Probability

What is Probability?

- We measured “how often” using

Relative frequency = *f/n*

- As *n* gets larger,

Sample → **Population**
And “How often”
= Relative frequency → **Probability**

Basic Concepts

- An **experiment** is the process by which an observation (or measurement) is obtained.
- An **event** is an outcome of an experiment, usually denoted by a capital letter.
- **Experiment: Record an age**
  - A: person is 30 years old
  - B: person is older than 65
- **Experiment: Toss a die**
  - A: observe an odd number
  - B: observe a number greater than 2

Basic Concepts

- Two events are **mutually exclusive** if, when one event occurs, the other cannot, and vice versa.

- **Experiment: Toss a die**
  - A: observe an odd number
  - B: observe a number greater than 2
  - C: observe a 6
  - D: observe a 3

Not Mutually Exclusive

Mutually Exclusive

B and C?
B and D?

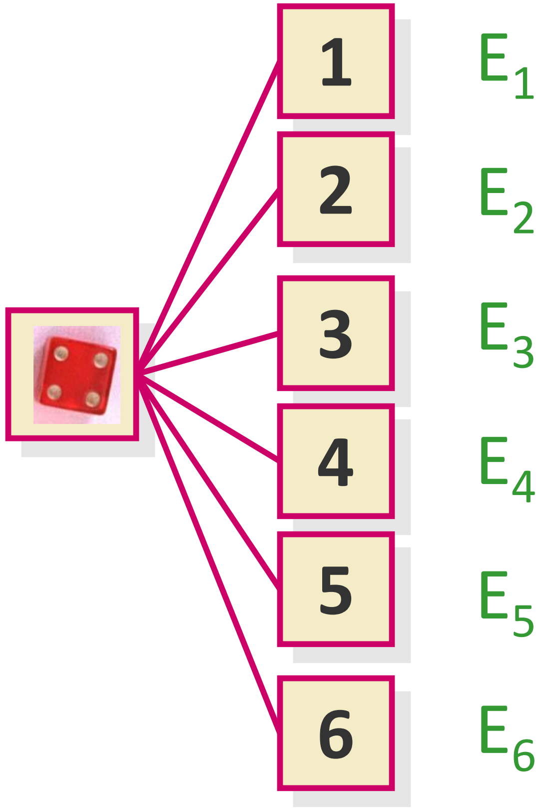
Basic Concepts

- An event that cannot be decomposed is called a **simple event.**
- Denoted by E with a subscript.
- Each simple event will be assigned a probability, measuring “how often” it occurs.
- The set of all simple events of an experiment is called the **sample space, S.**

Example

- **The die toss:**
- Simple events:

Sample space:



