Practice for Non-Homogeneous Equations

Why?

Exercise 1 *Find a particular solution of* $y'' - y' - 6y = e^{2x}$.

Exercise 2 *Find a particular solution of* $y'' - 4y' + 4y = e^{2x}$.

Exercise 3 *Find a particular solution to* $y'' - y' + y = 2\sin(3x)$

Exercise 4 *Solve the initial value problem* $y'' + 9y = \cos(3x) + \sin(3x)$ *for* $y(0) = 2$, $y'(0) = 1$.

Exercise 5 *Set up the form of the particular solution but do not solve for the coefficients for* $y^{(4)} - 2y''' + y'' = e^x$.

Exercise 6 *Set up the form of the particular solution but do not solve for the coefficients for* $y^{(4)} - 2y''' + y'' = e^x + x + \sin x$.

Exercise 7 *Solve* $y'' + 2y' + y = x^2$, $y(0) = 1$, $y'(0) = 2$.

Exercise 8 *Use the method of undetermined coefficients to solve the DE* $y'' + 4y' = 2t + 30$.

Exercise 9

a) *Using variation of parameters find a particular solution of* $y'' - 2y' + y = e^x$.

b) *Find a particular solution using undetermined coefficients.*

c) *Are the two solutions you found the same? See also* Exercise .

Exercise 10

a) *Find a particular solution to* $y'' + 2y = e^x + x^3$.

b) *Find the general solution.*

Exercise 11 *Find the general solution to* $y'' - 3y' - 4y = e^{2t} + 1$.

Exercise 12 *Find the general solution to* $y'' - 2y' + 5y = \sin(3t) + 2\cos(3t)$.

Exercise 13 *Find the general solution to* $y'' - 4y' - 21y = e^{-3t} + e^{4t}$.

Exercise 14 *Find the general solution to* $y'' - 2y' + y = e^{t} - t$.

Exercise 15 *Find the general solution to* $y'' + 4y = \sec(2t)$ *using variation of parameters.*

Exercise 16 *Find the solution of the initial value problem* $y'' - 2y' - 15y = e^{5t} + 3$, $y(0) = 2$, $y'(0) = -1$.

Exercise 17 *Find the solution of the initial value problem* $y'' + 4y' + 5y = \cos(3t) + t$, $y(0) = 0$, $y'(0) = 2$.

Exercise 18 *The following differential equations are all related. Find the general solution to each of them and compare and contrast the different solutions and the methods used to approach them.*

a) $y'' - 2y' - 15y = e^{t} + 5e^{-4t}$

b) $y'' - 2y' - 15y = 2e^{2t} + 3e^{-t}$

c) $y'' - 2y' - 15y = 3\cos(2t)$

d) $y'' - 2y' - 15y = 2e^{5t} - \sin(t)$

Exercise 19 *The following differential equations are all related. Find the general solution to each of them and compare and contrast the different solutions and the methods used to approach them.*

a) $y'' + 4y' + 3y = e^{2t} + 3e^{4t}$

b) $y'' - 2y' + 5y = e^{2t} + 3e^{4t}$

c) $y'' + 3y' - 10y = e^{2t} + 3e^{4t}$

d) $y'' - 8y' + 16y = e^{2t} + 3e^{4t}$

Exercise 20 *Find a particular solution of* $y'' - 2y' + y = \sin(x^2)$. *It is OK to leave the answer as a definite integral.*

Exercise 21 *Use variation of parameters to find a particular solution of* $y'' - y = \dfrac{1}{e^{x} + e^{-x}}$.

Exercise 22 *Recall that a homogeneous* Euler equation *is one of the form* $t^2y'' + aty' + by = 0$ *and is solved by using the guess* $y(t) = t^r$ *and solving for the potential values of* r.

a) *Solve* $t^2y'' - 2ty' - 10y = 0$.

b) *Let* y_1 *and* y_2 *be a fundamental set for the above equation. Use the variation of parameters equations* $u_1 = -\displaystyle\int \frac{y_2\, g(t)}{y_1y_2' - y_2y_1'}\,dt$, $y_2 = \displaystyle\int \frac{y_1\, g(t)}{y_1y_2' - y_2y_1'}\,dt$ *to solve the non-homogeneous equation* $y'' - \dfrac{2}{t} - \dfrac{10}{t^2} = t^3$.
(Do not attempt method of undetermined coefficients instead; it won't work.)

Exercise 23 *For an arbitrary constant c find the general solution to $y'' - 2y = \sin(x + c)$.*

Exercise 24 *For an arbitrary constant c find a particular solution to $y'' - y = e^{cx}$. Hint: Make sure to handle every possible real c.*

Exercise 25

a) *Using variation of parameters find a particular solution of $y'' - y = e^x$.*

b) *Find a particular solution using undetermined coefficients.*

c) *Are the two solutions you found the same? What is going on?*