Practice for Autonomous Equations

Why?

Exercise 1 *Consider* $x' = x^2$.

a) *Draw the phase diagram, find the critical points, and mark them asymptotically stable, semistable, or unstable.*

b) *Sketch typical solutions of the equation.*

c) *Find* $\lim_{t\to\infty} x(t)$ *for the solution with the initial condition* $x(0) = -1$.

Exercise 2 *Consider* $x' = \sin x$.

a) *Draw the phase diagram for* $-4\pi \le x \le 4\pi$. *On this interval mark the critical points asymptotically stable, semistable, or unstable.*

b) *Sketch typical solutions of the equation.*

c) *Find* $\lim_{t\to\infty} x(t)$ *for the solution with the initial condition* $x(0) = 1$.

Exercise 3 *Let* $x' = (x-1)(x-2)x^2$.

a) *Sketch the phase diagram and find critical points.*

b) *Classify the critical points.*

c) *If* $x(0) = 0.5$, *then find* $\lim_{t\to\infty} x(t)$.

Exercise 4 *Let* $y' = (y-2)(y^2+1)(y+3)$. *Sketch a phase diagram for this differential equation. Find and classify all critical points. If* $y(0) = 0$, *what will happen to the solution as* $t \to \infty$*?*

Exercise 5 *Find and classify all equilibrium solutions for the differential equation* $x' = (x-2)^2(x+1)(x+3)^3(x+2)$.

Exercise 6 *Let* $y' = (y-3)(y+2)^2 e^y$. *Sketch a phase diagram for this differential equation. Find and classify all critical points. If* $y(0) = 0$, *what will happen to the solution as* $t \to \infty$*?*

Exercise 7 *Consider the DE* $\frac{dy}{dt} = y^5 - 3y^4 + 3y^3 - y^2$. *Find and classify all equilibrium solutions of this DE. Then sketch a representative selection of solution curves.*

Exercise 8 *Let* $x' = e^{-x}$.

Learning outcomes:

a) *Find and classify all critical points.*

b) *Find* $\lim_{t\to\infty} x(t)$ *given any initial condition.*

Exercise 9 *Suppose* $f(x)$ *is positive for* $0 < x < 1$, *it is zero when* $x = 0$ *and* $x = 1$, *and it is negative for all other* x.

a) *Draw the phase diagram for* $x' = f(x)$, *find the critical points, and mark them asymptotically stable, semistable, or unstable.*

b) *Sketch typical solutions of the equation.*

c) *Find* $\lim_{t\to\infty} x(t)$ *for the solution with the initial condition* $x(0) = 0.5$.

Exercise 10 *Suppose* $\frac{dx}{dt} = (x-\alpha)(x-\beta)$ *for two numbers* $\alpha < \beta$.

a) *Find the critical points, and classify them.*

For b), c), d), find $\lim_{t\to\infty} x(t)$ *based on the phase diagram.*

b) $x(0) < \alpha$,

c) $\alpha < x(0) < \beta$,

d) $\beta < x(0)$.

Exercise 11 *A disease is spreading through the country. Let* x *be the number of people infected. Let the constant* S *be the number of people susceptible to infection. The infection rate* $\frac{dx}{dt}$ *is proportional to the product of already infected people,* x, *and the number of susceptible but uninfected people,* $S - x$.

a) *Write down the differential equation.*

b) *Supposing* $x(0) > 0$, *that is, some people are infected at time* $t = 0$, *what is* $\lim_{t\to\infty} x(t)$.

c) *Does the solution to part b) agree with your intuition? Why or why not?*