally fair PageRank**

- **Key idea:** jump with

- Equal probability to 1-hop neighbors
    - A residual probability  $\delta$  to the other red nodes

- **Example**

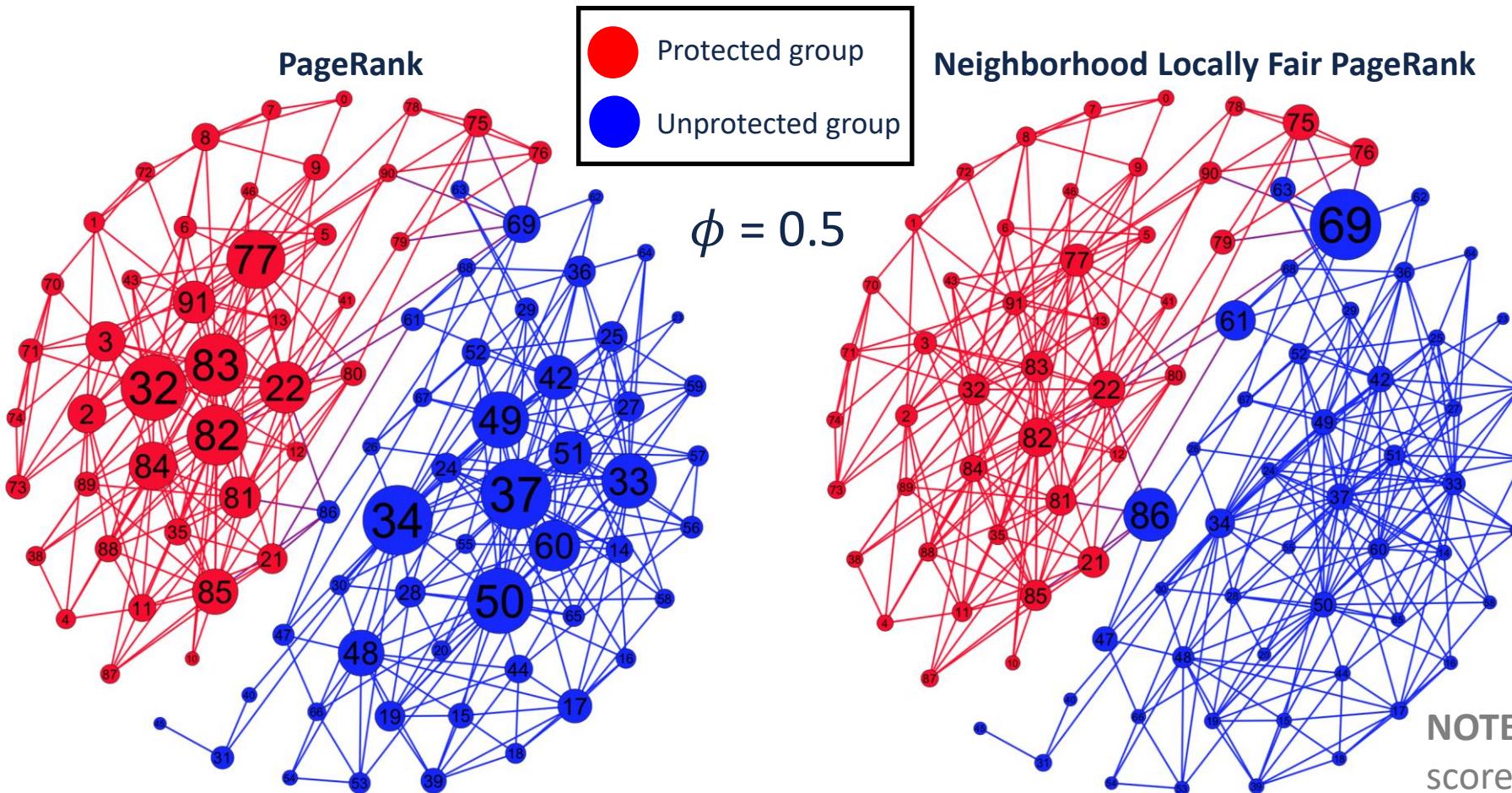


- **Residual allocation policies:** neighborhood allocation, uniform allocation, proportional allocation, optimized allocation

- **Neighborhood allocation:** allocate the residual to protected neighbors, equivalent to neighborhood locally fair PageRank
- **Uniform allocation:** uniformly allocate the residual to all protected nodes
- **Proportional allocation:** allocated the residual to all protected nodes proportionally to their PageRank score
- **Optimized allocation:** allocate the residual to all protected nodes while minimizing the difference with original PageRank score

# Locally Fair PageRank: Experiment

- **Observation:** PageRank weight is shifted to the blue nodes at boundary



# Fair PageRank: Solutions

- **Recap:** closed-form solution for PageRank

$$\mathbf{r} = (1 - c)(\mathbf{I} - c\mathbf{A}^T)^{-1}\mathbf{e}$$

- **Parameters in PageRank**

- **Damping factor  $c$**  avoids sinks in the random walk (i.e., nodes without outgoing links)
- **Teleportation vector  $e$**  controls the starting node where a random walker restarts
  - Can we control where the walker teleports to?
- **Transition matrix  $A$**  controls the next step where the walker goes to
  - Can we modify the transition probabilities?
  - Can we modify the graph structure? ← Solution #3: best fair edge identification

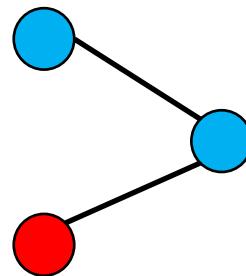
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# Solution #3: Best Fair Edge Identification

- **Intuition:** add edges that can improve the PageRank fairness to the graph
- **Example**

-  = protected node
- $\phi = 1/3$

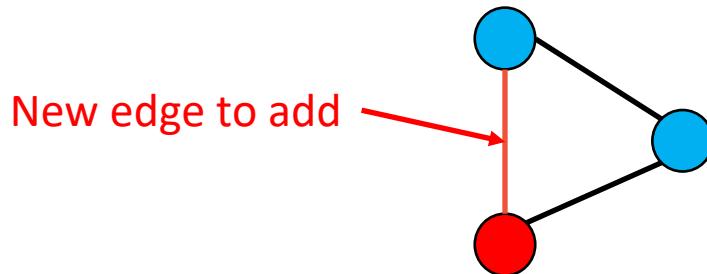


$$\mathbf{r} = \mathbf{Q}^T \mathbf{e} = \begin{matrix} 0.257 \\ 0.486 \\ 0.257 \end{matrix}$$



Not  $\phi$ -fair!

$$\frac{0.257}{0.257 + 0.486 + 0.257} < \frac{1}{3}$$



$$\tilde{\mathbf{r}} = \tilde{\mathbf{Q}}^T \mathbf{e} = \begin{matrix} 0.333 \\ 0.333 \\ 0.333 \end{matrix}$$



$\phi$ -fair!

$$\frac{0.333}{0.333 + 0.333 + 0.333} = \frac{1}{3}$$

- **Question:** how to find the edges with the highest improvement?

