

Practice for Forced Oscillations

Why?

Exercise 1 On a piece of graph paper draw the vectors:

a) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ -4 \end{bmatrix}$

c) $(3, -4)$

Exercise 2 On a piece of graph paper draw the vector $(1, 2)$ starting at (based at) the given point:

a) based at $(0, 0)$

b) based at $(1, 2)$

c) based at $(0, -1)$

Exercise 3 On a piece of graph paper draw the following operations. Draw and label the vectors involved in the operations as well as the result:

a) $\begin{bmatrix} 1 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} -3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

c) $3 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Exercise 4 Compute the magnitude of

a) $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

c) $(1, 3, -4)$

Exercise 5 Compute the magnitude of

a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

c) $(-2, 1, -2)$

Exercise 6 Compute

a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 7 \\ -8 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 6 \\ -4 \end{bmatrix}$

c) $-\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

d) $4 \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

e) $5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 9 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

f) $3 \begin{bmatrix} 1 \\ -8 \end{bmatrix} - 2 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Exercise 7 Compute

a) $\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

c) $-\begin{bmatrix} -5 \\ 3 \end{bmatrix}$

d) $2 \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

e) $3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 7 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

f) $2 \begin{bmatrix} 2 \\ -3 \end{bmatrix} - 6 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Exercise 8 Find the unit vector in the direction of the given vector

a) $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

c) $(3, 1, -2)$

Exercise 9 Find the unit vector in the direction of the given vector

a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

c) $(2, -5, 2)$

Exercise 10 If $\vec{x} = (1, 2)$ and \vec{y} are added together, we find $\vec{x} + \vec{y} = (0, 2)$. What is \vec{y} ?

Exercise 11 If $\vec{v} = (1, -4, 3)$ and $\vec{w} = (-2, 3, -1)$, compute $3\vec{v} - 2\vec{w}$ and $4\vec{w} + \vec{v}$.

Exercise 12 Write $(1, 2, 3)$ as a linear combination of the standard basis vectors \vec{e}_1 , \vec{e}_2 , and \vec{e}_3 .

Exercise 13 Determine if the following sets of vectors are linearly independent.

a) $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$

f) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

Exercise 14 If the magnitude of \vec{x} is 4, what is the magnitude of

a) $0\vec{x}$

b) $3\vec{x}$

c) $-\vec{x}$

d) $-4\vec{x}$

e) $\vec{x} + \vec{x}$

f) $\vec{x} - \vec{x}$

Exercise 15 If the magnitude of \vec{x} is 5, what is the magnitude of

a) $4\vec{x}$

b) $-2\vec{x}$

c) $-4\vec{x}$

Exercise 16 Suppose a linear mapping $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ takes $(1, 0)$ to $(2, -1)$ and it takes $(0, 1)$ to $(3, 3)$. Where does it take

a) $(1, 1)$

b) $(2, 0)$

c) $(2, -1)$

Exercise 17 Suppose a linear mapping $F: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ takes $(1, 0, 0)$ to $(2, 1)$ and it takes $(0, 1, 0)$ to $(3, 4)$ and it takes $(0, 0, 1)$ to $(5, 6)$. Write down the matrix representing the mapping F .

Exercise 18 Suppose that a mapping $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ takes $(1, 0)$ to $(1, 2)$, $(0, 1)$ to $(3, 4)$, and it takes $(1, 1)$ to $(0, -1)$. Explain why F is not linear.

Exercise 19 Suppose a linear mapping $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ takes $(1, 0)$ to $(1, -1)$ and it takes $(0, 1)$ to $(2, 0)$. Where does it take

a) $(1, 1)$

b) $(0, 2)$

c) $(1, -1)$

Exercise 20 Let P represent the space of quadratic polynomials in t : a point (a_0, a_1, a_2) in P represents the polynomial $a_0 + a_1t + a_2t^2$. Consider the derivative $\frac{d}{dt}$ as a mapping of P to P , and note that $\frac{d}{dt}$ is linear. Write down $\frac{d}{dt}$ as a 3×3 matrix.