Practice for Classifying Equations

Why?

Exercise 1 *Classify the following equations. Are they ODE or PDE? Is it an equation or a system? What is the order? Is it linear or nonlinear, and if it is linear, is it homogeneous, constant coefficient? If it is an ODE, is it autonomous?*

a) $\sin(t)\dfrac{d^2x}{dt^2} + \cos(t)x = t^2$

b) $\dfrac{\partial u}{\partial x} + 3\dfrac{\partial u}{\partial y} = xy$

c) $y'' + 3y + 5x = 0, \quad x'' + x - y = 0$

d) $\dfrac{\partial^2 u}{\partial t^2} + u\dfrac{\partial^2 u}{\partial s^2} = 0$

e) $x'' + tx^2 = t$

f) $\dfrac{d^4x}{dt^4} = 0$

Exercise 2 *Classify the following equations. Are they ODE or PDE? Is it an equation or a system? What is the order? Is it linear or nonlinear, and if it is linear, is it homogeneous, constant coefficient? If it is an ODE, is it autonomous?*

a) $\dfrac{\partial^2 v}{\partial x^2} + 3\dfrac{\partial^2 v}{\partial y^2} = \sin(x)$

b) $\dfrac{dx}{dt} + \cos(t)x = t^2 + t + 1$

c) $\dfrac{d^7F}{dx^7} = 3F(x)$

d) $y'' + 8y' = 1$

e) $x'' + tyx' = 0, \quad y'' + txy = 0$

f) $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial s^2} + u^2$

Exercise 3 *If* $\vec{u} = (u_1, u_2, u_3)$ *is a vector, we have the divergence* $\nabla \cdot \vec{u} = \dfrac{\partial u_1}{\partial x} + \dfrac{\partial u_2}{\partial y} + \dfrac{\partial u_3}{\partial z}$ *and curl* $\nabla \times \vec{u} = \left(\dfrac{\partial u_3}{\partial y} - \dfrac{\partial u_2}{\partial z}, \dfrac{\partial u_1}{\partial z} - \dfrac{\partial u_3}{\partial x}, \dfrac{\partial u_2}{\partial x} - \dfrac{\partial u_1}{\partial y}\right)$. *Notice that curl of a vector is still a vector. Write out Maxwell's equations in terms of partial derivatives and classify the system.*

Exercise 4 *Suppose* F *is a linear function, that is,* $F(x, y) = ax + by$ *for constants* a *and* b*. What is the classification of equations of the form* $F(y', y) = 0$*.*

Exercise 5 *Write down an explicit example of a third order, linear, nonconstant coefficient, nonautonomous, nonhomogeneous system of two ODE such that every derivative that could appear, does appear.*

Exercise 6 *Write down the general zeroth order linear ordinary differential equation. Write down the general solution.*

Exercise 7 *For which* k *is* $\dfrac{dx}{dt} + x^k = t^{k+2}$ *linear. Hint: there are two answers.*

Learning outcomes:

Exercise 8 *Write out an explicit example of a non-homogeneous fourth order, linear, constant coefficient differential equation. where all possible derivatives of the unknown function y appear.*

Exercise 9 *Let x, y, and z be three functions of t defined by the system of differential equations*

$$x' = y \quad y' = z \quad z' = 3x - 2y + 5z + e^t$$

with initial conditions $x(0) = 3$, $y(0) = -2$ and $z(0) = 1$, and let $u(t)$ be the function defined by the solution to

$$u''' - 5u'' + 2u' - 3u = e^t$$

with initial conditions $u(0) = 3$, $u'(0) = -2$, and $u''(0) = 1$.

a) *Use the substitution $u = x$, $u' = y$, and $u'' = z$ to verify that $x(t) = u(t)$ because they solve the same initial value problem.*

b) *What is the order of the system defining x, y, and z and how many components does it have?*

c) *What is the order of the equation defining u? How many components does that have?*

d) *How do these numbers relate to each other?*