# Best Fair Edge Identification: Problem Definition

- **Given**

- $G = (\mathcal{V}, \mathcal{E})$
- $\mathcal{S} \subseteq \mathcal{V}$ : protected node set
- $r_{\mathcal{E}}(\mathcal{S}) = \sum_{i \in \mathcal{V}} r_{\mathcal{E}}(i)$ : total PageRank mass of nodes in  $\mathcal{S}$  on graph with edge set  $\mathcal{E}$

- **Fairness gain of edge addition**

$$\text{gain}(x, y) = r_{\mathcal{E} \cup (x, y)}(\mathcal{S}) - r_{\mathcal{E}}(\mathcal{S})$$

**Naive method**  
Exhaustively recompute  
PageRank with the  
addition of **each** node pair

- **Goal:** find the edge  $(x, y), \forall x, y \in \mathcal{V}$ , such that

$$\operatorname{argmax}_{(x,y)} \text{gain}(x, y)$$

- **Question:** how to **efficiently** compute the gain?

# Best Fair Edge Identification: Fairness Gain

- **Main result:** Adding an edge to the graph is a **rank-1 perturbation** of the transition matrix

- We can estimate the gain as:

$$\text{gain}(x, y) = r_{\mathcal{E}}(x) \frac{\frac{c}{1-c} \left( r_{\mathcal{E}}(\mathcal{S}|y) - \frac{1}{d_x} \sum_{u \in \mathcal{N}_x} r_{\mathcal{E}}(\mathcal{S}|u) \right)}{d_x + \frac{c}{1-c} \left( \frac{1}{d_x} \sum_{u \in \mathcal{N}_x} r_{\mathcal{E}}(x|u) - r_{\mathcal{E}}(x|y) \right) + 1}$$

degree of source node →

Closeness of target node  $y$  to  $\mathcal{S}$  →

The average ‘closeness’ of neighbors of  $x$  to  $\mathcal{S}$  →

Average proximity of node  $x$ ’s neighbors to  $x$  ←

- $r_{\mathcal{E}}(x|y)$ : personalized PageRank (PPR) score of node  $x$ , with query node  $y$ , based on edge set  $\mathcal{E}$
- $r_{\mathcal{E}}(\mathcal{S}|y) = \sum_{i \in \mathcal{S}} r_{\mathcal{E}}(i|y)$ : total PPR mass of nodes in  $\mathcal{S}$ , with query node  $y$ , based on edge set  $\mathcal{E}$

- $r_{\mathcal{E}}(x)$  : node  $x$  should have high PageRank score

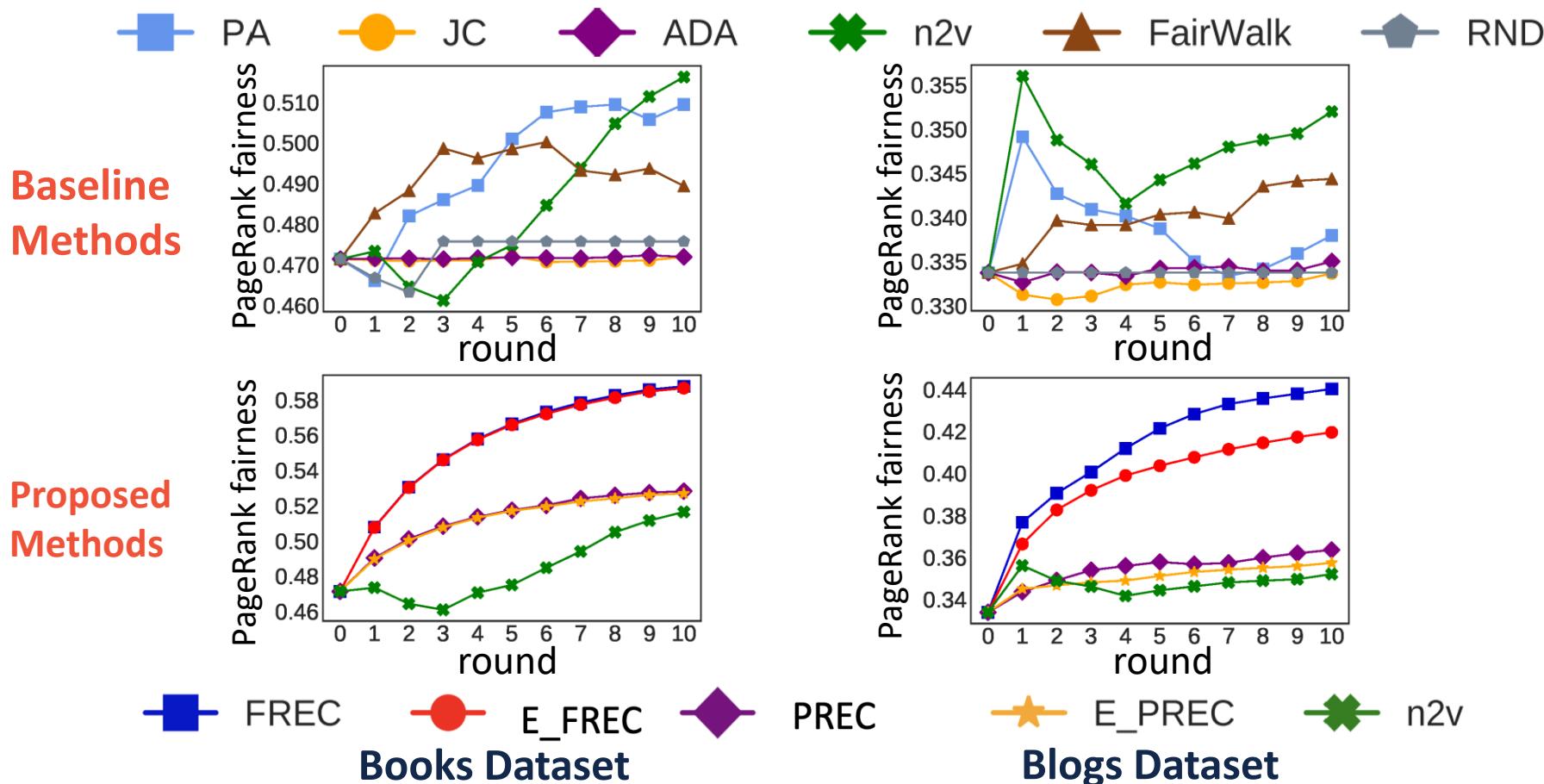
- $d_x$ : node  $x$  should have small degree

- $r_{\mathcal{E}}(x|y) - \frac{1}{d_x} \sum_{u \in \mathcal{N}_x} r_{\mathcal{E}}(x|u)$ : node  $y$  is close to node  $x$

- $r_{\mathcal{E}}(\mathcal{S}|y) - \frac{1}{d_x} \sum_{u \in \mathcal{N}_x} r_{\mathcal{E}}(\mathcal{S}|u)$ : node  $y$  is closer to  $\mathcal{S}$  than the neighborhood of  $x$

# Best Fair Edge Identification: Experiment

- **Observation:** the proposed method find the best edges to improve PageRank fairness



- FREC: select edge  $(x, y)$  with highest  $\text{gain}(x, y) = \Lambda(x, y)p_{\mathcal{E}}(x)$
- PREC: select edge  $(x, y)$  with highest  $\text{gain}(x, y | x) = \Lambda(x, y)p_{\mathcal{E}}(x|x)$

- E\_FREC: select edge  $(x, y)$  with highest  $\text{gain}(x, y)p_{\text{acc}}(x, y)$
- E\_PREC: select edge  $(x, y)$  with highest  $\text{gain}(x, y | x)p_{\text{acc}}(x, y)$