1 E_1

2 E_2

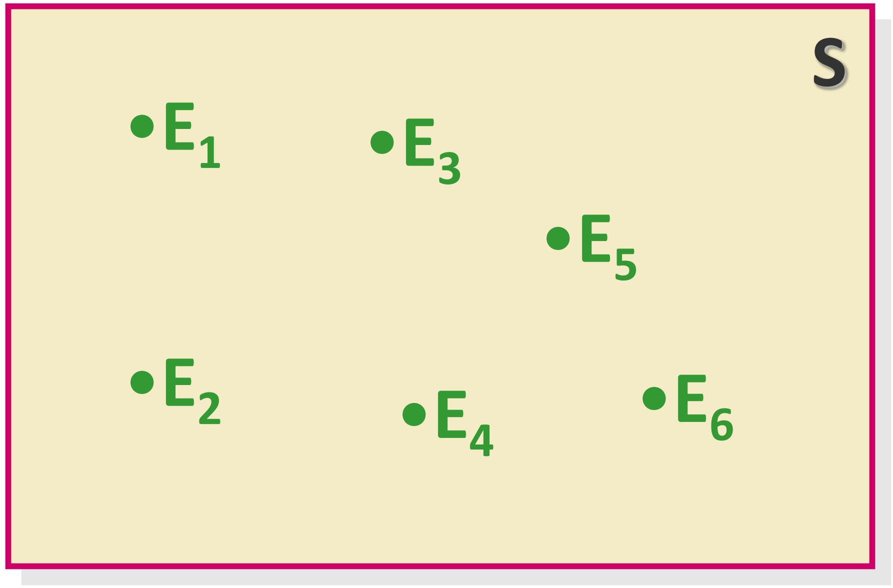
3 E_3

4 E_4

5 E_5

6 E_6

$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$



S

$\bullet E_1$ $\bullet E_3$

$\bullet E_5$

$\bullet E_2$ $\bullet E_4$ $\bullet E_6$

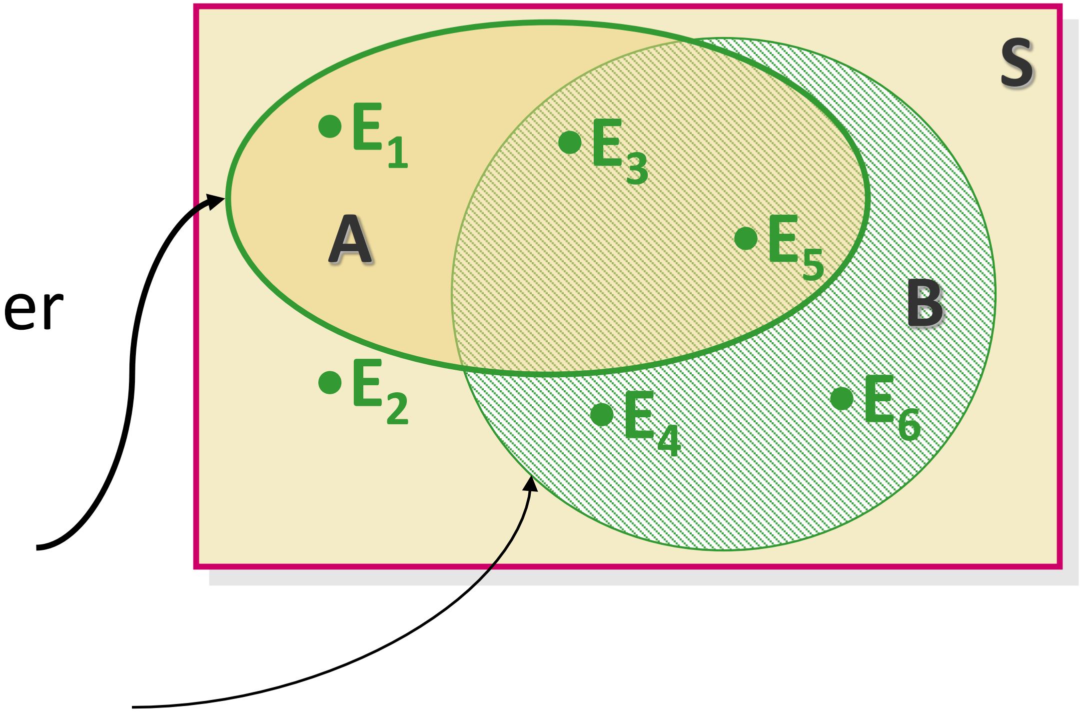
Basic Concepts

- An **event** is a collection of one or more **simple events.**

- **The die toss:**
  - –A: an odd number
  - –B: a number > 2

A =$\{E_1, E_3, E_5\}$

B =$\{E_3, E_4, E_5, E_6\}$



The Probability of an Event

- The probability of an event A measures “how often” A will occur. We write **P(A).**
- Suppose that an experiment is performed *n* times. The relative frequency for an event A is

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{n}$$

The Probability of an Event

- P(A) must be between 0 and 1.
  - If event A can never occur, P(A) = 0. If event A always occurs when the experiment is performed, P(A) =1.
- The sum of the probabilities for all simple events in S equals 1.

- The **probability of an event A** is found by adding the probabilities of all the simple events contained in A.

Finding Probabilities

- Probabilities can be found using
  - Estimates from empirical studies
  - Common sense estimates based on equally likely events.

- **Examples:**

–Toss a fair coin.

