Introduction to Deep Learning

SIG Research Meeting

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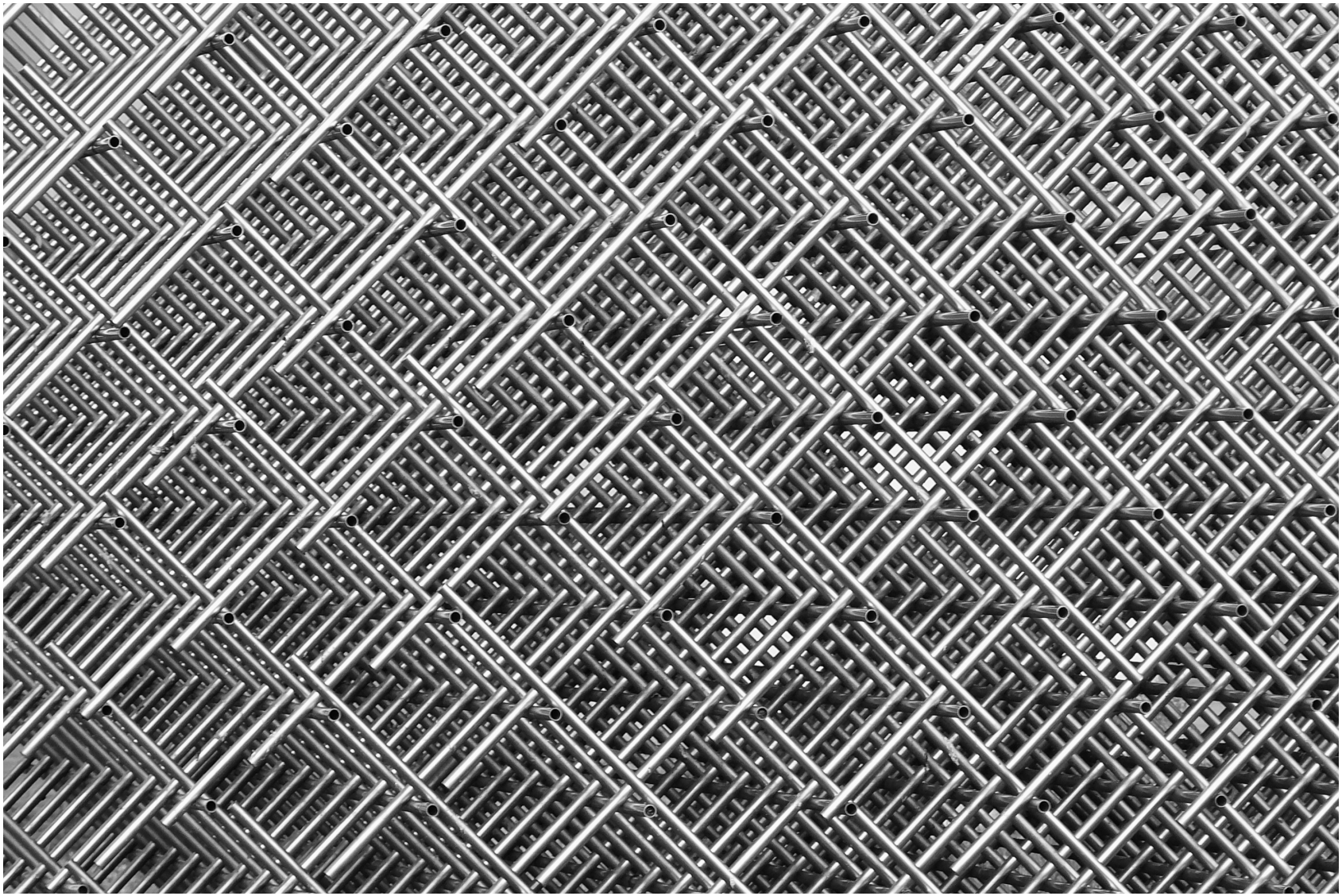
Contents

Just another tool

For identifying patterns



What is a pattern?