# Roadmap

- Network Centrality Fairness
- Spectral Clustering Fairness
- Fair Graph Embeddings
- Graph Neural Network Fairness

# Preliminary: Spectral Clustering (SC)

- **Goal:** partition the nodes into  $k$  subsets such that

maximize intra-connectivity  
minimize inter-connectivity

- **Ratio cut**

$$RatioCut(C_1, C_2, \dots, C_k) = \sum_{i=1}^k \frac{Cut(C_i, V \setminus C_i)}{|C_i|}$$

- **Spectral Clustering**

$$\min_{\mathbf{U}} \quad \boxed{\text{Tr}(\mathbf{U}^T \mathbf{L} \mathbf{U})} \quad \begin{matrix} \xrightarrow{\text{Ratio cut}} \\ \text{s.t.} \end{matrix} \quad \mathbf{U}^T \mathbf{U} = \mathbf{I}$$

where  $\mathbf{L}$  is Laplacian matrix of  $\mathbf{A}$ ,  $\mathbf{L} = \mathbf{D} - \mathbf{A}$

$\mathbf{U}$  is a matrix with  $k$  orthonormal column vectors

- **Solution:** rank- $k$  eigen-decomposition

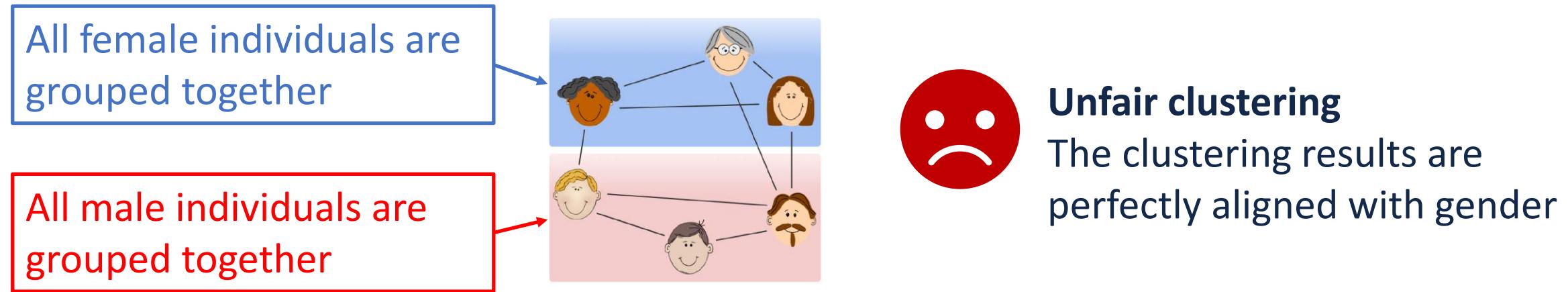
–  $\mathbf{U}$  = eigenvectors with  $k$  smallest eigenvalues

[1] Ng, A. Y., Jordan, M. I., & Weiss, Y. (2002). On Spectral Clustering: Analysis and an Algorithm. NeurIPS 2002.

[2] Shi, J., & Malik, J. (2000). Normalized Cuts and Image Segmentation. TPAMI 2000.

# Preliminary: Spectral Clustering (SC)

- **Example:** Nodes are partitioned into groups based on a sensitive attribute (gender)



- We would like groups to have proportional representation in the clusters

[1] Ng, A. Y., Jordan, M. I., & Weiss, Y. (2002). On Spectral Clustering: Analysis and an Algorithm. NeurIPS 2002.

[2] Shi, J., & Malik, J. (2000). Normalized Cuts and Image Segmentation. TPAMI 2000.

# Fairness Measure: Balance Score

- **Intuition:** fairness as balance among clusters

- **Given:** a node set  $V$  with

- $h$  demographic groups:  $V = V_1 \cup V_2 \dots \cup V_h$
- $k$  clusters:  $V = C_1 \cup C_2 \dots \cup C_k$

- **Definition**

$$\text{balance}(C_l) = \min_{s \neq s' \in [h]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|} \in [0, 1], \quad \forall l \in [1, 2, \dots, k]$$

$$\text{balance}(C_1, C_2, \dots, C_k) = \min_l \text{balance}(C_l),$$

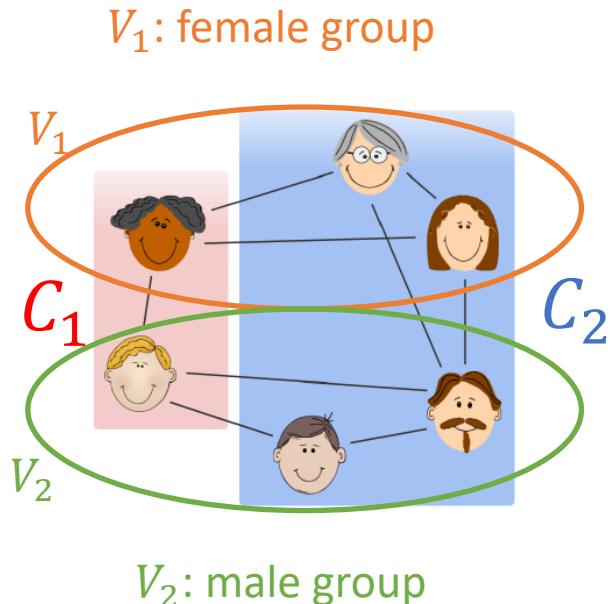
- **Intuition:** higher balance  $\rightarrow$  fairer

- Each demographic group is presented with similar fractions as in the whole dataset for every cluster

# Fairness Measure: Balance Score

- Example

$$\begin{aligned}\text{balance}(C_1) &= \min\left(\frac{|V_1 \cap C_1|}{|V_2 \cap C_1|}, \frac{|V_2 \cap C_1|}{|V_1 \cap C_1|}\right) \\ &= \min\left(\frac{\left|\begin{array}{|c|} \hline \text{black woman} \\ \hline \text{white woman} \\ \hline \end{array}\right|}{\left|\begin{array}{|c|} \hline \text{white woman} \\ \hline \text{black woman} \\ \hline \end{array}\right|}, \frac{\left|\begin{array}{|c|} \hline \text{white woman} \\ \hline \text{black woman} \\ \hline \end{array}\right|}{\left|\begin{array}{|c|} \hline \text{black woman} \\ \hline \text{white woman} \\ \hline \end{array}\right|}\right) \\ &= 1\end{aligned}$$



$$\begin{aligned}\text{balance}(C_2) &= \min\left(\frac{|V_1 \cap C_2|}{|V_2 \cap C_2|}, \frac{|V_2 \cap C_2|}{|V_1 \cap C_2|}\right) \\ &= \min\left(\frac{\left|\begin{array}{|c|} \hline \text{black woman} \\ \hline \text{white woman} \\ \hline \end{array}\right|}{\left|\begin{array}{|c|} \hline \text{white man} \\ \hline \text{black man} \\ \hline \end{array}\right|}, \frac{\left|\begin{array}{|c|} \hline \text{white man} \\ \hline \text{black man} \\ \hline \end{array}\right|}{\left|\begin{array}{|c|} \hline \text{black woman} \\ \hline \text{white woman} \\ \hline \end{array}\right|}\right) \\ &= 1\end{aligned}$$

