Practice for Complex Roots

Why?

Exercise 1 *Write* $3\cos(2x) + 3\sin(2x)$ *in the form* $R\cos(\beta x - \delta)$.

Exercise 2 *Write* $2\cos(3x) + \sin(3x)$ *in the form* $R\cos(\beta x - \delta)$.

Exercise 3 *Write* $3\cos(x) - 4\sin(x)$ *in the form* $R\cos(\beta x - \delta)$.

Exercise 4 *Show that* $e^{2x}\cos(x)$ *and* $e^{2x}\sin(x)$ *are linearly independent.*

Exercise 5 *Find the general solution of* $2y'' + 50y = 0$.

Exercise 6 *Find the general solution of* $y'' - 6y' + 13y = 0$.

Exercise 7 *Find the solution to* $y'' - 2y' + 5y = 0$ *with* $y(0) = 3$ *and* $y'(0) = 2$.

Exercise 8 *Find the general solution of* $y'' + 2y' - 3y = 0$.

Exercise 9 *Find the solution to* $2y'' + y' + y = 0$, $y(0) = 1$, $y'(0) = -2$.

Exercise 10 *Find the solution to* $z''(t) = -2z'(t) - 2z(t)$, $z(0) = 2$, $z'(0) = -2$.

Exercise 11 *Let us revisit the Cauchy–Euler equations of* ***Normally a reference to a previous exercise goes here.****. Suppose now that* $(b-a)^2 - 4ac < 0$. *Find a formula for the general solution of* $ax^2y'' + bxy' + cy = 0$. *Hint: Note that* $x^r = e^{r\ln x}$.

Exercise 12 *Construct an equation such that* $y = C_1e^{-2x}\cos(3x) + C_2e^{-2x}\sin(3x)$ *is the general solution.*

Exercise 13 *Find a second order, constant coefficient differential equation with general solution given by* $y(t) = C_1e^{x}\cos(2x) + C_2e^{2x}\sin(x)$ *or explain why there is no such thing.*

Exercise 14 *Find a second order, constant coefficient differential equation with general solution given by* $y(t) = C_1 e^x \cos(2x) + C_2 e^x \sin(2x)$ *or explain why there is no such thing.*

Exercise 15 *Find the solution to the initial value problem*

$$y'' + 4y' + 5y = 0 \qquad y(0) = 3,\ y'(0) = -1.$$

Determine a value T *so that* $|y(x)| < 0.02$ *for all* $x > T$.

Exercise 16 *Find the solution to the initial value problem*

$$y'' + 6y' + 13y = 0 \qquad y(0) = 4,\ y'(0) = 7.$$

Determine a value T *so that* $|y(x)| < 0.01$ *for all* $x > T$.