Practice for Modeling Parameters

Why?

Exercise 1 *Bob is getting coffee from a restaurant and knows that the temperature of the coffee will follow Newton's Law of Cooling, which says that*

$$\frac{dT}{dt} = k(T_0 - T)$$

where T_0 is the ambient temperature and k is a constant depending on the object and geometry. His car is held at a constant 20° C, and when he receives the coffee, he measures the temperature to be 90° C. Two minutes later, the temperature is 81°C.

a) *Use these two points of data to determine the value of k for this coffee.*

b) *Bob only wants to drink his coffee once it reaches 65° C. How long does he have to wait for this to happen?*

c) *If the coffee is too cold for Bob's taste once it reaches 35° C, how long is the acceptable window for Bob to drink his coffee?*

Exercise 2 *Assume that a falling object has a velocity (in meters per second) that obeys the differential equation*

$$\frac{dv}{dt} = 9.8 - \alpha v$$

where α represents the drag coefficient of the object.

a) *Solve this differential equation with initial condition $v(0) = 0$ to get a solution that depends on α.*

b) *Assume that you drop an object from a height of 10 meters and it hits the ground after 3 seconds. What is the value of α here? (Note: You solved for $v(t)$ in the previous part, and now you need to get to position. What does that require?)*

c) *Assume that a second object hits the ground in 6 seconds. How does this change the value of α?*

Exercise 3 *A restaurant is trying to analyze the to-go coffee cups that it uses in order to best serve their customers. They assume that the coffee follows Newton's Law of Cooling and place it in a room with ambient temperature 22° C. They record the following data for the temperature of the coffee as a function of time.*

t (min)	**T** (° C)
0	80
0.5	74
1.1	67
1.7	61
2.3	56

a) *Use code to determine the best-fit value of k for this data.*

b) *The restaurant determines that to avoid any potential legal issues, the coffee can be no warmer than 60 °C when it is served. If the coffee comes out of the machine at 90 °C, how long do they have to wait before they can serve the coffee?*

Exercise 4 *Assume that an object falling has a velocity that follows the differential equation*

$$\frac{dv}{dt} = 9.8 - \alpha v^2$$

where the velocity is in $^{m}/_{s}$ *and* α *represents the drag coefficient. During a physics experiment, a student measures data for the velocity of a falling object over time given in the table below.*

Use this data (and code) to estimate the value of α *for this object.*

t (s)	**v** (m/s)
0	0
0.1	1.0
0.2	1.9
0.4	3.6
0.6	5.2
0.9	6.8
1.1	7.4
1.3	7.9
1.5	8.2
1.8	8.5
2.1	8.8

Table 1: Data for Exercise .

t (d)	**P** (thousands)
0	50
7	60
14	70
28	97
37	117
50	148
78	220
100	268

Table 2: Data for Exercise .

Exercise 5 *Assume that a species of fish in a lake has a population that is modeled by the differential equation*

$$\frac{dP}{dt} = \frac{1}{100} r P(K - P) - \alpha$$

where r*,* K*, and* α *are parameters,* r *representing the growth rate,* K *the carrying capacity, and* α *the harvesting rate, and the population* P *is in thousands., with* t *given in years. From previous studies, you know that the best value of* r *is* 3.12*. After studying the population over a period of time, you get the data given above.*

a) *Your friend tells you that in a previous study, he found that the value of* K *for this particular lake is* 450*. Use code to determine the best value of* α *for this situation. Note that the equation expects* t *in years, but the data is given in days. Search for* α *in the range* $(0, 400)$*.*

b) *That answer doesn't look great. Plot the solution with these parameters along with the data and compare them.*

c) *The fit does not look great, so maybe your friend's value was not quite right. Run code to find best values for* K *and* α *simultaneously. Use the range* $(0, 400)$ *for both* α *and* K*.*