Practice for Second Order Repeated Roots

Why?

Exercise 1 *Find the general solution to* $y'' + 4y' + 4y = 0$.

Exercise 2 *Find the general solution to* $y'' - 6y' + 9y = 0$.

Exercise 3 *Find the solution to* $y'' + 6y' + 9y = 0$ *with* $y(0) = 3$ *and* $y'(0) = -1$.

Exercise 4 *Solve* $y'' - 8y' + 16y = 0$ *for* $y(0) = 2$, $y'(0) = 0$.

Exercise 5 *Find the general solution of* $y'' = 0$ *using the methods of this section.*

Exercise 6 *The method of this section applies to equations of other orders than two. We will see higher orders later. Try to solve the first order equation* $2y' + 3y = 0$ *using the methods of this section.*

Exercise 7 *Consider the second-order DE*

$$ty'' + (4t + 2)y' + (4t + 4)y = 0. \tag{1}$$

a) *Does the superposition principle apply to this DE? Give a one- or two-sentence explanation wither way.*

b) *Find a value of* r *so that* $y = e^{rt}$ *is a solution to* (1)

c) *Using your result from the previous page, apply* **reduction of order** *to find the general solution to* (1).

Exercise 8 *Consider the differential equation* $x^2y'' + 3xy' - 3y = 0$.

a) *Verify that* $y_1(x) = x$ *is a solution.*

b) *Use reduction of order to find a second linearly independent solution.*

c) *Write out the general solution.*

Exercise 9 *Consider the differential equation* $x^2y'' + 4xy' + 2y = 0$.

a) *Verify that* $y_1(x) = \dfrac{1}{x}$ *is a solution.*

b) *Use reduction of order to find a second linearly independent solution.*

c) *Write out the general solution.*

Exercise 10 *Consider the differential equation $x^2y'' - 6xy' + 10y = 0$.*

a) *Verify that $y_1(x) = x^2$ is a solution.*

b) *Use reduction of order to find a second linearly independent solution.*

c) *Write out the general solution.*

Exercise 11 *Write down a differential equation with general solution $y = at^2 + bt^{-3}$, or explain why there is no such thing.*

Exercise 12 *Find the solution to $y'' - (2\alpha)y' + \alpha^2 y = 0$, $y(0) = a$, $y'(0) = b$, where α, a, and b are real numbers.*

Exercise 13 *Suppose y_1 is a solution to $y'' + p(x)y' + q(x)y = 0$. By directly plugging into the equation, show that*

$$y_2(x) = y_1(x) \int \frac{e^{-\int p(x)\,dx}}{\left(y_1(x)\right)^2}\,dx$$

is also a solution.

Exercise 14 *Take $(1 - x^2)y'' - xy' + y = 0$.*

a) *Show that $y = x$ is a solution.*

b) *Use reduction of order to find a second linearly independent solution.*

c) *Write down the general solution.*

Exercise 15 *Take $y'' - 2xy' + 4y = 0$.*

a) *Show that $y = 1 - 2x^2$ is a solution.*

b) *Use reduction of order to find a second linearly independent solution. (It's OK to leave a definite integral in the formula.)*

c) *Write down the general solution.*

The rest of these exercises can be solved using any of the methods discussed in the last three sections. Pick the appropriate method in order to solve the problem.

Exercise 16 *Find the general solution of $y'' + 5y' - 6y = 0$.*

Exercise 17 *Find the general solution of $y'' - 2y' + 2y = 0$.*

Exercise 18 *Find the general solution of* $y'' + 4y' + 4y = 0$.

Exercise 19 *Find the general solution of* $y'' + 4y' + 5y = 0$.

Exercise 20 *Find the solution to* $y'' - 6y' + 13y = 0$ *with* $y(0) = 2$ *and* $y'(0) = 1$.

Exercise 21 *Find the solution to* $y'' + 4y' - 12y = 0$ *with* $y(0) = -1$ *and* $y'(0) = 3$.

Exercise 22 *Find the solution to* $y'' - 6y' + 9y = 0$ *with* $y(0) = -4$ *and* $y'(0) = -1$.