Computers & Operations Research 139 (2022) 105641

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Big data driven order-up-to level model: Application of machine learning

Johan Bjerre Bach Clausen *, Hongyan Li

Department of Economics and Business Economics, School of Business and Social Science, Aarhus University, Fuglesangs Allé 4, Aarhus, 8210, Denmark

ARTICLE INFO

Keywords:
Inventory models
Machine learning
Big data
Data driven operations research
Empirical risk minimisation
Neural networks

ABSTRACT

Data driven optimisation has become one of the research frontiers in operations management and operations research. Likewise, the recent academic interest in big data has created a desire for big data driven operations research. A new data driven methodology, which employs the empirical risk minimisation (ERM) principle, has recently been introduced in the inventory management literature. It has been used to formulate data driven inventory models which can take multiple features into account and do not need classical distributional assumptions. However, the research on big data driven inventory models is currently confined to the newsvendor model. In this paper, we aim to generalise the previous results on the big data driven newsvendor model and to expand the research by solving a big data driven dynamic order-up-to level inventory model. We show how the ERM methodology is employed to formulate a big data driven order-up-to level inventory model and design a machine learning algorithm to solve the model. The performance of our big data driven inventory model and solution algorithm is demonstrated by an experimental study based on real business data. The numerical results show that our integrated big data driven model generates up to 60% cost savings compared to the best performing univariate benchmark model, and up to 6.37% cost savings compared to the best performing big data driven benchmark model.

1. Introduction

In the past decades companies have invested heavily in information technologies and digitalisation. Increased computing power and the explosion of data have changed the way organisations capture data, analyse information, and make decisions. These changes provide opportunities for the operations management (OM) community to develop new models for data driven decision-making (Simchi-Levi, 2014). A few recent studies investigate the applications of big data analytics in logistics and supply chain management and have proclaimed the arrival of the big data era in the field of OM (Maheshwari et al., 2020; Nguyen et al., 2018; Wang et al., 2016; Yu et al., 2018). The seminal insight in Davenport et al. (2006), that aggressive firms on the analytical edge are often clear leaders in their industry, might become more and more applicable to logistics and supply chain management.

In practice, big data sources generally include enterprise resource planning (ERP) systems, distributed manufacturing environments, and social media feeds, customer buying patterns, global position systems, and radio frequency based identification tracking, etc. In order to benefit from the insights of big data within the field of OM, it requires bringing together statistics, computational science, and operations research (OR) techniques. Bertsimas and Kallus (2020) combine ideas from machine learning (ML) and OR to develop a framework for using data to determine optimal decisions in OR problems. However, the research on how to integrate the big data analysis and OR models to drive business performance improvements is still in its infancy.

The classic OR models assume that demand is deterministic or follows certain known statistical distributions. While the deterministic and stochastic OR models are an effective type of analysis in operations management, the rapid development of data science and company data availability has created a need for new data driven OR methods to be explored. The data driven methods seek to improve operational performance by analysing observed data instead of making distributional assumptions. It is therefore interesting to see when the data driven methods will outperform the classic OR methods, and vice versa, so that we can choose the right methods for a specific case and model. Many studies try to relax the assumption of probability distributions about demands (Cheung and Simchi-Levi, 2019; Huh et al., 2011; Shi et al., 2016, etc.). A subset of these studies, e.g., Ban (2020), Cheung and Simchi-Levi (2019), Ding et al. (2002) and Levi et al. (2015) can be denoted as data driven papers because the papers not only relax distribution assumptions but also solve their problems based on observed data.

Our research aims to contribute to the literature on big data driven inventory models. Big data is characterised by 4 V's: Volume, Velocity, Variety, and Veracity. A truly big data framework must be able to

* Corresponding author.
E-mail addresses: joca@econ.au.dk (J.B.B. Clausen), hojl@econ.au.dk (H. Li).

https://doi.org/10.1016/j.cor.2021.105641
Received 4 March 2021; Received in revised form 11 October 2021; Accepted 12 November 2021
Available online 27 November 2021

handle these four characteristics (Chen and Zhang, 2014). However, dealing with all 4 V's at the same time is too broad in scope to be in one paper. Therefore, we primarily focus on the Volume and Variety characteristics, since they are often discussed together when researchers denote big data as feature rich. We will adopt the terminology established in Ban and Rudin (2018) and define big data as data that takes features into account. Features are synonymous with the terms of independent variables, exogenous variables or explanatory variables. For example, product demand (which in an OR context is a common dependent variable) is often in a real-world case dependent on a set of features. We will later in our experimental study explore the demand for orange juice, which is dependent on features such as price, if a promotion is present, the number of other stores selling orange juice nearby, the average age of consumers, the income of consumers, etc. We define the opposite of feature rich data as univariate data. Univariate demand data is often considered when one observes product demand but nothing else. However, in reality product demand is often dependent on a set of features, but one has not spent resources to observe those features. Therefore, it is necessary to study if OR models based on feature rich demand data offer improved decision-making compared to univariate models.

The concept of including multiple features into business analysis is not a new concept within the field of (business) statistical research, but it is a much more scarce topic within the field of OR. Several papers see big data as the next big leap in OR but integrating multiple features into existing OR models remains a challenging task. To overcome this challenge, we focus on big data driven inventory models, and we expand upon the new methodology presented in Ban and Rudin (2018). The only big data driven inventory problem treated in detail in literature is the big data driven newsvendor model (Ban and Rudin, 2018; Huber et al., 2019; Zhang and Gao, 2017). The paper by Ban and Rudin (2018) may become a seminal paper because of their introduction and application of the empirical risk minimisation (ERM) principle to solve the big data newsvendor model. The newsvendor model is chosen as the breakthrough point for the big data driven methodology in OR because of its simplicity and well-defined solution structure.

The main contribution of our study is that we formulate and solve a big data driven dynamic inventory model. Nguyen et al. (2018) state that further extending the use of big data analysis for inventory models is a current important research question for operations research. We do this by extending the use of the ERM principle to an order-up-to level model. This is done by using a custom loss function together with a neural network (NN) that enables us to make inventory proscriptions directly. Then we demonstrate the performance of our integrated big data driven inventory model using a real-world data set and compare the results with those from a set of benchmark models from literature. The numerical results show that our integrated big data driven model generates up to 60% cost savings compared to the best performing univariate benchmark model, and up to 6.37% cost savings compared to the best performing big data driven benchmark model.

Based on our results and discussions, we present three key managerial insights. First, when faced with an inventory management problem where feature rich demand data is available, performing standard multiple feature analysis can greatly improve inventory performance compared to univariate models. Second, if one can identify the optimal predictor of the big data driven model, it is often possible to identify well understood estimation methods that can solve the big data driven problem. Third, in the absence of a known optimal predictor, it is still possible to design a problem specific ML model. The performance of the model depends on both the complexity level of the model and the problem.

The rest of the paper is organised as follows: The key literature is reviewed, and the research gaps are identified in Section 2. In Section 3 we introduce the ERM principle and the concept of an optimal predictor to generalise the results established in the literature about big data driven newsvendor models. In Section 4 our ML methodology and our big data driven order-up-to level inventory model are developed. In Section 5 we test our model on a real business data set and compare the performance of our model and solution algorithm with a set of benchmark models from the literature. Section 6 summarises our research and proposes future research directions.

2. Literature review

Big data driven inventory models is a very recent area of research. It is closely related to the literature on classic inventory models and the more recent literature on data driven inventory models. We will focus on reviewing the most recent developments about data driven inventory models and identify the research gaps.

We classify the related literature as shown in Fig. 1. For the classic subjects of *Deterministic Inventory models* and *Stochastic Inventory models* we refer to comprehensive textbooks on the subjects by Zipkin (2000) and Axsäter (2015).

2.1. Univariate data driven newsvendor models

The classic newsvendor model assumes that demand is deterministic or follows an assumed probability distribution (Zipkin, 2000). However, business managers may need to make decisions without complete knowledge of the demand distribution. For example, the manager might know an approximation of the demand distribution or know some distributional moments. This has been treated in the literature by Gallego and Moon (1993) where the authors solve a class of newsvendor models where only the mean and variance are known by using and expanding on the Scarf ordering rule (Scarf, 1958). Similarly, Perakis and Roels (2008) only assume knowledge of distributional moments such as mean and variance in their treatment of the newsvendor model. Corlu et al. (2019) use the newsvendor model to derive a closed form solution for the confidence interval around the simulated service level in the absence of complete information about the demand distribution. Zheng et al. (2016) study newsvendor order quantities when demand forecasts can be updated within the full time horizon of the model. Looking into data driven papers, we can identify different solution methodologies. Ding et al. (2002) take a Bayesian approach where the demand is assumed to follow a family of distributions. The assumption is then used to establish an optimal ordering policy by determining the distributional parameters through a data driven Bayesian methodology. Carrizosa et al. (2016) develop a robust solution to the newsvendor model based on observed univariate data modelled as an AR(P) process. Zhang and Yang (2016) study a multi-item newsvendor model where historical univariate demand is observed. The study by Ding et al. (2002) is interesting because it models the problem based on univariate censored data. Godfrey and Powell (2001) use a CAVE algorithm on univariate censored data to determine an optimal newsvendor order quantity. More recent papers such as Levi et al. (2015) investigate the data driven newsvendor model with a random, independent sample drawn from the demand distribution. The sample average approximation (SAA) approach is analysed, and a new analytical bound on the probability that the relative regret of the SAA solution exceeds a threshold is found. The new bound is significantly tighter than the existing bounds. Harsha et al. (2019) also use a quantile based approach based on observed demand data and develop a data driven algorithm which considers the problem of both order quantity and price setting simultaneously.

2.2. Univariate data driven dynamic inventory models

The simplicity of the newsvendor model makes it a good model for developing new methodological insights, but data driven inventory models and solution algorithms are also studied in the context of data driven dynamic inventory models. For example, Bollapragada and Morton (1999) derive a heuristic that can solve the (s, S) inventory

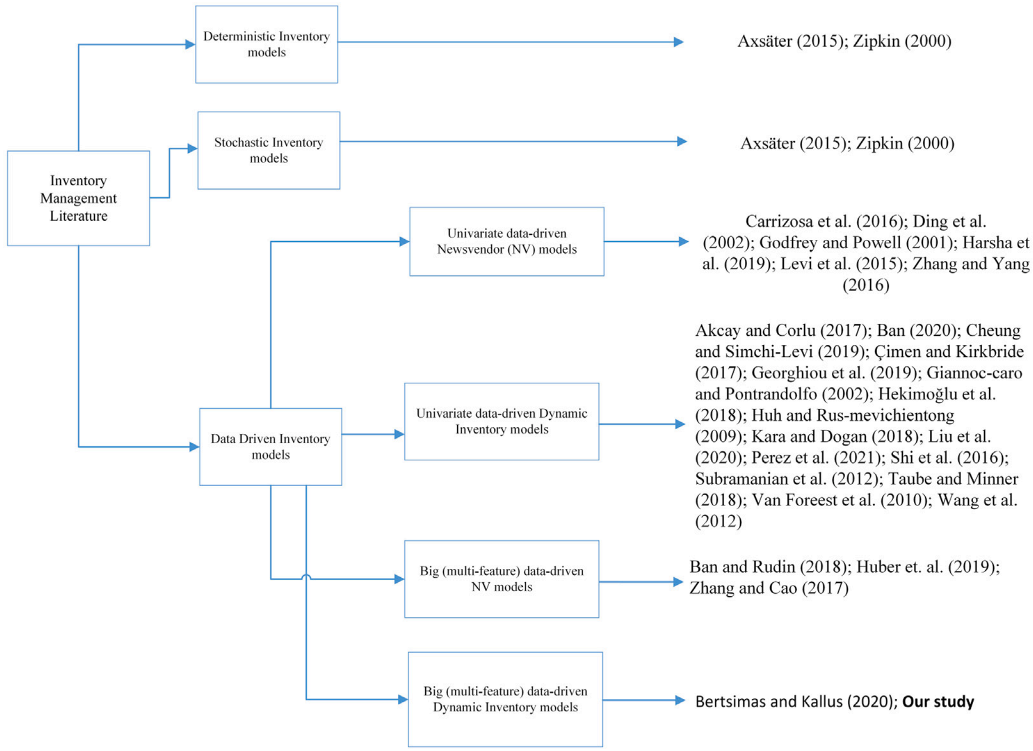

Fig. 1. Literature structure.

model under non-IID demand. This work is a continuation and improvement on the earlier work of Askin (1981) which seeks to solve the same problem. Recently, Ban (2020) develops a data driven solution of the (s, S) inventory model which can handle both censored and non-censored univariate independent and identically distributed (IID) demand data. Taube and Minner (2018) examine a univariate data driven multi-item multi-echelon order-up-to level model. Bayesian solutions methodologies is explored in Akcay and Corlu (2017) where the authors develop a data driven Bayesian methodology which can solve a base stock inventory problem, and Liu et al. (2020) present an empirical Bayesian analysis of the base-stock list-price inventory policy. Cheung and Simchi-Levi (2019) propose a sampling-based approximation scheme for a capacitated stochastic inventory control model. Of specific interest to our study is Huh and Rusmevichientong (2009). They are inspired by the newsvendor model to design a data driven multiple period order-up-to level inventory control model. The order-up-to level inventory model objective is to minimise (maximise) the total cost (profit) over the planning horizon when taking order cost, holding cost, and lost sales cost into account. They formulate a univariate algorithm which is asymptotically optimal when applied on IID data. Shi et al. (2016) also discuss the same order-up-to level model but extend the data driven results to the multi-product case. The IID assumption is still needed for establishing an asymptotic optimal order-up-to level.

In addition, a number of studies employ solution algorithms based on dynamic programming, Markov processes, or reinforcement learning to solve data driven dynamic inventory problems. Subramanian et al. (2012) study a univariate closed loop multi-echelon distribution inventory supply chain model. Van Foreest et al. (2010) use univariate demand simulation to evaluate different ordering strategies in a more complex customised stochastic lot scheduling problem. Hekimoğlu et al. (2018) examine an inventory control model that can handle stochastic lead times and disruptions by modelling the stochastic elements through a Markov process, which is integrated into the authors' solution algorithm. Lately, Georghiou et al. (2019) develop a methodology they call robust dual dynamic programming to solve multistage optimisation problems. The authors show the efficiency of their methodology in a single echelon multi-item inventory problem with observed stochastic demands and fixed holding and back order costs. Çimen and Kirkbride (2017) also look into solving a multi-item single echelon inventory model with observed demand, but employ approximate dynamic programming algorithms.

Table 1
Big data driven papers.

Study	Inventory model	Demand function	Methodology	Regression type
Ban and Rudin (2018)	Newsvendor	Linear	ERM	Global regression
Huber et al. (2019)	Newsvendor	Non-linear	ERM	Global regression
Zhang and Gao (2017)	Newsvendor	Non-linear	ERM	Global regression
Bertsimas and Kallus (2020)	Multi-item	Non-linear	Multistep regression	Local regression
Our study	Order-up-to level (dynamic)	Non-linear	ERM	Global regression

Reinforcement learning is a data driven methodology that is gaining traction in many economic research areas, and it is theoretically connected to dynamic programming and Markov processes (Nguyen et al., 2020). Giannoccaro and Pontrandolfo (2002) provide an early introduction of reinforcement learning methods to inventory management. Many theoretical developments within the field of reinforcement learning have been achieved since then, and Kara and Dogan (2018), Perez et al. (2021), and Wang et al. (2012) apply newer reinforcement learning methods to solve inventory management problems based on observed univariate demand data.

2.3. Big data driven inventory models

We have summarised the papers we will review in this section in Table 1.

The first column lists the key literature. The second column lists the focal problem addressed in the literature. The third column presents how demand is assumed to be related to the observed features. The linear demand model is discussed in Ban and Rudin (2018). The ERM model proposed in Ban and Rudin (2018) only works when a linear relationship is assumed. Whereas, the papers following Ban and Rudin (2018) have proposed ERM models for the newsvendor model, which can be solved for both linear and non-linear demand relationships. The fourth column describes the solution methodology used in each paper. The key takeaway is that the papers on the newsvendor model use the ERM methodology. Whereas, Bertsimas and Kallus (2020) employ a multistep regression methodology. The fifth column summarises what regression type is used to make operational proscriptions. In the ERM methodology global regression can be used, which means all data points are considered when making operational proscriptions. Instead, Bertsimas and Kallus (2020) use local regression, which means that only a neighbourhood of data points are used when making operational proscriptions. For an illustration of the differences between global and local regression, see Shao et al. (2017).

The big data driven inventory model is a rather new research area and few studies have been published. Ban and Rudin (2018) investigate the newsvendor model when a business has access to multiple historical demand observations as well as a potentially large number of features related to demand. They solve the big data-driven newsvendor model via distribution free ML algorithms which can handle feature rich data and derive finite-sample performance bounds on their out-of sample costs. Two types of ML algorithms are presented: ERM based algorithms (linear stochastic programming) and a Kernel optimisation approach. Their experiment shows that the integrated ML algorithms outperform the benchmark SAA approach.

Huber et al. (2019) is inspired by the methodology in Ban and Rudin (2018) and classifies data driven approaches in inventory management by introducing three levels of analysis. The first level is demand estimation, the second level is that one's inventory decision is made based on demand estimation and the associated regression errors, and the third level is the integration of demand estimation and inventory optimisation. Huber et al. (2019) test algorithms across the three levels in their empirical experiment and find that the multiple feature approaches consistently outperforms the univariate approaches. Within the multiple feature algorithms on the second and third level, results are not as conclusive, but the integrated approaches perform better in most of their tested cost instances. Using the framework from Huber et al. (2019), the work of Ban and Rudin (2018) would be on the third level. Another recent paper on the third level is by Zhang and Gao (2017). This paper only looks at an NN formulation of the big data driven newsvendor model, using the ERM principle. Instead of the ERM methodology, Bertsimas and Kallus (2020) propose another methodology rather than the ERM methodology and examine a big data driven multi-item dynamic inventory model. Comparing (Bertsimas and Kallus, 2020) to the big data driven newsvendor models reviewed above, the authors employ a methodology that uses local regression methods in the last step in a multiple step methodology. It is in the last step of the methodology the authors model operational decision-making, and it is a clear limitation of the multistep methodology that the operational decision-making only can be based on data points within a local neighbourhood. The ERM methodology on the other hand uses all data points to determine the optimal operational decision, but it is challenging to formulate ERM models of advanced operational problems.

