Practice for Using the EU-Thm

Why?

Exercise 1 *Is it possible to solve the equation* $y' = \dfrac{xy}{\cos x}$ *for* $y(0) = 1$*? Justify.*

Exercise 2 *Is it possible to solve the equation* $y' = y\sqrt{|x|}$ *for* $y(0) = 0$*? Is the solution unique? Justify.*

Exercise 3 *Consider the differential equation* $y' + \dfrac{1}{t-2}y = \dfrac{1}{t+3}$.

a) *Is this equation linear or non-linear?*

b) *What is the maximum guaranteed interval of existence for the solution to this equation with initial condition* $y(0) = 3$*?*

c) *What if we start with the initial condition* $y(4) = 0$*?*

Exercise 4 *Consider the differential equation* $y' + \dfrac{1}{t+2}y = \dfrac{\ln(|t|)}{t-4}$.

a) *Is this equation linear or non-linear?*

b) *What is the maximum guaranteed interval of existence for the solution to this equation with initial condition* $y(-3) = 1$*?*

c) *What if we start with the initial condition* $y(2) = 5$*?*

d) *What happens if we want to start with* $y(4) = 3$*?*

Exercise 5 *Consider the differential equation* $(t+3)y' + t^2 y = \dfrac{1}{t-2}$.

a) *Is this equation linear or non-linear?*

b) *What is the maximum guaranteed interval of existence for the solution to this equation with initial condition* $y(-2) = 1$*?*

c) *What if we start with the initial condition* $y(-4) = 5$*?*

d) *What happens if we want to start with* $y(4) = 2$*?*

Exercise 6 *Consider the differential equation* $y' = y^2$.

Learning outcomes:

a) *Is this equation linear or non-linear?*

b) *What is the most we can say about the interval of existence for the solution to this equation with initial condition* $y(0) = 1$*?*

c) *Find the solution to this differential equation with* $y(0) = 1$*. Over what values of* x *does this solution exist?*

d) *Find the solution to this differential equation with* $y(0) = 4$*. Over what values of* x *does this solution exist?*

e) *Find the solution to this differential equation with* $y(0) = -2$*. Over what values of* x *does this solution exist?*

f) *Do any of these contradict your answer in (b)?*

Exercise 7 *Consider the differential equation* $y' = y^2 + 4$*.*

a) *Is this equation linear or non-linear?*

b) *What is the most we can say about the interval of existence for the solution to this equation with initial condition* $y(0) = 0$*?*

c) *Find the solution to this differential equation with* $y(0) = 0$*. Over what values of* x *does this solution exist?*

Exercise 8 *Consider the differential equation* $y' = x(y+1)^2$*.*

a) *Is this equation linear or non-linear?*

b) *If we set* $f(x, y) = x(y+1)^2$*, for what values of* x *and* y *are* f *and* $\dfrac{\partial f}{\partial y}$ *continuous?*

c) *What is the most we can say about the interval of existence for the solution to this equation with initial condition* $y(0) = 1$*?*

d) *Find the solution to this differential equation with* $y(0) = 1$*. Over what values of* x *does this solution exist?*

Exercise 9 *Take* $(y - x)y' = 0$, $y(0) = 0$*.*

a) *Find two distinct solutions.*

b) *Explain why this does not violate Picard's theorem.*

Exercise 10 *Find a solution to* $y' = |y|$, $y(0) = 0$*. Does Picard's theorem apply?*

Exercise 11 *Consider the IVP* $y' \cos t + y \sin t = 1$; $y(\pi/6) = 1$*.*

a) *The Existence and Uniqueness Theorem guarantees a unique solution to this IVP on what interval?*

b) *Find this solution explicitly.*

Exercise 12 *Take an equation $y' = (y - 2x)g(x, y) + 2$ for some function $g(x, y)$. Can you solve the problem for the initial condition $y(0) = 0$, and if so what is the solution?*

Exercise 13 *Consider the differential equation $y' = e^x(2 - y)$.*

a) *Verify that $y = 2$ is a solution to this differential equation.*

b) *Assume that we look for the solution with $y(0) = 0$. Is it possible that $y(x) = 3$ for some later time x? Why or why not?*

c) *Based on this, what do we know about the solution with $y(0) = 5$?*

Exercise 14 *Suppose $y' = f(x, y)$ is such that $f(x, 1) = 0$ for every x, f is continuous and $\dfrac{\partial f}{\partial y}$ exists and is continuous for every x and y.*

a) *Guess a solution given the initial condition $y(0) = 1$.*

b) *Can graphs of two solutions of the equation for different initial conditions ever intersect?*

c) *Given $y(0) = 0$, what can you say about the solution. In particular, can $y(x) > 1$ for any x? Can $y(x) = 1$ for any x? Why or why not?*

Exercise 15 *Consider the differential equation $y' = y^2 - 4$.*

a) *Verify that $y = 2$ and $y = -2$ are both solutions to this differential equation.*

b) *Verify that the hypotheses of Picard's theorem are satisfies for this equation.*

c) *Assume that we solve this differential equation with $y(0) = 1$. Is it possible for the solution to reach $y = 3$ at any point? Why or why not?*

d) *Assume that we solve this differential equation with $y(0) = -1$. Is it possible for the solution to reach $y = -4$ at any point? Why or why not?*

Exercise 16 *Is it possible to solve $y' = xy$ for $y(0) = 0$? Is the solution unique?*

Exercise 17 *Is it possible to solve $y' = \dfrac{x}{x^2 - 1}$ for $y(1) = 0$?*

Exercise 18 *Suppose*

$$f(y) = \begin{cases} 0 & \text{if } y > 0, \\ 1 & \text{if } y \leq 0. \end{cases}$$

Does $y' = f(y)$, $y(0) = 0$ have a continuously differentiable solution? Does Picard apply? Why, or why not?

Exercise 19 *Consider an equation of the form $y' = f(x)$ for some continuous function f, and an initial condition $y(x_0) = y_0$. Does a solution exist for all x? Why or why not?*
