Practice for bifurcation diagrams

Why?

Exercise 1 *Start with the logistic equation* $\frac{dx}{dt} = kx(M-x)$*. Suppose we modify our harvesting. That is we will only harvest an amount proportional to current population. In other words, we harvest* hx *per unit of time for some* $h > 0$ *(Similar to earlier example with* h *replaced with* hx*).*

a) *Construct the differential equation.*

b) *Show that if* $kM > h$*, then the equation is still logistic.*

c) *What happens when* $kM < h$*?*

Exercise 2 *Assume that a population of fish in a lake satisfies* $\frac{dx}{dt} = kx(M-x)$*. Now suppose that fish are continually added at* A *fish per unit of time.*

a) *Find the differential equation for* x*.*

b) *What is the new limiting population?*

Exercise 3 *Consider the differential equation with parameter* α *given by* $y' = y(y - \alpha + 1)$*.*

a) *Sketch a phase diagram for this differential equation with* $\alpha = -3$*,* $\alpha = 1$*, and* $\alpha = 3$*.*

b) *Draw a bifurcation diagram for this differential equation with parameter.*

c) *What is the bifurcation point for this equation? What changes when* α *passes over the bifurcation point?*

Exercise 4 *Consider the differential equation with parameter* α *given by* $y' = y^2(y^2 - \alpha)$*.*

a) *Sketch a phase diagram for this differential equation with* $\alpha = -3$*,* $\alpha = 0$*, and* $\alpha = 3$*.*

b) *Draw a bifurcation diagram for this differential equation with parameter.*

c) *What is the bifurcation point for this equation? What changes when* α *passes over the bifurcation point?*

Exercise 5 *Consider the differential equation with parameter* α *given by* $y' = y(\alpha - y)$*.*

a) *Sketch a phase diagram for this differential equation with* $\alpha = -3$*,* $\alpha = 0$*, and* $\alpha = 3$*.*

b) *Draw a bifurcation diagram for this differential equation with parameter.*

c) *What is the bifurcation point for this equation? What changes when* α *passes over the bifurcation point?*

Learning outcomes: