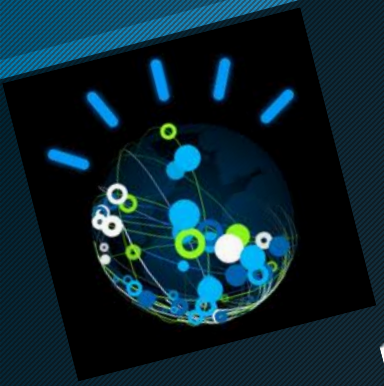
"When the Facts Change, I Change My Mind. What Do You Do, Sir?" – Attributed to Keynes or Samuelson

BAYESIAN MACHINE LEARNING – NAÏVE BAYES

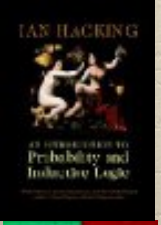
THE ROAD LIES PLAIN BEFORE ME;--'TIS A THEME SINGLE AND OF DETERMINED BOUNDS; …
-WORDSWORTH, THE PRELUDE

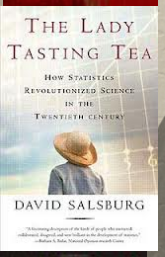


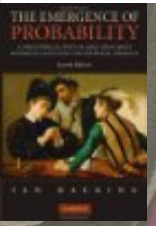
PyData | Silicon Valley 2013

Krishna Sankar,
http://doubleclix.wordpress.com
March 18,2013


Bayesian Topics
Applications
Problems
Agenda
Focus on a small slice of the Bayesian Domain
Classification
Model Evaluation
Work on a couple of problems using Python libraries
"Learn you a Bayesian for Great Good !"
Non parametric Bayesian
Bayesian Signal Processing
Classification Naïve Bayes
Reversible Jump Markov Chain Monte Carlo
Bayesian Hypothesis testing
Bayesian Networks







THANKS TO ...

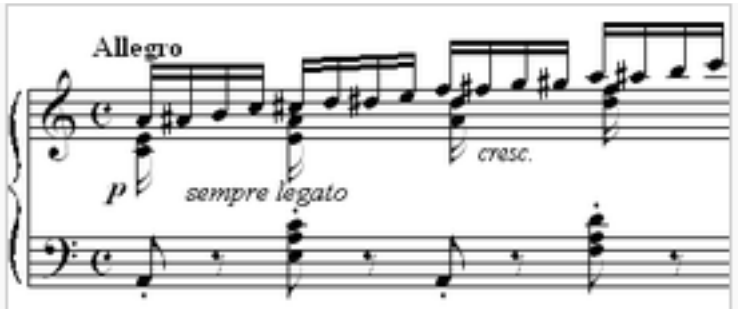
THE GIANTS WHOSE SHOULDERS I AM STANDING ON

- Prof. Allen Downey
  - Pycon Bayesian tutorial as well as his excellent books
- Prof. Andrew Moore
- Prof. Tom Mitchell/CMU
- Prof. Andrew Gelman/Columbia
- Corey Chivers/Montreal R Users Group
- Oscar Bonilla
- Calum Miller
- Olivier Grisel
- Paul Graham
- Prof. Todd Ebert
- Jacob Perkins a.k.a. StreamHacker for his excellent code
- Ref. URLs at the end
- Any omission is inadvertent & I will correct them ...

etude

An **étude** (/ ˈeɪtjuːd /; French pronunciation: [eˈtyd], a French word meaning *study*) is an instrumental musical composition, usually short and of considerable difficulty, usually designed to provide practice material for perfecting a particular musical skill. The

We will focus on "short", "acquiring skill" & "having fun"!



Frédéric Chopin's Étude Op. 10, No. 2: a rapid chromatic scale in the right hand is used to develop the weaker fingers of the right hand. Most études are written to perfect a particular technical skill.

http://en.wikipedia.org/wiki/%C3%89tude, http://www.etudesdemarche.net/articles/etudes-sectorielles.htm, http://upload.wikimedia.org/wikipedia/commons/2/26/La_Cour_du_Palais_des_%C3%A9tudes_de_l%E2%80%99%C3%89cole_des_beaux-arts.jpg

Agenda

1:15-1:40 : Bayesian Fundamentals (25)

- Bayesian Theorem, Probability (15)
- Solve 3 problems by hand (10)

1:40-1:55 : Classification Using Bayes (15)

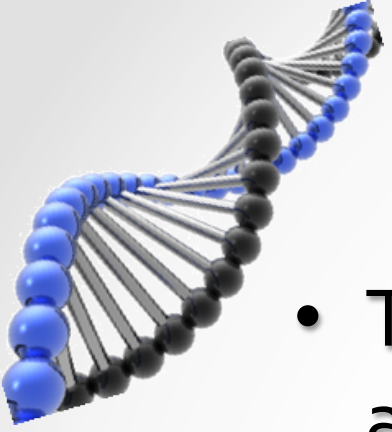
1:55-2:10 : Dr.Seuss Classification-Hands on (5)

Break (10)

2:10-2:40 : Spam Filter – Naïve Bayes (30)

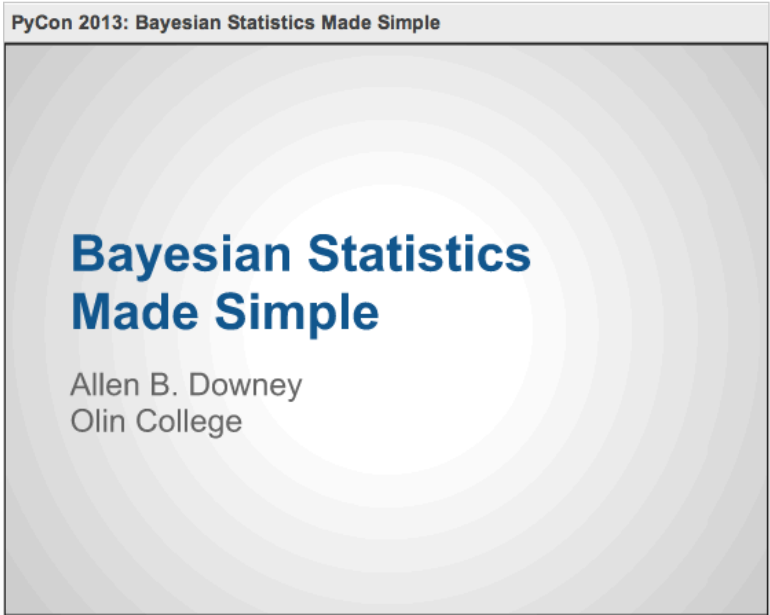
- Hands on – Bayes Spam Classifier (5+10)
- Hands on – Model Evaluation, Precision & Recall (5+10)

2:40-2:50 : Q&A(10)



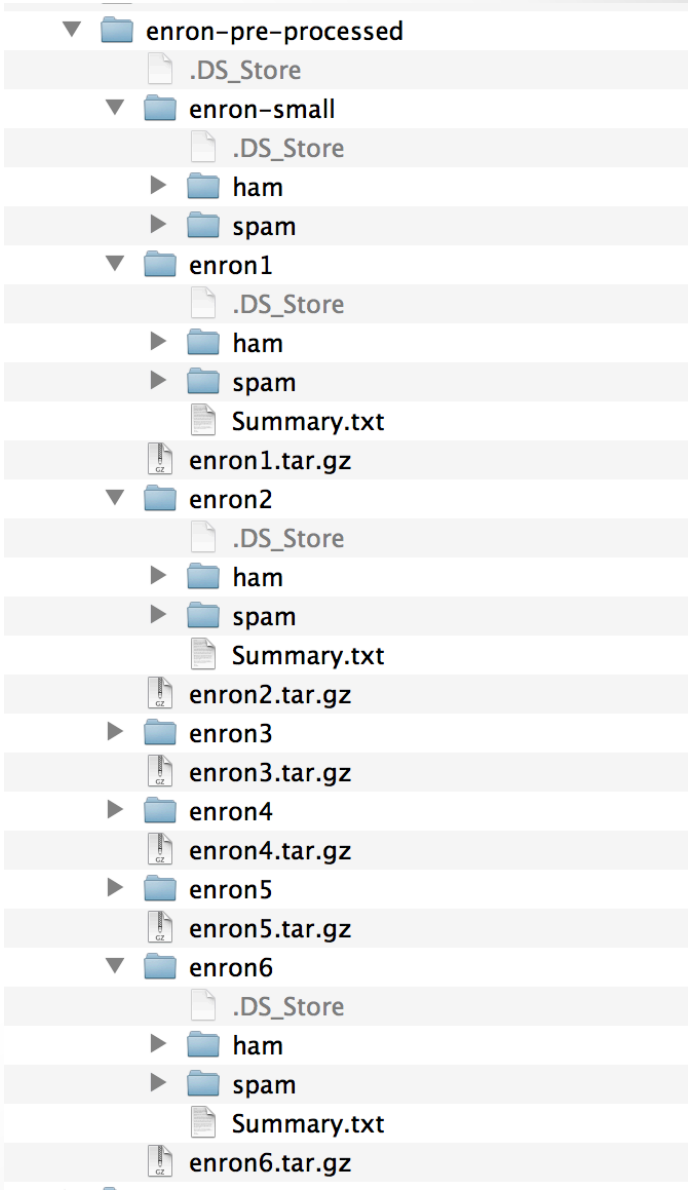
Pragmatix (1 of 3)

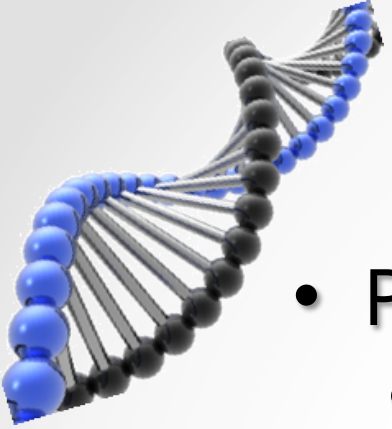
- The hands-on programs use Python 2.x and the nltk stack
  - Installing epd free is a good idea
  - http://www.enthought.com/products/epd_free.php
  - But just pip install nltk should work
- Did you attend Prof.Downey's tutorial last week ?



Pragmatix (2 of 3)

- Data
  - We will use the Enron spam dataset
  - Background : http://csmining.org/index.php/enron-spam-datasets.html
  - The files are also available at http://labs-repos.iit.demokritos.gr/skel/i-config/
- We will work with the data in the enron-pre-processed folder
- *FYI, full Enron dataset is available at http://www.cs.cmu.edu/~enron/*
- *With attachments is available at http://www.edrm.net/resources/data-sets/edrm-enron-email-data-set-v2*
  - *It is ~210 GB, aws public dataset*





Pragmatix (3 of 3)

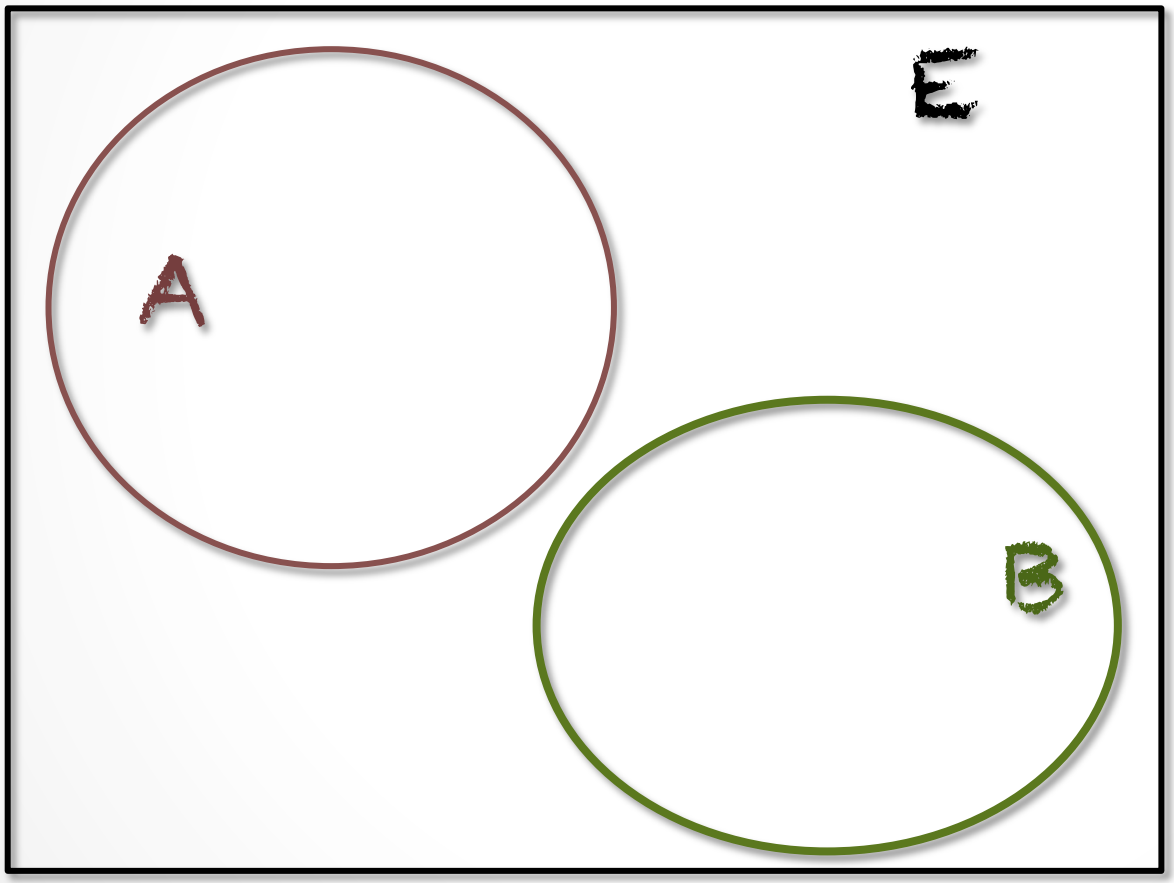
- Programs
  - The python code is in Github
  - git@github.com:xsankar/pydata.git
  - https://github.com/xsankar/pydata
  - Three files
    - cat_in_the_hat-xx.py
    - spam-01.py
    - Spam-02.py
- This slide set is in slideshare
  - *http://www.slideshare.net/ksankar/pydata-19*
  - as well as in the github repository

http://www.google.com/doodles/douglas-adams-61st-birthday

Bayesian Fundamentals

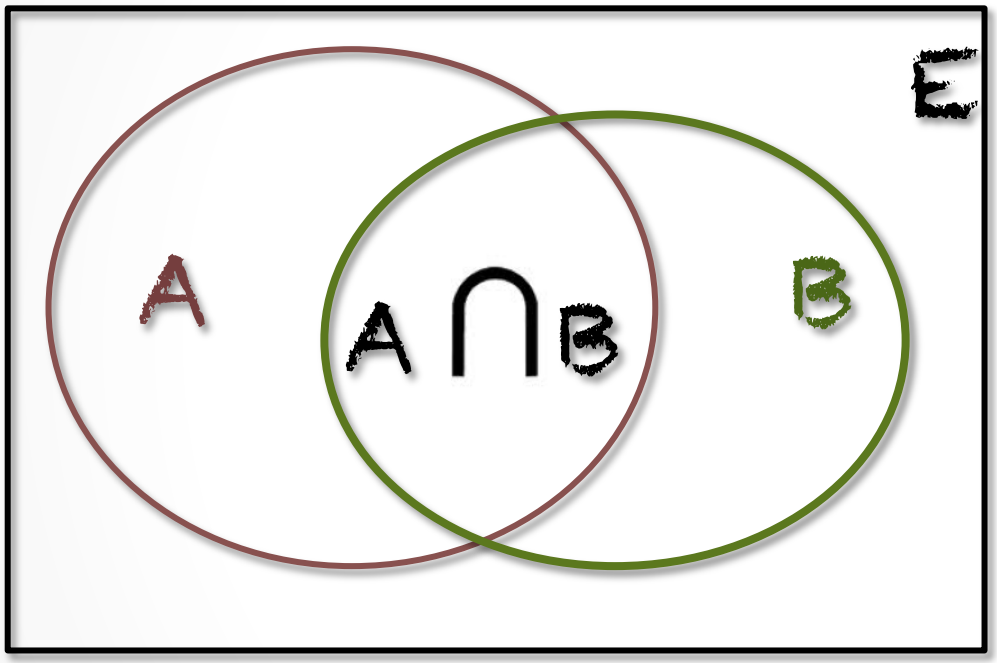
"Probability theory is nothing but common sense reduced to calculation"
- Pierre Laplace, 1812

Independent Events



- E = Event Space
- A,B = Event sets
- No possibility of both A & B occurring

Overlapping Event Sets



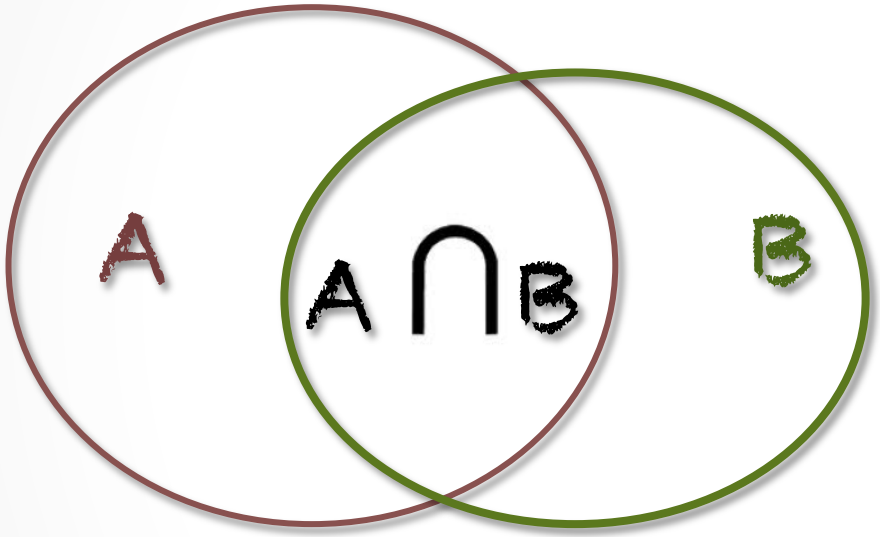
- E = Event Space
- A,B = Event sets
- Area A & B where both occur

Basics

- $\mathcal{P}(A)$ = Probability of event A
- $\mathcal{P}(B)$ = Probability of event B
- $\mathcal{P}(A \wedge B)$ = Probability of events A & B
- $\mathcal{P}(A|B)$ = Conditional probability of event A given that B has happened
- Definitions (Very important to internalize)
  - Joint Probability - $\mathcal{P}(A \wedge B)$
  - Conditional Probability - $\mathcal{P}(A|B)$
  - Marginal Probability – Unconditional Probability of one event occurring – $\sum \mathcal{P}(A|B) * \mathcal{P}(B)$

https://controls.engin.umich.edu/wiki/index.php/Bayes_Rule,_conditional_probability,_independence

Overlapping Event Sets



- E = Event Space
- A,B = Event sets

$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$= P(B) \cdot P(A|B)$$

$$P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

2

Bayes Theorem

"Inverse Inference Rule"

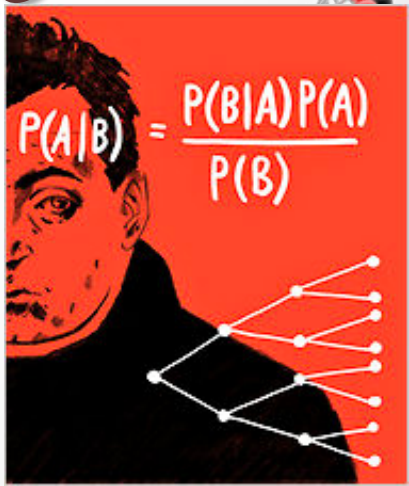
Thomas Bayes

Portrait used of Bayes in the 1936 book *History of Life Insurance*; it is dubious whether it actually depicts Bayes.[1] No earlier portrait or claimed portrait survived.

Born	c. 1701 London, England
Died	7 April 1761 (aged 59) Tunbridge Wells, Kent, England
Nationality	English

Signature

T. Bayes.



The paper that started it all …

PHILOSOPHICAL
TRANSACTIONS:

Royal Society Publishing
Informing the science of the future

LII. *An Eſſay towards ſolving a Problem in the Doctrine of Chances. By the late Rev. Mr.* Bayes, *F. R. S. communicated by Mr.* Price, *in a Letter to* John Canton, *A. M. F. R. S.*

Dear Sir,

Read Dec. 23, 1763. I Now ſend you an eſſay which I have found among the papers of our deceaſed friend Mr. Bayes, and which, in my opinion, has great merit, and well deſerves to be preſerved. Experimental philoſophy, you will find, is nearly intereſted in the ſubject of it; and on this account there ſeems to be particular reaſon for thinking that a communication of it to the Royal Society cannot be im-

PROBLEM.

Given the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a ſingle trial lies ſomewhere between any two degrees of probability that can be named.

- http://rstl.royalsocietypublishing.org/content/53/370.short
- http://en.wikipedia.org/wiki/An_Essay_towards_solving_a_Problem_in_the_Doctrine_of_Chances

Utility Of Bayes Theorem

- "Inference is a Big deal"[1]
  - I have this evidence, what is the probability that this conclusion is true (based on previous experience)
  - Establish
    - Probabilities & Relationships
  - For
    - Inference & Prediction
- Bayes Classifiers[1]
  - Rather Technical Complaint: Bayes Classifiers don't try to be maximally discriminative---they merely try to honestly model what's going on
  - Zero probabilities are painful for Joint and Naïve. A hack (justifiable with the magic words "Dirichlet Prior") can help
  - Naïve Bayes is wonderfully cheap. And survives 10,000 attributes cheerfully!

[1] *http://www.autonlab.org/tutorials/prob18.pdf, Prof. Andrew Moore*

Bayes Theorem - Formal

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

2

Assume we have observed occurrences of χ *&* θ *in an event set. Later we want to know given* χ *(Data), what is the probability that* θ *(Hypothesis) has occurred.*

Posterior

Likelihood

Prior

$$P(\theta|\chi) = \frac{P(\chi|\theta) \cdot P(\theta)}{P(\chi)}$$

3

Normalizing Constant (a.k.a Marginal Probability)

Bayes Theorem – 2nd form

$$P(\theta|x) = \frac{P(x|\theta)\cdot P(\theta)}{P(x)} \quad (3)$$

$$P(\theta|x) = \frac{P(x|\theta)\cdot P(\theta)}{P(x|\theta)\cdot P(\theta)+P(x|\sim\theta)\cdot P(\sim\theta)} \quad (4)$$

- Marginal Probability ie unconditional probability
- Properties
  - Exclusive & Exhaustive

Bayes Theorem – 2nd (extended) form

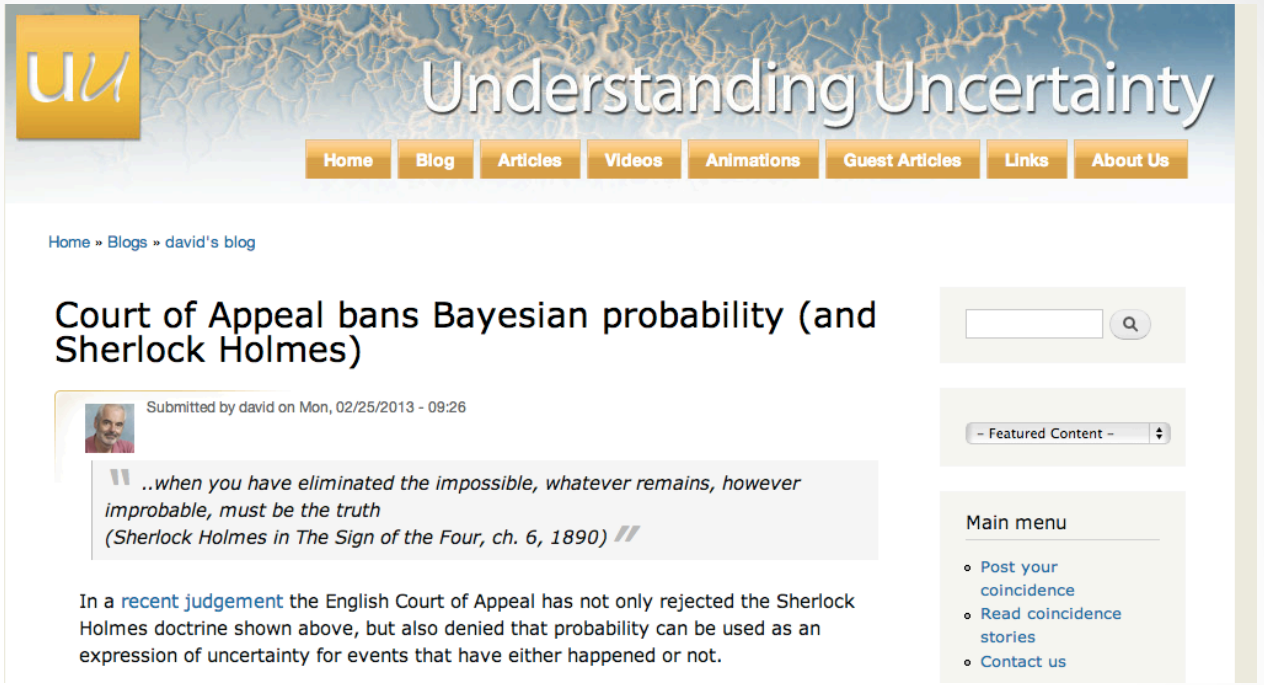
$$P(\theta|x) = \frac{P(x|\theta)\cdot P(\theta)}{P(x)} \quad \text{---------- 3}$$

$$P(\theta|x) = \frac{P(x|\theta)\cdot P(\theta)}{\sum_m P(x|\theta_m)\cdot P(\theta_m)} \quad \text{---------- 5}$$

✓ *When m = 2, we get equation 4.*

In many cases (eg.argmax), P(X) calculation can be avoided

$$P(\theta|x) \propto P(x|\theta)\cdot P(\theta) \quad \text{---------- 6}$$



"..Whilst the chance of a future event happening may be expressed in percentage terms the same is not true of past events. It cannot be said that there is a particular percentage chance that a particular event happened. The event either happened or it did not..."

" .. the "balance of probabilities" standard when expressed mathematically as "50 + % probability" carried with it a "danger of pseudo-mathematics .."

Ref : http://understandinguncertainty.org/court-appeal-bans-bayesian-probability-and-sherlock-holmes
http://www.lexology.com/library/detail.aspx?g=471d7904-20d2-4fdb-a061-8d49b10de60d&l=7HVPC65
http://www.12kbw.co.uk/cases/commentary/id/161/,

Good References

- http://yudkowsky.net/rational/bayes has an excellent write-up on Bayes theorem.
- Branden Fitelson's PhD thesis "Studies in Bayesian Confirmation Theory [http://fitelson.org/thesis.pdf] is a good read
- http://plato.stanford.edu/entries/bayes-theorem/
- http://plato.stanford.edu/entries/bayes-theorem/supplement.html

Let us solve some problems by hand

- Strategy
  - Choose representation of hypothesis
  - Choose representation for Data
  - Write appropriate likelihood function
  - Calculate
- I don't like cancer problems, let us try to get some happier problems

Strategy : http://www.greenteapress.com/thinkbayes/thinkbayes.pdf
Problems : http://people.hofstra.edu/Stefan_Waner/RealWorld/tutorialsf3/frames6_5.html
http://people.hofstra.edu/Stefan_Waner/RealWorld/tutorialsf3/unit6_6.html

Problem #1

"Of Colossal Conglomerate's 16,000 clients, 3200 own their own business, 1600 are "gold class" customers, and 800 own their own business and are also "gold class" customers. What is the probability that a randomly chosen client who owns his or her own business is a "gold class" customer?"

Hint : Create a 4 X 4 Matrix

Problem #1

"Of Colossal Conglomerate's 16,000 clients, 3200 own their own business, 1600 are "gold class" customers, and 800 own their own business and are also "gold class" customers. What is the probability that a randomly chosen client who owns his or her own business is a "gold class" customer?"

	Owner	!Owner	
Gold	800		1600
! Gold			
	3200		16,000

Solution – Intuition i.e.Table form

	Owner	!Owner	
Gold	800	800	1600
! Gold	2400	12,000	
	3200	12,800	16,000

$$\mathcal{P}(O \wedge G) = 800/3200 = 0.25$$

Solution – Bayes

	Owner	!Owner	
Gold	800		1600
! Gold			
	3200		16,000

$$P(G|O) = \frac{P(O|G) \cdot P(G)}{P(O)}$$

3

$$= \frac{(800/1600)(1600/16000)}{(3200/16000)}$$

$$= 800/3200$$

$$= 0.25$$

Problem #2

A test for a disorder has the following properties. If you have the disorder, the probability that the test returns positive is 0.999. If you don't have the disorder the probability that the test returns positive is 0.004. Assume that 3% of the population has the disorder. If a person is randomly chosen from the population and tested, and the test returns positive, what is the probability that the person has the disorder?

Hint : Use Bayes 2nd form

$$P(\theta|X) = \frac{P(X|\theta)\cdot P(\theta)}{P(X|\theta)\cdot P(\theta)+P(X|\sim\theta)\cdot P(\sim\theta)}$$

4

θ = Has disorder,
X = Positive result

http://www.cecs.csulb.edu/~ebert/teaching/lectures/551/bayes/bayes.pdf

Solution – Problem #2

$$P(\theta|X) = \frac{P(X|\theta)\cdot P(\theta)}{P(X|\theta)\cdot P(\theta)+P(X|\sim\theta)\cdot P(\sim\theta)}$$

4

θ = Has disorder, X = Positive result

$$\frac{0.999 * 0.03}{(0.999 * 0.03) + (0.004 * 0.97)} = 0.885$$

Intuition:

- Initial Prior Belief = 0.03 same as general population
- New data/observation = The test ;o(
- After test, increased to 0.885, given the likelihood 0.999 & factor 0.004 * 0.97

Problem #3

- When observing a blue vehicle driving by on a dimly-lit night, there is a 25% chance that its true color is green. The same can be said when observing a green vehicle under the same conditions: there is a 25% chance that its true color is blue.
- Suppose that 90% of taxis on the streets have blue color, while the other 10% have green color.
- On a dimly-lit night you see a taxi randomly drive by and observe its color as blue.
- What is the probability that its true color is blue?
- Hint: *Condition the event that the true color is blue on the event that you've observed it to have blue color*.

Solution – Problem #3

$$P(\theta|X) = \frac{P(X|\theta)\cdot P(\theta)}{P(X|\theta)\cdot P(\theta)+P(X|\sim\theta)\cdot P(\sim\theta)}$$

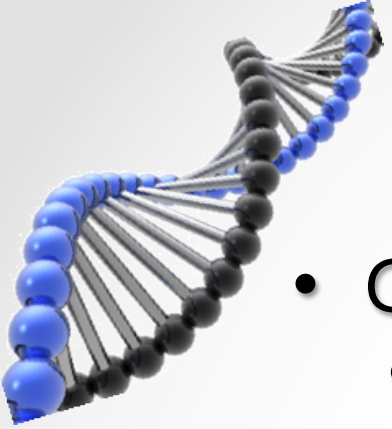
4

θ = Color is Blue, X = Observation is blue

$$\frac{(.75 * 0.9)}{(0.75*0.9)+(0.25*.1)} = 0.964$$

http://www.google.com/doodles/nicolaus-copernicus-540th-birthday

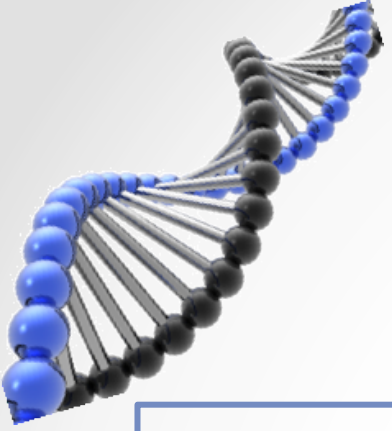
Classification



Classification

- Classifiers
  - Label Documents – Binary classification or multi-class classification
  - Mechanics
    - Learn from labeled feature sets
    - Then during classification extract feature set from an unknown document & map to known label-features
- Uses
  - Categorization – Topics, Languages, Type, News Feed…
  - Spam Filtering
  - Sentiment Analysis
  - Security
  - Topic Modeling – LDA -> 3 level Bayesian
  - Recommender Systems
  - Ad placement based on web page visits
  - Authorship – The Federalist Papers

Classifying Classifiers



- Statistical
  - Regression
  - Boosting
  - SVM
  - Naïve Bayes
  - Bayesian Networks
  - Logistic Regression[1]
- Structural
  - Neural Networks
    - Multi-layer Perception
  - Rule-based
    - Production Rules
    - Decision Trees
  - Distance-based
    - Functional
      - Linear
      - Spectral Wavelet
    - Nearest Neighbor
      - kNN
      - Learning vector Quantization
  - Ensemble
    - Random Forests

[1]Max Entropy Classifier

Ref: Algorithms of the Intelligent Web, Marmanis & Babenko

Bayes Classifier

Definitions :

d = Document, $\mathcal{D}$ = Document space

c = class, C = Class space

t = token, $\mathcal{T}$ = Token space

$$d \in D \quad c \in \{c_1, c_2, \ldots c_n\} \quad t \in \{t_1, t_2, \ldots t_n\}$$

Problem:

Given a document d,

Map it to a class in C

http://nlp.stanford.edu/IR-book/pdf/13bayes.pdf

Bayes Classifier

Bayes Theorem says that

$$\mathcal{P}(c|d) \propto \mathcal{P}(c)\ \mathcal{P}(d|c) \quad \text{----------} \quad (6)$$

Our problem reduces to finding $\mathcal{P}(c)$ & $\mathcal{P}(d|c)$

If we have a training set of N′ documents, without any prior knowledge our estimate would be

$$\mathcal{P}(c) = N_c/N' \quad \text{---------} \quad (7)$$

where N_c = Number of document in class c

http://nlp.stanford.edu/IR-book/pdf/13bayes.pdf

Bayes Classifier

We can define a document in terms of a set of features, here the tokens it has; and then

$$\mathcal{P}(d|c) = \prod_{1 \to k} \mathcal{P}(t_k|c) \quad \text{---------} \; 8$$

- k = Number of tokens &
- $\mathcal{P}(t_k|c)$ = The probability of t_k occurring in class c, likelihood as inferred from the training set, "conditional attribute probability"

Note : Since multiplication of small numbers can result in floating point underflow, we take log & add them

Bayes Classifier

We still are left with calculating the likelihood probability $\mathcal{P}(t_k|c)$:

$$\mathcal{P}(t_k|c) = \frac{N_{tc}}{\sum N_{tc}} \quad (8)$$

Which is the relative frequency of t occurring in class c in the training set, a.k.a. conditional attribute probability

In order to make this work for unseen tokens, ie if N = 0; we add the Laplace Smoothing as follows:

$$\mathcal{P}(t_k|c) = \frac{N_{tc} + 0.5}{\sum N_{tc} + (N_t/2)} \quad (9)$$

Bayes Classifier

This is exactly what nltk does!

```
class ELEProbDist(LidstoneProbDist):
    """
    The expected likelihood estimate for the probability distribution
    of the experiment used to generate a frequency distribution.  The
    X{expected likelihood estimate} approximates the probability of a
    sample with count M{c} from an experiment with M{N} outcomes and
    M{B} bins as M{(c+0.5)/(N+B/2)}.  This is equivalant to adding 0.5
    to the count for each bin, and taking the maximum likelihood
    estimate of the resulting frequency distribution.
    """
```

One more thing ….

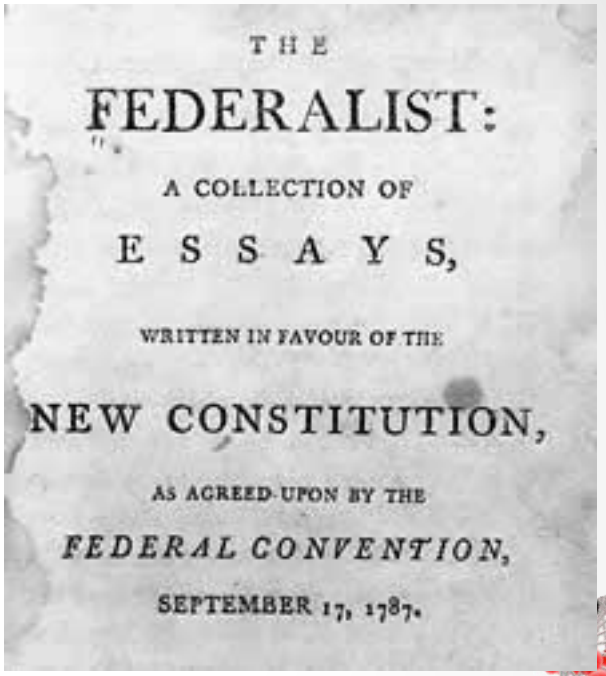
Combining all the equations 6 & 9, we get:

$$\mathcal{P}(c|d) \propto \mathcal{P}(c) \prod_{1 \to k} \frac{N_{kc} + 0.5}{\sum N_{kc} + (k/2)} \quad \text{---------} \quad 10$$

Our work is to find the best class, so we select the class that maximizes equation 10

The Federalist Papers

- 85 Essays published anonymously 1787-88 to persuade New Yorkers to ratify the US constitution
- Authorship of 12 papers was not clear – between Hamilton and Madison
- In 1962, Mosteller & Wallace undertook a study to fingerprint the authorship based on word frequencies in previous writings
- Their work revived Bayesian Classification Methods

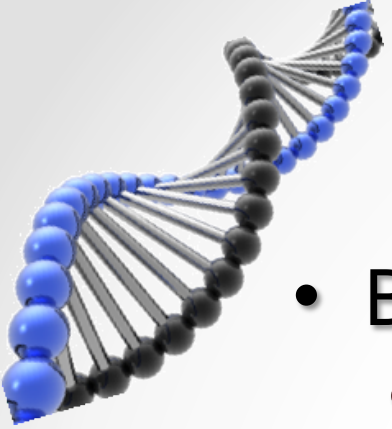

THE
FEDERALIST:
A COLLECTION OF
ESSAYS,
WRITTEN IN FAVOUR OF THE
NEW CONSTITUTION,
AS AGREED UPON BY THE
FEDERAL CONVENTION,
SEPTEMBER 17, 1787.

Education: Madison's Avenue

Sep 21, 1962

Oh Hamilton, Poor Hamilton, Madison Wrote 'Em and You're Feeling So Sad. That, in effect, was the title of the story, sketched out last week before a joint-meeting of the American Statistical Association, the Biometric Society and the Institute of Mathematical Statistics. Plotting the story were two smart mathematicians, Harvard's Frederick Mosteller, and the University ...

http://www.freerepublic.com/focus/f-news/1386274/posts



Feature Extraction

- Bag Of Words
  - Tokenize into unigrams (split et al)
  - Binary Occurrence (True/False)
  - Frequencies (as opposed to set of words)
  - TF-IDF
  - Bigram of words (Better : e.g. "New York", "Very Bad")
  - N-gram characters (for language classification)
- Cleanup
  - Lowercase
  - Take out accents
  - At least 2 characters
  - Remove stop words
  - Order doesn't matter

http://blip.tv/pycon-us-videos-2009-2010-2011/pycon-2011-statistical-machine-learning-for-text-classification-with-scikit-learn-4898362

Bayesian Analytics Data Pipeline Story Board

Stages, Strategies & Tasks

Stage 1 - Collect

- Collect & Store
- Store as Text
- Could ne MongoDB or HDFS

Stage 2 - Annotate

- Cleanup & Annotate
- Transform, Validate Data & Refresh
- Setup stream & trickle feeds

Stage 3 – Build Model

- Extract Feature Vector (numerical representation)
- Label Training Set
- Reason

Stage 4 – Train the Dragon

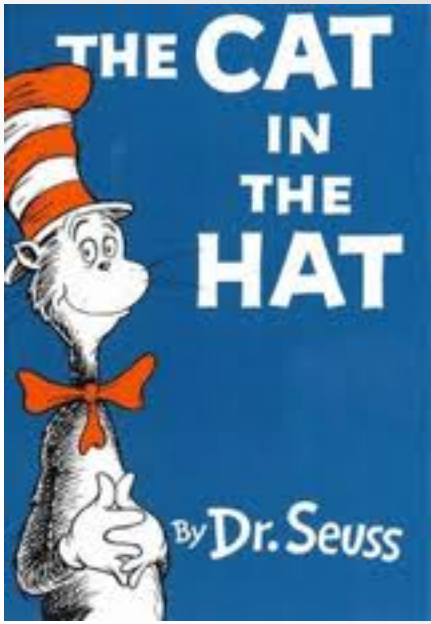
- Train Algorithms
- Cross validation - Dev Test set
- Validate Model

Stage 5 - Evaluate

- Evaluate F-score on unseen Test set
- Inspect Confusion Matrix
- Tweak

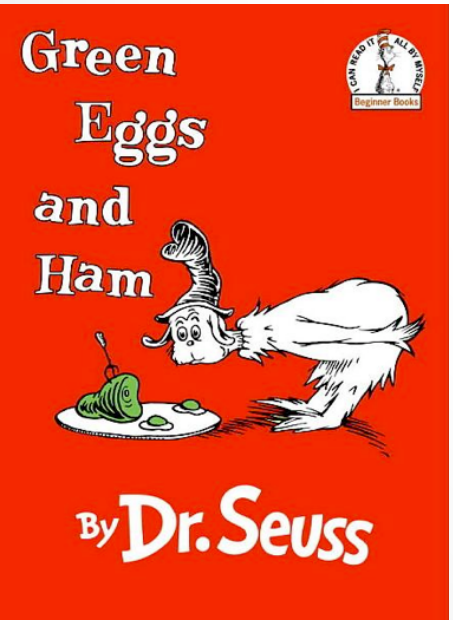
Stage 6 - Deploy

- Deploy into prod
- Monitor & Adjust
- Update with priors

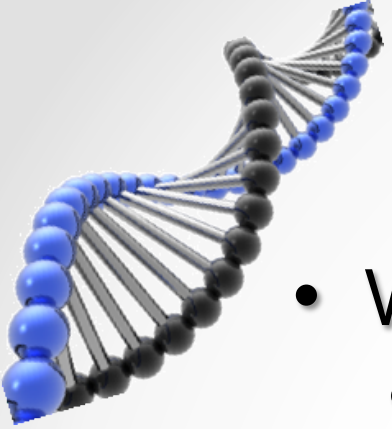


A Classification of the Seuss Kind

. . .



Ham I am Thing1 & Thing2



Problem Statement

- We have two small paragraphs
  - One from "Cat In The Hat" &
  - Another from "Green Eggs and Ham"
- We train the classifiers with them
- Then we test our classifier with other lines from the books
- Gives us a starting point for Bayesian Classifier
- Hands on : Load "cat_in_the_hat-xx.py"

Code Walk thru (1 of 2)

```
#!/usr/bin/env python
#
# pydata Tutorial March 18, 2013
# Thanks to StreamHacker a.k.a. Jacob Perkins
# Thanks to Prof.Todd Ebert
#
import nltk.classify.util
from nltk.classify import NaiveBayesClassifier
from nltk import tokenize
#
# Data
#
label_1 = "Cat In The Hat"

train_text_1 = "So we sat in the house all that cold, wet day. \
And we saw him! The Cat in the Hat! \
Your mother will not mind. \
He should not be here when your mother is out. \
With a cake on the top of my hat! \
I can hold the ship."

test_text_1 = "That cat is a bad one,\
That Cat in the Hat. \
He plays lots of bad tricks. \
Don't you let him come near."

label_2 = "Green Eggs and Ham"
train_text_2 = "I am Sam. \
Sam I am.\
I do not like green eggs and ham. \
I would not like them anywhere. \
Would you like them in a house? \
I will not eat them on a train. \
Thank you, thank you, Sam I am."

test_text_2 = "I would not eat them\
here or there.\
I would not eat them anywhere.\
I would not eat green eggs and ham."
#
# For testing classification
#
classify_cith = "We saw the cat in the house."
classify_geah = "And I will eat them in a house."
classify_other = "A man a plan a canal Panama!"
```

Code Walk thru (2 of 2)

```
#
# Take a list of words and turn it into a Bag Of Words
#
def bag_of_words(words):
    return dict([(word, True) for word in words])
#
# Program Starts Here
#
#
# Step 1 : Feature Extraction
#
train_words_1 = tokenize.word_tokenize(train_text_1)
train_words_2 = tokenize.word_tokenize(train_text_2)
train_cith = [(bag_of_words(train_words_1), label_1)]
train_geah = [(bag_of_words(train_words_2), label_2)]

test_words_1 = tokenize.word_tokenize(test_text_1)
test_words_2 = tokenize.word_tokenize(test_text_2)
test_cith = [(bag_of_words(test_words_1), label_1)]
test_geah = [(bag_of_words(test_words_2), label_2)]

train_features = train_cith + train_geah
test_features = test_cith + test_geah
#
# Step 2: Train The Dragon er ... classifier ;o)
#
classifier = NaiveBayesClassifier.train(train_features)
print 'Accuracy : %d' %
nltk.classify.util.accuracy(classifier, test_features)
#print classifier.most_informative_features()
#print classifier.labels()
#
# Step 3 :Test Classification
#
print
classifier.classify(bag_of_words(tokenize.word_tokenize
(classify_cith)))
print
classifier.classify(bag_of_words(tokenize.word_tokenize
(classify_geah)))
print
classifier.classify(bag_of_words(tokenize.word_tokenize
(classify_other)))
#
# That's all Folks!
#
```

Run the program

Break

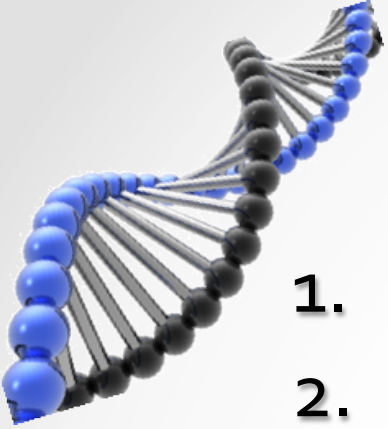
Aftermath/Postmortem

Discussions ?

Questions ?

Spam Classification

Spam I am
I do not like them
Not in my Outlook
Nor in my Gmail
I will not take them in my Inbox
I like them trashed sooner they arrive



Problem Statement

1. Train the classifier with ham & spam
2. Then classify unseen data set
3. Evaluate the model

- Programs:
  - spam-01.py
  - spam-02.py
- Dataset
  - Enron
- Following the work by Metsis, Androusopoulos & Paliouras
- http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.61.5542

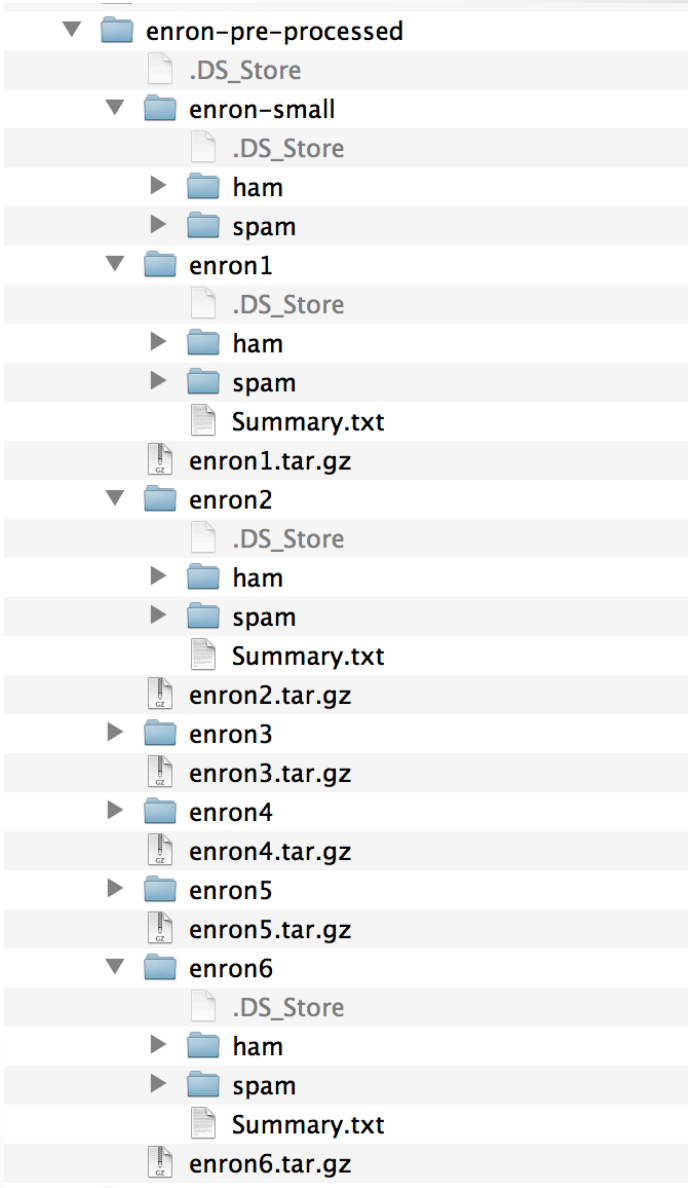
Background

- Spam & Ham data are hard to come by
- In 2005, the Enron investigations made public e-mails of 150 employees from Enron
- That gave impetus to spam research work
- The data we are using are the processed e-mails of 6 employees, mixing with 3 spam collections
- 16,545 Ham, 17,171 spam for a total of 33,716 items

	Ham	Spam
enron1	3672	1500
enron2	4361	1496
enron3	4012	1500
enron4	1500	4500
enron5	1500	3675
enron6	1500	4500

Data Prep

- Unzip the data
- Each user has ham & spam directory
- The program spam-01.py runs over these files.
- You should change the directory name [root_path variable] to where you downloaded the data
- *Caution :*
  - *The program takes 10 minutes to run over the entire dataset.*
  - *I created a directory enron-small with 20 ham & spam e-mails to test during development*



Code walk-thru

```
!/usr/bin/env python
#
# pydata Tutorial March 18, 2013
# Thanks to StreamHacker a.k.a. Jacob Perkins
# Thanks to Metsis, Androutsopoulos & Paliouras
#
import os
import sys
import time
#
import nltk.classify.util
from nltk.classify import NaiveBayesClassifier
from nltk import tokenize

#
# Take a list of words and turn it into a Bag Of Words
#
def bag_of_words(words):
   return dict([(word, True) for word in words])

start_time = time.clock()

#
# Data
#
label_1 = "ham"
label_2 = "spam"
ham_list=[]
spam_list=[]
#
root_path = '/Users/ksankar/Documents/Erlang/Bayesian/spam/enron-spam/enron-pre-processed/'
directories = ['enron5']
#['enron1','enron2','enron3','enron4','enron5','enron6']
labels=['ham','spam']
#
# Need separate lists so that we can vary the proportions
#
ham_features=[]
spam_features=[]
tot_files=0
for directory in directories:
   label = labels[0] # ham
   dir_name = os.path.join(root_path,directory,label)
   #print dir_name
   for file in os.listdir(dir_name):
      file_name_full = os.path.join(dir_name,file)
      #print 'Reading file %s' % (file_name_full)
      #print '.',
      in_buffer = open(file_name_full,'r').read()
      ham_features.append((bag_of_words(tokenize.word_tokenize(in_buffer)),
label)) # ham
      tot_files += 1
   print "%s %s Total Files = %d Feature Count = %d" % (directory, label, tot_files,
len(ham_features))
   #
   label = labels[1] # spam
   dir_name = os.path.join(root_path,directory,label)
   for file in os.listdir(dir_name):
      file_name_full = os.path.join(dir_name,file)
      in_buffer = open(file_name_full,'r').read()
      spam_features.append((bag_of_words(tokenize.word_tokenize(in_buffer)),
label)) # spam
      tot_files += 1
   print "%s %s Total Files = %d Feature Count = %d" % (directory, label, tot_files,
len(spam_features))
read_time = time.clock()
print "Read Time : %1.3f" % (read_time-start_time)
```

```
#
# Now ceate the training and test data sets
#
ham_cutoff=len(ham_features) * 9/10 #3/4
spam_cutoff=len(spam_features) * 9/10 # 3/4
#
print ham_features[0]
print spam_features[0]
print ham_features[1]
print spam_features[1]
trainfeats = ham_features[:ham_cutoff] + spam_features[:spam_cutoff]
testfeats = ham_features[ham_cutoff:] + spam_features[spam_cutoff:]
print 'train on %d instances, test on %d instances' % (len(trainfeats), len(testfeats))
classifier = NaiveBayesClassifier.train(trainfeats)
print classifier.labels()
print 'Accuracy:', nltk.classify.util.accuracy(classifier, testfeats)
classifier.show_most_informative_features()
#
end_time = time.clock()
print "Run Time : %1.3f" % (end_time-start_time)
print "That's all Folks! All Good Things have to come to an end !"
#sys.exit()
#
# That's all Folks!
#
```

Hands-On

Run spam-01.py

Inspect variables

Try changing Training Set vs. Validation Set

enron5 ham Total Files = 1500 Feature Count = 1500

enron5 spam Total Files = 5175 Feature Count = 3675

Read Time : 78.967

, 'remove': True, '2002': True, 'i': True, 'write': True, '2020': True, 'have': True, 'wish': True, 'at': True, 'the': True, 'inc': True, 'or': True, 'Subject': True}, 'spam')

({'operations': True, 'help': True, 'global': True, 'already': True, 'causey': True, 'harrison': True, 'still': True, 'staff': True, 'had': True, ',': True, 'send': True, 'should': True, 'to': True, 'going': True, 'rick': True, 'mail': True, 'sent': True, '@': True, 'possible': True, 'early': True, 'know': True, 'shared': True, 'not': True, 'regarding': True, 'notes': True, 'leave': True, 'glover': True, 'Subject': True, 'energy': True, 'some': True, 'apologies': True, 'brent': True, 'subject': True, '01': True, '06': True, 'for': True, 'away': True, '/': True, 're': True, 'new': True, 'before': True, 'shults': True, 'be': True, 'we': True, 'beck': True, 'ect': True, 'corp': True, 'wanted': True, 'by': True, 'enron': True, 'on': True, 'addtional': True, 'of': True, 'bob': True, 'create': True, 'first': True, 'sheila': True, 'via': True, 'solmonson': True … 'my': True, 'and': True, 'an': True, 'vp': True, 'as': True, 'at': True, 'in': True, 'any': True, '!': True, 'again': True, 'functions': True, 'end': True, 'provide': True, 'personnel': True, 'discussion': True, 'hou': True, 'role': True, 'responsibility': True, 'you': True, 'ena': True, 'price': True, 'yesterday': True, 'validate': True, 'america': True, 'sally': True, 'a': True, '04': True, 'susan': True, 'i': True, 'more': True, '2000': True, 'so': True, 'organization': True, 'the': True, 'having': True}, 'ham')

({'through': True, 'edt': True, 'now': True, 'll': True, ',': True, 'to': True, 'usage': True, 'checking': True, 'unlimited': True, 'one': True, 'name': True, 'list': True, 'email': True, 'try': True, 'Subject': True, 'activate': True, 'rate': True, 'expect': True, 'our': True, 'click': True, 'what': True, 'for': True, 'please': True, 'daytime': True, 'state': True, 'toll': True, '529': True, '?': True, 'be': True, 'we': True, 'who': True, 'friday': True, 'free': True, 'here': True, 'by': True, 'on': True, 'of': True, 'trial': True, 'connection': True, 'or': True, 'period': True … 'is': True, 'received': True, 'am': True, 'it': True, 'have': True, 'if': True, '!': True, 'information': True, '877': True, 'offer': True, '-': True, '1': True, 'valid': True, '25711': True, '9': True, 'you': True, 'week': True, 'monday': True, 'after': True, 'never': True, 'a': True, "'": True, 'clear': True, 'nighttime': True, 'the': True, 'mailing': True, 'spend': True}, 'spam')

train on 4657 instances, test on 518 instances

['spam', 'ham']

Accuracy: 0.994208494208

Most Informative Features

beck = True	ham : spam	=	485.6 : 1.0
ect = True	ham : spam	=	427.5 : 1.0
sally = True	ham : spam	=	359.2 : 1.0
713 = True	ham : spam	=	167.3 : 1.0
causey = True	ham : spam	=	154.3 : 1.0
sheila = True	ham : spam	=	136.3 : 1.0
cc = True	ham : spam	=	123.3 : 1.0
ted = True	ham : spam	=	120.0 : 1.0
mike = True	ham : spam	=	107.5 : 1.0
leslie = True	ham : spam	=	100.4 : 1.0

