Practice for Solving Exact ODEs

Why?

*Note: Exercises marked with * have answers in the back of the book.*

Exercise 1 *Show that $x = e^{4t}$ is a solution to $x''' - 12x'' + 48x' - 64x = 0$.*

Exercise 2 *Show that $x = e^{-2t}$ is a solution to $x'' + 4x' + 4x = 0$.*

Exercise 3 *Show that $x = e^{t}$ is not a solution to $x''' - 12x'' + 48x' - 64x = 0$.*

Exercise 4 *Is $y = \sin t$ a solution to $\left(\frac{dy}{dt}\right)^2 = 1 - y^2$? Justify.*

Exercise 5 *Is $y = x^2$ a solution to $x^2y'' - 2y = 0$? Justify.*

Exercise 6 *Let $y'' + 2y' - 8y = 0$. Now try a solution of the form $y = e^{rx}$ for some (unknown) constant r. Is this a solution for some r? If so, find all such r.*

Exercise 7 *Let $xy'' - y' = 0$. Try a solution of the form $y = x^r$. Is this a solution for some r? If so, find all such r.*

Exercise 8 *Verify that $x = Ce^{-2t}$ is a solution to $x' = -2x$. Find C to solve for the initial condition $x(0) = 100$.*

Exercise 9 *Verify that $x = C_1e^{-t} + C_2e^{2t}$ is a solution to $x'' - x' - 2x = 0$. Find C_1 and C_2 to solve for the initial conditions $x(0) = 10$ and $x'(0) = 0$.*

Exercise 10 *Verify that $x = C_1e^{t} + C_2$ is a solution to $x'' - x' = 0$. Find C_1 and C_2 so that x satisfies $x(0) = 10$ and $x'(0) = 100$.*

Exercise 11 *Find a solution to $(x')^2 + x^2 = 4$ using your knowledge of derivatives of functions that you know from basic calculus.*

Exercise 12 *Solve $\frac{d\varphi}{ds} = 8\varphi$ and $\varphi(0) = -9$.*

Learning outcomes:

Exercise 13 *Solve:*

a) $\dfrac{dA}{dt} = -10A, \quad A(0) = 5$

b) $\dfrac{dH}{dx} = 3H, \quad H(0) = 1$

c) $\dfrac{d^2y}{dx^2} = 4y, \quad y(0) = 0, \quad y'(0) = 1$

d) $\dfrac{d^2x}{dy^2} = -9x, \quad x(0) = 1, \quad x'(0) = 0$

Exercise 14 *Solve:*

a) $\dfrac{dx}{dt} = -4x, \quad x(0) = 9$

b) $\dfrac{d^2x}{dt^2} = -4x, \quad x(0) = 1, \quad x'(0) = 2$

c) $\dfrac{dp}{dq} = 3p, \quad p(0) = 4$

d) $\dfrac{d^2T}{dx^2} = 4T, \quad T(0) = 0, \quad T'(0) = 6$

Exercise 15 *Is there a solution to $y' = y$, such that $y(0) = y(1)$?*

Exercise 16 *The population of city X was 100 thousand 20 years ago, and the population of city X was 120 thousand 10 years ago. Assuming constant growth, you can use the exponential population model (like for the bacteria). What do you estimate the population is now?*

Exercise 17 *Suppose that a football coach gets a salary of one million dollars now, and a raise of 10% every year (so exponential model, like population of bacteria). Let s be the salary in millions of dollars, and t is time in years.*

a) *What is $s(0)$ and $s(1)$.*

b) *Approximately how many years will it take for the salary to be 10 million.*

c) *Approximately how many years will it take for the salary to be 20 million.*

d) *Approximately how many years will it take for the salary to be 30 million.*
