# Practice for Separable ODEs

*Why?*

**Exercise 1** *Solve* $y' = y^3$ *for* $y(0) = 1$.

**Exercise 2** *Solve* $x' = \dfrac{1}{x^2}$, $x(1) = 1$.

**Exercise 3** *Solve* $y' = (y-1)(y+1)$ *for* $y(0) = 3$. *(Note: Requires partial fractions)*

**Exercise 4** *Solve* $x' = \dfrac{1}{\cos(x)}$, $x(0) = \dfrac{\pi}{2}$.

**Exercise 5** *Solve* $\dfrac{dy}{dx} = \dfrac{1}{y+1}$ *for* $y(0) = 0$.

**Exercise 6** *Solve* $y' = {}^{x}/_{y}$.

**Exercise 7** *Solve* $y' = x^2 y$.

**Exercise 8** *Consider the differential equation*

$$\frac{dy}{dx} = \frac{2x}{y}$$

a) *Find the general solution as an implicit function.*

b) *Find the solution to this differential equation as an* explicit *function with* $y(1) = 4$.

c) *Find the solution to this differential equation as an* explicit *function with* $y(0) = -2$.

**Exercise 9** *Solve* $y' = y^n$, $y(0) = 1$, *where* $n$ *is a positive integer. Hint: You have to consider different cases.*

**Exercise 10** *Solve* $\dfrac{dx}{dt} = (x^2 - 1)\, t$, *for* $x(0) = 0$. *(Note: Requires partial fractions)*

**Exercise 11** *Solve* $\dfrac{dx}{dt} = x \sin(t)$, *for* $x(0) = 1$.

Learning outcomes:

**Exercise 12** *Solve* $y' = 2xy$.

**Exercise 13** *Solve* $y' = ye^{2x}$ *with* $y(0) = 4$.

**Exercise 14** *Solve* $\dfrac{dy}{dx} = xy + x + y + 1$. *Hint: Factor the right-hand side.*

**Exercise 15** *Solve* $x' = 3xt^2 - 3t^2$, $x(0) = 2$.

**Exercise 16** *Find the general solution of* $y' = e^x$, *and then* $y' = e^y$.

**Exercise 17** *Solve* $xy' = y + 2x^2y$, *where* $y(1) = 1$.

**Exercise 18** *Find an implicit solution for* $x' = \dfrac{1}{3x^2 + 1}$, $x(0) = 1$.

**Exercise 19** *Solve* $\dfrac{dy}{dx} = \dfrac{y^2 + 1}{x^2 + 1}$, *for* $y(0) = 1$.

**Exercise 20** *Find an implicit solution for* $\dfrac{dy}{dx} = \dfrac{x^2 + 1}{y^2 + 1}$, *for* $y(0) = 1$.

**Exercise 21** *Find an implicit solution to* $y' = \dfrac{\sin(x)}{\cos(y)}$.

**Exercise 22** *Find an implicit solution for* $xy' = \dfrac{x^2 + 1}{y^2 - 1}$ *with* $y(3) = 2$.

**Exercise 23** *Find an explicit solution for* $y' = xe^{-y}$, $y(0) = 1$.

**Exercise 24** *Find an explicit solution to* $xy' = y^2$, $y(1) = 1$.

**Exercise 25** *Find an explicit solution for* $xy' = e^{-y}$, *for* $y(1) = 1$.

**Exercise 26** *Find an explicit solution for* $y' = y^2(x^4 + 1)$ *with* $y(1) = 2$.

**Exercise 27** *Find an explicit solution for* $y' = \dfrac{\cos(x)+1}{y}$ *with* $y(0) = 4$.

**Exercise 28** *Find an explicit solution for* $y' = ye^{-x^2}$, $y(0) = 1$. *It is alright to leave a definite integral in your answer.*

**Exercise 29** *Is the equation* $y' = x + y + 1$ *separable? If so, find the general solution, if not, explain why.*

**Exercise 30** *Is the equation* $y' = ty^2 + t$ *separable? If so, find the general solution, if not, explain why.*

**Exercise 31** *Is the equation* $y' = xy^2 + 3y^2 - 4x - 12$ *separable? If so, find the general solution, if not, explain why. (Note: Requires partial fractions)*

**Exercise 32** *Suppose a cup of coffee is at 100 degrees Celsius at time* $t = 0$, *it is at 70 degrees at* $t = 10$ *minutes, and it is at 50 degrees at* $t = 20$ *minutes. Compute the ambient temperature.*

**Exercise 33** *Take* ***Normally a reference to a previous example goes here.*** *with the same numbers: 89 degrees at* $t = 0$, *85 degrees at* $t = 1$, *and ambient temperature of 22 degrees. Suppose these temperatures were measured with precision of* $\pm 0.5$ *degrees. Given this imprecision, the time it takes the coffee to cool to (exactly) 60 degrees is also only known in a certain range. Find this range. Hint: Think about what kind of error makes the cooling time longer and what shorter.*

**Exercise 34** *A population* $x$ *of rabbits on an island is modeled by* $x' = x - \left(1/1000\right)x^2$, *where the independent variable is time in months. At time* $t = 0$, *there are 40 rabbits on the island.*

a) *Find the solution to the equation with the initial condition.*

b) *How many rabbits are on the island in 1 month, 5 months, 10 months, 15 months (round to the nearest integer).*