Practice for Substitution

Why?

Hint: Answers need not always be in closed form.

Exercise   1 *Solve $y' + y(x^2 - 1) + xy^6 = 0$, with $y(1) = 1$.*

Exercise   2 *Solve $xy' + y + y^2 = 0$, $y(1) = 2$.*

Exercise   3 *Solve $2yy' + 1 = y^2 + x$, with $y(0) = 1$.*

Exercise   4 *Solve $xy' + y + x = 0$, $y(1) = 1$.*

Exercise   5 *Solve $y' + xy = y^4$, with $y(0) = 1$.*

Exercise   6 *Solve $y' + 3y = 2xy^4$.*

Exercise   7 *Solve $xy' - 2y = (3x^2 - x^{-3})y^5$ with $y(1) = 2$.*

Exercise   8 *Solve $y' + 5y = \dfrac{e^{2x}}{y^2}$.*

Exercise   9 *Solve $y^2 y' = y^3 - 3x$, $y(0) = 2$.*

Exercise   10 *Solve $yy' + x = \sqrt{x^2 + y^2}$. (Hint: What is $\dfrac{d}{dx}(x^2 + y^2)$)*

Exercise   11 *Solve $y' = (x + y - 1)^2$.*

Exercise   12 *Solve $y' = \dfrac{x^2 - y^2}{xy}$, with $y(1) = 2$.*

Exercise   13 *Solve $2yy' = e^{y^2 - x^2} + 2x$.*

Learning outcomes:

Exercise   14   *Consider the DE*

$$\frac{dy}{dt} = \left(y - \frac{1}{t}\right)^2 - \frac{1}{t^2}. \tag{1}$$

a) Explain why (1) is not a linear equation.

b) Use a Bernoulli substitution to solve (1).