- a *form* or *model* proposed for *imitation*[1]
- artistic, musical, literary *design* or *form*[1]
- a reliable sample of traits, acts, tendencies, or other *observable characteristics* of a . . . [1]

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$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$y = f(\boldsymbol{x}, \boldsymbol{\theta})$$

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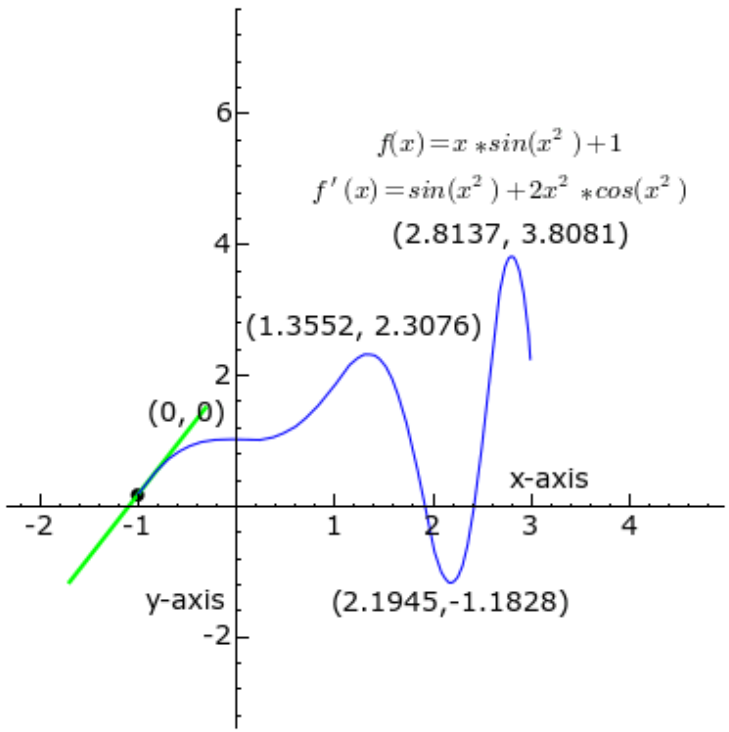
Rate of change

Definition

The derivative of a function $y = f(x)$ of variable x is a measure of the *rate* at which the value y *changes* with respect to the change of x:

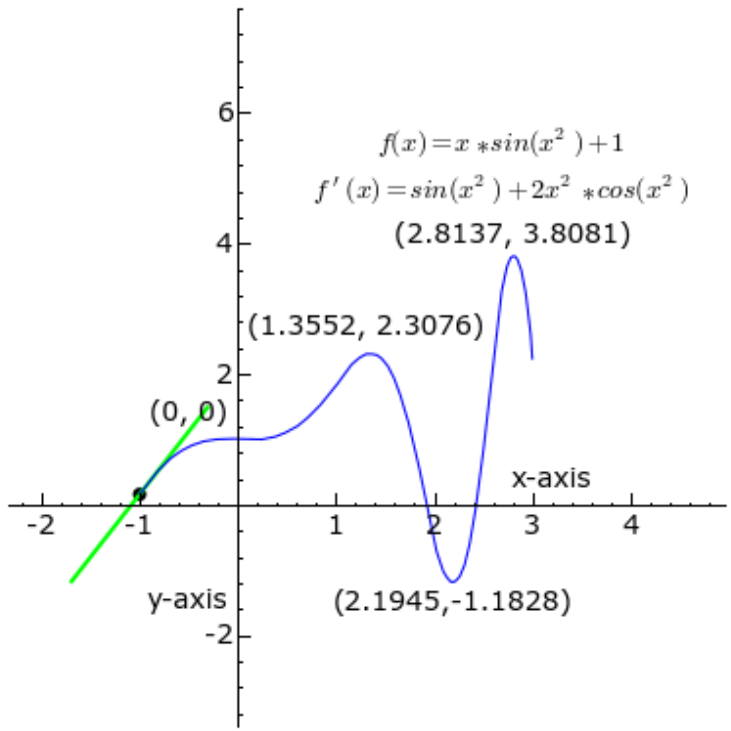
$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$



Searching for Minimum

Definition

A function $f(x)$ is *increasing* on any interval in which $f'(x) > 0$ and it is *decreasing* on any interval in which $f'(x) < 0$.



Gradient

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

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$$f : \mathbb{R}^n \to \mathbb{R}$$

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Definition

The gradient is a multi-variable generalisation of the derivative.

Things get hairy: The Jacobian Matrix

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Definition

The Jacobian Matrix is the matrix of all *first-order* partial derivatives of a *vector-valued* function.

Machines that learn - a metaphor

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Computer programs learn from *experience* E with respect to some class of *tasks* T and *performance measure* P, if its performance at tasks in T as measured by P *improves* with experience E [2].

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1. Task - understand language, move boxes, etc.
2. Experience - interpret *data*
3. Performance measure i.e. error/cost/objective functions - number of words known, number of boxes moved, etc.

Learning is used analogously to *modelling* i.e. finding the best function $f(x, \theta)$ that can approximate a task.

A brief classification

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- Unsupervised learning
- Supervised learning
- Reinforcement learning

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By task:

- Classification
- Regression
- Focused tasks i.e. machine translation, anomaly detection, software analysis

A simple example

Problem setting:

- Task: Classify irises (flowers) in two categories.
- Experience: Labeled measurements: $(x, y) = ([height, color, \dots], Virginica)$
- Performance measure?
- *Model* $[f(x, \theta)]$?

Linear Models

Performance: Mean Squared Error (MSE)

Model: $\hat{y} = \boldsymbol{W}^{\boldsymbol{T}} \boldsymbol{x}$

Where:

$\boldsymbol{W} \in \mathbb{R}^n$ is a vector of parameters

$MSE = \frac{1}{m} \sum_i (\hat{y}^{(test)} - y^{(test)})_i^2$

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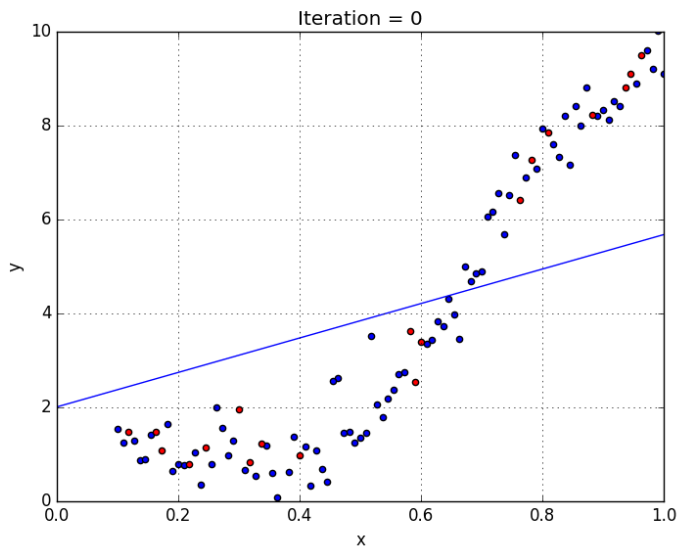
Solution: Compute derivatives!

Linear Models

Solution:

$$\nabla_w MSE_{train} = 0$$

$$\nabla_w \frac{1}{m} \sum_i (\hat{y}^{(test)} - y^{(test)})_i^2$$



The power of derivatives

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Challenges Motivating Deep Learning

The development of deep learning was motivated in part by the failure of traditional algorithms to generalise well on central problems of AI such as recognising speech or objects [1].

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Reasons?

- High Dimensionality [the number of possible configuration of a set of variables increases exponentially with the number of variables]
- Hard to learn functions with interesting variations across all of $\mathbb{R}$ [Manifold Learning]

In search for better approximations

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- Represent highly nonlinear functions in large hyper-spaces

