Practice for Solving by Direct Integration

Why?

Exercise 1 *Solve* $\dfrac{dy}{dx} = x^2 + x$ *with* $y(1) = 3$.

Exercise 2 *Solve* $\dfrac{dy}{dx} = \sin(5x)$ *with* $y(0) = 2$.

Exercise 3 *Solve* $\dfrac{dy}{dx} = e^x + x$ *with* $y(0) = 10$.

Exercise 4 *Solve* $\dfrac{dy}{dx} = 2xe^{3x}$ *with* $y(0) = 1$.

Exercise 5 *Solve* $\dfrac{dx}{dt} = e^t \cos(2t) + t$ *with* $x(0) = 3$.

Exercise 6 *Solve* $\dfrac{dy}{dx} = \dfrac{1}{x^2+1} + 3e^{2x}$ *with* $y(0) = 2$.

Exercise 7 *Solve* $\dfrac{dy}{dx} = \dfrac{1}{x^2-1}$ *for* $y(0) = 0$. *(This requires partial fractions or hyperbolic trigonometric functions.)*

Exercise 8 *Solve* $y'' = \sin x$ *for* $y(0) = 0$, $y'(0) = 2$.

Exercise 9 *A spaceship is traveling at the speed* $2t^2 + 1$ $^{\text{km}}/_{\text{s}}$ *(*t *is time in seconds). It is pointing directly away from earth and at time* $t = 0$ *it is 1000 kilometers from earth. How far from earth is it at one minute from time* $t = 0$*?*

Exercise 10 *Sid is in a car traveling at speed* $10t + 70$ *miles per hour away from Las Vegas, where* t *is in hours. At* $t = 0$*, Sid is 10 miles away from Vegas. How far from Vegas is Sid 2 hours later?*

Exercise 11 *Solve* $\dfrac{dx}{dt} = \sin(t^2) + t$, $x(0) = 20$. *It is OK to leave your answer as a definite integral.*

Exercise 12 *Solve* $\dfrac{dy}{dt} = e^{t^2} + \sin(t)$, $y(0) = 4$. *The answer can be left as a definite integral.*

Learning outcomes:

Exercise 13 *A dropped ball accelerates downwards at a constant rate 9.8 meters per second squared. Set up the differential equation for the height above ground h in meters. Then supposing $h(0) = 100$ meters, how long does it take for the ball to hit the ground.*

Exercise 14 *The rate of change of the volume of a snowball that is melting is proportional to the surface area of the snowball. Suppose the snowball is perfectly spherical. The volume (in centimeters cubed) of a ball of radius r centimeters is $(4/3)\pi r^3$. The surface area is $4\pi r^2$. Set up the differential equation for how the radius r is changing. Then, suppose that at time $t = 0$ minutes, the radius is 10 centimeters. After 5 minutes, the radius is 8 centimeters. At what time t will the snowball be completely melted?*

Exercise 15 *Find the general solution to $y'''' = 0$. How many distinct constants do you need?*