Summary of Current Work - Source of Entropy

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This is a summary of work done by J. P. Hou, NUAA. Two main topics are included:

1)Vaydia black holes in f(R) gravity and its thermodynamics, a conjecture - there's a possible relation between uniqueness of solutions of motion equations with a specific $T_{\mu\nu}$ and the sources of entropy - is also discussed.

2)A investigation of the definitions of temperature in GR and alternative theories.

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I. THERMODYNAMICS AND ENTROPY

The general spheric symmetric metric is assumed in most situations in this work

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + b(r)^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{I.1}$$

or

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{I.2}$$

These two metrics are considered the same as one can always change one of them into anther by coordinates transactions, however, we find that it is possible to distinguish them in thermodynamics. The following demonstration includes GR, f(R) and Gauss-Bonnet gravities and the $T_{\mu\nu}$ is also defined as $T_{\mu\nu} = 0$.

A. Simplest case

To begin with, we consider the simplest case - GR:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} \tag{I.3}$$

Thus, we have

$$T_t^t = -\frac{1}{8\pi b(r)^2}(1 - f(r)b'(r)^2 - b(r)b'(r)f'(r) - 2b(r)f(r)b''(r)) \tag{I.4}$$

$$T_r^r = \frac{1}{8\pi b(r)^2}(-1 + f(r)b'(r)^2 + b(r)b'(r)f'(r)) \tag{I.5}$$

In this work, it is always presumed that all black holes involved are non-extrem black holes, that is, in the horizon $r = r_+$ $f(r_+) = 0$ and $f'(r_+) \neq 0$. Hence,

$$T_t^t|_{r=r_+} = T_r^r|_{r=r_+} = -\frac{1}{8\pi b(r_+)^2}(1 - b(r_+)b'(r_+)f'(r_+)) \tag{I.6}$$

The equation above is obvious constraint. To the ideal fluent, T_r^r can be treated as pressure

$$4\pi P b(r_+)^2 = \frac{1}{2}(-1 + b(r_+)b'(r_+)f'(r_+)) \tag{I.7}$$

B. Gauss-Bonnet

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \alpha H_{\mu\nu} = 8\pi T_{\mu\nu} \tag{I.8}$$

where

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\lambda}R^{\lambda\nu} - 2R^{\alpha\beta}) \quad \text{(I.9)}$$

$$T^t_t \;=\; -\frac{1}{16\pi b(r)^2}(2 - 2f(r)b'(r)^2 - 2b(r)b'(r)f'(r) + 2\alpha b'(r)^2 f'(r)^2 \quad \text{(I.10)}$$

$$-4b(r)f(r)b''(r) + \alpha b(r)^2 f''(r)^2) \quad \text{(I.11)}$$

$$T^r_r \;=\; \frac{1}{16\pi b(r)^2}(-2 + 2f(r)b'(r)^2 + 2b(r)b'(r)f'(r) - 2\alpha b'(r)^2 f'(r)^2 \quad \text{(I.12)}$$

$$-8\alpha f(r)b'(r)f'(r)b''(r) - 8\alpha f(r)^2 b''(r)^2 - \alpha b(r)^2 f''(r)^2) \quad \text{(I.13)}$$

Following the same step, one has

$$T^t_t|_{r=r_+} = T^r_r|_{r=r_+} = \frac{1}{16\pi b(r)^2}(-2 + b(r)b'(r)f'(r) - 2\alpha b'(r)^2 f'(r)^2 - \alpha b(r)^2 f''(r)^2) \quad \text{(I.14)}$$

That is

$$4\pi P b(r)^2 = \frac{1}{4}(-2 + b(r)b'(r)f'(r) - 2\alpha b'(r)^2 f'(r)^2 - \alpha b(r)^2 f''(r)^2) \quad \text{(I.15)}$$

C. Non-Minmum Coupled