2.4. Research gaps

The literature review above shows that several research gaps need to be abridged.

First, the use of the ERM methodology to solve big data driven inventory models is only applied to the newsvendor model. In our study, we will therefore extend the application of the ERM methodology to formulate and solve a big data driven dynamic inventory control model. Inventory models are not the only place where the use of a ERM based methodology is being further explored. New insights have recently been published in Notz and Pibernik (2021) which solve a big data formulation of a static two-stage capacity planning problem using the ERM principle and a novel regression tree based machine learning algorithm. The authors also note the difficulty of generalising their approach to other problems, but together with our paper the applicability of ERM based analysis in OR is being advanced.

Second, solving (complex) big data driven dynamic operations research problems remains a challenging task. A core element of our study is a novel application and modification of ML algorithms. This approach is seeing increased interest in papers such as Bravo and Shaposhnik (2020) and Ciocan and Mišić (2020). The authors in both of the papers do not take an ERM approach. Instead, the authors examine the state space of different operational problems and their associated action spaces (determined by a given policy on the state space) and then try to improve and understand the problem by applying the ML algorithm known as regression trees. Therefore, future research should investigate if insights from ERM based modelling and state space based modelling can be used to generate new insights.

3. An ERM based big data driven methodology

With the introduction of the ERM methodology into big data driven inventory models by Ban and Rudin (2018), a new methodological approach to solve big data driven inventory models has begun to be investigated. The methodology consists of two steps. The first step is to construct a big data driven model, where the objective function is constructed using the ERM principle. Second, one solves the big data driven objective function by estimating the conditional quantity one seeks to either minimise or maximise. In order to generalise the research results from the big data driven newsvendor models, we introduce the ERM principle in the context of the newsvendor model first.

Performing ERM in a dynamic inventory context, one has access to historical demand and feature values for N periods. We then seek to determine that solution the minimises cost given the historical demand and feature values. Each observation i is therefore a period in time.

To show how we arrive at our solution algorithm, we will start by discussing the ERM principle. The ERM principle states that when estimating a function g that maps a P-dimensional input to a scalar value:

$$g : X \to D \tag{1}$$

One chooses the estimate of g that minimises the empirical risk (Vapnik, 1998). The empirical risk function is defined as

$$R(d_i, \mathbf{x}_i) = \frac{1}{N}\sum_{i=1}^{N} L(\hat{g}(\mathbf{x}_i), d_i) \tag{2}$$

where $R(\cdot)$ is the empirical risk function, $\hat{g}(\cdot)$ is the estimate of g, and $L(\cdot)$ is denoted as a loss function. Since the joint distribution of D and X is unknown or difficult to be modelled, we therefore seek to minimise $R(\cdot)$ by finding the best estimate of $\hat{g}(\cdot)$ given the realised demand and features. The best estimate is dependent on the choice of loss function $L(\cdot)$, because the loss function is how we measure how well $\hat{g}(\cdot)$ estimates g. The central innovation in Ban and Rudin (2018) is to choose $L(\cdot)$ such that when the multiple feature model is solved, it also solves the multiple feature inventory control problem we are seeking to solve. In the case of the newsvendor model, we seek to estimate the multiple feature order quantity $\hat{q}(\mathbf{x}_i)$. This is done by using the newsvendor cost function as the loss function in Eq. (2):

$$R(d_i, \hat{q}(\mathbf{x}_i)) = \frac{1}{N}\sum_{i=1}^{N}[u(d_i - \hat{q}(\mathbf{x}_i))^+ + o(\hat{q}(\mathbf{x}_i) - d_i)^+] \tag{3}$$

Because we have used the newsvendor cost function as the loss function, when minimising Eq. (3) we get the multiple feature newsvendor order quantity $\hat{q}(\mathbf{x}_i)$. The central difference between a classic newsvendor order quantity and $\hat{q}(\mathbf{x}_i)$, is that $\hat{q}(\mathbf{x}_i)$ depends on the values of $\mathbf{x}_i$.

We follow the terminology established in Ban and Rudin (2018) and denote the newsvendor model introduced above as a big data driven model. A big data driven model means that the inventory decision is dependent on a set of observed features. The opposite of a big data driven model is a univariate data driven model (Huh and Rusmevichientong, 2009; Ban, 2020). The challenge of a big data driven problem is that if one does not take the features that influence demand into account (treating observed demand as univariate) one would in most interesting cases be dealing with non-IID data. This is because the univariate demand data would not be identically distributed, since either the mean and variance (or both) of the data would depend on the realised features. The solution methodology in Huh and Rusmevichientong (2009) and Ban (2020) is based on the common assumption of IID demand data. Even if a univariate data driven model can handle non-IID demand data (Bollapragada and Morton, 1999), not taking features into account likely leads to worse performance compared to a big data driven model. This is because the univariate data driven model does not use the full demand information available. In the next sections, we will discuss how we solve a big data driven order-up-to level inventory model.

3.1. The optimal predictor

There is however a type of big data driven inventory model which we now can solve. Ban and Rudin (2018) observe that the loss function in Eq. (3) is mathematically equivalent to the loss function used in a traditional quantile regression (QR). In fact, the observation can be used to generalise the big data driven newsvendor model. In this section we introduce the theoretical concept of an optimal predictor to generalise the findings in the big data driven newsvendor literature.

The ERM principle states that one should pick the estimate of g that minimises the empirical risk, but the empirical risk changes when data is changed. Therefore, we would also like to say what is the best estimator on average. Understanding what would be the best estimator on average is the topic of statistical decision theory. We refer to Hastie et al. (2009) for a full theoretical development, but within statistical decision theory the concept of an optimal predictor, $g^*(X)$, is developed. $g^*(X)$ is the theoretical predictor that minimises the expected prediction error. The optimal predictor depends on the choice of loss function, and the most commonly used loss function is the squared error loss function:

$$L(D, g(X)) = (D - g(X))^2 \tag{4}$$

where $g(X)$ is a general functional form defined in Eq. (1). The theoretical optimal predictor $g^*(X)$ for the squared error loss function is the conditional mean of D given $X = x$ (Hastie et al., 2009) which is also called the regression function $E(D|X = x)$. There are countless methods to estimate the regression function given an assumed relationship between D and X, but we can analyse how each estimator tries to estimate the theoretical optimal predictor. Similar results can be derived for other loss functions, for example, the absolute value loss function $L(D, g(X)) = |D-g(X)|$, where $g^*(X)$ is the conditional median of D given $X = x$ (Hastie et al., 2009).

What is especially relevant for our study is the observation that the loss function in Eq. (3) is mathematically equivalent to the loss function used in traditional QR. It means that we can find the optimal predictor for the big data driven newsvendor model by using Eq. (5), called the LinLin loss function (Christoffersen and Diebold, 1997).

$$L(D, g(X)) = \begin{cases} u|D - g(X)| & \text{if } (D - g(X)) > 0 \\ o|D - g(X)| & \text{if } (D - g(X)) \leq 0 \end{cases} \tag{5}$$

Moreover, Christoffersen and Diebold (1997) prove that the optimal predictor $g^*(X)$ for the LinLin loss function is the conditional quantile in the cdf of D.

$$F(D|X = x) = \frac{u}{u + o} \tag{6}$$

Therefore, when using the newsvendor cost minimisation function as a loss function, the optimal predictor is a conditional quantile in the cdf of D given $X = x$. This is an interesting result because it shows an accordance with the traditional newsvendor univariate result, where the optimal order quantity is an unconditional quantile in the cdf of D. The remaining question is how to best estimate the conditional relationship between D and X in a way that estimates the optimal predictor given in Eq. (6). This question has already received academic attention in the literature on QR where both computational and asymptotic characteristics is discussed in depth in Koenker (2005).

The insight that the big data driven newsvendor model can be solved by QR means that one seeking to estimate $\hat{q}(\mathbf{x}_i)$ can use any tractable QR estimation method to estimate $\hat{q}(\mathbf{x}_i)$. Ban and Rudin (2018) and Zhang and Gao (2017) do not use this finding in their modelling. Ban and Rudin (2018) solve a model specific convex optimisation problem to estimate $\hat{q}(\mathbf{x}_i)$. Zhang and Gao (2017) employ a model specific NN to estimate $\hat{q}(\mathbf{x}_i)$. With our introduction of the optimal predictor for the newsvendor model, we identify that the two different solution algorithms are actually also two problem specific QR estimators.

