Practice for Solving Exact ODEs

Why?

Exercise 1 *Solve the following exact equations, implicit general solutions will suffice:*

a) $(2xy + x^2)\,dx + (x^2 + y^2 + 1)\,dy = 0$

b) $x^5 + y^5 \dfrac{dy}{dx} = 0$

c) $e^x + y^3 + 3xy^2 \dfrac{dy}{dx} = 0$

d) $(x + y)\cos(x) + \sin(x) + \sin(x)y' = 0$

Exercise 2 *Solve the following exact equations, implicit general solutions will suffice:*

a) $\cos(x) + ye^{xy} + xe^{xy}y' = 0$

b) $(2x + y)\,dx + (x - 4y)\,dy = 0$

c) $e^x + e^y \dfrac{dy}{dx} = 0$

d) $(3x^2 + 3y)\,dx + (3y^2 + 3x)\,dy = 0$

Exercise 3 *Solve the differential equation* $(2ye^{2xy} - 2x) + (2xe^{2xy} + \cos(y))y' = 0$

Exercise 4 *Solve the differential equation* $(-y\sin(xy) - 2xe^{x^2}) + (-x\sin(xy) + 1)y' = 0$

Exercise 5 *Solve the differential equation* $(2x + 3y\sin(xy)) + (3x\sin(xy) - e^y)y' = 0$ *with* $y(2) = 0$.

Exercise 6 *Solve the differential equation* $x + yy' = 0$ *with* $y(0) = 8$. *Write this as an explicit function and determine the interval of* x *values where the solution is valid.*

Exercise 7 *Solve the differential equation* $2x - 2 + (8y + 16)y' = 0$ *with* $y(2) = 0$. *Write this as an explicit function and determine the interval of* x *values where the solution is valid.*

Exercise 8 *Find the integrating factor for the following equations making them into exact equations. You can either use the formulas in this section or guess what the integrating factor should be.*

a) $e^{xy}\,dx + \dfrac{y}{x}e^{xy}\,dy = 0$

b) $\dfrac{e^x + y^3}{y^2}\,dx + 3x\,dy = 0$

c) $(4x^5 + 9x^4y^2 + 10\dfrac{y}{x})\,dx + (6x^5y + 3x^2y^2 - 5)\,dy = 0$

d) $2\sin(y)\,dx + x\cos(y)\,dy = 0$

Learning outcomes:

Exercise 9 *Find the integrating factor for the following equations making them into exact equations:*

a) $\frac{1}{y}\,dx + 3y\,dy = 0$

b) $dx - e^{-x-y}\,dy = 0$

c) $\left(\frac{\cos(x)}{y^2} + \frac{1}{y}\right)dx + \frac{x}{y^2}\,dy = 0$

d) $\left(2y + \frac{y^2}{x}\right)dx + (2y + x)\,dy = 0$

Exercise 10 *Suppose you have an equation of the form:* $f(x) + g(y)\frac{dy}{dx} = 0$.

a) *Show it is exact.*

b) *Find the form of the potential function in terms of f and g.*

Exercise 11 *Suppose that we have the equation* $f(x)\,dx - dy = 0$.

a) *Is this equation exact?*

b) *Find the general solution using a definite integral.*

Exercise 12 *Find the potential function $F(x, y)$ of the exact equation* $\frac{1+xy}{x}\,dx + \left({}^{1}/_{y} + x\right)dy = 0$ *in two different ways.*

a) *Integrate M in terms of x and then differentiate in y and set to N.*

b) *Integrate N in terms of y and then differentiate in x and set to M.*

Exercise 13 *A function $u(x, y)$ is said to be a* harmonic function *if* $u_{xx} + u_{yy} = 0$.

a) *Show if u is harmonic, $-u_y\,dx + u_x\,dy = 0$ is an exact equation. So there exists (at least locally) the so-called* harmonic conjugate *function $v(x, y)$ such that $v_x = -u_y$ and $v_y = u_x$.*

Verify that the following u are harmonic and find the corresponding harmonic conjugates v:

b) $u = 2xy$

c) $u = e^x \cos y$

d) $u = x^3 - 3xy^2$

Exercise 14

a) *Show that every separable equation $y' = f(x)g(y)$ can be written as an exact equation, and verify that it is indeed exact.*

b) *Using this rewrite $y' = xy$ as an exact equation, solve it and verify that the solution is the same as it was in* **Normally a reference to a previous example goes here.**.