$A^*/\approx_L$ mulțimea cât

Fie $\hat{a}, \hat{b} \in A^*/\approx_L$ $\quad \cdot \subseteq A^*/\approx_L \times A^*/\approx_L$ operație internă ~~(bine def.)~~ : $\boxed{\hat{a}\cdot\hat{b} \overset{def}{=} \widehat{ab}}$ concatenare

- bine definită

dc. $x, y \in \hat{a}$, $z, t \in \hat{b}$ $\Rightarrow$ $\hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{t}$. (vezi sem.)

- asociativă
- element neutru : $\hat{e}$

$\Big\} \Rightarrow \boxed{(A^*/\approx_L, \cdot)}$ monoidul sintactic

Automate finite (în sens Eilenberg)

Cursul 4

Def.: A - alfabet, $\mathcal{A}$ - A-automat

$\boxed{\mathcal{A} = (S, \delta, s_0, T)}$ s.n. A-automat dc.

$S = \{s_0, s_1, \dots, s_n\}$ (s.n. mulțimea stărilor) mult. finită

$s_0 \in S$ s.n. starea inițială

$T \subseteq S$ s.n. mulțimea stărilor terminale

$\delta : S \times A \to S$ s.n. funcția de tranziție a automatului

$\boxed{\delta(s, a) = s'}$, $s, s' \in S$, $a \in A$.

generalizare:

$\delta^* : S \times A^* \to S$

$\boxed{\delta^*(s, e) = s}$, $\forall s \in S$

$\boxed{\delta^*(s, ax) = \delta^*(\delta(s,a), x)}$, $\forall s \in S$, $a \in A$, $x \in A^*$.

$\left(\boxed{x = e} \Rightarrow \underline{\delta^*(s, a)} = \delta^*(\delta(s,a), e) = \underline{\delta(s,a)} \right)$

ex. fie $a = a_1 \dots a_n \in A^*$, $a_i \in L$

$n = 2$; $s \in S$ $\quad \delta^*(s, a_1 a_2) = \delta^*(\underbrace{\delta(s, a_1)}_{s'}, a_2) = \delta^*(s', a_2) =$

$= \delta(s', a_2) = s''$.

δ^* generalizează pe δ (vom folosi δ pt. δ^*). !!!

Def.: $a \in A^*$ recunoscut de A-automatul $\mathcal{A}$ dc. $\boxed{\delta(s_0, a) \in T}$

$\boxed{L(\mathcal{A})} = \{ a \in A^* \mid \delta(s_0, a) \in T \}$ limbajul ~~format~~ generat de automatul $\mathcal{A}$ (mult. cuvintelor recunoscute de automat)

$L \subseteq A^*$ limbaj recunoscut dc. $\exists$ A-automat $\mathcal{A}$ a.î. $L = L(\mathcal{A})$.

Not. $\boxed{\mathrm{Rec}\,A} = \{ L \subseteq A^* \mid \exists\, \mathcal{A} = A\text{-automat a.î. } L = L(\mathcal{A}) \}$ mulț. limbajelor recun.

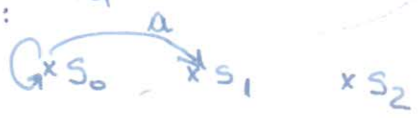
Exemple de automate:

δ dată 1) explicit: $\delta(\ ) =$

2) tabel:

δ	a	b	c
s_1			
s_2			

3) graf:



$\delta(s_0, a) = s_1$

$\delta(s_0, a) = s_0$

1. $\mathcal{A} = A$-automat

$\mathcal{A} = (S, \delta, s_0, T)$

$A = \{x_0, x_1, \dots, x_n\}$

$S = \{s_0, s_1, \dots, s_{n+1}\}$

$\delta(s_0, x_0) = s_1$

$\delta(s_1, x_1) = s_2$

$- - - -$

$\delta(s_n, x_n) = s_{n+1}$

$T = \{s_{n+1}\}$

$\boxed{L(\mathcal{A}) = \{x_0 x_1 \dots x_n\}}$

2. $A = \{a\}$, $S = \{s_0, s_1\} = T$, $\delta : S \times A \to A$, $\delta(s_0, a) = s_1$

