Practice for Separable ODEs

Why?

Exercise 1 *Solve* $y' = y^3$ *for* $y(0) = 1$.

Exercise 2 *Solve* $x' = \dfrac{1}{x^2}$, $x(1) = 1$.

Exercise 3 *Solve* $y' = (y-1)(y+1)$ *for* $y(0) = 3$. *(Note: Requires partial fractions)*

Exercise 4 *Solve* $x' = \dfrac{1}{\cos(x)}$, $x(0) = \dfrac{\pi}{2}$.

Exercise 5 *Solve* $\dfrac{dy}{dx} = \dfrac{1}{y+1}$ *for* $y(0) = 0$.

Exercise 6 *Solve* $y' = {}^{x}/_{y}$.

Exercise 7 *Solve* $y' = x^2 y$.

Exercise 8 *Consider the differential equation*

$$\frac{dy}{dx} = \frac{2x}{y}$$

a) *Find the general solution as an implicit function.*

b) *Find the solution to this differential equation as an* explicit *function with* $y(1) = 4$.

c) *Find the solution to this differential equation as an* explicit *function with* $y(0) = -2$.

Exercise 9 *Solve* $y' = y^n$, $y(0) = 1$, *where* n *is a positive integer. Hint: You have to consider different cases.*

Exercise 10 *Solve* $\dfrac{dx}{dt} = (x^2 - 1)\, t$, *for* $x(0) = 0$. *(Note: Requires partial fractions)*

Exercise 11 *Solve* $\dfrac{dx}{dt} = x \sin(t)$, *for* $x(0) = 1$.

Learning outcomes:

Exercise 12 *Solve* $y' = 2xy$.

Exercise 13 *Solve* $y' = ye^{2x}$ *with* $y(0) = 4$.

Exercise 14 *Solve* $\dfrac{dy}{dx} = xy + x + y + 1$. *Hint: Factor the right-hand side.*

Exercise 15 *Solve* $x' = 3xt^2 - 3t^2$, $x(0) = 2$.

Exercise 16 *Find the general solution of* $y' = e^x$, *and then* $y' = e^y$.

Exercise 17 *Solve* $xy' = y + 2x^2y$, *where* $y(1) = 1$.

Exercise 18 *Find an implicit solution for* $x' = \dfrac{1}{3x^2 + 1}$, $x(0) = 1$.

Exercise 19 *Solve* $\dfrac{dy}{dx} = \dfrac{y^2 + 1}{x^2 + 1}$, *for* $y(0) = 1$.

Exercise 20 *Find an implicit solution for* $\dfrac{dy}{dx} = \dfrac{x^2 + 1}{y^2 + 1}$, *for* $y(0) = 1$.

Exercise 21 *Find an implicit solution to* $y' = \dfrac{\sin(x)}{\cos(y)}$.

Exercise 22 *Find an implicit solution for* $xy' = \dfrac{x^2 + 1}{y^2 - 1}$ *with* $y(3) = 2$.

Exercise 23 *Find an explicit solution for* $y' = xe^{-y}$, $y(0) = 1$.

Exercise 24 *Find an explicit solution to* $xy' = y^2$, $y(1) = 1$.

Exercise 25 *Find an explicit solution for* $xy' = e^{-y}$, *for* $y(1) = 1$.

Exercise 26 *Find an explicit solution for* $y' = y^2(x^4 + 1)$ *with* $y(1) = 2$.

Exercise 27 *Find an explicit solution for* $y' = \dfrac{\cos(x)+1}{y}$ *with* $y(0) = 4$.

Exercise 28 *Find an explicit solution for* $y' = ye^{-x^2}$, $y(0) = 1$. *It is alright to leave a definite integral in your answer.*

Exercise 29 *Is the equation* $y' = x + y + 1$ *separable? If so, find the general solution, if not, explain why.*

Exercise 30 *Is the equation* $y' = ty^2 + t$ *separable? If so, find the general solution, if not, explain why.*

Exercise 31 *Is the equation* $y' = xy^2 + 3y^2 - 4x - 12$ *separable? If so, find the general solution, if not, explain why. (Note: Requires partial fractions)*

Exercise 32 *Suppose a cup of coffee is at 100 degrees Celsius at time* $t = 0$, *it is at 70 degrees at* $t = 10$ *minutes, and it is at 50 degrees at* $t = 20$ *minutes. Compute the ambient temperature.*

Exercise 33 *Take* ***Normally a reference to a previous example goes here.*** *with the same numbers: 89 degrees at* $t = 0$, *85 degrees at* $t = 1$, *and ambient temperature of 22 degrees. Suppose these temperatures were measured with precision of* ± 0.5 *degrees. Given this imprecision, the time it takes the coffee to cool to (exactly) 60 degrees is also only known in a certain range. Find this range. Hint: Think about what kind of error makes the cooling time longer and what shorter.*

Exercise 34 *A population* x *of rabbits on an island is modeled by* $x' = x - \left(1/1000\right)x^2$, *where the independent variable is time in months. At time* $t = 0$, *there are 40 rabbits on the island.*

a) *Find the solution to the equation with the initial condition.*

b) *How many rabbits are on the island in 1 month, 5 months, 10 months, 15 months (round to the nearest integer).*