Huber et al. (2019) are aware of the connection between QR and the big data driven newsvendor model since they use different types of QR in their empirical study. The authors provide a brief discussion of the equivalence of the loss function in Ban and Rudin (2018) and the loss function employed in QR. We further include a thorough discussion in this paper so that the theoretical work of the earlier papers is generalised. Moreover, the discussion allows us to summarise the following finding. If the optimal predictor can be established (e.g., for the newsvendor model) explicitly, the big data driven model should be solved by a solution algorithm (e.g, QR) based on the optimal predictor or an equivalent model specific solution algorithm. If the optimal predictor cannot be established explicitly, model specific solution algorithms may be designed to solve the model.

4. The big data driven dynamic inventory model

As stated in the literature review, the big data driven research has primarily been confined to the newsvendor model. In this section, we extend the research by solving a big data driven dynamic inventory model. We employ the two step big data driven methodology described in Section 3. First, we make an ERM formulation of a suitable inventory model. Second, we design a solution algorithm.

4.1. The big data driven order-up-to level inventory model

We will study an order-up-to level inventory model which describes a dynamic multiple period inventory management problem. The product is non-perishable, and due to this there is a starting inventory in each period. Inventory replenishment may occur at each period and the decision variable is how many units to order in each period.

The univariate order-up-to level inventory model can be described as:

$$\Pi(D) = \sum_{i=1}^{N} [c(y_i - s_i) + h(y_i - d_i)^+ + b(d_i - y_i)^+] \tag{7}$$

In addition to the already established notation in Table 2, y_i is the order-up-to level for period i, s_i is the starting inventory at period i, d_i is the demand in period t, b is the lost sales cost per unit per period, h is the holding cost per unit per period, and c is the order cost per unit. Similar to the newsvendor model, the cost function includes an underage term $(d_i - y_i)^+$ associated with a unit lost sales cost b, and an

Table 2
Notations.

N	The number of observations.
P	The number of features.
D	A stochastic variable of demand.
X	A P-dimensional stochastic input variable (our features) expected to influence demand.
x	A realisation of the stochastic variable X.
$\mathbf{X}$	An N times P matrix. The N rows hold the observed features corresponding to N demand observations. Each of the P columns holds N observations of each feature.
$\mathbf{x}_i$	The ith row in $\mathbf{X}$, $i = 1, \dots, N$.
$\mathbf{x}^p$	The pth column in $\mathbf{X}$, $i = 1, \dots, N$.
d_i	The ith demand observation.
u	The underage cost per unit.
o	The overage cost per unit.

overage term $(y_i - d_i)$ associated with a unit holding cost h. The non-perishable characteristic of the product is captured by the term $(y_i - s_i)$, where s_i is the inventory level at the beginning of the period t. In the model, $s_1 = 0$ and $s_i = (y_{i-1} - d_{i-1})^+$ for all periods $i \neq 1$.

Huh and Rusmevichientong (2009) propose a univariate data driven solution algorithm to the model. In their modelling set-up, the decision maker is in period i and can only observe the past order-up-to levels $y_i, \dots, y_{i-1}$ and the past demands $d_i, \dots, d_{i-1}$, and one seeks to determine y_i. The authors develop a solution algorithm (AIM) under the assumption that the demands $d_1, d_2, d_3, \dots, d_N$ are IID. The algorithm gives an asymptotically optimal sequence of the order-up-to levels $y_1, \dots, y_i$. The optimal solution that the AIM algorithm approaches is the newsvendor solution. However, the solution is only optimal under the IID assumption.

In this paper, we consider demand as a function of multiple features and do not require any assumptions about demand following a specific distribution. Moreover, the problem is modelled based on the ERM principle. The strength of the ERM approach is that using an NN we do not need to assume that our observed demand is IID commonly assumed in the univariate data driven literature. Instead, we consider demand to be dependent on a set of features, and the relationship between demand and the features can be linear or non-linear. Additionally, the features are allowed to follow different unknown probability distributions. The central idea of a big data driven model is to take feature information into consideration in order to improve the inventory decision compared to univariate models. The ERM formulation of the order-up-to level inventory model is:

$$\hat{R}(\hat{y}(\mathbf{x}_i), \mathbf{D}) = \frac{1}{N} \sum_{i=1}^{N} [c(\hat{y}(\mathbf{x}_i) - s_i)^+ + h(\hat{y}(\mathbf{x}_i) - d_i)^+ + b(d_i - \hat{y}(\mathbf{x}_i))^+] \tag{8}$$

where $\hat{y}(\mathbf{x}_i)$ is the order-up-to level as a function of $\mathbf{x}_i$, meaning that the order-up-to level varies from period to period as $\mathbf{x}_i$ takes different values. The term $(\hat{y}(\mathbf{x}_i) - s_i)$ is the order quantity in period i, and in the ERM formulation $\hat{y}(\mathbf{x}_i) < s_i$ may occur, therefore we need to rectify the term into $(\hat{y}(\mathbf{x}_i) - s_i)^+$.

4.2. *The solution algorithm*

With regard to the big data driven order-up-to loss function defined in Eq. (8), we cannot establish a closed form expression of the optimal predictor. Moreover, we are not aware of any other literature that employs an equivalent loss function. Therefore, we design a problem specific solution algorithm to estimate the conditional order-up-to level $\hat{y}(\mathbf{x}_i)$. Our solution algorithm employs an NN to estimate $\hat{y}(\mathbf{x}_i)$. In designing our estimator we follow an ML solution methodology consisting of three main steps: (i) Construct a relevant loss function, (ii) design an NN architecture, and (iii) employ a suitable optimiser (Chollet and Allaire, 2018).

Chollet and Allaire (2018) discuss the importance of choosing a relevant loss function in the context of very different ML problems, and they state that the choice of loss function is central to achieve the desired results when using ML algorithms. In this paper our loss function is the loss function established in Section 4.1, which is an ERM formulation of the order-up-to level cost function such that when solving the ML optimisation problem, we also solve our inventory decision problem.

The architecture of our NN is a standard feed forward design with RELU activation functions in each hidden nodes. The layer depth and the number of nodes in each hidden layer are determined by cross-validation, as is the best practice when doing hyperparameter optimisation (Chollet and Allaire, 2018).

The choice of optimiser will also be given special attention because our loss function is not suitable for a standard gradient descent optimisation method. Therefore, in the next section, we address the choice of optimiser in details.

4.3. *The choice of optimiser*

Using an NN architecture, $\hat{y}(\mathbf{x}_i)$ is the estimated order-up-to level, and a gradient based optimisation algorithm is traditionally used when training an NN. In traditional regression problems, where the squared error loss function is used, a first order gradient based optimiser, such as the batch gradient descent method or the stochastic gradient descent method, is the standard optimiser (Sun et al., 2019). The choice of optimiser is often not complex, but more research into different forms of gradient based optimisers, designed to solve specific optimisation problems, have been published (Sun et al., 2019). While using traditional optimisers may in many cases result in both good predictive and computational performance, our use of a customised loss function makes the choice of an optimiser non-trivial.

The complexity lies in the non-linearity introduced by the use of (piecewise) rectified linear terms in the loss function. This same problem is also faced in the simpler ERM newsvendor model. Zhang and Gao (2017) solve this problem by using indicator functions that take the value 1 when the rectified linear terms are larger than 0, and 0 otherwise. This allows Zhang and Gao (2017) to use a standard stochastic gradient descent algorithm, but when the rectified linear terms are 0 the gradient is undefined. The advantage of this method is that it is simple, but the undefined gradient at the origin may or may not be a problem in a specific optimisation problem.

In the QR literature, the problem of an undefined gradient at the origin can be solved by smoothing the function at the origin (Cannon, 2011) or by using subgradients. In this paper we choose to use subgradients which allow our problem to be differentiable at the origin because under certain conditions it allows us to establish a set of subgradients at a point where a gradient would not be defined. Determining the entire set of subgradients can be difficult, but in our case only one subgradient is needed in the optimisation algorithm. For theoretical proofs and details, we refer to Shalev-Shwartz and Ben-David (2014). Furthermore, we use the subgradient descent algorithm *ADAGRAD* developed by Duchi et al. (2011).

We use this specific optimisation algorithm because we have three rectified linear terms in our loss function, and Duchi et al. (2011) report good computational and predictive performance of their algorithm in the case of rectified linear terms. We have made one modification to the algorithm. Since *ADAGRAD* is a subgradient descent algorithm, in each iteration one needs to determine the current value of the function and the direction (and length) of descent. In neural networks, the first part is done by making a forward pass through the network. When the algorithm makes the forward pass, the current value of $\hat{y}(\mathbf{x_i})$ is determined for all our observations, we also calculate s_i for all observations since $s_1 = 0$ and $s_i = (\hat{y}(\mathbf{x}_{i-1}) - y_{i-1})^+$ for all observations $i \neq 1$. Since the *ADAGRAD* algorithm uses subdifferentials to determine the descent, one must evaluate the subdifferential of the loss function in each iteration. The evaluation of the subdifferential of the loss function depends on the current value (for the iteration) of $\hat{y}(\mathbf{x}_i)$ and s_i. We apply the R implementation of *ADAGRAD* to do the subdifferential evaluation of the loss function based on our custom loss function and the values of the forward pass.

