Practice for Higher Order ODEs

Why?

Exercise 1 *Find the general solution for* $y''' - y'' + y' - y = 0$.

Exercise 2 *Find the general solution of* $y^{(5)} - y^{(4)} = 0$.

Exercise 3 *Find the general solution for* $y^{(4)} - 5y''' + 6y'' = 0$.

Exercise 4 *Find the general solution for* $y''' + 2y'' + 2y' = 0$.

Exercise 5 *Suppose the characteristic equation for an ODE is* $(r-1)^2(r-2)^2 = 0$.

a) *Find such a differential equation.*

b) *Find its general solution.*

Exercise 6 *Suppose that a fourth order equation has a solution* $y = 2e^{4x}x\cos x$.

a) *Find such an equation.*

b) *Find the initial conditions that the given solution satisfies.*

Exercise 7 *Suppose that the characteristic equation of a third order differential equation has roots* $\pm 2i$ *and 3.*

a) *What is the characteristic equation?*

b) *Find the corresponding differential equation.*

c) *Find the general solution.*

Exercise 8 *Find the general solution for the equation of* Exercise .

Exercise 9 *Find the general solution of*

$$y^{(4)} - y''' - 5y'' - 23y' - 20y = 0.$$

Exercise 10 *Find the general solution of*

$$y''' - 6y'' + 13y' - 10y = 4e^x + 5e^{3x} - 20.$$

Exercise 11 *Find the general solution of*

$$y''' - 3y' + 2y = 2e^x - e^{3x}.$$

Exercise 12 *Find the general solution of*

$$y''' + 2y'' + y' + 2y = 3\cos(x) + x.$$

Exercise 13 *Find the general solution of*

$$y^{(4)} + 2y'' + y = 4x\cos(x) - e^{3x} + 1$$

Hint: Remember, the guess needs to make sure that no terms in it solve the homogeneous equation.

Exercise 14 *Show that $y = \cos(2t)$ is a solution to $y^{(4)} + 2y''' + 9y'' + 8y' + 20y = 0$. This tells us something about the factorization of the characteristic polynomial of this DE. Factor the characteristic polynomial completely, and solve the DE.*

Exercise 15 *Consider*

$$y''' - y'' - 8y' + 12y = 0. \tag{1}$$

a) *Show that $y = e^{2t}$ is a solution of (1).*

b) *Find the general solution to (1).*

c) *Solve $y''' - y'' - 8y' + 12y = e^{2t}$.*

Exercise 16 *Let $f(x) = e^x - \cos x$, $g(x) = e^x + \cos x$, and $h(x) = \cos x$. Are $f(x)$, $g(x)$, and $h(x)$ linearly independent? If so, show it, if not, find a linear combination that works.*

Exercise 17 *Let $f(x) = 0$, $g(x) = \cos x$, and $h(x) = \sin x$. Are $f(x)$, $g(x)$, and $h(x)$ linearly independent? If so, show it, if not, find a linear combination that works.*

Exercise 18 *Are e^x, e^{x+1}, e^{2x}, $\sin(x)$ linearly independent? If so, show it, if not find a linear combination that works.*

Exercise 19 *Are x, x^2, and x^4 linearly independent? If so, show it, if not, find a linear combination that works.*

Exercise 20 *Are e^x, xe^x, and x^2e^x linearly independent? If so, show it, if not, find a linear combination that works.*

Exercise 21 *Are $\sin(x)$, x, $x\sin(x)$ linearly independent? If so, show it, if not find a linear combination that works.*

Exercise 22 *Show that $\{e^t, te^t, e^{-t}, te^{-t}\}$ is a linearly independent set.*

Exercise 23 *Solve $1001y''' + 3.2y'' + \pi y' - \sqrt{4}y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$.*

Exercise 24 *Could $y = t^2 \cos t$ be a solution of a homogeneous DE with constant real coefficients? If so, give the minimum possible order of such a DE, and state which functions must also be solutions. If not, explain why this is impossible.*

Exercise 25 *Find a linear DE with constant real coefficients whose general solution is*

$$y = c_1e^{2t} + c_2e^{-t}\cos(4t) + c_3e^{-t}\sin(2t),$$

or explain why there is no such thing.

Exercise 26 *Find an equation such that $y = xe^{-2x}\sin(3x)$ is a solution.*

Exercise 27 *Find an equation of minimal order such that $y = \cos(x)$, $y = \sin(x)$, $y = e^x$ are solutions.*

Exercise 28 *Find an equation of minimal order such that $y = \cos(x)$, $y = \sin(2x)$, $y = e^{3x}$ are solutions.*

Exercise 29 *Find a homogeneous DE with general solution*

$$y = c_1e^t + c_2e^{-t} + c_3\cos t + c_4\sin t + c_5te^t + c_6te^{-t} + c_7t\cos t + c_8t\sin t.$$