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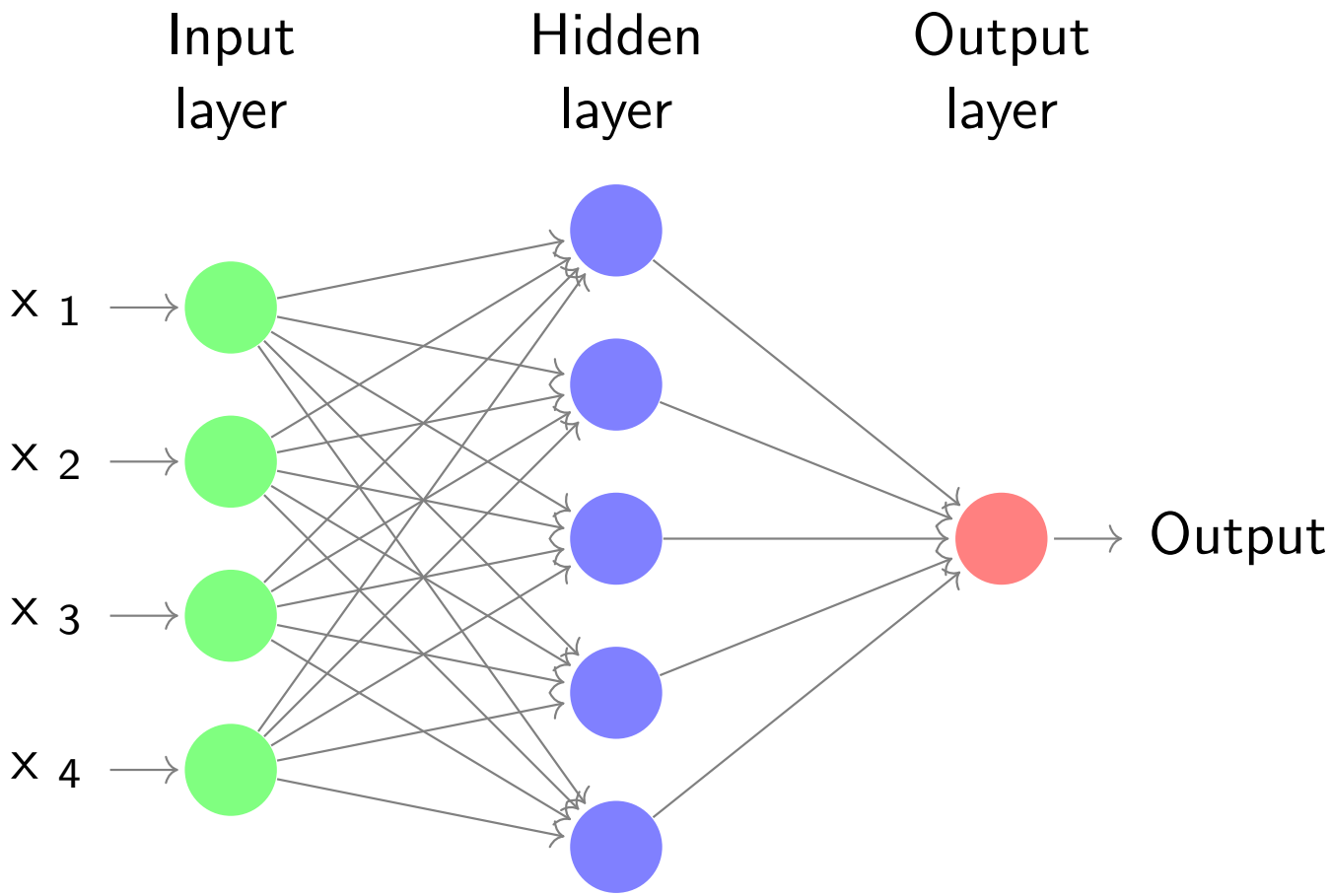
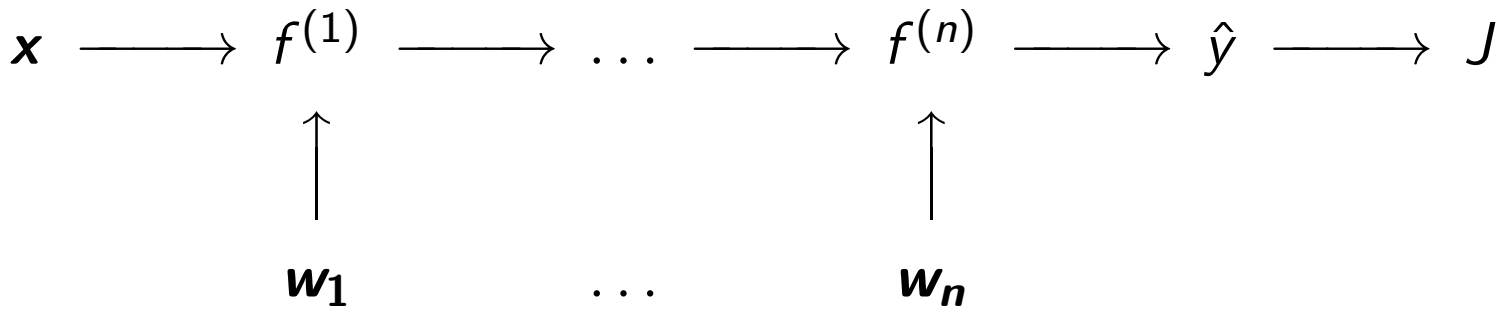
Solution? Functional *composition*!

