Exercise_2_Xiru Lyu

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Problem 1

Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = 0$ where 0 is a 3×3 matrix with every entry equal to 0.

```
A <- cbind(c(1,5,-2),c(1,2,-1),c(3,6,-3))
A %*% A %*% A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

(b) Replace the third column of A by the sum of the second and third columns.

```
x <- A[,2] + A[,3]
A[,3] <- x
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

Problem 2

Create the following matrix B with 15 rows:

$$\begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \dots & \dots & \dots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3 × 3 matrix $B^T B$.

```
B <- matrix(nrow=15,ncol=3)
B[,1] <- 10; B[,2] <- -10; B[,3] <- 10
t(B) %*% B
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

Problem 3

Create a 6 × 6 matrix `matE` with every entry equal to 0. Check what the functions row and col return when applied to `matE`. Hence create the 6 × 6 matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

```
matE <- matrix(0,nrow=6,ncol=6)
matE[abs(col(matE)-row(matE)) == 1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

Problem 4

Create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
outer(0:4,0:4,"+")
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

Problem 5: Create the following patterned matrices

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```
outer(0:4,0:4,"+") %% 5
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
outer(0:9,0:9,"+") %% 10
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
e <- outer(0:8,0:8,"+")
f <- row(e) - col (e)
for (i in 1:9){
  for (j in 1:9){
    if (f[i,j] < 0){
      f[i,j] <- f[i,j] + nrow(f)
    }}}
f
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

Problem 6: Solve the System

```
m <- matrix(0,nrow=5,ncol=5)
a <- abs(col(m)-row(m))+1
b <- rbind(7,-1,-3,5,17)
solve(a,b)
```

```
##      [,1]
## [1,]   -2
## [2,]    3
## [3,]    5
## [4,]    2
## [5,]   -4
```

Problem 7

```
set.seed(75)
aMat <- matrix( sample(10, size=60, replace=T), nr=6)
```

(a) Find the number of entries in each row which are greater than 4.

```
apply(aMat,1,function(x){sum(x>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
which(apply(aMat,1,function(x){sum(x==7)==2}))
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater than 75.

```
# Find sums for each column
aMat_colsum <- apply(aMat,2,sum)

# Find the outer product for the sum
outer <- outer(aMat_colsum,aMat_colsum,"+")

# Exclude cases when the value of the outer product is computed by the sum of the same column
for (i in 1:nrow(outer)) {
  for (j in 1:ncol(outer)) {
    if (i == j) {
      outer[i,j] <- 0
    }
  }
}

# Find col numbers whose total is greater than 75
which(outer>75,arr.ind=TRUE)
```

```
##      row col
## [1,]   6   2
## [2,]   8   2
## [3,]   2   6
## [4,]   8   6
## [5,]   2   8
## [6,]   6   8
```

Problem 8

(a) Calculate $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```
# Create a matrix that contains all possible values with i varies between 1
# and 20, j varies between 1 and 5
mat1 <- matrix(nrow=20,ncol=5)
for (i in 1:nrow(mat1)){
  for (j in 1:ncol(mat1)){
    mat1[i,j] <- i^4/(3+j)
  }
}

# Find the sum of all these values in the matrix mat1
sum(mat1)
```

```
## [1] 639215.3
```

```
# OR

sum((1:20)^4) * sum(1/(4:8))
```

```
## [1] 639215.3
```

(b) Calculate $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```
# Create a matrix that contains all possible values with i varies between 1
# and 20, j varies between 1 and 5
mat2 <- matrix(nrow=20,ncol=5)
for (i in 1:nrow(mat2)) {
  for (j in 1:ncol(mat2)) {
    mat2[i,j] <- i^4/(3+i*j)
  }
}

# Find the sum of all these values in the matrix mat2
sum(mat2)
```

```
## [1] 89912.02
```

```
# OR
i <- seq(from=1,to=20)
j <- seq(from=1,to=5)
num <- 1/(outer(i,j,"*")+3)    # find the numerator
denom <- matrix(i^4,nrow=1,byrow=FALSE)  # find the denominator
sum(denom %*% num)
```

```
## [1] 89912.02
```

(c) Calculate $\sum_{i=1}^{10}\sum_{j=1}^{i}\frac{i^4}{(3+ij)}$

```
# Create a matrix that contains all possible values with i varies between 1
# and 10, j varies between 1 and 10 (i)
mat3 <- matrix(0,nrow=10,ncol=10)
for (i in 1:nrow(mat3)) {
  for (j in 1:i) {
    mat3[i,j] <- i^4/(3+i*j)
  }
}

# Find the sum all of these values in the matrix mat3
sum(mat3)
```

```
## [1] 6944.743
```

```
# OR

i <- seq(from=1,to=10)
j <- seq(from=1,to=10)

# write a function that would compute the product of i and j only if i >= j
func <- function(i,j) {
  ifelse(i>=j,1/(i*j+3),0)
}

denom <- outer(i,j,func) # find the denominator
num <- matrix(i^4,nrow=1,byrow=FALSE) # find the numerator
sum(num%*%denom)
```

```
## [1] 6944.743
```