The pseudo-code of the algorithm is:

Algorithm 1 Big data driven order-up-to level solution algorithm pseudocode

1: Initialise $\hat{y}(\mathbf{x}_i)$ for $i = 1, 2, ..., N$ to determine s_i for $i = 1, 2, ..., N$ and initialise random NN weights.
2: **for** $iteration = 1, 2, \dots, M$ **do**
3: Update NN weights using the optimiser *ADAGRAD* and the loss function in Eq. (8)
4: Calculate the empirical risk using Eq. (8)
5: Save NN weights
6: Make forward pass to determine $\hat{y}(\mathbf{x}_i)$ and to determine s_i for $i = 1, 2, ..., N$ using the loss function in Eq. (8)
7: **end for**

Where the number of iterations, M, can be determined through cross-validation. Line (3) in the pseudo-code is where the training of our model takes place, we use the known optimiser *ADAGRAD*. Please refer to Duchi et al. (2011) for more algorithmic details.

5. Experimental study

An important element of successful big data driven methods is that the performance of the method can be verified in an actual business decision-making context. Moreover, only real business data displays the complex relationship between the multiple features and the dependent variable. Although it is possible to simulate data with multiple features, it is hard to capture the feature richness and randomness of real business data sets. Of course, real business data may not have the expected quality, and extra resources are normally needed to preprocess and maintain data bases. In order to best test the performance of our algorithm, we choose to use real business data.

5.1. Data

In this paper, we use a publicly available data set of orange juice sales in the US. The data set is made available in the digital appendix to the book by Ledolter (2013). Using the data set, we will demonstrate experimentally that an integrated multiple feature approach outperforms univariate and multiple feature benchmark models. The data set is collected from 83 stores in the Chicago area for 3 brands of orange juice sold during 120 weeks. In the data set there exists some missing data that has been removed by listwise deletion. The data has also been used in Montgomery (1997) and Ledolter (2013), as well as an empirical experiment in Mišić (2020), and thus the missing data do not cause significant problems.

As we seek to model an inventory problem, we determine the order-up-to level for each of the 3 brands in each period. One can therefore view our problem as 3 independent problems, where we observe data for the same features for each problem, but the values observed might differ from problem to problem. Moreover, the relationship between demand and features are allowed to vary from problem to problem. In Table 3, we describe the features used to model orange juice sales. These features have been used in Montgomery (1997) and Ledolter (2013), and Mišić (2020) to model orange juice sales in a non inventory management context. This data set is one empirical example of the feature matrix $\mathbf{X}$ discussed generally in Sections 3 and 4. Actually, for different problems or situations, different features may be considered, for example, Huber et al. (2019) model bakery demand using calendar, weather and other features. An advantage of the data used in this study is that we choose to use the publicly available data, specifically for the sake of replication.

Table 3
Description of demographic features.

Feature	Description
$\mathbf{x}^1$	The per unit sales price in dollars.
$\mathbf{x}^2$	Binary variable indicating if a promotion is present.
$\mathbf{x}^3$	Percentage of population aged 60 or older.
$\mathbf{x}^4$	Percentage of population with a college degree.
$\mathbf{x}^5$	Percent of the population that is African Americans or Hispanic.
$\mathbf{x}^6$	Median income.
$\mathbf{x}^7$	Percentage of households with 5 or more persons.
$\mathbf{x}^8$	Percentage of women with full-time jobs.
$\mathbf{x}^9$	Percentage of households worth more than 150,000.
$\mathbf{x}^{10}$	Distance to the nearest warehouse store.
$\mathbf{x}^{11}$	Ratio of sales of a given store to the nearest warehouse store.
$\mathbf{x}^{12}$	Average distance in miles to the nearest 5 supermarkets.
$\mathbf{x}^{13}$	Ratio of sales of a given store to the average of the nearest five stores.

It is the inclusion of both price and demographic data which makes the data collected for Ledolter (2013) particularly interesting, because a core thesis of micro economics is that people change buying behaviour based on price changes. Moreover, different people (or groups of people) have different buying preferences at a given price point. Expanding the analysis from univariate analysis (that only takes previous sales amounts into consideration) to multiple feature analysis can capture changes in sales that are driven by a change in price, promotional strategy, or customer demographics. We employ an NN because we seek to use the complexity of the estimator to capture cross relationships in the data. An NN can capture complex cross relationships, because the non-parametric architecture does not require assumptions about the functional form of the relationship between demand and the features and between the features themselves.

The data set is not originally created for our big data driven inventory model. However, the dependent variable is the sales quantity, which is the type of dependent variable that we model because we want to determine the order quantity as a function of the sales quantity. The astute reader may already have observed that orange juice is a perishable product (but some juices can have long perish times). That is of course an unfortunate characteristic with the data. However, we do not see that as a hindrance for using the otherwise very interesting data because we are performing an empirical experiment to test the performance of our model compared to several benchmark models. Therefore, we assume that the product is non-perishable within the planning horizon. If a real world store uses our model to make juice inventory decisions, it only needs to combine the model with simple tracking and deduction of expired stock.

In Ledolter (2013), the data is described as uncensored data, which means the data is directly useable for our analysis. Uncensored demand data means in this case that one is observing the true demand and no stock outs has occurred. If data is censored, one could artificially uncensor the data as done in Huber et al. (2019). In general, the existence of censored data is a challenge which is especially relevant in a big data driven OR context, and it deserves more academic attention, but that is beyond the scope of this paper.

In Fig. 2 we illustrate the data partitioning and cross-validation procedure. First, we split our 83 stores into a training data set of 68 stores and a test data set of 15 stores. Each store has up to 120 weeks of sales observations. It is to ensure that test performance is measured using data which the models have not been trained on, in order to get an as realistic estimate of the out of sample performance as possible. The data partitioning simulates the situation where the company plans to open a new store. The company knows the price at which each brand of juice is sold in a given period, the promotional strategy for a given period, and demographic data of the area for a given period, but the company does not have any historic sales data for the store.

Since the effective NN architecture differs from problem to problem (Zhang and Gao, 2017), we need a method to determine the hyperparameters of our NN architecture. For that, we use the commonly

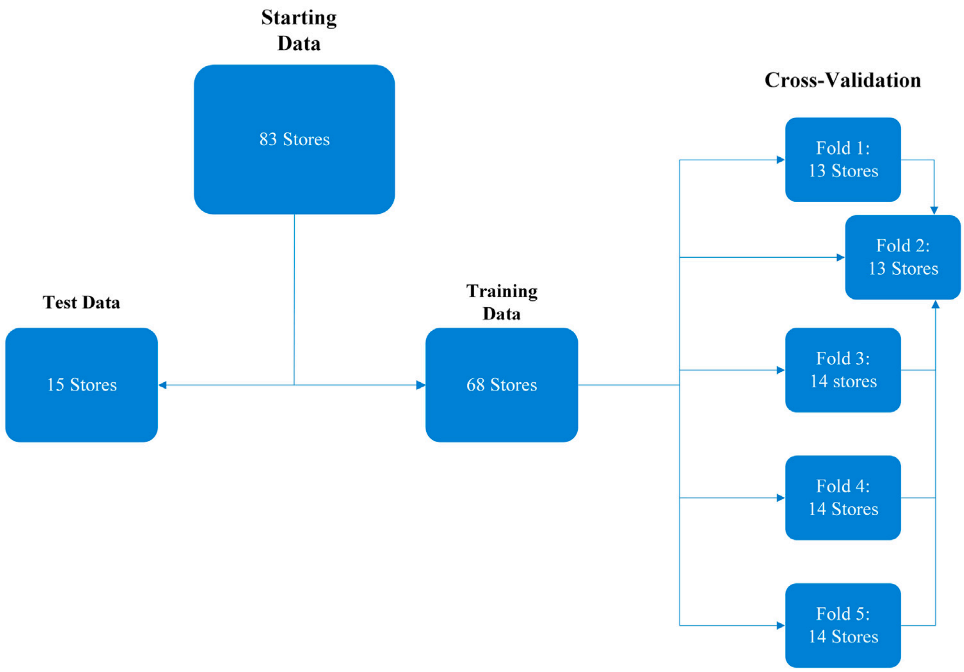

Fig. 2. Data partitioning and cross-validation procedure.

employed method of grid search cross-validation. On the training data set of 68 stores, we therefore do 5-fold grid search cross-validation to determine the hyperparameters of our NN architecture. The hyperparameters we determine through cross-validation are network depth and number of nodes in each hidden layer of the NN. Inspired by the NN architecture employed in Zhang and Gao (2017) we allowed up to 3 layers and either 64, 128, or 256 nodes in each layer. This gave us 39 combinations to check using grid search cross-validation. The cross-validation procedure is also shown in Fig. 2. We split our 68 stores in the training data into two folds with 13 stores and three folds with 14 stores. The cross-validation procedure consists of iteratively using one fold as a validation data set and using the remaining (in our case four folds) to train the model. In Fig. 2, the iteration where fold 2 is used to validate a model trained on fold $1, 3, 4, 5$ is illustrated. When each fold has been used as a validation set, we can calculate and save the average loss across the 5 folds. The entire cross-validation procedure is repeated for each of the 39 combinations of layers and nodes, and the combination with the minimum average loss is chosen as the NN architecture.

In Fig. 3, the weekly sales data for the three brands of orange juice from a representative store (store 48) is displayed. We have two important observations about the data. First, sales amounts are stable in most periods, and it indicates that a univariate data driven model may work well on these periods. However, we also observe that there are numerous periods with high sales. To ensure that the periods with high sales are not outliers, we have performed a univariate studentized outlier test (James et al., 2013) on all store and brand combinations. This test identified between 0 and 3 potential outliers in each brand and store combination. Combining the low number of potential outliers with a visual inspection of multiple stores, we conclude that a few true outlier periods may exist, but we cannot describe all periods of high sales as outliers. By taking information about prices, promotions and more into account, a period of high sales will be better captured by the model. Second, no seasonal patterns are found in the data because the periods with high sales do not fit any seasonal patterns. In addition, in his treatment of the data, Ledolter (2013) did not identify any seasonal patterns either. If seasonal patterns were suspected to be the reason for periods of high sales, we could deseason the data and use a univariate algorithm on the data. Instead, use our big data driven algorithm to account for the variation in sales.

5.2. Benchmark models and big data driven models

In this section, we compare the performance of big data driven models with the performance of a set of benchmark models used in the reviewed literature. The non-big data driven benchmark models are the following:

- Naive Mean (NM): The naive mean univariate solution where the mean of the training data is used as the order-up-to level. This gives a baseline cost if one only used the mean of the training data in one's ordering decisions.
- Newsvendor (NV): The newsvendor solution where the ordering cost is also taken into account. This benchmark is chosen because the newsvendor solution is the optimal solution of the order-up-to level inventory model with IID demands.
- AIM Algorithm (AIM): The solution based on the AIM algorithm of Huh and Rusmevichientong (2009). We have run the AIM algorithm on all store and brand combinations in the training data, and the mean of those solutions is used on the test stores.
- Neural Network Mean Estimation (NNME): The solution of a best in class NN (same architecture as our integrated model) used to estimate the conditional mean. The loss function in this case is the mean squared error and the optimiser is a stochastic gradient descent as implemented by *rmsprop* in R (Zeiler, 2012).

Furthermore, we have two big data driven models which employ multiple features and take our order-up-to level model costs into account. The first big data driven model is a two-step estimation and optimisation model (E+O). This model is chosen to test the two-step estimation and optimisation model proposed in Huber et al. (2019). In the two-step approach we first perform a mean estimating regression and then do problem specific optimisation on the regression errors. In our case we use the NNME model in the first step and determine a SAA newsvendor solution on the regression errors in the second step.

The second big data driven model is our integrated big data driven model based on the ERM principle. The methodology, including the choice of loss function and optimiser, for formulating our integrated big data driven model (IM) is introduced in Section 4. The experiment specific NN architecture is a feedforward model where each layer

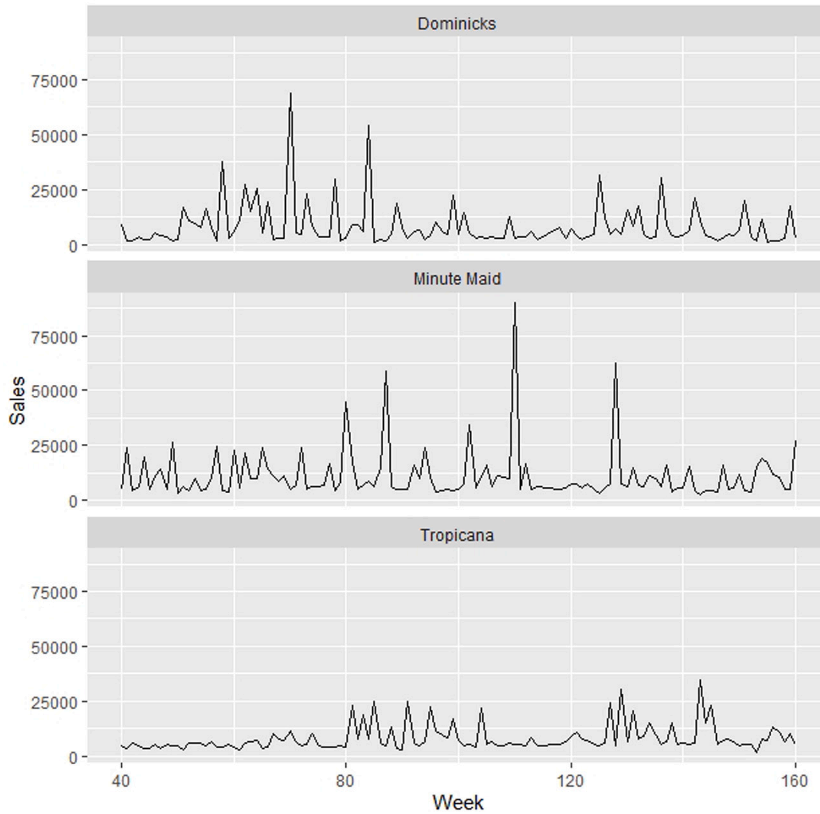

Fig. 3. Example sales data from a representative store.

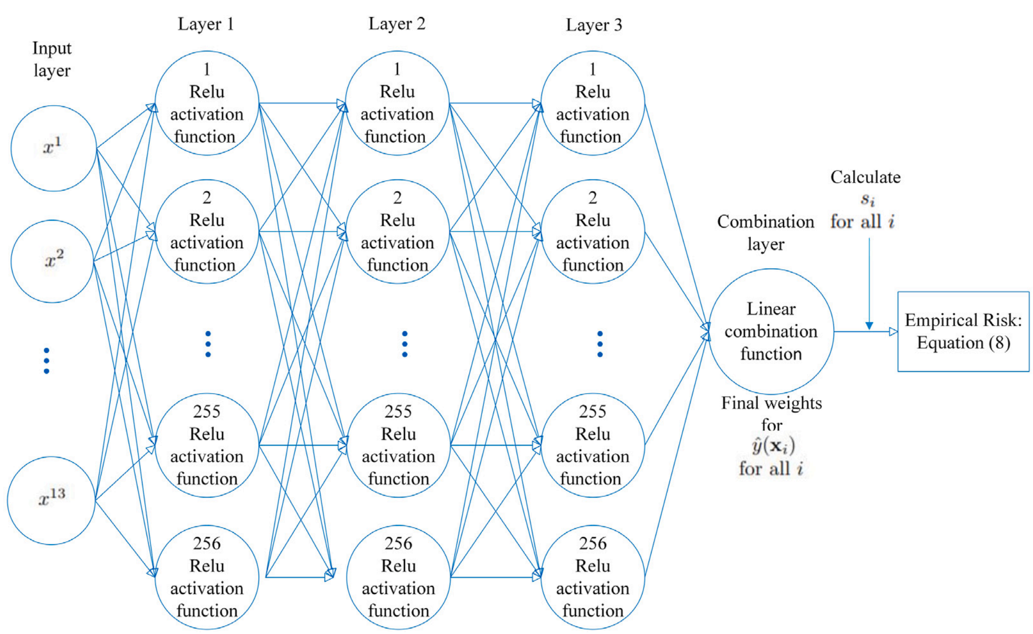

Fig. 4. NN Architecture.

is fully connected. The number of layers and nodes is determined using cross-validation as described in Section 5.1. We have visualised our NN architecture in Fig. 4. The input layer contains our multiple features $x^1, x^2, \dots, x^{11}$. Each of our three hidden layers is denoted $Layer1, Layer2, Layer3$ and each layer contains 256 nodes with $ReLu$ activation functions. To construct $\hat{y}(\mathbf{x}_i)$ an output layer is used which is a standard linear combination function. The estimated $\hat{y}(\mathbf{x}_i)$ can then be used in Eq. (8).

5.3. Experimental results

The results from our experiment are shown in Table 4. The results are calculated by taking the estimated order quantities from our trained models and calculating the total cost for up to 120 weeks of sales for each of the 3 products for each of the 15 stores in the test data. We follow Huber et al. (2019) and use the total cost which is the sum of the 45 different product and store combinations in our test data. We

Table 4
Experimental results.

	CPU time (s)	$b = 1$		$b = 1.25$		$b = 1.5$		$b = 1.75$		$b = 2$	
		Δ cost (%)	Cost ($)	Δ cost (%)	Cost ($)	Δ cost (%)	cost ($)	Δ cost (%)	Cost ($)	Δ Cost (%)	Cost ($)
Benchmark											
NM	0.74	18.10	52.78	29.07	61.59	42.54	70.40	53.60	79.21	64.76	88.02
NV	0.79	19.41	53.37	24.21	59.28	29.15	63.79	30.95	67.53	32.21	70.63
AIM	1.12	20.04	53.65	27.64	60.91	37.30	67.81	44.64	74.59	51.93	53.42
NNME	25.23	8.70	48.58	10.18	52.58	8.87	53.77	16.64	60.15	29.91	69.40
Big data											
E+O	25.97	0	44.69	0.1	48.19	2.58	50.67	4.34	53.81	6.37	56.82
IM	242.23	2.95	46.01	0	47.72	0	49.39	0	51.57	0	53.42

report the percentage increase in the total cost compared to the model with the lowest cost. Similar to Shi et al. (2016), we assume that b is larger than h, and that h is a small percentage of c. Hence, below we will report the performance results for different b values when keeping h and c constant. Furthermore, in the univariate case, a single order quantity is determined in the feature rich training data and the order quantity is then used for all the test data. In the multivariate case, the models are trained on the training data, and the multiple features observed in the test data are used as the only input for the multiple feature models. All computations were performed in R version 4.0.2 on a HP Elitebook with an Intel $i5-8250U$ processor and 8.0 GB of RAM.

The results show that big data driven models considerably outperform the univariate benchmarks. When $b = 1$, the best performing big data driven model is the estimation and optimisation method. The best univariate benchmark model has a 18.10% higher total cost. Comparing our big data driven model to the univariate benchmarks, we observe that our model has the best performance in four out of five cost instances, and the performance of the benchmarks compared to our big data driven model decreases as the lost sales cost, b, increases. When $b = 2$, the best performing univariate benchmark model has a 32.21% higher total cost than our big data driven model. The results show the value of using a big data driven model compared to a univariate data driven model in cases where relevant features are available. If one uses a univariate data driven model without considering the feature data, the inventory decision (solution) is not based on the full information available. The consequence thereof is that one risks making considerably worse inventory decisions (measured by the cost). Our performance gap between the big data driven model and the univariate models, which is the percentage difference in total cost between our model and the benchmarks, is higher than the results reported in Ban and Rudin (2018) but in line with the results reported in Huber et al. (2019). We have tested that our results are robust to changes in the h and c values while keeping the other cost values constant.

Our results confirm the conclusions found in Huber et al. (2019), namely that one can gain a large performance improvement only by using a standard multiple feature mean estimation method compared to a sophisticated univariate method. We also observe a relatively poor performance of the AIM algorithm (Huh and Rusmevichientong, 2009), which is expected given our earlier discussion of the non-IID characteristics of our data. Therefore, our study further verifies that if a company faces an inventory control problem and has access to multiple relevant features for the demand, it should consider whether a standard multiple feature model may outperform novel and sophisticated univariate models. The loss of performance by using a non-specialised multiple feature model may be compensated by the increase in the predictive accuracy gained by including multiple features into the analysis.

Comparing our big data driven model with a similar multiple feature mean estimation model, we see an increasing performance gap as b increases and h and c are held constant. When $b = 1.25$, the benchmark mean estimating NN model has a 10.18% higher total cost, and when $b = 2$, the benchmark mean estimating NN model has 29.91% higher total cost. This is an expected result since an increase in the lost sales cost can be captured in our big data driven algorithm but not in a multiple feature mean estimation model.

We also follow the testing methodology of Huber et al. (2019) by including a two-step estimation and optimisation model. We observe that when $b = 1$ the benchmark estimation and optimisation model outperforms our big data driven model with our model having 2.95% higher total cost. However, when b increases our big data driven model becomes the best performing model and we observe a consistent increase in the performance gap between our big data driven model and estimation and optimisation approach. When $b = 2$ the benchmark estimation and optimisation model has a 6.37% higher total cost. This is expected because we follow the literature by using a SAA method to do our optimisation on the regression error terms. It is documented that the performance of the SAA method decreases as one seeks to approximate either low or high distributional quantiles (Levi et al., 2015).

Our results confirm the findings in Huber et al. (2019) that it is important to choose the right big data driven model for a specific problem. For example, the two-step estimation and optimisation model uses a best in class mean estimating NN in the estimation step. Whereas, our integrated model uses a customised loss function and a novel optimiser, which are more complex. A trap that an analyst can fall into is always employing the more complex model. The results indicate that the integrated model does not always give better solutions than those of the two-step model. This is not surprising, because if the goal is only to estimate the conditional relationship between the dependent variable and the multiple features, the simpler best in class NN is likely preferred. However, when solving a big data driven OR model, such as our order-up-to level model, estimating the relationship between the dependent variable and the multiple features is only part of the solution. We also need to take our problem specific costs into account. In the two-step model the performance of the model is dependent on how well the relationship between the dependent variable and the multiple features is estimated, and how well the SAA optimisation performs. In our experiment we suspect, that when b is relatively small the SAA method performs relatively well, but when b increases the performance of the SAA method decreases. Therefore, we see that our experiment confirms the importance of choosing a big data driven model at the right complexity level for a specific problem.

6. Conclusions and future research

In this paper we strive to contribute to the new research area of big data driven OR by extending the most recent big data newsvendor model to a dynamic order-up-to level model. The main innovation of our study is the application of the ERM principle and ML to a dynamic inventory model. Before introducing our own order-up-to level inventory model, we present the concept of an optimal predictor from statistical decision theory. It enables us to better understand and generalise the methodology employed in the existing literature. Moreover, we use the ERM principle to formulate and solve a big data driven version of the order-up-to level inventory model. The model is an interesting extension of the big data driven newsvendor model because the products addressed in the model are non-perishable and the decision is dynamic. From a practitioner's standpoint, this is valuable because we have showed how to use the ERM methodology to solve a dynamic inventory model. Our study provides a simple and applicable big data driven dynamic inventory model and solutions. It can be integrated into the artificial intelligence or business intelligence packages used in the OM field. Furthermore, the experimental results show that big data driven models outperform univariate benchmark models by a large margin, and thus reconfirming the value of performing big data analysis. Moreover, our results also show that even in cases where an integrated model might be too complex for a given problem, one could still get considerably higher costs by using a univariate model compared to a simpler big data model. Comparing our big data driven model to the two-step big data driven model, we observe a consistent performance increase as the lost sales cost increases, which is expected based on theoretical results in literature.

Our study is just the first step towards extending the ERM methodology to more complex inventory models. More research on using the ERM methodology to solve complex inventory management problems should be investigated. For example, using the ERM methodology to jointly solve multi-item inventory models or solving multi-echelon inventory models needs more research. The core academic implication of this paper is the possibility of using the ERM methodology to solve dynamic inventory problems. The research question for the future is to develop a general ERM based methodology/framework for different complex big data driven inventory problems. The insights from new advances in the use of the ERM methodology such as our paper and Notz and Pibernik (2021), as well as the methodology developed by Bertsimas and Kallus (2020) could be the foundation of that work. The current challenge of the ERM methodology is how to construct tractable solution algorithms for more complex inventory problems. It would be interesting to apply algorithms from advanced ML methods such as reinforcement learning etc.

CRediT authorship contribution statement

Johan Bjerre Bach Clausen: Conceptualisation, Methodology, Software, Validation, Formal analysis, Data curation, Writing - Original Draft, Writing - review & editing, Visualisation. **Hongyan Li:** Methodology, Writing - Original Draft, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data set, used with permission, in this paper is found in the digital ap- pendix to Ledolter (2013); URL: https://www.biz.uiowa.edu/faculty/jledolter/DataMining

References

Akcay, A., Corlu, C.G., 2017. Simulation of inventory systems with unknown input models: a data-driven approach. Int. J. Prod. Res. 55 (19), 5826–5840.
Askin, R.G., 1981. A procedure for production lot sizing with probabilistic dynamic demand. AIIE Trans. 13 (2), 132–137.
Axsäter, S., 2015. Inventory Control, Vol. 225. Springer.
Ban, G.-Y., 2020. Confidence intervals for data-driven inventory policies with demand censoring. Oper. Res. 68 (2), 309–326.
Ban, G.-Y., Rudin, C., 2018. The big data newsvendor: Practical insights from machine learning. Oper. Res. 67 (1), 90–108.
Bertsimas, D., Kallus, N., 2020. From predictive to prescriptive analytics. Manage. Sci. 66 (3), 1025–1044.
Bollapragada, S., Morton, T.E., 1999. A simple heuristic for computing nonstationary (s, S) policies. Oper. Res. 47 (4), 576–584.
Bravo, F., Shaposhnik, Y., 2020. Mining optimal policies: A pattern recognition approach to model analysis. INFORMS J. Optim. 2 (3), 145–166.
Cannon, A.J., 2011. Quantile regression neural networks: Implementation in R and application to precipitation downscaling. Comput. Geosci. 37 (9), 1277–1284.
Carrizosa, E., Olivares-Nadal, A.V., Ramírez-Cobo, P., 2016. Robust newsvendor problem with autoregressive demand. Comput. Oper. Res. 68, 123–133.
Chen, C.P., Zhang, C.-Y., 2014. Data-intensive applications, challenges, techniques and technologies: A survey on Big Data. Inform. Sci. 275, 314–347.
Cheung, W.C., Simchi-Levi, D., 2019. Sampling-based approximation schemes for capacitated stochastic inventory control models. Math. Oper. Res. 44 (2), 668–692.
Chollet, F., Allaire, J.J., 2018. Deep Learning with R. Manning Publications Company, Shelter Island.
Christoffersen, P.F., Diebold, F.X., 1997. Optimal prediction under asymmetric loss. Econom. Theory 13 (6), 808–817.
Çimen, M., Kirkbride, C., 2017. Approximate dynamic programming algorithms for multidimensional flexible production-inventory problems. Int. J. Prod. Res. 55 (7), 2034–2050.
Ciocan, D.F., Mišić, V.V., 2020. Interpretable optimal stopping. Manage. Sci. 0 (0).
Corlu, C.G., Biller, B., Tayur, S., 2019. Driving inventory system simulations with limited demand data: Insights from the newsvendor problem. J. Simul. 13 (2), 152–162.
Davenport, T.H., et al., 2006. Competing on analytics. Harv. Bus. Rev. 84 (1), 98–107.
Ding, X., Puterman, M.L., Bisi, A., 2002. The censored newsvendor and the optimal acquisition of information. Oper. Res. 50 (3), 517–527.
Duchi, J., Hazan, E., Singer, Y., 2011. Adaptive subgradient methods for online learning and stochastic optimization. J. Mach. Learn. Res. 12 (7), 2121–2159.
Gallego, G., Moon, I., 1993. The distribution free newsboy problem: review and extensions. J. Oper. Res. Soc. 44 (8), 825–834.
Georghiou, A., Tsoukalas, A., Wiesemann, W., 2019. Robust dual dynamic programming. Oper. Res. 67 (3), 813–830.
Giannoccaro, I., Pontrandolfo, P., 2002. Inventory management in supply chains: a reinforcement learning approach. Int. J. Prod. Econ. 78 (2), 153–161.
Godfrey, G.A., Powell, W.B., 2001. An adaptive, distribution-free algorithm for the newsvendor problem with censored demands, with applications to inventory and distribution. Manage. Sci. 47 (8), 1101–1112.
Harsha, P., Natarajan, R., Subramanian, D., 2019. A Prescriptive Machine Learning Framework to the Price-Setting Newsvendor Problem. Working paper, Watson Research Center, New York.
Hastie, T., Tibshirani, R., Friedman, J., 2009. The Elements of Statistical Learning: Data Mining, Inference, and Prediction. Springer Science & Business Media, Berlin.
Hekimoğlu, M., van der Laan, E., Dekker, R., 2018. Markov-modulated analysis of a spare parts system with random lead times and disruption risks. European J. Oper. Res. 269 (3), 909–922.
Huber, J., Müller, S., Fleischmann, M., Stuckenschmidt, H., 2019. A data-driven newsvendor problem: From data to decision. European J. Oper. Res. 278 (3), 904–915.
Huh, W.T., Levi, R., Rusmevichientong, P., Orlin, J.B., 2011. Adaptive data-driven inventory control with censored demand based on Kaplan-Meier estimator. Oper. Res. 59 (4), 929–941.
Huh, W.T., Rusmevichientong, P., 2009. A nonparametric asymptotic analysis of inventory planning with censored demand. Math. Oper. Res. 34 (1), 103–123.
James, G., Witten, D., Hastie, T., Tibshirani, R., 2013. An Introduction to Statistical Learning, Vol. 112. Springer.
Kara, A., Dogan, I., 2018. Reinforcement learning approaches for specifying ordering policies of perishable inventory systems. Expert Syst. Appl. 91, 150–158.
Koenker, R., 2005. Computational Aspects of Quantile Regression. In: Econometric Society Monographs, Cambridge University Press, pp. 173–221. http://dx.doi.org/10.1017/CBO9780511754098.007.
Ledolter, J., 2013. Data Mining and Business Analytics with R. John Wiley & Sons, Hoboken.
Levi, R., Perakis, G., Uichanco, J., 2015. The data-driven newsvendor problem: new bounds and insights. Oper. Res. 63 (6), 1294–1306.
Liu, J., Pang, Z., Qi, L., 2020. Dynamic pricing and inventory management with demand learning: A Bayesian approach. Comput. Oper. Res. 124, 105078.
Maheshwari, S., Gautam, P., Jaggi, C.K., 2020. Role of Big Data Analytics in supply chain management: current trends and future perspectives. Int. J. Prod. Res. Advance online publication.
Mišić, V.V., 2020. Optimization of tree ensembles. Oper. Res. 68 (5), 1605–1624.
Montgomery, A.L., 1997. Creating micro-marketing pricing strategies using supermarket scanner data. Mark. Sci. 16 (4), 315–337.
Nguyen, T., Li, Z., Spiegler, V., Ieromonachou, P., Lin, Y., 2018. Big data analytics in supply chain management: A state-of-the-art literature review. Comput. Oper. Res. 98, 254–264.
Nguyen, T.T., Nguyen, N.D., Nahavandi, S., 2020. Deep reinforcement learning for multiagent systems: A review of challenges, solutions, and applications. IEEE Trans. Cybern. 50 (9), 3826–3839.
Notz, P.M., Pibernik, R., 2021. Prescriptive analytics for flexible capacity management. Manage. Sci..

Perakis, G., Roels, G., 2008. Regret in the newsvendor model with partial information. Oper. Res. 56 (1), 188–203.
Perez, H.D., Hubbs, C.D., Li, C., Grossmann, I.E., 2021. Algorithmic approaches to inventory management optimization. Processes 9 (1), 102.
Scarf, H., 1958. A min-max solution of an inventory problem. In: Arrow, K.J., Karlin, S., Scarf, H. (Eds.), Studies in the Mathematical Theory of Inventory and Production. Stanford Univ. Press, Stanford, pp. 201–209.
Shalev-Shwartz, S., Ben-David, S., 2014. Understanding Machine Learning: From Theory to Algorithms. Cambridge University Press, Cambridge.
Shao, L., Mahajan, A., Schreck, T., Lehmann, D.J., 2017. Interactive regression lens for exploring scatter plots. In: Computer Graphics Forum, Vol. 36. Wiley Online Library, pp. 157–166.
Shi, C., Chen, W., Duenyas, I., 2016. Nonparametric data-driven algorithms for multiproduct inventory systems with censored demand. Oper. Res. 64 (2), 362–370.
Simchi-Levi, D., 2014. OM forum—OM research: From problem-driven to data-driven research. Manuf. Serv. Oper. Manage. 16 (1), 2–10.
Subramanian, P., Ramkumar, N., Narendran, T., Ganesh, K., 2012. A technical note on 'Analysis of closed loop supply chain using genetic algorithm and particle swarm optimisation'. Int. J. Prod. Res. 50 (2), 593–602.
Sun, S., Cao, Z., Zhu, H., Zhao, J., 2019. A survey of optimization methods from a machine learning perspective. IEEE Trans. Cybern. 50 (8), 3668–3681.
Taube, F., Minner, S., 2018. Data-driven assignment of delivery patterns with handling effort considerations in retail. Comput. Oper. Res. 100, 379–393.
Van Foreest, N.D., Wijngaard, J., van der Vaart, T., 2010. Scheduling and order acceptance for the customised stochastic lot scheduling problem. Int. J. Prod. Res. 48 (12), 3561–3578.
Vapnik, V., 1998. Statistical Learning Theory. Wiley, New York.
Wang, G., Gunasekaran, A., Ngai, E.W., Papadopoulos, T., 2016. Big data analytics in logistics and supply chain management: Certain investigations for research and applications. Int. J. Prod. Econ. 176, 98–110.
Wang, J., Li, X., Zhu, X., 2012. Intelligent dynamic control of stochastic economic lot scheduling by agent-based reinforcement learning. Int. J. Prod. Res. 50 (16), 4381–4395.
Yu, W., Chavez, R., Jacobs, M.A., Feng, M., 2018. Data-driven supply chain capabilities and performance: A resource-based view. Transp. Res. E 114, 371–385.
Zeiler, M.D., 2012. Adadelta: An Adaptive Learning Rate Method. Cornell University, Published Online. URL: arXiv:1212.5701.
Zhang, Y., Gao, J., 2017. Assessing the performance of deep learning algorithms for newsvendor problem. In: International Conference on Neural Information Processing. Springer, pp. 912–921.
Zhang, Y., Yang, X., 2016. Online ordering policies for a two-product, multi-period stationary newsvendor problem. Comput. Oper. Res. 74, 143–151.
Zheng, M., Wu, K., Shu, Y., 2016. Newsvendor problems with demand forecast updating and supply constraints. Comput. Oper. Res. 67, 193–206.
Zipkin, P.H., 2000. Foundations of Inventory Management. McGraw-Hill, New York.