# Fair Spectral Clustering: Formulation

- Key idea: fairness as linear constraint

- Given

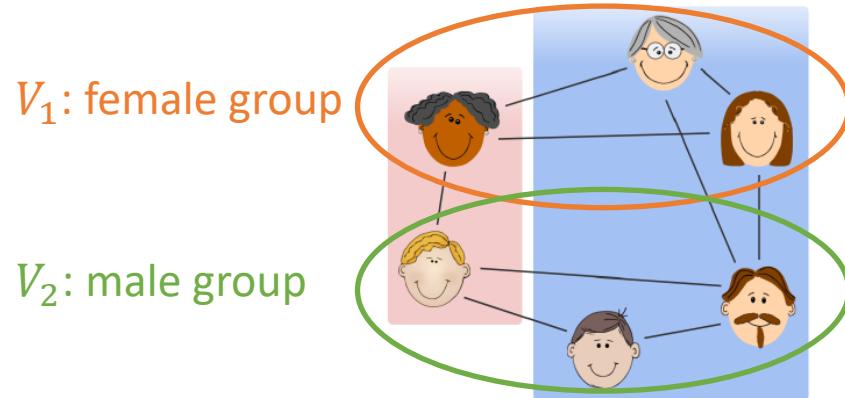
- The spectral embedding  $\mathbf{U}$  of  $n$  nodes in  $l$  clusters ( $C_1, \dots, C_l$ )
    - $h$  demographic groups ( $V_1, \dots, V_s$ )

- Define

- $\mathbf{f}^{(s)}[i] = 1$  if  $i \in V_s$  and 0 otherwise
    - $\mathbf{F}$  = a matrix with  $\mathbf{f}^{(s)} - \left(\frac{|V_s|}{n}\right) \mathbf{1}_n$  ( $s \in [1, \dots, h - 1]$ ) as column vectors

- Observation:  $\mathbf{F}^T \mathbf{U} = \mathbf{0} \Leftrightarrow$  balanced clusters (i.e., fair clusters)

- Example



	$\mathbf{f}^{(1)}$	$\mathbf{f}^{(2)}$	Fair fraction
1	1	0	0.5
2	1	0	0.5
3	1	0	0.5
4	0	1	0.5
5	0	1	0.5
6	0	1	0.5
7	0.5	0.5	0.5
8	0.5	0.5	0.5
9	0.5	0.5	0.5
10	-0.5	0.5	-0.5
11	-0.5	0.5	-0.5
12	-0.5	0.5	-0.5

$\mathbf{F} =$

# Fair Spectral Clustering: Solution

- Optimization problem

$$\min_{\mathbf{U}} \quad \text{Tr}(\mathbf{U}^T \mathbf{L} \mathbf{U}) \quad \text{s. t.} \quad \mathbf{U}^T \mathbf{U} = \mathbf{I}, \boxed{\mathbf{F}^T \mathbf{U} = \mathbf{0}}$$

- Solution

– Observation:  $\mathbf{F}^T \mathbf{U} = \mathbf{0} \rightarrow \mathbf{U}$  is in the null space of  $\mathbf{F}^T$

– Steps

- Define  $\mathbf{Z}$  = orthonormal basis of null space of  $\mathbf{F}^T$
- Rewrite  $\mathbf{U} = \mathbf{ZY}$

$$\min_{\mathbf{Y}} \quad \text{Tr}(\mathbf{Y}^T \mathbf{Z}^T \mathbf{L} \mathbf{Z} \mathbf{Y}) \quad \text{s. t.} \quad \mathbf{Y}^T \mathbf{Y} = \mathbf{I}$$

– Method: rank- $k$  eigen-decomposition on  $\mathbf{Z}^T \mathbf{L} \mathbf{Z}$

How to solve?

# Fair Spectral Clustering: Correctness

- **Given**

- A random graph with nodes  $V$  by a variant of the Stochastic Block Model (SBM)
- Edge probability between two nodes  $i$  and  $j$

$$P(i,j) = \begin{cases} a, & i \text{ and } j \text{ in same cluster and in same group} \\ b, & i \text{ and } j \text{ not in same cluster but in same group} \\ c, & i \text{ and } j \text{ in same cluster but not in same group} \\ d, & i \text{ and } j \text{ not in same cluster and not in same group} \end{cases}$$

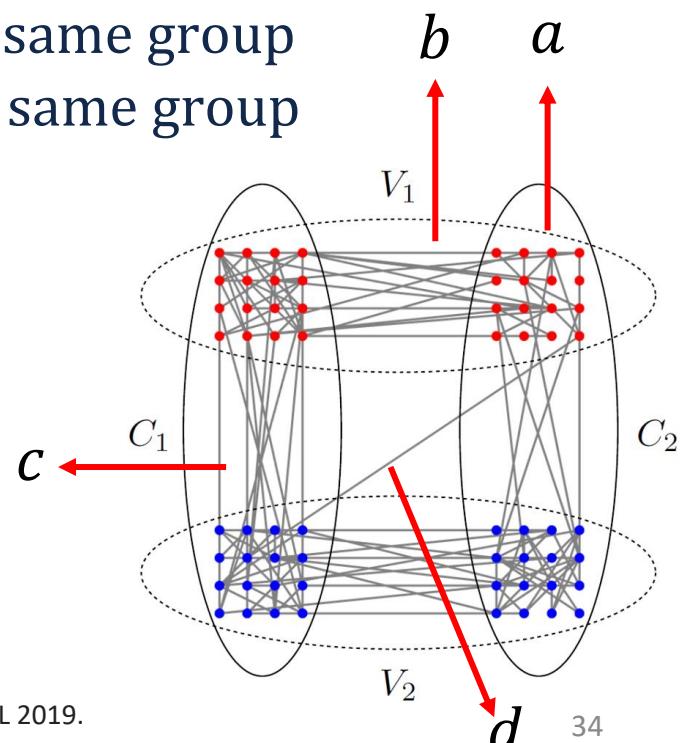
for some  $a > b > c > d$

- A fair ground-truth clustering  $V = C_1 \cup C_2$

- **Theorem:** Fair SC recovers the ground-truth clustering  $C_1 \cup C_2$

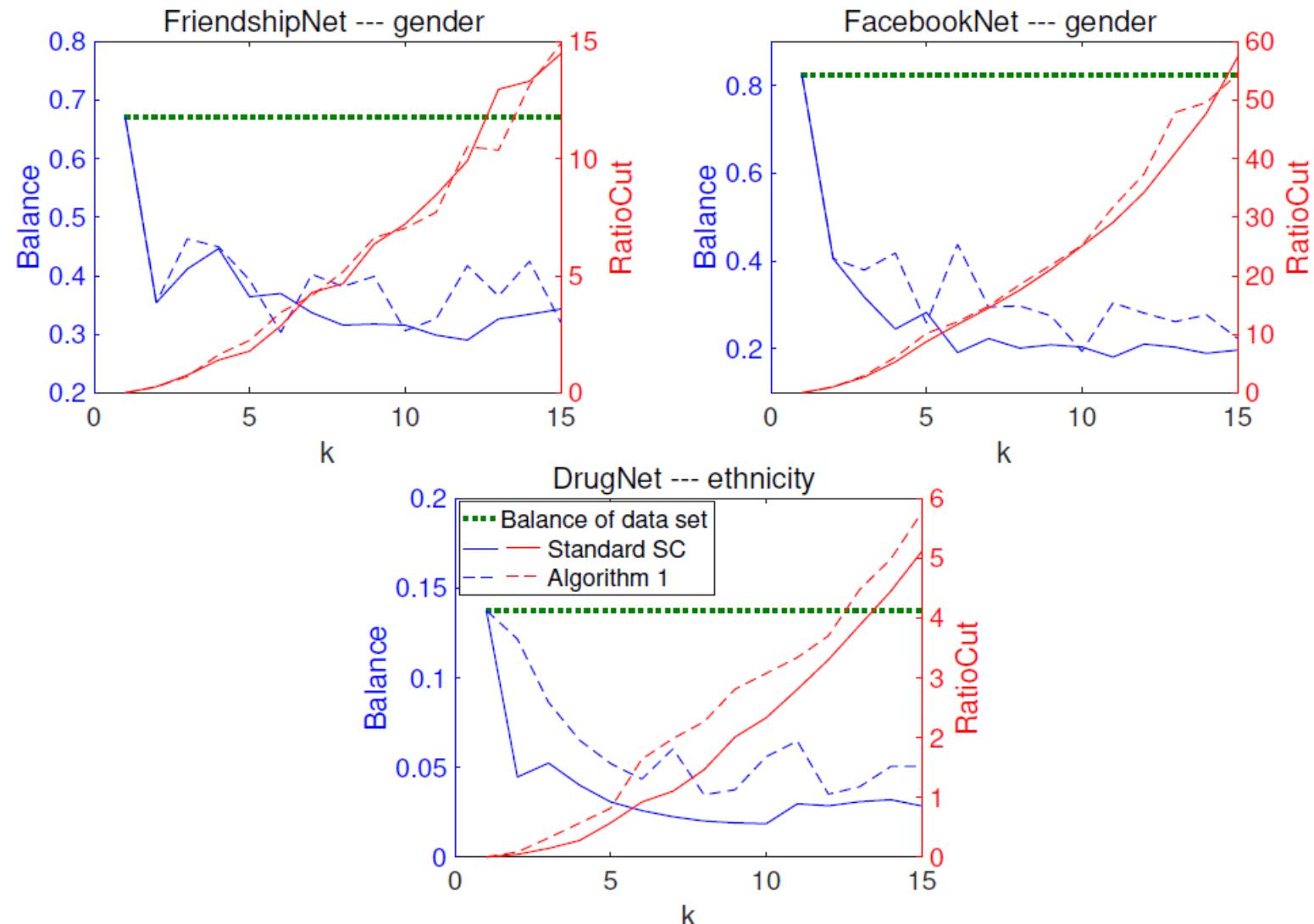
- **Example**

- Standard SC is likely to return  $V_1 \cup V_2$
- Fair SC will return  $C_1 \cup C_2$



# Fair Spectral Clustering: Experiment

- **Observation:** Fairer (higher balance score) with similar ratio cut values for the proposed method (Algorithm 1 in the figure)

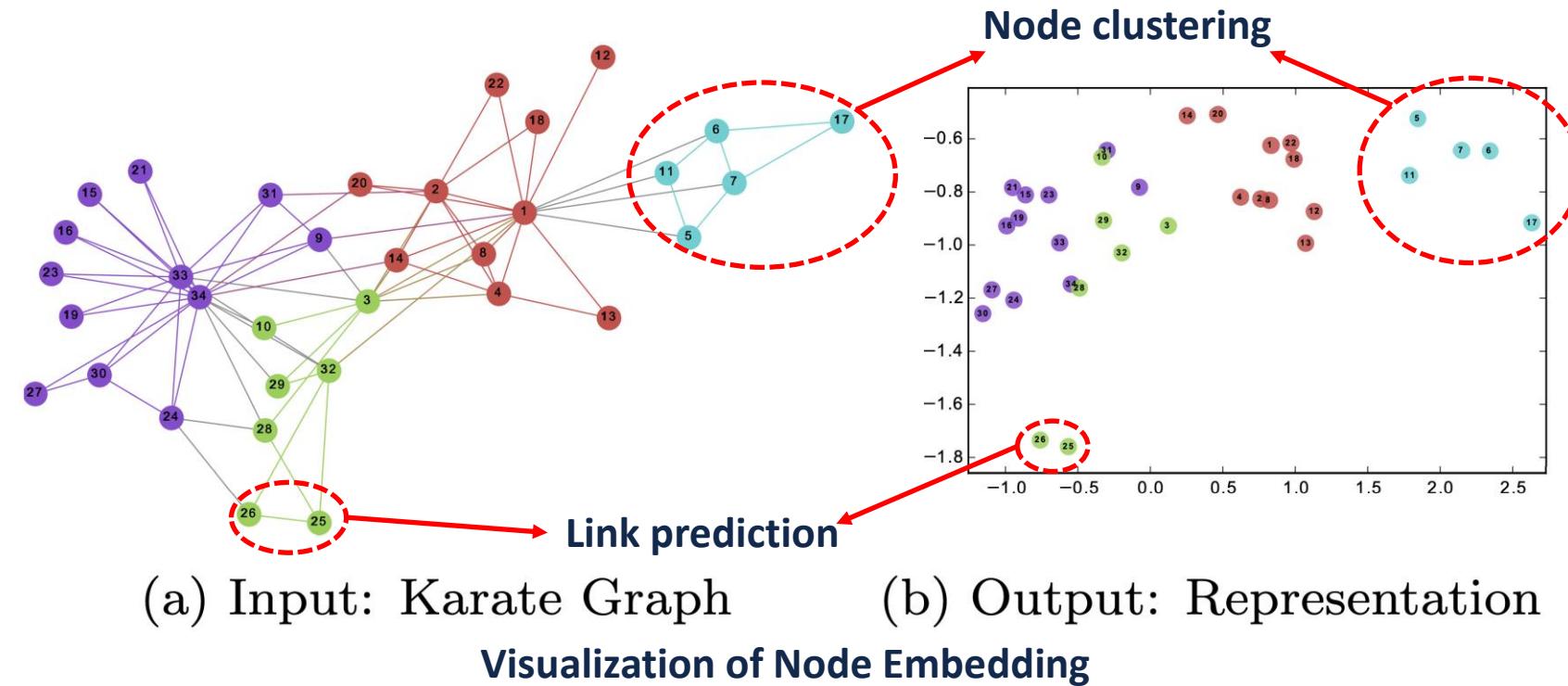


# Roadmap

- Network Centrality Fairness
- Spectral Clustering Fairness
- Fair Graph Embeddings
- Graph Neural Network Fairness

# Preliminary: Node Embedding

- **Motivation:** learn low-dimensional node representations that preserve structural/attributive information
- **Applications**
  - Node classification
  - Link prediction
  - Node visualization



[1] Perozzi, B., Al-Rfou, R., & Skiena, S. (2014). Deepwalk: Online Learning of Social Representations. KDD 2014.

[2] Grover, A., & Leskovec, J. (2016). node2vec: Scalable Feature Learning for Networks. KDD 2016.

[3] Bordes, A., Usunier, N., Garcia-Duran, A., Weston, J., & Yakhnenko, O. (2013). Translating Embeddings for Modeling Multi-relational Data. NeurIPS 2013.

