

Practice for Slope Fields

Why?

Exercise 1 Sketch slope field for $y' = e^{x-y}$. How do the solutions behave as x grows? Can you guess a particular solution by looking at the slope field?

Exercise 2 Sketch the slope field of $y' = y^3$. Can you visually find the solution that satisfies $y(0) = 0$?

Exercise 3 Sketch slope field for $y' = x^2$.

Exercise 4 Sketch slope field for $y' = y^2$.

Exercise 5 For each of the following differential equations, sketch out a slope field on $-3 < x < 3$ and $-3 < y < 3$ and determine the overall behavior of the solutions to the equation as $t \rightarrow \infty$. If this fact depends on the value of the solution at $t = 0$, explain how it changes.

a) $\frac{dy}{dx} = 3 - 2y$

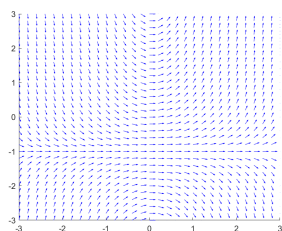
b) $\frac{dy}{dx} = 1 + y$

c) $\frac{dy}{dx} = y - 1$

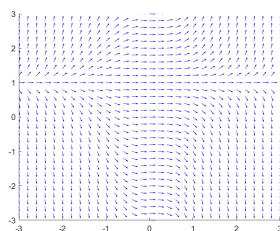
d) $\frac{dy}{dx} = -2 - y$

Exercise 6 Which of the following slope fields corresponds to the differential equation $\frac{dy}{dt} = t(y - 1)$. Explain your reasoning.

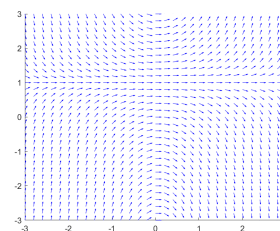
a)



b)

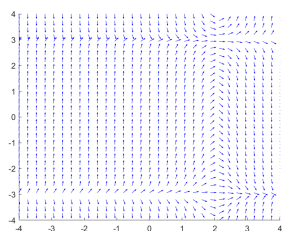


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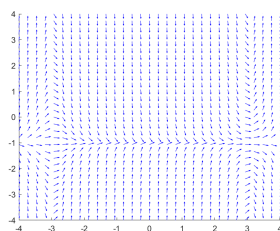


Exercise 7 Which of the following slope fields corresponds to the differential equation $\frac{dy}{dt} = (2 - t)(y^2 - 9)$. Explain your reasoning.

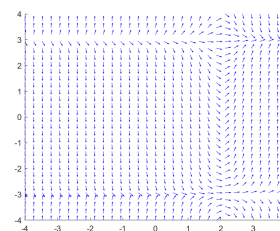
a)



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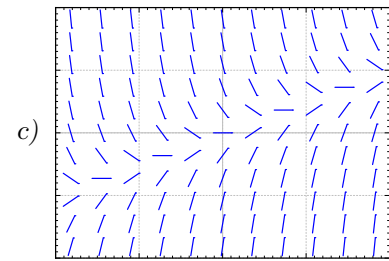
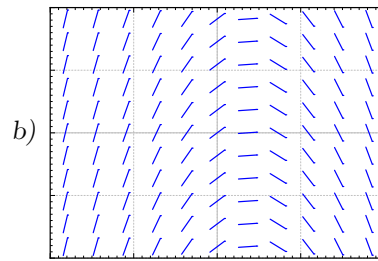
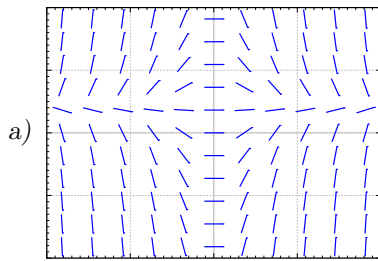


c)

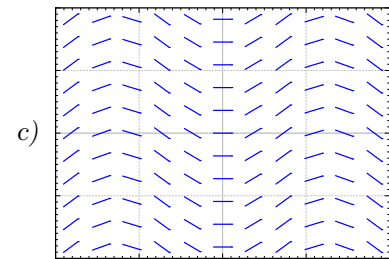
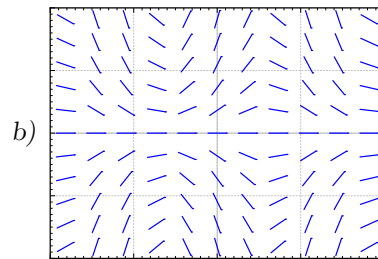
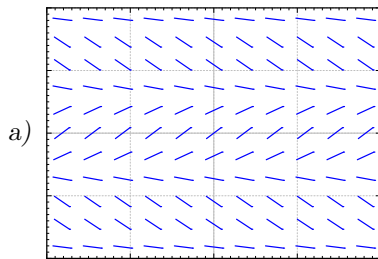


Learning outcomes:

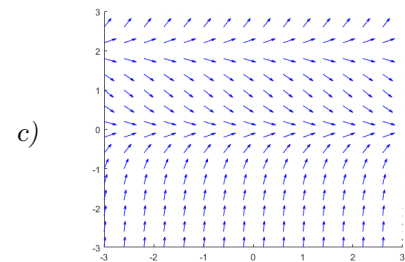
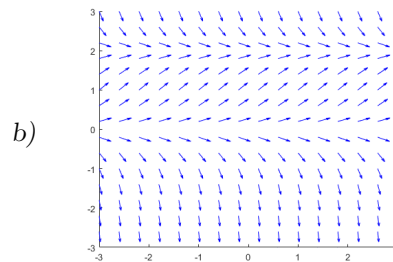
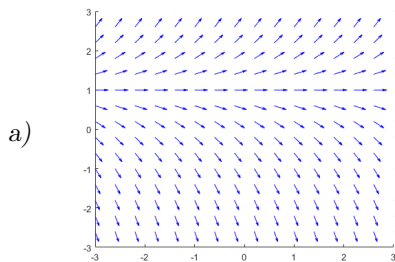
Exercise 8 Match equations $y' = 1 - x$, $y' = x - 2y$, $y' = x(1 - y)$ to slope fields. Justify.