$\delta(s_1, a) = s_0$

~~$L(\mathcal{A}) = \{\ \}$~~

$\delta(s_0, aa) = \delta(\delta(s_0, a), a) =$

$= \delta(s_1, a) = s_0$

$\Rightarrow \boxed{L(\mathcal{A}) = \{e, a, a^{[2]}, \dots\} = \{a^{[n]} \mid n \geq 0\} = A^*.}$

~~CURS 4~~

26.10.2001

$\mathcal{A} = (S, \delta, s_0, T) = A$-automat

$L(\mathcal{A}) = \{x \in A^* \mid \delta(s_0, x) \in T\}$

$\delta(s, x) = s' \Rightarrow$ automatele finite (a.f.) sunt deterministe (trece dintr-o stare în altă stare)

$\mathcal{L}_0 \supset \mathcal{L}_1 \supset \mathcal{L}_2 \supset \mathcal{L}_3$ (MT - mașina Turing)

$\widehat{MT}$ $\widehat{ALM}$ $\widehat{APD}$ $\widehat{AFD}$ (automate finite deterministe)

D N D N D N D N

prob. desch.

A, $\mathcal{A}_n = (S, \delta, s_0, T)$, $S = \{\ \}$ m.f. $s_0 \in S$

$\boxed{\delta : S_0 \times A^* \to \mathcal{P}(S)}$, $\boxed{\delta(s, a) = \{s', s'', \dots\} \in \mathcal{P}(S)}$ automate finite nedeterministe

$\delta(s, ax) = \bigcup\limits_{s' \in \delta(s,a)} \delta(s', x)$, $a \in A$, $x \in A^*$

$\delta(s, e) = \{s\}$

$T \subseteq S$ $\boxed{\mathcal{L}(A_n) = \{ x \in A^* \mid \delta(s_0, x) \cap T \neq \emptyset \}}$

$\boxed{Rec_n(A)}$ = mult. lbj recunoscute de a.f. nedeterministe

$Rec_n(A) = \{ L \subseteq A^* \mid \exists A_n = A$ - a.f. nedeterminist, $L(A_n) = L \}$.

Ex: **1.** $A = \{0, 1\}$, $S = \{s_0, s_1, s_2, s_3, s_4\}$, $T = \{s_2, s_4\}$

δ	s_0	s_1	s_2	s_3	s_4
0	$\{s_0, s_3\}$	$\emptyset$	$\{s_2\}$	$\{s_4\}$	$\{s_4\}$
1	$\{s_0, s_1\}$	$\{s_2\}$	$\{s_2\}$	$\emptyset$	$\{s_4\}$

$\delta(s_0, 010) = \delta(\delta(s_0, 0), 10) = \delta(s_0, 10) \cup \delta(s_3, 10) =$

$= \delta(s_0, 0) \cup \delta(s_1, 0) \cup \underbrace{\delta(\emptyset, 0)}_{\emptyset} = \{s_0, s_3\} \left(\cap T = \emptyset \right) \Rightarrow 010 \notin L(A_n)$.

$\delta(s_0, 011) = \delta(s_0, 11) \cup \delta(s_3, 11) = \delta(s_0, 1) \cup \delta(s_1, 1) \cup \emptyset = \{s_0, s_1\} \cup \{s_2\} = \{s_0, s_1, s_2\}$

$\{s_0, s_1, s_2\} \cap T = \{s_2\} \neq \emptyset \Rightarrow 011 \in L(A_n)$.

$L(A_n) = ?$ mult. succ. care conțin 2 simboluri consecutive de 0 sau 2 simboluri consecutive de 1 (2 simb. identice consecutive).

Teorema 1 $\boxed{Rec(A) = Rec_n(A)}$

Ex. $A = \{0, 1\}$, $B = \{a, b\}$ nu facem dif. calitativă (tot 2 simb.)

Dem: 1°) $\boxed{Rec_n(A) \subseteq Rec(A)}$ //

fie $L \in Rec_n(A) \Rightarrow \exists A_n = (S, \delta, s_0, T)$ a.î. $L(A_n) = L$.

Construim automatul finit det. care recunoaște acest limbaj.

construcție:

Fie $\boxed{A = (\mathcal{P}(S), \delta', \{s_0\}, T')}$

$\boxed{\delta'(s', a) = \begin{cases} \emptyset, & s' = \emptyset \\ \bigcup\limits_{s \in s'} \delta(s, a), & s' \neq \emptyset \end{cases}}$

$\delta'(\{x, y\}, a) = \delta(x, a) \cup \delta(y, a) = \{z, t\}$

$\boxed{T' = \{ Z \mid Z \in \mathcal{P}(S), Z \cap T \neq \emptyset \}}$.

dem. că rec. ac. lbj:

$L(A) = L = L(A_n)$ //

fie $p \in L(A_n)$, $\underbrace{\delta(s_0, p)}_{Z} \cap T \neq \emptyset \Rightarrow \exists Z \in \mathcal{P}(S)$ a.î. $Z \cap T \neq \emptyset$

$Z \in T'$!

$\delta'(\{s_0\}, p) = \delta(s_0, p) = Z \cap T \neq \emptyset \Rightarrow Z \in T' \Rightarrow p \in L(A)$

fie $p \in L(A)$, $\delta'(\{s_0\}, p) \in T' \Rightarrow \exists Z = \delta(\{s_0\}, p) \cap T \neq \emptyset$

Dar $\delta'(\{s_0\}, p) = \delta(s_0, p)$ $(\cap T \neq \emptyset)$ => $p \in L(A_N)$ => $\boxed{L(A_N) = L(A)}$.

2°) **Reciproca** $\boxed{Rec(A) \subseteq Rec_M(A)}$ ev. pt că ∀ a.f. nedet. e un caz particular de a.f. det.

constr.

Fie $L \in Rec(A)$ => $\exists\ \mathcal{A} = A$ - automat finit det. $L(\mathcal{A}) = L$

$\mathcal{A} = (S, \delta, s_0, T)$

Fie $\boxed{A_N = (S, \delta', s_0, T)}$ $\boxed{\delta' : S \times A^* \to \mathcal{P}(S),\ \delta'(s, a) = \{\delta(s, a)\}}$

dem că rec ac. lb.

Evident $L(\mathcal{A}) = L(A_N)$.

$p \in L(\mathcal{A})$ => $\delta(s_0, p) \in T$ <=> $\{\delta(s_0, p)\} \subset T$ => $\delta'(s_0, p) \cap T \neq \emptyset$

$\delta'(s_0, p)$

$L(A_N) \subset L(\mathcal{A})$

fie $p \in L(A_N)$ => $\delta'(s_0, p) \cap T \neq \emptyset$

Dar $\delta'(s_0, p) = \{\delta(s_0, p)\}$ $(\cap T \neq \emptyset)$

=> $\delta(s_0, p) \in T$ => $p \in L(\mathcal{A})$

=> $L(A_N) = L(\mathcal{A})$. => $\boxed{Rec(A) = Rec_M(A)}$

QED.

Def aut. finite a fost dată de Robin, Scott.

Ex. 2. Fie $A_N = (\{s_0, s_1\}, \delta, s_0, \{s_1\})$, $A = \{0, 1\}$

δ	s_0	s_1
0	$\{s_0, s_1\}$	$\{s_1\}$
1	$\{s_1\}$	$\{s_0, s_1\}$

$\mathcal{A} = (\mathcal{P}(S), \delta', \{s_0\}, T')$

$\mathcal{P}(S) = \{\emptyset, \{s_0\}, \{s_1\}, \{s_0, s_1\}\}$

$\delta'(\emptyset, a) = \emptyset$

$\delta'(\{s_0\}, 0) = \delta(s_0, 0) = \{s_0, s_1\}$

$\delta'(\{s_0\}, 1) = \delta(s_0, 1) = \{s_1\}$

$\delta'(\{s_1\}, 0) = \{s_1\}$

$\delta'(\{s_1\}, 1) = \{s_0, s_1\}$

$\delta'(\{s_0, s_1\}, 0) = \delta(s_0, 0) \cup \delta(s_1, 0) = \{s_0, s_1\} \cup \{s_1\} = \{s_0, s_1\}$

δ'	$\emptyset$	$\{s_0\}$	$\{s_1\}$	$\{s_0, s_1\}$
0	$\emptyset$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_0, s_1\}$
1	$\emptyset$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_1\}$

$T' = \{z \in \mathcal{P}(S) \mid z \cap T \neq \emptyset\}$

$T' = \{\{s_1\}, \{s_0, s_1\}\}$

$T = \{s_1\}$

δ'	U	T	X
0	X	T	X
1	T	X	X

$\mathcal{A} = (\{U, T, X\}, \delta', U, \{T, X\})$