Run Time : 103.863

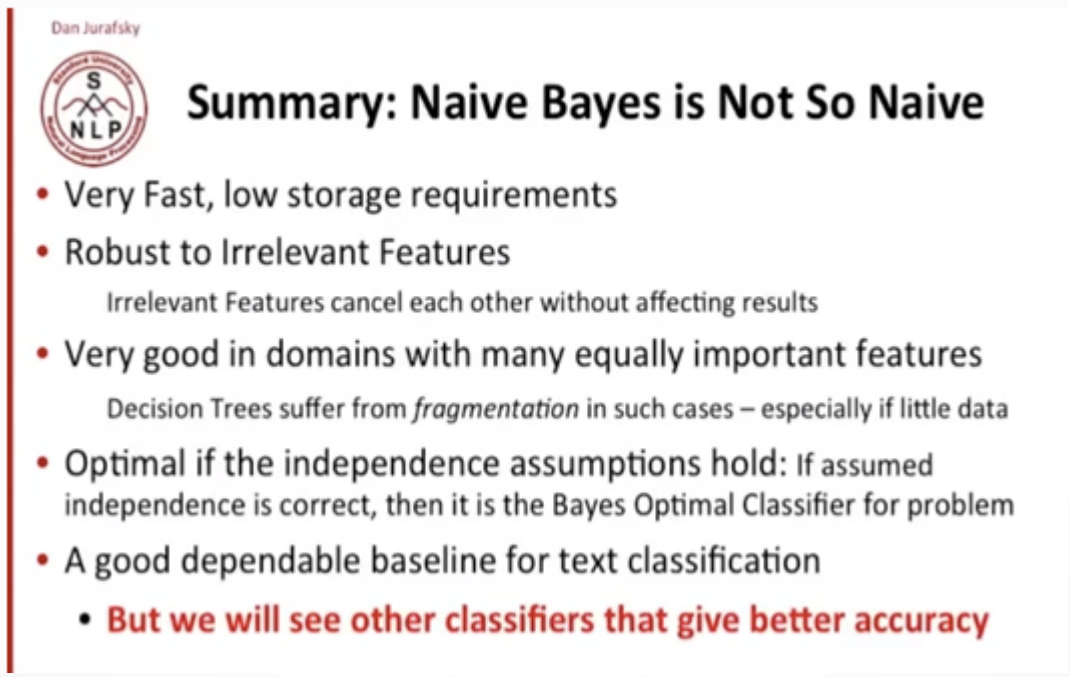
That's all Folks! All Good Things have to come to an end !

```
enron1 ham Total Files = 3672 Feature Count = 3672
enron1 spam Total Files = 5172 Feature Count = 1500
enron2 ham Total Files = 9533 Feature Count = 8033
enron2 spam Total Files = 11029 Feature Count = 2996
enron3 ham Total Files = 15041 Feature Count = 12045
enron3 spam Total Files = 16541 Feature Count = 4496
enron4 ham Total Files = 18041 Feature Count = 13545
enron4 spam Total Files = 22541 Feature Count = 8996
enron5 ham Total Files = 24041 Feature Count = 15045
enron5 spam Total Files = 27716 Feature Count = 12671
enron6 ham Total Files = 29216 Feature Count = 16545
enron6 spam Total Files = 33716 Feature Count = 17171
Read Time : 568.714
({'farm': True, 'pictures': True, 'tree': True, ':': True,
'christmas': True, 'Subject': True}, 'ham')
({'restore': True, 'less': True, '-': True, 'young': True,
'stability': True, 'sleep': True, 'human': True, 'skin':
True, 'density': True, 'with': True, ',': True, 'texture':
True, 'to': True, 'production': True, 'has': True,
'hormone': True, 'increased': True, 'disappearance': True,
'very': True, 'fat': True, 'resistance': True, 'nearly':
True, 'loss': True, 'level': True, 'deficient': True,
'reduces': True, 'common': True, 'cholesterol': True,
'vision': True, 'referred': True, 'strengthened': True,
'everyone': True, 'mental': True, 'energy': True,
'dobmeos': True, 'growth': True, 'are': True, 'our': True,
…octor': True, 'when': True, 'also': True, 'quickens':
True, 'hgh': True, 'begin': True, 'eighty': True,
'produce': True, 'blood': True, 'wrinkle': True, 'lower':
True, 'potency': True, 'read': True, 'age': True, 'time':
True, 'the': True, 'muscle': True, 'normally': True,
'bodies': True}, 'spam')
train on 30343 instances, test on 3373 instances
['spam', 'ham']
```

```
enron1 ham ['spam', 'ham']
Accuracy: 0.981322265046
Most Informative Features
                    php = True   spam : ham    =    503.9 : 1.0
                    hpl = True    ham : spam   =    485.1 : 1.0
                stinson = True    ham : spam   =    388.5 : 1.0
               crenshaw = True    ham : spam   =    374.0 : 1.0
                   meds = True   spam : ham    =    367.1 : 1.0
                  corel = True   spam : ham    =    301.0 : 1.0
             scheduling = True    ham : spam   =    296.1 : 1.0
            medications = True   spam : ham    =    290.0 : 1.0
                 louise = True    ham : spam   =    286.7 : 1.0
              macromedia = True   spam : ham    =    274.0 : 1.0
Run Time : 708.233
That's all Folks! All Good Things have to come to an end !
```

Summary

- Naïve Bayes
  - Assumes independence between the features
  - Assumes order is irrelevant
  - Assumes all features have equal importance
  - Skewed Data Bias (Bias for class with more data)
  - http://www.stanford.edu/class/cs276/handouts/rennie.icml03.pdf has some good ideas
  - http://courses.ischool.berkeley.edu/i290-dm/s11/SECURE/Optimality_of_Naive_Bayes.pdf is another good paper



http://www.youtube.com/watch?v=pc36aYTP440

http://www.google.com/doodles/st-patricks-day-2013
http://www.pcmag.com/article2/0,2817,2416709,00.asp

Doodle Slide Show : http://www.pcmag.com/slideshow_viewer/0,3253,l=308206&a=262220&po=1,00.asp

Model Evaluation

Model Evaluation - Accuracy

	Correct	Not Correct
Selected	True+ (tp)	False+ (fp)
Not Selected	False- (fn)	True- (tn)

- Accuracy = $\frac{tp + tn}{tp+fp+fn+tn}$
- For cases where tn is large compared tp tp, a simple degenerate classifier 'return(false)' will be very accurate !
- Hence the F-measure is a better reflection of the model strength

Model Evaluation – Precision & Recall

	Correct	Not Correct
Selected	True +ve - tp	False +ve - fp
Not Selected	False –ve - fn	True –ve - tn

- Precision
- Accuracy
- Relevancy

$$\frac{tp}{tp+fp}$$

- Recall
- True +ve Rate
- Coverage
- Sensitivity

$$\frac{tp}{tp+fn}$$

- Precision = How many items we identified are relevant
- Recall = How many relevant items did we identify
- Inverse relationship – Tradeoff depends on situations
  - Legal – Coverage is important than correctness
  - Search – Accuracy is more important
  - Fraud
    - Support cost (high fp) vs. wrath of credit card co.(high fn)

http://nltk.googlecode.com/svn/trunk/doc/book/ch06.html

Model Evaluation : F-Measure

	Correct	Not Correct
Selected	True+ (tp)	False+ (fp)
Not Selected	False- (fn)	True- (tn)

Precision = tp / (tp+fp) : Recall = tp / (tp+fn)

F-Measure = Balanced, Combined, Weighted Harmonic Mean, measures effectiveness

$$\frac{1}{\alpha \frac{1}{P} + (1-\alpha)\frac{1}{R}} = \frac{(\beta^2 + 1)\,PR}{\beta^2 P + R}$$

Common Form : $\beta=1$ $\alpha = ½$

$$F1 = 2PR / P+R$$

Confusion Matrix

Actual	Predicted				
		C1	C2	C3	C4
	C1	10	5	9	3
	C2	4	20	3	7
	C3	6	4	13	3
	C4	2	1	4	15

Correct Ones (c_{ii})

$$\text{Precision} = \frac{c_{ii}}{\sum_{\text{Columns } i} c_{ij}}$$

$$\text{Recall} = \frac{c_{ii}}{\sum_{\text{Rows } j} c_{ij}}$$

Dan Jurafsky

Confusion matrix c

- For each pair of classes <c_1,c_2> how many documents from c_1 were incorrectly assigned to c_2?
  - $c_{3,2}$: 90 wheat documents incorrectly assigned to poultry

Docs in test set	Assigned UK	Assigned poultry	Assigned wheat	Assigned coffee	Assigned interest	Assigned trade
True UK	95	1	13	0	1	0
True poultry	0	1	0	0	0	0
True wheat	10	90	0	1	0	0
True coffee	0	0	0	34	3	7
True interest	-	1	2	13	26	5
True trade	0	0	2	14	5	10

6

Video 6-8 Text Classification Evaluation Stanford NLP has very good discussion
http://www.youtube.com/watch?v=OwwdYHWRB5E

Hands On

Let us try Precision & Recall for our classifier

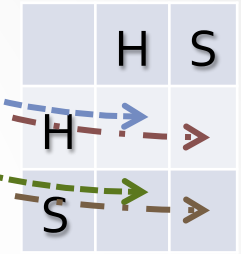
setup-02.py

Code Walk-Thru

```
.. .. .. Classifier & above – same as before
#
# Now create the data structure for model evaluation
#
refsets = collections.defaultdict(set)
testsets = collections.defaultdict(set)
for i, (feats, label) in enumerate(testfeats):
   refsets[label].add(i)
   observed = classifier.classify(feats)
   testsets[observed].add(i)
#print len(refsets)
#print len(testsets)
#print refsets
precisions = {}
recalls = {}
for label in classifier.labels():
   precisions[label] = metrics.precision(refsets[label],testsets[label])
   recalls[label] = metrics.recall(refsets[label], testsets[label])
#
# Let us calculate Precision & Recall and compare with nltk
#
# Luckily the data structures are symmetric
#
c_00=len(refsets[labels[0]].intersection(testsets[labels[0]]))
c_01=len(refsets[labels[0]].intersection(testsets[labels[1]]))
c_10=len(refsets[labels[1]].intersection(testsets[labels[0]]))
c_11=len(refsets[labels[1]].intersection(testsets[labels[1]]))
```

Code Walk-Thru

```
c_00=len(refsets[labels[0]].intersection(testsets[labels[0]]))
c_01=len(refsets[labels[0]].intersection(testsets[labels[1]]))
c_10=len(refsets[labels[1]].intersection(testsets[labels[0]]))
c_11=len(refsets[labels[1]].intersection(testsets[labels[1]]))
#
print ' |  H  |  S  |'
print '--|-------|-------|'
print 'H | %5d | %5d |' % (c_00,c_01)
print '--|-------|-------|'
print 'S | %5d | %5d |' % (c_10,c_11)
print '--|-------|-------|'
#
ham_p = float(c_00) / (c_00 + c_10)
ham_r = float(c_00) / (c_00 + c_01)
spam_p = float(c_11) / (c_01 + c_11)
spam_r = float(c_11) / (c_10 + c_11)
#
print 'P:{ham:%f,spam:%f}' % (ham_p,spam_p)
print 'R:{ham:%f,spam:%f}' % (ham_r,spam_r)
#
print '(nltk)P:',precisions
print '(nltk)R:',recalls
```

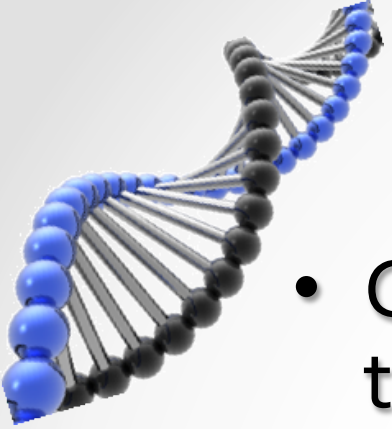


$$\text{Precision} = \frac{c_{ii}}{\sum_{\substack{\text{Columns} \\ i}} c_{ij}}$$

$$\text{Recall} = \frac{c_{ii}}{\sum_{\substack{\text{Rows} \\ j}} c_{ij}}$$

Run Capture & Discussion

```
enron1 ham Total Files = 3672 Feature Count = 3672
enron1 spam Total Files = 5172 Feature Count = 1500
enron2 ham Total Files = 9533 Feature Count = 8033
enron2 spam Total Files = 11029 Feature Count = 2996
enron3 ham Total Files = 15041 Feature Count = 12045
enron3 spam Total Files = 16541 Feature Count = 4496
enron4 ham Total Files = 18041 Feature Count = 13545
enron4 spam Total Files = 22541 Feature Count = 8996
enron5 ham Total Files = 24041 Feature Count = 15045
enron5 spam Total Files = 27716 Feature Count = 12671
enron6 ham Total Files = 29216 Feature Count = 16545
enron6 spam Total Files = 33716 Feature Count = 17171
Read Time : 574.077
Train on 30343 instances, Test on 3373 instances
   |   H    |   S    |
--|--------|--------|
H |  1630  |    25  |
--|--------|--------|
S |    38  |  1680  |
--|--------|--------|
P:{ham:0.977218,spam:0.985337}
R:{ham:0.984894,spam:0.977881}
(nltk)P: {'ham': 0.9772182254196643, 'spam': 0.9853372434017595}
(nltk)R: {'ham': 0.9848942598187311, 'spam': 0.9778812572759022}
Run Time : 694.238
That's all Folks! All Good Things have to come to an end !
```



Future Vectors to Follow …

- Calculate for spam with different sizes of training & testing datasets
- Delete words with low information gain
- Keep only high information gain words
- Stop words elimination (even though spam has lot more stop words)
- Sequence classifier
- Bigram & Trigram models, collocations

"Run multiple variants & measure relative lift" -- Paco Nathan

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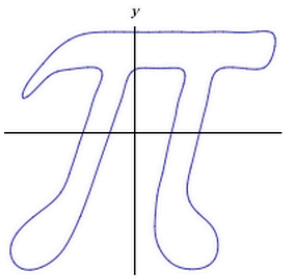
Input interpretation:

pi curve (plane curve)

Plot:

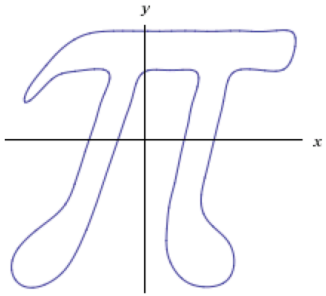


(plotted for t from 0 to 2π)

Ena

pi curve

Plot:



(plotted for t from 0 to 2π)

Equations:

Parametric equations:

$$x(t) = \frac{17}{31}\sin\left(\frac{235}{57} - 32t\right) + \frac{19}{17}\sin\left(\frac{192}{55} - 30t\right) + \frac{47}{32}\sin\left(\frac{69}{25} - 29t\right) + \frac{35}{26}\sin\left(\frac{75}{34} - 27t\right) + \frac{6}{31}\sin\left(\frac{23}{10} - 26t\right) + \frac{35}{43}\sin\left(\frac{10}{33} - 25t\right) + \frac{126}{43}\sin\left(\frac{421}{158} - 24t\right) + \frac{143}{57}\sin\left(\frac{35}{22} - 22t\right) + \frac{106}{27}\sin\left(\frac{84}{29} - 21t\right) + \frac{88}{25}\sin\left(\frac{23}{27} - 20t\right) + \frac{74}{27}\sin\left(\frac{53}{22} - 19t\right) + \frac{44}{53}\sin\left(\frac{117}{25} - 18t\right) + \frac{126}{25}\sin\left(\frac{88}{49} - 17t\right) + \frac{79}{11}\sin\left(\frac{43}{26} - 16t\right) + \frac{43}{12}\sin\left(\frac{41}{17} - 15t\right) + \frac{47}{27}\sin\left(\frac{244}{81} - 14t\right) + \frac{8}{5}\sin\left(\frac{79}{19} - 13t\right) + \frac{373}{46}\sin\left(\frac{109}{38} - 12t\right) + \frac{1200}{31}\sin\left(\frac{133}{74} - 11t\right) + \frac{67}{24}\sin\left(\frac{157}{61} - 10t\right) + \frac{583}{28}\sin\left(\frac{13}{8} - 8t\right) + \frac{772}{35}\sin\left(\frac{59}{16} - 7t\right) + \frac{3705}{46}\sin\left(\frac{117}{50} - 6t\right) + \frac{862}{13}\sin\left(\frac{19}{8} - 5t\right) + \frac{6555}{34}\sin\left(\frac{157}{78} - 3t\right) + \frac{6949}{13}\sin\left(\frac{83}{27} - t\right) - \frac{6805}{54}\sin\left(2t + \frac{1}{145}\right) - \frac{5207}{37}\sin\left(4t + \frac{49}{74}\right) - \frac{1811}{58}\sin\left(9t + \frac{55}{43}\right) - \frac{63}{20}\sin\left(23t + \frac{2}{23}\right) - \frac{266}{177}\sin\left(28t + \frac{13}{18}\right) - \frac{2}{21}\sin\left(31t + \frac{7}{16}\right)$$

$$y(t) = \frac{70}{37}\sin\left(\frac{65}{32} - 32t\right) + \frac{11}{12}\sin\left(\frac{98}{41} - 31t\right) + \frac{26}{29}\sin\left(\frac{35}{12} - 30t\right) + \frac{54}{41}\sin\left(\frac{18}{7} - 29t\right) + \frac{177}{71}\sin\left(\frac{51}{19} - 27t\right) + \frac{59}{34}\sin\left(\frac{125}{33} - 26t\right) + \frac{49}{29}\sin\left(\frac{18}{11} - 25t\right) + \frac{151}{75}\sin\left(\frac{59}{22} - 24t\right) + \frac{52}{9}\sin\left(\frac{118}{45} - 22t\right) + \frac{52}{33}\sin\left(\frac{133}{52} - 21t\right) + \frac{37}{45}\sin\left(\frac{61}{14} - 20t\right) + \frac{143}{46}\sin\left(\frac{144}{41} - 19t\right) + \frac{254}{47}\sin\left(\frac{19}{52} - 18t\right) + \frac{246}{35}\sin\left(\frac{92}{25} - 17t\right) + \frac{722}{111}\sin\left(\frac{176}{67} - 16t\right) + \frac{136}{23}\sin\left(\frac{3}{19} - 15t\right) + \frac{273}{25}\sin\left(\frac{32}{21} - 13t\right) + \frac{229}{33}\sin\left(\frac{117}{28} - 12t\right) + \frac{19}{4}\sin\left(\frac{43}{11} - 11t\right) + \frac{135}{8}\sin\left(\frac{23}{10} - 10t\right) + \frac{205}{6}\sin\left(\frac{33}{23} - 8t\right) + \frac{679}{45}\sin\left(\frac{55}{12} - 7t\right) + \frac{101}{8}\sin\left(\frac{11}{12} - 6t\right) + \frac{2760}{59}\sin\left(\frac{40}{11} - 5t\right) + \frac{1207}{18}\sin\left(\frac{21}{23} - 4t\right) + \frac{8566}{27}\sin\left(\frac{39}{28} - 3t\right) + \frac{12\,334}{29}\sin\left(\frac{47}{37} - 2t\right) + \frac{15\,410}{39}\sin\left(\frac{185}{41} - t\right) - \frac{596}{17}\sin\left(9t + \frac{3}{26}\right) - \frac{247}{28}\sin\left(14t + \frac{25}{21}\right) - \frac{458}{131}\sin\left(23t + \frac{21}{37}\right) - \frac{41}{36}\sin\left(28t + \frac{7}{8}\right)$$

Was it a vision, or a waking dream?
Fled is that music:—do I wake or sleep?
-Keats, Ode to a Nightingale