P(Head) = 1/2

– Suppose that 10% of the U.S. population has red hair. Then for a person selected at random,

P(Red hair) = .10

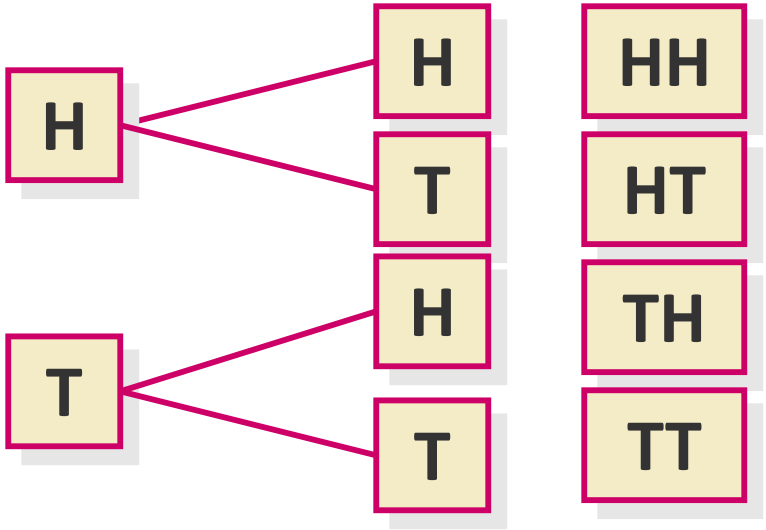
Using Simple Events

- The probability of an event A is equal to the sum of the probabilities of the simple events contained in A
- If the simple events in an experiment are **equally likely,** you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

Example 1

Toss a fair coin twice. What is the probability of observing at least one head?



1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

มีทั้งหมด 3 เหตุการณ์จาก 4

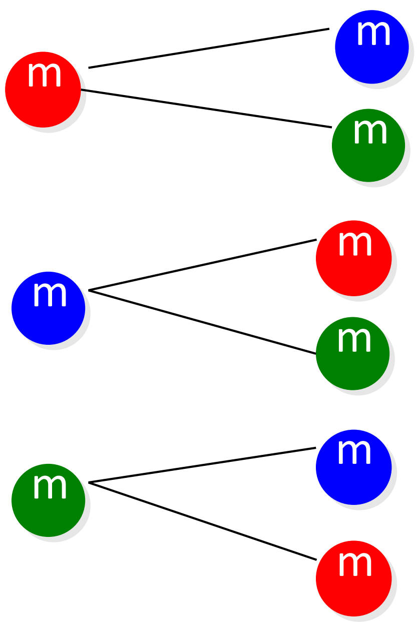
P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

Example 2

A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms at random. What is the probability that at least one is red?



1st M&M	2nd M&M	E_i	$P(E_i)$
		RB	1/6
		RG	1/6
		BR	1/6
		BG	1/6
		GB	1/6
		GR	1/6

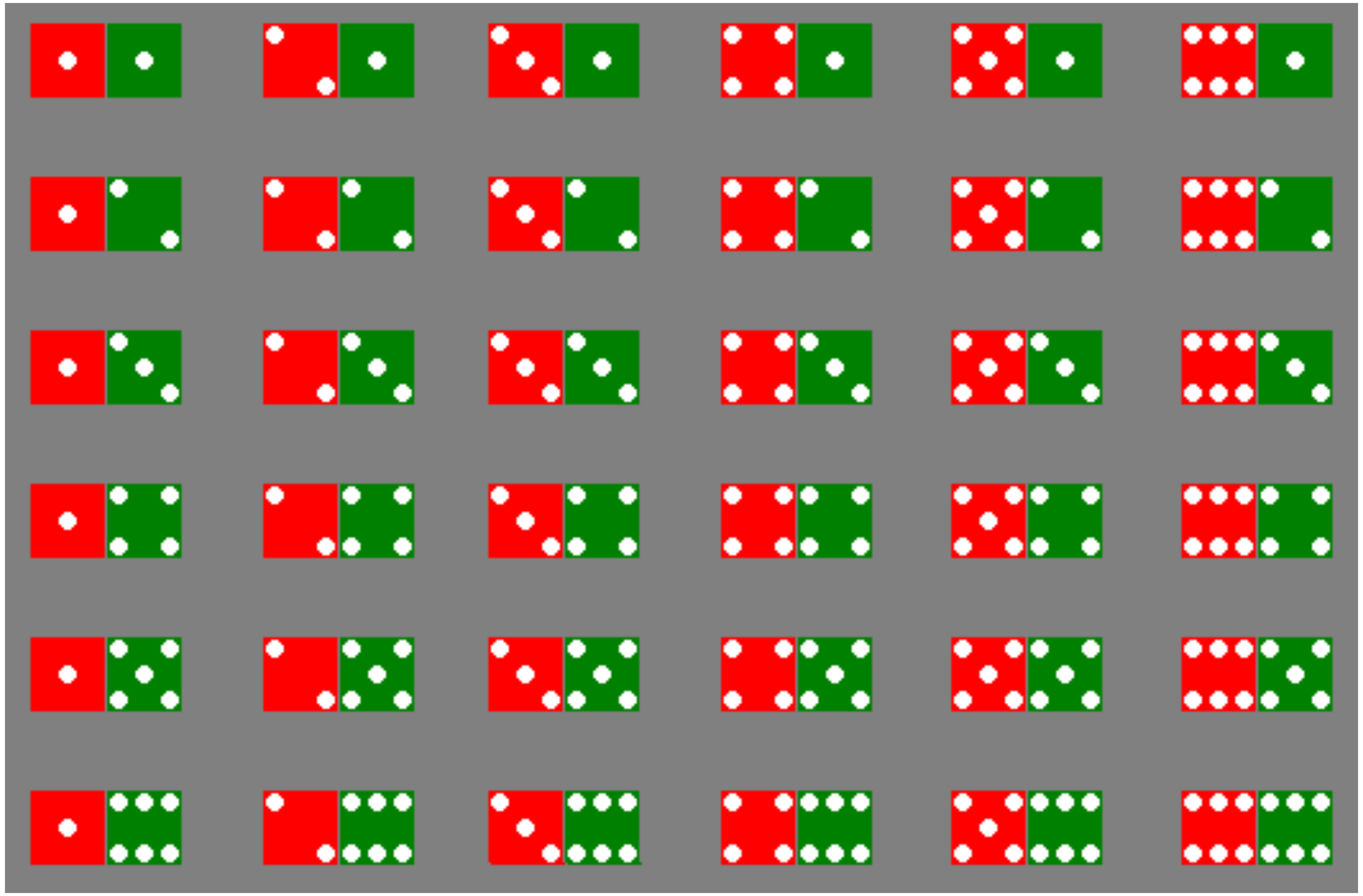
P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Example 3

The sample space of throwing a pair of dice is



Example 3

Event	Simple events	Probability
Dice add to 3	(1,2),(2,1)	2/36
Dice add to 6	(1,5),(2,4),(3,3), (4,2),(5,1)	5/36
Red die show 1	(1,1),(1,2),(1,3), (1,4),(1,5),(1,6)	6/36
Green die show 1	(1,1),(2,1),(3,1), (4,1),(5,1),(6,1)	6/36

Counting Rules

- Sample space of throwing 3 dice has 216 entries, sample space of throwing 4 dice has 1296 entries, …
- At some point, we have to stop listing and start thinking …
- We need some counting rules
  - Rule of sum
  - Rule of product
  - Permutation
  - Combination

Examples

Example: Toss three coins. The total number of simple events is:

$2 \times 2 \times 2 = 8$

Example: Toss two dice. The total number of simple events is:

$6 \times 6 = 36$

Example: Toss three dice. The total number of simple events is:

$6 \times 6 \times 6 = 216$

Example: Two M&Ms are drawn from a dish containing two red and two blue candies. The total number of simple events is:

$4 \times 3 = 12$

Permutations

เลือกหยิบออกมาก่อน (C) แล้วนำมาเรียงเอง

$\binom{5}{2} = \frac{5\times4}{2\times1}$

- The number of ways you can arrange ***n*** distinct objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)\ldots(2)(1)$ and $0! \equiv 1$.

$\binom{6}{4} = \frac{6\times5\times4\times3}{4\times3\times2\times1} = 15$

$\binom{6}{2} = \frac{6\times5}{2\times1} = 15$

เท่ากัน

#ดูเทคนิคนะครับ

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important! →

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Examples

หยิบมาทั้งหมด

$$\binom{5}{5} 5! = 1 \times 5!$$

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

$$\boxed{P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120}$$

Combinations

this more than permutation.

- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is

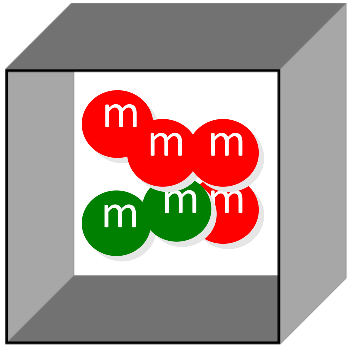
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example

- A box contains six M&Ms®, four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?



$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

4 × 2 =8 ways to choose 1 red and 1 green M&M.

P(exactly one red) = 8/15

Example

A deck of cards consists of 52 cards, 13 "kinds" each of four suits (spades, hearts, diamonds, and clubs). The 13 kinds are Ace (A), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q), King (K). In many poker games, each player is dealt five cards from a well shuffled deck.

There are $C_5^{52} = \dfrac{52!}{5!(52-5)!} = \dfrac{52(51)(50)(49)48}{5(4)(3)(2)1} = 2{,}598{,}960$

possible hands

Example

Four of a kind: 4 of the 5 cards are the same "kind". What is the probability of getting four of a kind in a five card hand?

There are 13 possible choices for the kind of which to have four, and 52-4=48 choices for the fifth card. Once the kind has been specified, the four are completely determined: you need all four cards of that kind. Thus there are 13×48=624 ways to get four of a kind.

The probability=624/2598960=.000240096

Example

One pair: two of the cards are of one kind, the other three are of three different kinds.
What is the probability of getting one pair in a five card hand?

There are 13 possible choices for the kind of which to have a pair; given the choice, there are $C_2^4 = 6$ possible choices of two of the four cards of that kind

Example

There are 12 kinds remaining from which to select the other three cards in the hand. We must insist that the kinds be different from each other and from the kind of which we have a pair, or we could end up with a second pair, three or four of a kind, or a full house.

Example

There are $C_3^{12} = 220$ ways to pick the kinds of the remaining three cards. There are 4 choices for the suit of each of those three cards, a total of $4^3 = 64$ choices for the suits of all three.

Therefore the number of "one pair" hands is $13 \times 6 \times 220 \times 64 = 1{,}098{,}240$.

The probability $= 1098240/2598960 =$

$= .422569$

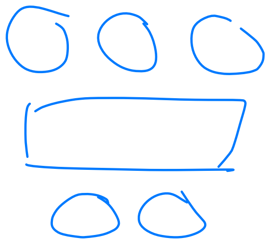
Question

- \- You are dealt a hand of four cards from a well-shuffled deck of 52 cards. Specify an appropriate sample space and determine the probability that you receive the four cards J, Q, K, A in any order, with suit irrelevant.

$$\binom{52}{1}\binom{48}{1}\binom{44}{1}\binom{40}{1}$$

ไม่ชัวคำตอบนะ

Question



2 ชก 3 ทแถว

- \- Five people are sitting at a table in a restaurant. Two of them order coffee and the other three order tea. The waiter forgot who ordered what and puts the drinks in a random order for the five persons. Specify an appropriate sample space and determine the probability that each person gets the correct drink

หาที่แต่ละคนได้เครื่องดื่มถูกชนิด $\binom{2}{1}\binom{1}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1}$

$= 2\times3\times2$

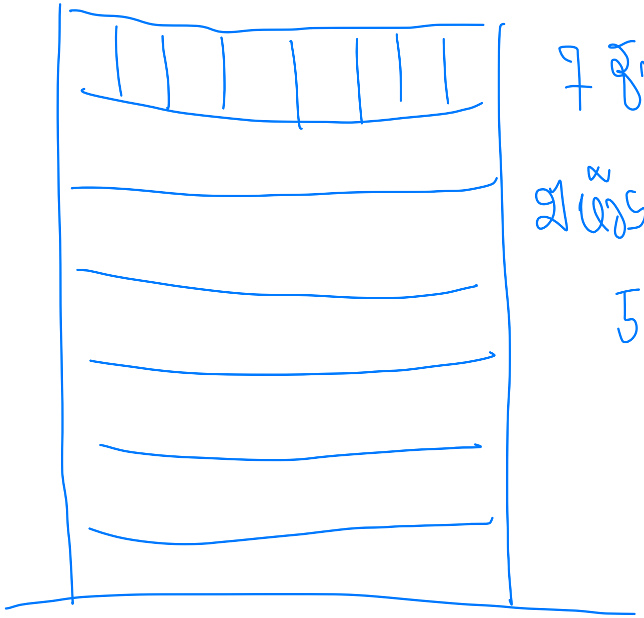
$= 12$ แบบ

$\cancel{5}\times4\times3\times\cancel{2}$

$n(S) = 120$ แบบ

Question

- \- Somebody is looking for a top-floor apartment. She hears about two vacant apartments in a building with 7 floors x 8 apartments per floor. What is the probability that there is a vacant apartment on the top floor?



