# Practice for Mechanical Vibrations

*Why?*

**Exercise 1** *Consider a mass and spring system with a mass $m = 2$, spring constant $k = 3$, and damping constant $\gamma = 1$.*

a) *Set up and find the general solution of the system.*

b) *Is the system underdamped, overdamped or critically damped?*

c) *If the system is not critically damped, find a $\gamma$ that makes the system critically damped.*

**Exercise 2** *Do Exercise for $m = 3$, $k = 12$, and $\gamma = 12$.*

**Exercise 3** *Using the mks units (meters-kilograms-seconds), suppose you have a spring with spring constant $4\,\mathrm{N/m}$. You want to use it to weigh items. Assume no friction. You place the mass on the spring and put it in motion.*

a) *You count and find that the frequency is $0.8\,\mathrm{Hz}$ (cycles per second). What is the mass? (Be careful with the units here, the frequency is given in cycles per second, not radians per second.)*

b) *Find a formula for the mass $m$ given the frequency $\omega$ in Hz.*

**Exercise 4** *A mass of 2 kilograms is on a spring with spring constant $k$ newtons per meter with no damping. Suppose the system is at rest and at time $t = 0$ the mass is kicked and starts traveling at 2 meters per second. How large does $k$ have to be to so that the mass does not go further than 3 meters from the rest position?*

**Exercise 5** *Suppose we add possible friction to Exercise . Further, suppose you do not know the spring constant, but you have two reference weights $1\,\mathrm{kg}$ and $2\,\mathrm{kg}$ to calibrate your setup. You put each in motion on your spring and measure the quasi-frequency. For the $1\,\mathrm{kg}$ weight you measured $1.1\,\mathrm{Hz}$, for the $2\,\mathrm{kg}$ weight you measured $0.8\,\mathrm{Hz}$.*

a) *Find $k$ (spring constant) and $\gamma$ (damping constant).*

b) *Find a formula for the mass in terms of the frequency in Hz.* Note that there may be more than one possible mass for a given frequency.

c) *For an unknown object you measured $0.2\,\mathrm{Hz}$, what is the mass of the object? Suppose that you know that the mass of the unknown object is more than a kilogram.*

**Exercise 6** *Suppose you wish to measure the friction a mass of $0.1\,\mathrm{kg}$ experiences as it slides along a floor (you wish to find $\gamma$). You have a spring with spring constant $k = 5\,\mathrm{N/m}$. You take the spring, you attach it to the mass and fix it to a wall. Then you pull on the spring and let the mass go. You find that the mass oscillates with quasi-frequency $1\,\mathrm{Hz}$. What is the friction?*

**Exercise 7** *A $5000\,\mathrm{kg}$ railcar hits a bumper (a spring) at $1\,\mathrm{m/s}$, and the spring compresses by $0.1\,\mathrm{m}$. Assume no damping.*

a) *Find* $k$.

b) *How far does the spring compress when a 10000 kg railcar hits the spring at the same speed?*

c) *If the spring would break if it compresses further than 0.3 m, what is the maximum mass of a railcar that can hit it at 1 m/s?*

d) *What is the maximum mass of a railcar that can hit the spring without breaking at 2 m/s?*

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**Exercise 8** *When attached to a spring, a 2 kg mass stretches the spring by 0.49 m.*

a) *What is the spring constant of this spring? Use 9.8 m/s² as the gravity constant.*

b) *This mass is allowed to come to rest, lifted up by 0.4 m and then released. If there is no damping, set up and solve an initial value problem for the position of the mass as a function of time.*

c) *For a next experiment, you attach a dampener of coefficient 16 Ns/m to the system, and give the same initial condition. Set up and solve an initial value problem for the position of the mass. What type of "dampening" would be used to characterize this situation?*

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**Exercise 9** *A mass of* $m$ *kg is on a spring with* $k = 3\,\mathrm{N/m}$ *and* $c = 2\,\mathrm{Ns/m}$. *Find the mass* $m_0$ *for which there is critical damping. If* $m < m_0$, *does the system oscillate or not, that is, is it underdamped or overdamped?*

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**Exercise 10** *Suppose we have an RLC circuit with a resistor of 100 milliohms (0.1 ohms), inductor of inductance of 50 millihenries (0.05 henries), and a capacitor of 5 farads, with constant voltage.*

a) *Set up the ODE equation for the current* $I$.

b) *Find the general solution.*

c) *Solve for* $I(0) = 10$ *and* $I'(0) = 0$.

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**Exercise 11** *For RLC circuits, we can use either charge or current to set up the equation. Let's see how the two of those compare.*

a) *Assume that we have an RLC circuit with a 30 millihenry inductor, a 120 milliohm resistor, and a capacitor with capacitance* $20/3$ *F. Set up a differential equation for the charge on the capacitor as a function of time.*

b) *Use the same circuit to set up a differential equation for the current through the circuit as a function of time. How do these equations relate?*

c) *Find the general solution to each of these equations.*

d) *Solve the initial value problem for the charge with* $Q(0) = 1/2C$ *and* $Q'(0) = 0$.

e) *Using the fact that* $I = Q'$, *determine the appropriate initial conditions needed for* $I$ *in order to solve for the current in this same setup (with those initial values for charge).*

f) *Now, we'll do the same in the other direction. Solve the initial value problem for current with* $I(0) = 2A$ *and* $I'(0) = 1\,\mathrm{A/s}$, *and see what the initial conditions would be for* $Q(t)$ *for this setup.*

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**Exercise 12** *Assume that the system* $my'' + \gamma y' + ky = 0$ *is either critically or overdamped. Prove that the solution can pass through zero* at most once, *regardless of initial conditions.* Hint: *Try to find all values of* $t$ *for which* $y(t) = 0$, *given the form of the solution.*

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