Neural Networks



Deep Representations

- A *deep representation* is a composition of many functions



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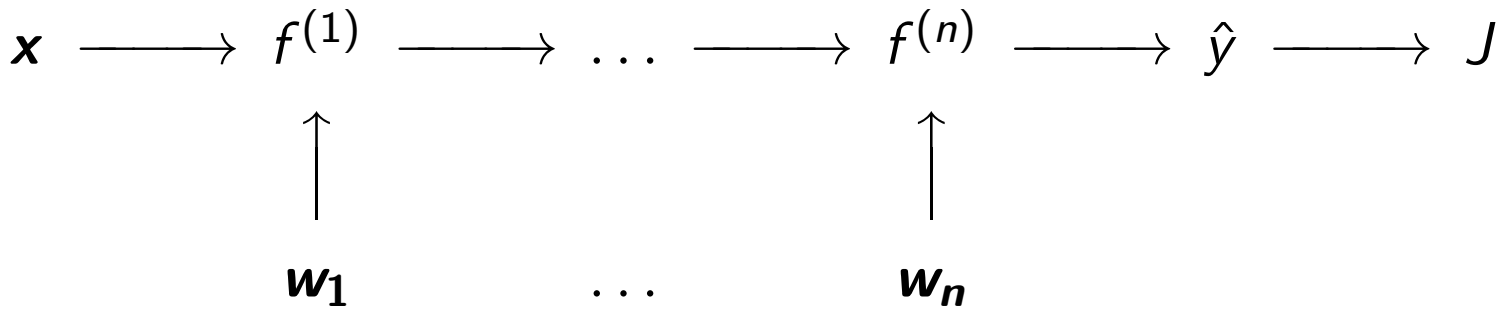
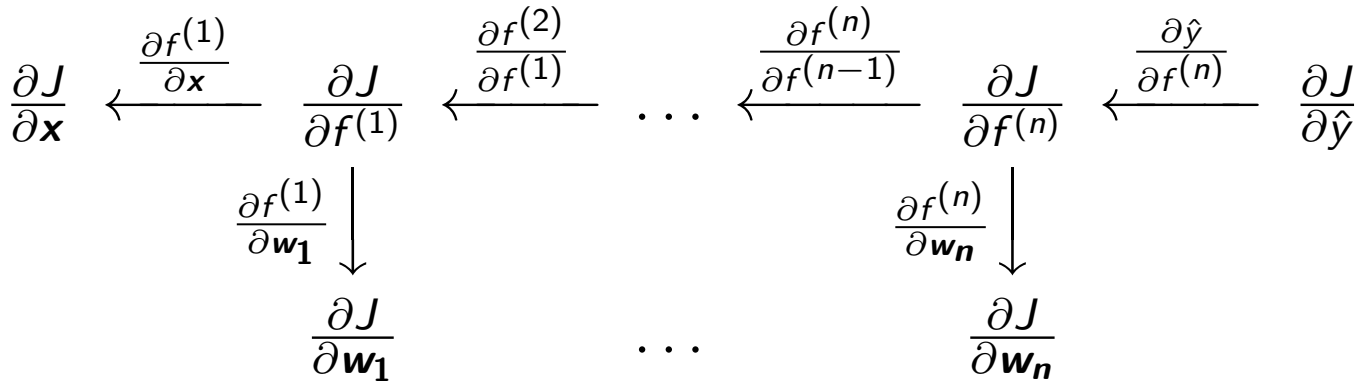


- Its gradient can be *back-propagated* by the chain rule



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- A loss function, e.g.
  - Mean-squared error $l = ||\hat{y} - y||^2$
  - Log likelihood $l = \log \mathbb{P}[\hat{y}]$

Universal Approximation Theorem

Definition

A feed-forward network with a *single hidden layer* containing a finite number of neurones can approximate any continuous function on compact subsets of $\mathbb{R}^n$.

Wrapping it up

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Key take-away(s):

- Computers love high dimensions
- It is easier to discover highly non-linear functions through *composition* than to *engineer* them
- Learning is just a tool

References I


Ian Goodfellow et al. *Deep learning*. Vol. 1. MIT press Cambridge, 2016.


Tom M Mitchell et al. "Machine learning. 1997". In: *Burr Ridge, IL: McGraw Hill* 45.37 (1997), pp. 870–877.