7 ชั้น 8 ห้อง

มีห้องทั้งหมด

56 ห้อง

มี 2 ห้องว่าง

หาโอกาสที่เป็นชั้นบน

ทั้งหมด $\binom{56}{2}$

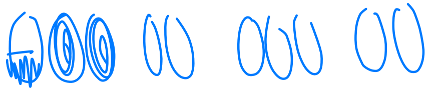
$= \frac{56 \times 55}{2}$

= 1540 แบบห้องว่าง

ห้องเดียวชั้นบน + 2 ห้องชั้นบน

หาทุกรูปแบบที่ห้องว่างอยู่ชั้นบน!

Question



แบ่งเป็นกลุ่มละ 5 คน

- \- You and two of your friends are in a group of 10 people. The group is randomly split up into two groups of 5 people each. Specify an appropriate sample space and determine the probability that you and your two friends are in the same group.

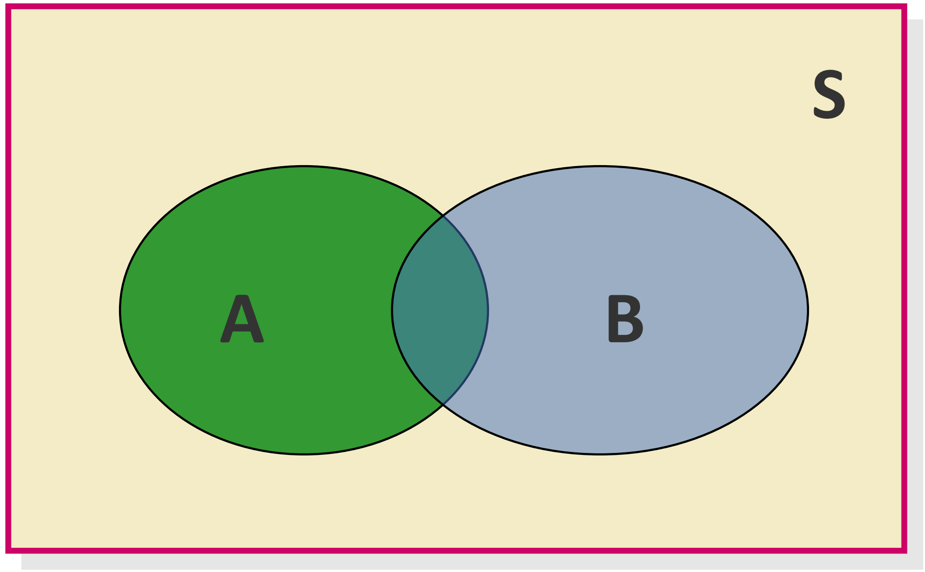
โอกาสที่เพื่อนทั้ง 2 คนกับเธออยู่ในกลุ่มเดียวกัน

Event Relations

The beauty of using events, rather than simple events, is that we can **combine** events to make other events using logical operations: and, or and not.

The **union** of two events, **A** and **B**, is the event that either **A or B or both** occur when the experiment is performed. We write

