Practice for Solving First Order Linear ODEs

Why?

In the exercises, feel free to leave answer as a definite integral if a closed form solution cannot be found. If you can find a closed form solution, you should give that.

Exercise 1 *Solve* $y' + xy = x$.

Exercise 2 *Solve* $y' + 6y = e^x$.

Exercise 3 *Solve* $y' + 4y = x^2 e^{-4x}$.

Exercise 4 *Solve* $y' - 3y = xe^x$.

Exercise 5 *Solve* $y' + 3y = e^{4x} - e^{-2x}$ *with* $y(0) = -3$.

Exercise 6 *Solve* $y' - 2y = x + 4$.

Exercise 7 *Solve* $xy' + 4y = x^2 - \dfrac{1}{x^2}$.

Exercise 8 *Solve* $xy' - 3y = x - 2$ *with* $y(1) = 3$.

Exercise 9 *Solve* $y' - 4y = \cos(3t)$.

Exercise 10 *Solve* $y' + 3x^2 y = x^2$.

Exercise 11 *Solve* $y' + 3x^2 y = \sin(x)\, e^{-x^3}$, *with* $y(0) = 1$.

Exercise 12 *Solve* $y' + \cos(x)y = \cos(x)$.

Exercise 13 *Solve the IVP* $4ty' + y = 24\sqrt{t};\ y(10000) = 100$.

Learning outcomes:

Exercise 14 *Solve the IVP* $(t^2+1)y' - 2ty = t^2 + 1;\ y(1) = 0.$

Exercise 15 *Solve* $\dfrac{1}{x^2+1}\,y' + xy = 3$, *with* $y(0) = 0$.

Exercise 16 *Solve* $y' + 2\sin(2x)y = 2\sin(2x)$, $y(\pi/2) = 3$.

Exercise 17 *Consider the initial value problem*

$$5y' - 3y = e^{-2t} \qquad y(0) = a$$

for an undetermined value a. *Solve the problem and determine the dependence on the value of* a. *How does the value of the solution as* $t \to \infty$ *depend on the value of* a*?*

Exercise 18 *Find an expression for the general solution to* $y' + 3y = \sin(t^2)$ *with* $y(0) = 2$. *Simplfy your answer as much as possible.*