# Graph embeddings

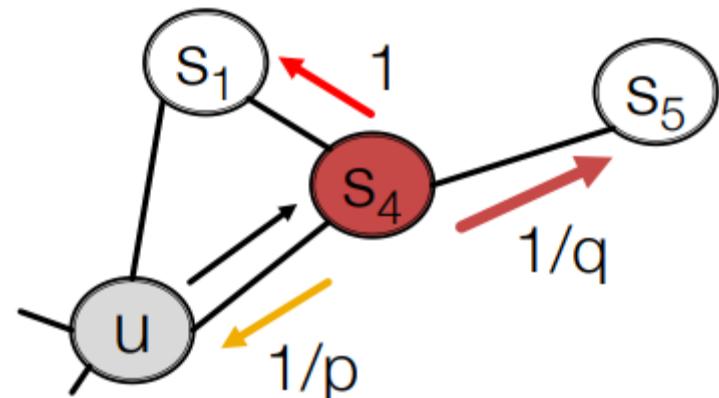
- Graph embeddings utilize only the graph structure to derive the node representation
- In broad terms, the embedding of a node depends on the embeddings of the k-hop neighborhood of the node
- Since neighboring nodes tend to have similar (sensitive) attributes, the embeddings are likely to encode information about the sensitive attributes
  - Therefore, they are **biased**
- How can we remove these biases?

# Graph unfairness

- Homophily-based metrics:
  - E.g., the fraction of edges that link nodes with the same sensitive attribute value
- Neighborhood metrics:
  - The entropy of the label distribution of the neighborhood of a node.
- Preprocessing approach:
  - Change the graph (e.g., via edge rewiring or edge additions) to improve fairness

# Preliminary: Random Walk-based Node Embedding

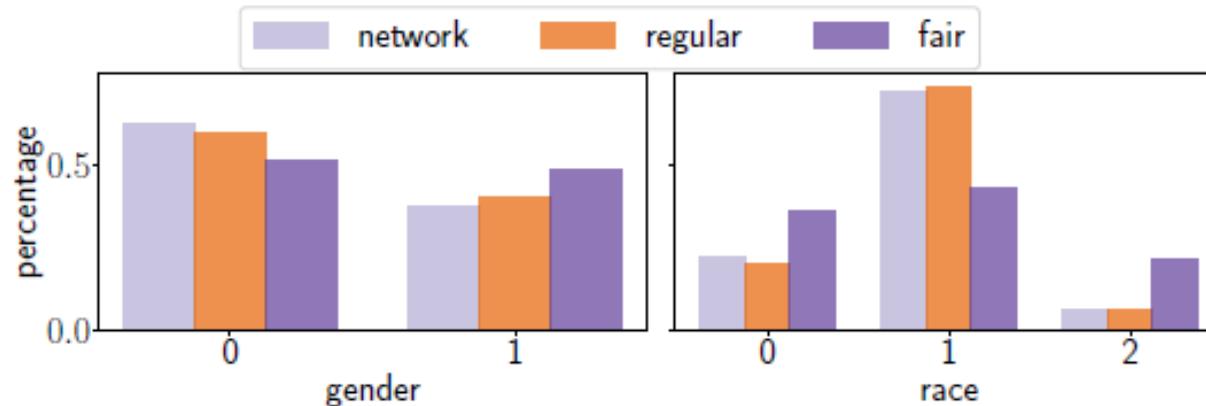
- **Goal:** learn node embeddings that are predictive of nodes in its neighborhood
- **Key idea**
  - Simulate random walk as a sequence of nodes
  - Apply skip-gram technique to predict the context node
- **Example**
  - **DeepWalk:** random walk for sequence generation
  - **Node2vec:** biased random walk for sequence generation
    - **Return parameter  $p$ :** how fast the walk **explores** the neighborhood of the starting node
    - **In-out parameter  $q$ :** how fast the walk **leaves** the neighborhood of the starting node



[1] Perozzi, B., Al-Rfou, R., & Skiena, S. (2014). Deepwalk: Online Learning of Social Representations. KDD 2014.  
[2] Grover, A., & Leskovec, J. (2016). node2vec: Scalable Feature Learning for Networks. KDD 2016.

# Fairwalk: Solution

- **Key idea:** modify the random walk procedure in node2vec
- **Steps of Fairwalk**
  - Partition neighbors into demographic groups
  - Uniformly sample a demographic group to walk to
  - Randomly select a neighboring node within the chosen demographic group
- **Example:** ratio of each demographic group
  - Original network vs. regular random walk vs. fair random walk



# Fairwalk vs. Existing Works

- **Fairwalk vs. node2vec**
  - **Node2vec:** skip-gram model + walk sequences by **original random walk**
  - **Fairwalk:** skip-gram model + walk sequences by **fair random walk**
- **Fairwalk vs. fairness-aware PageRank**
  - **Fairness-aware PageRank:** the minority group should have **a certain proportion** of PageRank probability mass
  - **Fairwalk:** all demographic group have **the same** random walk transition probability mass

[1] Rahman, T., Surma, B., Backes, M., & Zhang, Y. (2019). Fairwalk: Towards Fair Graph Embedding. IJCAI 2019.

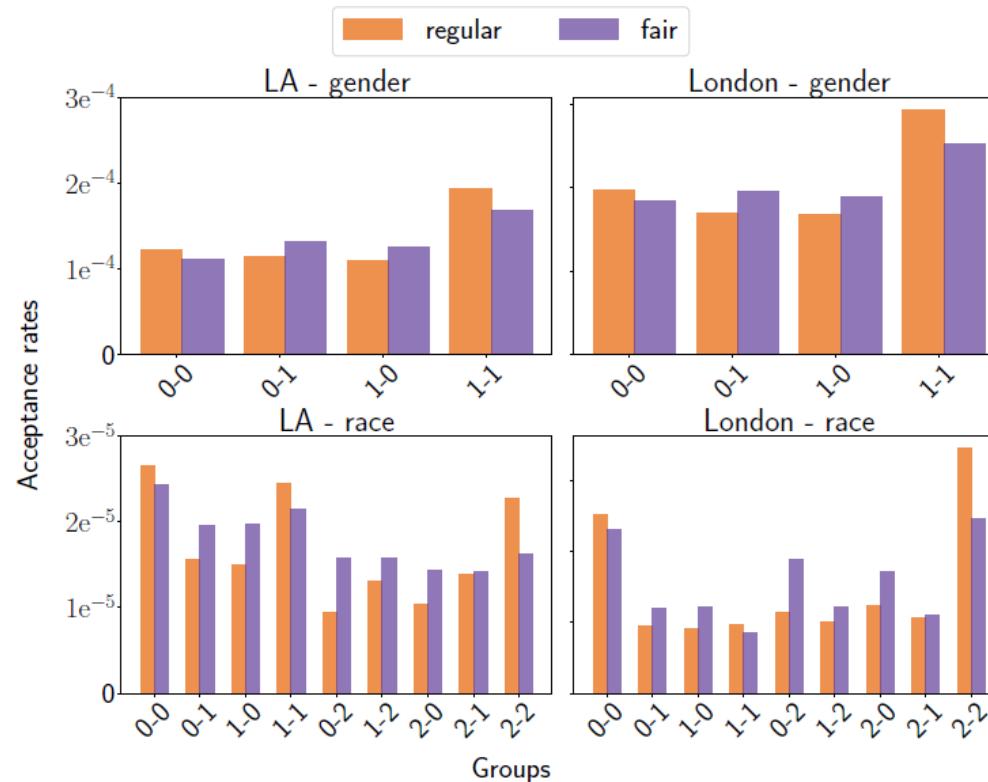
[2] Grover, A., & Leskovec, J. (2016). node2vec: Scalable Feature Learning for Networks. KDD 2016.