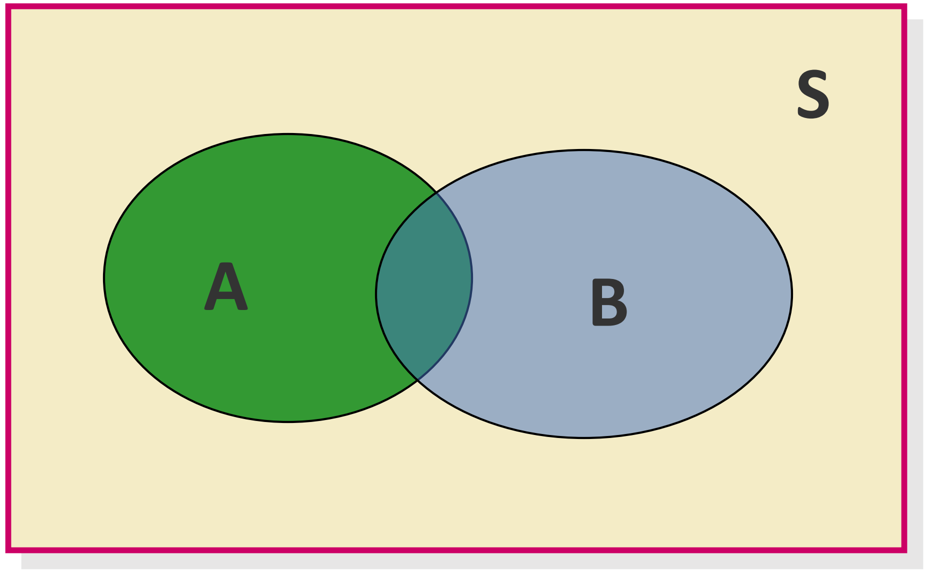
$$A \cup B$$



Event Relations

The **intersection** of two events, **A** and **B**, is the event that both A **and** B occur when the experiment is performed. We write **A ∩ B**.

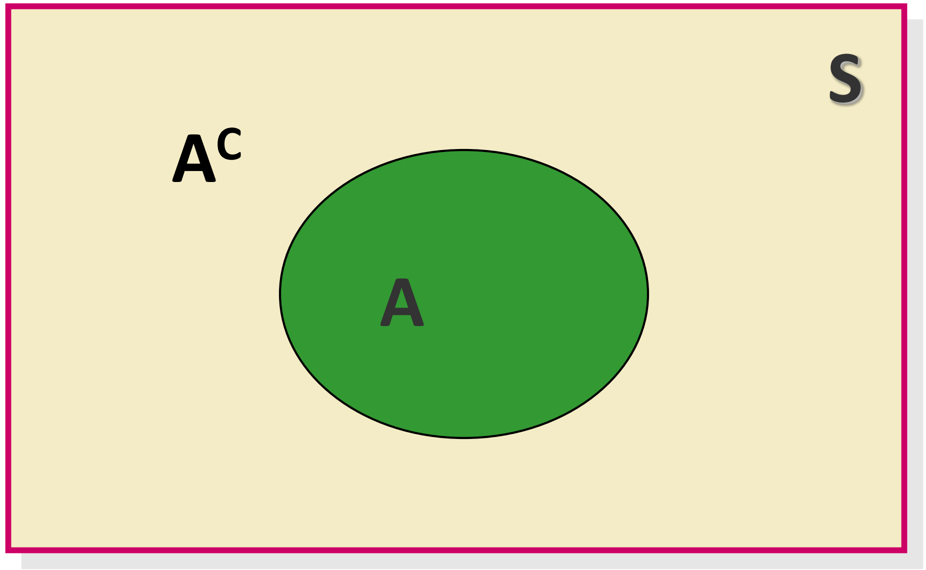
$A \cap B$



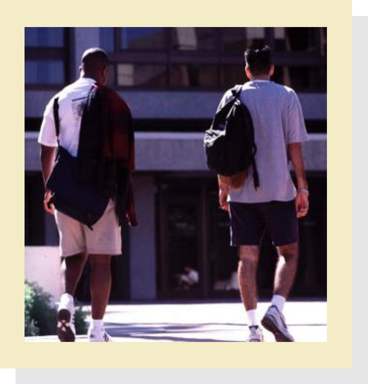
- If two events A and B are **mutually exclusive**, then **P(A ∩ B) = 0**.

Event Relations

The **complement** of an event **A** consists of all outcomes of the experiment that do not result in event **A**. We write **A^C**.



Example



Select a student from the classroom and record his/her **hair color** and **gender.**

- **A:** student has brown hair
- **B:** student is female
- **C:** student is male

Mutually exclusive; $B = C^C$

What is the relationship between events **B** and **C**?

