Practice for Determinants

Why?

Exercise 1 *Compute the determinant of the following matrices:*

a) $\begin{bmatrix} 3 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

e) $\begin{bmatrix} 2 & 1 & 0 \\ -2 & 7 & -3 \\ 0 & 2 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 2 & 1 & 3 \\ 8 & 6 & 3 \\ 7 & 9 & 7 \end{bmatrix}$ g) $\begin{bmatrix} 0 & 2 & 5 & 7 \\ 0 & 0 & 2 & -3 \\ 3 & 4 & 5 & 7 \\ 0 & 0 & 2 & 4 \end{bmatrix}$ h) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & -1 & 2 \\ 1 & 1 & 2 & 1 \\ 2 & -1 & -2 & 3 \end{bmatrix}$

Exercise 2 *% Compute the determinant of the following matrices:*

a) $\begin{bmatrix} -2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 9 & -11 \\ 0 & -1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

e) $\begin{bmatrix} 2 & 1 & 0 \\ -2 & 7 & 3 \\ 1 & 1 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 5 & 1 & 3 \\ 4 & 1 & 1 \\ 4 & 5 & 1 \end{bmatrix}$ g) $\begin{bmatrix} 3 & 2 & 5 & 7 \\ 0 & 0 & 2 & 0 \\ 0 & 4 & 5 & 0 \\ 2 & 1 & 2 & 4 \end{bmatrix}$ h) $\begin{bmatrix} 0 & 2 & 1 & 0 \\ 1 & 2 & -3 & 4 \\ 5 & 6 & -7 & 8 \\ 1 & 2 & 3 & -2 \end{bmatrix}$

Exercise 3 *For which x are the following matrices singular (not invertible).*

a) $\begin{bmatrix} 2 & 3 \\ 2 & x \end{bmatrix}$ b) $\begin{bmatrix} 2 & x \\ 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} x & 1 \\ 4 & x \end{bmatrix}$ d) $\begin{bmatrix} x & 0 & 1 \\ 1 & 4 & 2 \\ 1 & 6 & 2 \end{bmatrix}$

Exercise 4 *% For which x are the following matrices singular (not invertible).*

a) $\begin{bmatrix} 1 & 3 \\ 1 & x \end{bmatrix}$ b) $\begin{bmatrix} 3 & x \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} x & 3 \\ 3 & x \end{bmatrix}$ d) $\begin{bmatrix} x & 1 & 0 \\ 1 & 4 & 0 \\ 1 & 6 & 2 \end{bmatrix}$

Exercise 5 *% Consider the matrix*

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -5 & -4 & -5 \\ 2 & 3 & 4 \end{bmatrix}.$$

a) *Compute the determinant of A using cofactor expansion along row 1.*

b) *Compute the determinant of A using cofactor expansion along column 2.*

c) *Compute the determinant using row reduction.*

Exercise 6 *% Consider the matrix*

$$A = \begin{bmatrix} -1 & 0 & -3 \\ 1 & 2 & 1 \\ 3 & 3 & 3 \end{bmatrix}.$$

a) *Compute the determinant of A using cofactor expansion along row 1.*
b) *Compute the determinant of A using cofactor expansion along column 3.*
c) *Compute the determinant using row reduction.*

Exercise 7 *% Consider the matrix*

$$A = \begin{bmatrix} -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 \\ -5 & 3 & 1 & 3 \\ -3 & 4 & 1 & 3 \end{bmatrix}.$$

a) *Compute the determinant of A using cofactor expansion along row 1.*
b) *Compute the determinant of A using cofactor expansion along column 4.*
c) *Compute the determinant using row reduction.*

Exercise 8 *Is the matrix A below invertible? How do you know?*

$$A = \begin{bmatrix} 4 & 0 & 3 & 1 \\ 2 & 1 & -2 & 0 \\ 0 & 0 & 1 & -3 \\ 3 & 2 & 1 & -5 \end{bmatrix}$$

Exercise 9 *% Compute the determinant of the matrix*

$$A = \begin{bmatrix} 5 & 4 & 3 \\ -4 & -3 & -4 \\ -5 & -5 & 4 \end{bmatrix}$$

using row reduction. What does this say about the solutions to $A\vec{x} = 0$?

Exercise 10 *% Compute the determinant of the matrix*

$$A = \begin{bmatrix} -5 & -3 & -5 & -1 \\ 4 & 0 & -5 & 4 \\ 0 & -2 & -1 & -2 \\ -1 & -5 & -4 & -4 \end{bmatrix}$$

using row reduction. What does this say about the columns of A?

Exercise 11 *% Compute the determinant of the matrix*

$$A = \begin{bmatrix} 4 & 1 & -3 & 0 \\ -1 & 4 & 2 & -2 \\ -1 & -3 & 3 & 2 \\ -5 & -4 & 1 & 1 \end{bmatrix}$$

using row reduction. What does this say about the solutions to $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$.

Exercise 12 *Compute*

$$\det\left(\begin{bmatrix} 2 & 1 & 2 & 3 \\ 0 & 8 & 6 & 5 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}\right)$$

without computing the inverse.

Exercise 13 *% Compute*

$$\det\left(\begin{bmatrix} 3 & 4 & 7 & 12 \\ 0 & -1 & 9 & -8 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}^{-1}\right)$$

without computing the inverse.

Exercise 14 *Suppose*

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 7 & \pi & 1 & 0 \\ 2^8 & 5 & -99 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 5 & 9 & 1 & -\sin(1) \\ 0 & 1 & 88 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let $A = LU$. Compute $\det(A)$ in a simple way, without computing what is A. Hint: First read off $\det(L)$ and $\det(U)$.

Exercise 15 *Consider the linear mapping from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by the matrix $A = \left[\begin{smallmatrix} 1 & x \\ 2 & 1 \end{smallmatrix}\right]$ for some number x. You wish to make A such that it doubles the area of every geometric figure. What are the possibilities for x (there are two answers).*

Exercise 16 *% Consider the matrix*

$$A(x) = \begin{bmatrix} 1 & 2 \\ 1 & x \end{bmatrix}^{-1}$$

as a function of the variable x.

a) *Find all the x so that $A(x)$ and the matrix inverse $A(x)^{-1}$ have only integer entries (no fractions). Note that there are two answers.*

b) *Find all the x so that the matrix inverse $A(x)^{-1}$ has only integer entries (no fractions). (You should get more answers here than the previous part.)*

Exercise 17 *Suppose A and S are $n \times n$ matrices, and S is invertible. Suppose that $\det(A) = 3$. Compute $\det(S^{-1}AS)$ and $\det(SAS^{-1})$. Justify your answer using the theorems in this section.*

Exercise 18 *Let A be an $n \times n$ matrix such that $\det(A) = 1$. Compute $\det(xA)$ given a number x. Hint: First try computing $\det(xI)$, then note that $xA = (xI)A$.*