[3] Tsoutsouliklis, S., Pitoura, E., Tsaparas, P., Kleftakis, I., & Mamoulis, N. (2021). Fairness-Aware PageRank. WWW 2021.

# Fairwalk: Results on Statistical Parity

- **Observations**

- Fairwalk achieves a more balanced acceptance rates among groups
- Fairwalk increases the fraction of cross-group recommendations



# Roadmap

- Network Centrality Fairness
- Spectral Clustering Fairness
- Fair Graph Embeddings
- Graph Neural Network Fairness

# FairGNN: Fairness with Limited Sensitive Attribute Information

- **Key idea**

- Train a sensitive attribute estimator to infer pseudo sensitive attribute
- Train adversary to learn fair embedding using the pseudo sensitive attribute

- **FairGNN framework**

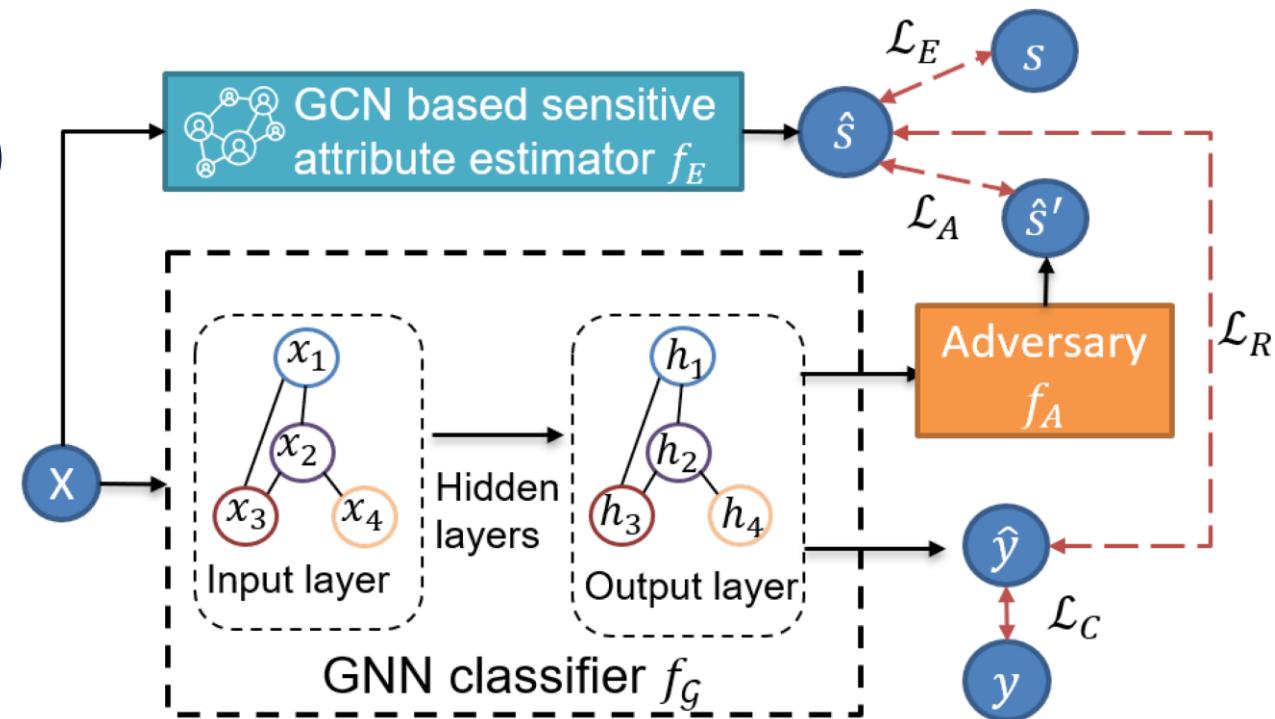
- A backbone graph neural network (GNN)
  - Any GNN can be the backbone

- Adversarial debiasing module

- GCN-based sensitive attribute estimator
- Adversary in the figure

- Covariance minimizer

Main focus



# FairGNN: Adversarial Debiasing Module

- **Adversary**
  - **Intuition:** maximize the error of predicting pseudo sensitive attribute information
  - **Loss function**
$$\mathcal{L}_A = \mathbb{E}_{\mathbf{h} \sim p(\mathbf{h}|\tilde{s}=1)} [\log f_A(\mathbf{h})] + \mathbb{E}_{\mathbf{h} \sim p(\mathbf{h}|\tilde{s}=0)} [\log(1 - f_A(\mathbf{h}))]$$
    - $\tilde{s}$ : pseudo sensitive attribute information
    - $\mathbf{h}$ : node embedding extracted from a graph neural network
    - $\mathbf{h} \sim p(\mathbf{h}|\tilde{s} = 1)$ : randomly sample a node embedding whose corresponding node has  $\tilde{s} = 1$
    - $f_A(\mathbf{h})$ : output of the adversary

# FairGNN: Covariance Minimizer

- **Observation:** adversarial learning is notoriously unstable to train
  - Failure to converge may cause discrimination
- **Key idea:** additional prerequisite of independence is needed to provide additional supervision signal
- **Solution:** absolute covariance between model prediction  $\hat{y}$  and pseudo sensitive attribute  $\hat{s}$  should be minimized
  - **Why absolute:** covariance can be negative

$$\mathcal{L}_R = |\text{cov}(\hat{s}, \hat{y})| = |\mathbb{E}[(\hat{s} - \mathbb{E}[\hat{s}])(\hat{y} - \mathbb{E}[\hat{y}])]|$$

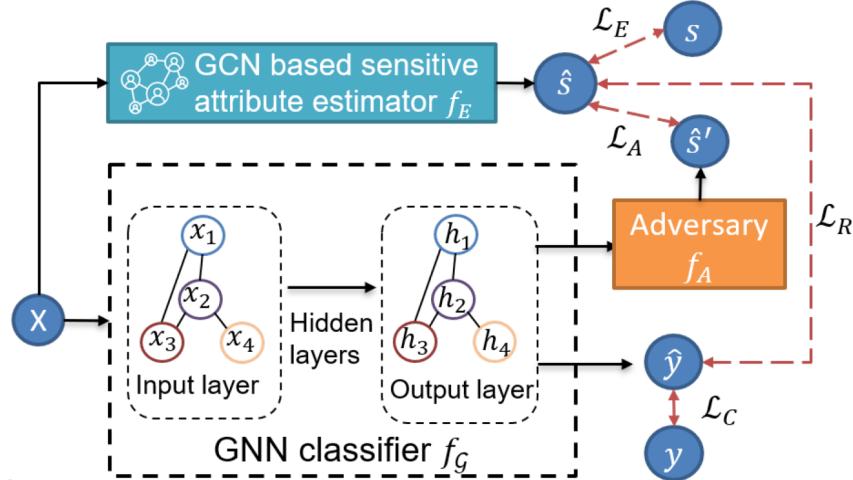
# FairGNN: Overall Loss Function

- Regularized learning

$$\min_{\theta_G, \theta_E} \max_{\theta_A} \mathcal{L}_C + \mathcal{L}_E + \alpha \mathcal{L}_R - \beta \mathcal{L}_A,$$