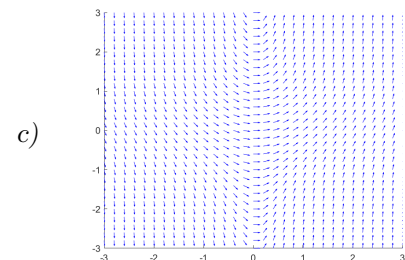
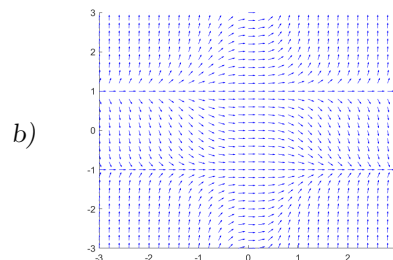
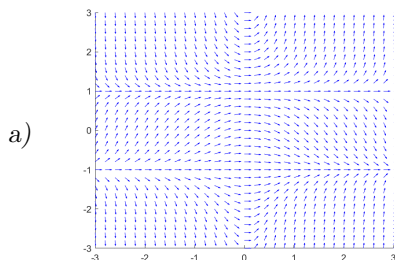
Exercise 9 Match equations $y' = \sin x$, $y' = \cos y$, $y' = y \cos(x)$ to slope fields. Justify.



Exercise 10 Match equations $y' = y(y - 2)$, $y' = y - 1$, $y' = y(2 - y)$ to slope fields. Justify.



Exercise 11 Match equations $y' = t(y^2 + 1)$, $y' = t(y^2 - 1)$, $y' = t^2(y^2 - 1)$ to slope fields. Justify.



Exercise 12 The slope field for the differential equation $y' = (3 - y)(y + 2)$ is below. If we find the solution to this differential equation with initial condition, $y(0) = 1$, what will happen to the solution as $t \rightarrow \infty$? Use the slope field and your knowledge of the equation to determine the long-time behavior of this solution.

Exercise 13 The slope field for the differential equation $y' = (t - 2)(y + 4)(y - 3)$ is below. If we find the solution to this differential equation with initial condition, $y(0) = 1$, what will happen to the solution as $t \rightarrow \infty$? Use the slope field and your knowledge of the equation to determine the long-time behavior of this solution.

Exercise 14 The slope field for the differential equation $y' = (y + 1)(y + 4)$ is below. If we find the solution to this differential equation with initial condition, $y(0) = 1$, what will happen to the solution as $t \rightarrow \infty$? Use the slope field and your knowledge of the equation to determine the long-time behavior of this solution.

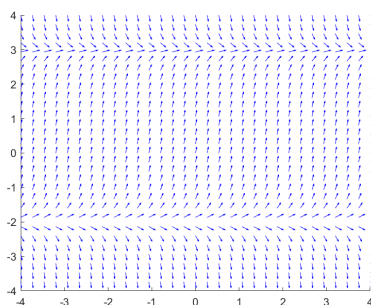


Figure 1: Exercise

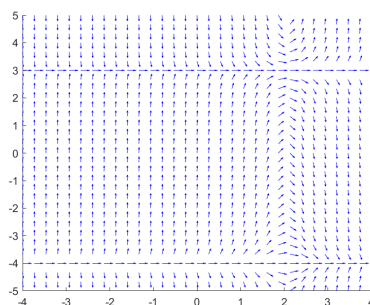


Figure 2: Exercise

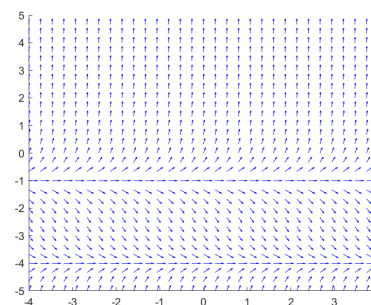


Figure 3: Exercise

Exercise 15 Take $y' = f(x, y)$, $y(0) = 0$, where $f(x, y) > 1$ for all x and y . If the solution exists for all x , can you say what happens to $y(x)$ as x goes to positive infinity? Explain.

Exercise 16 Suppose $y' = f(x, y)$. What will the slope field look like, explain and sketch an example, if you know the following about $f(x, y)$:

- | | |
|---------------------------------------|--|
| a) f does not depend on y . | b) f does not depend on x . |
| c) $f(t, t) = 0$ for any number t . | d) $f(x, 0) = 0$ and $f(x, 1) = 1$ for all x . |

Exercise 17 Describe what each of the following facts about the function $f(x, y)$ tells you about the slope field for the differential equation $y' = f(x, y)$.

- $f(2, y) = 0$ for all y
- $f(x, -x) = 0$ for all x
- $f(x, x) = 1$ for all x
- $f(x, -1) = 0$ for all x