Practice for Forced Oscillations

Why?

Exercise 1 *Compute the rank of the given matrices*

a) $\begin{bmatrix} 6 & 3 & 5 \\ 1 & 4 & 1 \\ 7 & 7 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 5 & -2 & -1 \\ 3 & 0 & 6 \\ 2 & 4 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 2 & 4 & 6 \end{bmatrix}$

Exercise 2 *Compute the rank of the given matrices*

a) $\begin{bmatrix} 7 & -1 & 6 \\ 7 & 7 & 7 \\ 7 & 6 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 3 & -1 \\ 6 & 3 & 1 \\ 4 & 7 & -1 \end{bmatrix}$

Exercise 3 *For the matrices in Exercise , find a linearly independent set of row vectors that span the row space (they don't need to be rows of the matrix).*

Exercise 4 *For the matrices in Exercise , find a linearly independent set of columns that span the column space. That is, find the pivot columns of the matrices.*

Exercise 5 *For the matrices in Exercise , find a linearly independent set of row vectors that span the row space (they don't need to be rows of the matrix).*

Exercise 6 *For the matrices in Exercise , find a linearly independent set of columns that span the column space. That is, find the pivot columns of the matrices.*

Exercise 7 *Compute the rank of the matrix*

$$\begin{bmatrix} 10 & -2 & 11 & -7 \\ -5 & -2 & -5 & 5 \\ 1 & 0 & -4 & -4 \\ 1 & 2 & 2 & -1 \end{bmatrix}$$

Exercise 8 *Compute the rank of the matrix*

$$\begin{bmatrix} 4 & -2 & 0 & -4 \\ 3 & -5 & 2 & 0 \\ 1 & -2 & 0 & 1 \\ -1 & 1 & 3 & -3 \end{bmatrix}$$

Exercise 9 *Find a linearly independent subset of the following vectors that has the same span.*

$$\begin{bmatrix}-1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\-2\\-4\end{bmatrix}, \begin{bmatrix}-2\\4\\1\end{bmatrix}, \begin{bmatrix}-1\\3\\-2\end{bmatrix}$$

Exercise 10 *Find a linearly independent subset of the following vectors that has the same span.*

$$\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}3\\1\\-5\end{bmatrix}, \begin{bmatrix}0\\3\\-1\end{bmatrix}, \begin{bmatrix}-3\\2\\4\end{bmatrix}$$

Exercise 11 *For the following sets of vectors, determine if the set is linearly independent. Then find a basis for the subspace spanned by the vectors, and find the dimension of the subspace.*

a) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\-1\\-1\end{bmatrix}$

b) $\begin{bmatrix}1\\0\\5\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\-1\\0\end{bmatrix}$

c) $\begin{bmatrix}-4\\-3\\5\end{bmatrix}, \begin{bmatrix}2\\3\\3\end{bmatrix}, \begin{bmatrix}2\\0\\2\end{bmatrix}$

d) $\begin{bmatrix}1\\3\\0\end{bmatrix}, \begin{bmatrix}0\\2\\2\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\end{bmatrix}$

e) $\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}-1\\-1\end{bmatrix}$

f) $\begin{bmatrix}3\\1\\3\end{bmatrix}, \begin{bmatrix}2\\4\\-4\end{bmatrix}, \begin{bmatrix}-5\\-5\\-2\end{bmatrix}$

Exercise 12 *For the following sets of vectors, determine if the set is linearly independent. Then find a basis for the subspace spanned by the vectors, and find the dimension of the subspace.*

a) $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$

b) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}1\\1\\2\end{bmatrix}$

c) $\begin{bmatrix}5\\3\\1\end{bmatrix}, \begin{bmatrix}5\\-1\\5\end{bmatrix}, \begin{bmatrix}-1\\3\\-4\end{bmatrix}$

d) $\begin{bmatrix}2\\2\\4\end{bmatrix}, \begin{bmatrix}2\\2\\3\end{bmatrix}, \begin{bmatrix}4\\4\\-3\end{bmatrix}$

e) $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}3\\0\end{bmatrix}$

f) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}$

Exercise 13 *Suppose that X is the set of all the vectors of $\mathbb{R}^3$ whose third component is zero. Is X a subspace? And if so, find a basis and the dimension.*

Exercise 14 *Consider a set of 3 component vectors.*

a) *How can it be shown if these vectors are linearly independent?*

b) *Can a set of 4 of these 3 component vectors be linearly independent? Explain your answer.*

c) *Can a set of 2 of these 3 component vectors be linearly independent? Explain.*

d) *How would it be shown if these vectors make up a spanning set for all 3 component vectors?*

e) *Can 4 vectors be a spanning set? Explain.*

f) *Can 2 vectors be a spanning set? Explain.*

Exercise 15 *Consider the vectors*

$$\vec{v}_1 = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}.$$

Let A be the matrix with these vectors as columns and $\vec{b}$ the vector $[1\ 0\ 0]$.

a) *Compute the rank of A to determine how many of these vectors are linearly independent.*

b) *Determine if $\vec{b}$ is in the span of the given vectors by using row reduction to try to solve $A\vec{x} = \vec{b}$.*

c) *Look at the columns of the row-reduced form of A. Is $\vec{b}$ in the span of those vectors?*

d) *What do these last two parts tell you about the span of the columns of a matrix, and the span of the columns of the row-reduced matrix?*

e) *Now, build a matrix D with these vectors as rows. Row-reduce this matrix to get a matrix D_2.*

f) *Is $\vec{b}$ in the span of the rows of D_2? You can't check this in using the matrix form; instead, just brute force it based on the form of D_2. What does this potentially say about the span of the rows of D and the rows of D_2?*

Exercise 16 *Complete* Exercise *with*

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -6 \\ 2 \\ 3 \\ -1 \end{bmatrix} \quad \begin{bmatrix} -13 \\ 3 \\ 1 \\ 1 \end{bmatrix} \quad \vec{v}_4 \begin{bmatrix} 11 \; - \; 1 \\ -5 \\ -1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$