- Intuition

- $\mathcal{L}_C$ : classification loss (e.g., cross entropy) for learning representative node representation
- $\mathcal{L}_E$ : sensitive attribute estimation loss for generating accurate pseudo sensitive attribute information
- $\mathcal{L}_A$ : adversarial loss for debiasing the learned node representation
- $\mathcal{L}_R$ : covariance for stabilizing the training of adversary



# FairGNN: Experiment

- Observation:** FairGNN achieves comparable node classification accuracy with a much smaller bias

Dataset	Metrics	GCN	GAT	ALFR	ALFR-e	Debias	Debias-e	FCGE	FairGCN	FairGAT
Pokec-z	ACC (%)	70.2 $\pm$ 0.1	70.4 $\pm$ 0.1	65.4 $\pm$ 0.3	68.0 $\pm$ 0.6	65.2 $\pm$ 0.7	67.5 $\pm$ 0.7	65.9 $\pm$ 0.2	70.0 $\pm$ 0.3	70.1 $\pm$ 0.1
	AUC (%)	77.2 $\pm$ 0.1	76.7 $\pm$ 0.1	71.3 $\pm$ 0.3	74.0 $\pm$ 0.7	71.4 $\pm$ 0.6	74.2 $\pm$ 0.7	71.0 $\pm$ 0.2	76.7 $\pm$ 0.2	76.5 $\pm$ 0.2
	$\Delta_{SP}$ (%)	9.9 $\pm$ 1.1	9.1 $\pm$ 0.9	2.8 $\pm$ 0.5	5.8 $\pm$ 0.4	1.9 $\pm$ 0.6	4.7 $\pm$ 1.0	3.1 $\pm$ 0.5	0.9 $\pm$ 0.5	0.5 $\pm$ 0.3
	$\Delta_{EO}$ (%)	9.1 $\pm$ 0.6	8.4 $\pm$ 0.6	1.1 $\pm$ 0.4	2.8 $\pm$ 0.8	1.9 $\pm$ 0.4	3.0 $\pm$ 1.4	1.7 $\pm$ 0.6	1.7 $\pm$ 0.2	0.8 $\pm$ 0.3
Pokec-n	ACC (%)	70.5 $\pm$ 0.2	70.3 $\pm$ 0.1	63.1 $\pm$ 0.6	66.2 $\pm$ 0.5	62.6 $\pm$ 0.9	65.6 $\pm$ 0.8	64.8 $\pm$ 0.5	70.1 $\pm$ 0.2	70.0 $\pm$ 0.2
	AUC (%)	75.1 $\pm$ 0.2	75.1 $\pm$ 0.2	67.7 $\pm$ 0.5	71.9 $\pm$ 0.3	67.9 $\pm$ 0.7	71.7 $\pm$ 0.7	69.5 $\pm$ 0.4	74.9 $\pm$ 0.4	74.9 $\pm$ 0.4
	$\Delta_{SP}$ (%)	9.6 $\pm$ 0.9	9.4 $\pm$ 0.7	3.05 $\pm$ 0.5	4.1 $\pm$ 0.5	2.4 $\pm$ 0.7	3.6 $\pm$ 0.2	4.1 $\pm$ 0.8	0.8 $\pm$ 0.2	0.6 $\pm$ 0.3
	$\Delta_{EO}$ (%)	12.8 $\pm$ 1.3	12.0 $\pm$ 1.5	3.9 $\pm$ 0.6	4.6 $\pm$ 1.6	2.6 $\pm$ 1.0	4.4 $\pm$ 1.2	5.5 $\pm$ 0.9	1.1 $\pm$ 0.5	0.8 $\pm$ 0.2
NBA	ACC (%)	71.2 $\pm$ 0.5	71.9 $\pm$ 1.1	64.3 $\pm$ 1.3	66.0 $\pm$ 0.4	63.1 $\pm$ 1.1	65.6 $\pm$ 2.4	66.0 $\pm$ 1.5	71.1 $\pm$ 1.0	71.5 $\pm$ 0.8
	AUC (%)	78.3 $\pm$ 0.3	78.2 $\pm$ 0.6	71.5 $\pm$ 0.3	72.9 $\pm$ 1.0	71.3 $\pm$ 0.7	72.9 $\pm$ 1.2	73.6 $\pm$ 1.5	77.0 $\pm$ 0.3	77.5 $\pm$ 0.7
	$\Delta_{SP}$ (%)	7.9 $\pm$ 1.3	10.2 $\pm$ 2.5	2.3 $\pm$ 0.9	4.7 $\pm$ 1.8	2.5 $\pm$ 1.5	5.3 $\pm$ 0.9	2.9 $\pm$ 1.0	1.0 $\pm$ 0.5	0.7 $\pm$ 0.5
	$\Delta_{EO}$ (%)	17.8 $\pm$ 2.6	15.9 $\pm$ 4.0	3.2 $\pm$ 1.5	4.7 $\pm$ 1.7	3.1 $\pm$ 1.9	3.1 $\pm$ 1.3	3.0 $\pm$ 1.2	1.2 $\pm$ 0.4	0.7 $\pm$ 0.3

# Resources

- **Datasets:** <https://github.com/yushundong/Graph-Mining-Fairness-Data>
- **Paper collection:** <https://github.com/EdisonLeeeee/Awesome-Fair-Graph-Learning>
- **Surveys**
  - Dong, Y., Ma, J., Chen, C., & Li, J. (2023). Fairness in Graph Mining: A Survey. TKDE 2023.
  - Zhang, W., Weiss, J. C., Zhou, S., & Walsh, T. (2022). Fairness Amidst Non-IID Graph Data: A Literature Review. arXiv preprint arXiv:2202.07170.
  - Zhang, H., Wu, B., Yuan, X., Pan, S., Tong, H., & Pei, J. (2022). Trustworthy Graph Neural Networks: Aspects, Methods and Trends. arXiv preprint arXiv:2205.07424.
  - Dai, E., Zhao, T., Zhu, H., Xu, J., Guo, Z., Liu, H., ... & Wang, S. (2022). A Comprehensive Survey on Trustworthy Graph Neural Networks: Privacy, Robustness, Fairness, and Explainability. arXiv preprint arXiv:2204.08570.
- **Related tutorials**
  - Algorithmic Fairness on Graphs: Methods and Trends
    - [http://jiank2.web.illinois.edu/tutorial/kdd22/algofair\\_on\\_graphs.html](http://jiank2.web.illinois.edu/tutorial/kdd22/algofair_on_graphs.html)
  - Fairness in Graph Mining: Metrics, Algorithms, and Applications
    - [https://yushundong.github.io/icdm\\_tutorial\\_2022.pdf](https://yushundong.github.io/icdm_tutorial_2022.pdf)
  - Fair Graph Mining
    - [http://jiank2.web.illinois.edu/tutorial/cikm21/fair\\_graph\\_mining.html](http://jiank2.web.illinois.edu/tutorial/cikm21/fair_graph_mining.html)
  - Fairness in Networks
    - <https://algofairness.github.io/kdd-2021-network-fairness-tutorial/>

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  - Xianfeng Tang (Amazon)
- If you would like to re-use these contents, please contact the original authors.

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Thank you!