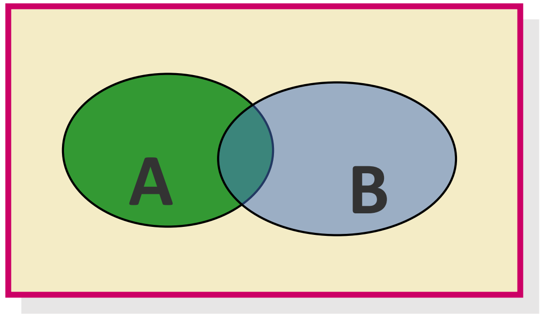
- **A^C:** Student does not have brown hair
- **$B \cap C$:** Student is both male and female = $\varnothing$
- **$B \cup C$:** Student is either male and female = all students = S

Calculating Probabilities for Unions and Complements

- There are special rules that will allow you to calculate probabilities for composite events.
- **The Additive Rule for Unions:**
- For any two events, **A** and **B**, the probability of their union, **P(A ∪ B)**, is

สูตรเหมือนแผนภาพเวนออยเลอร์

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



Example: Additive Rule

Example: Suppose that there were 120 students in the classroom, and that they could be classified as follows:

A: brown hair

$P(A) = 50/120$

B: female

$P(B) = 60/120$

	Brown	Not Brown
Male	20	40
Female	30	30

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 50/120 + 60/120 - 30/120$

$= 80/120 = 2/3$

Check: $P(A \cup B)$

$= (20 + 30 + 30)/120$

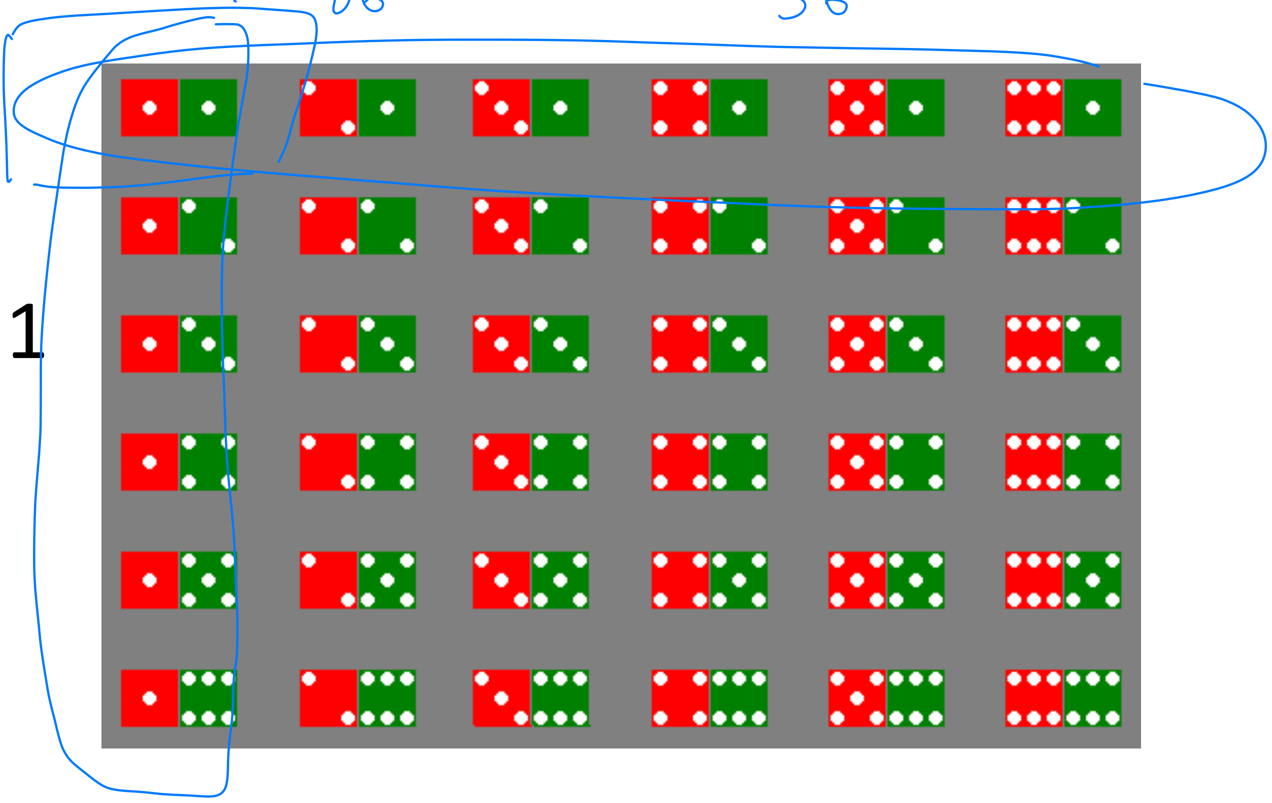
Example: Two Dice

$\frac{1}{36}$

A: red die show 1

B: green die show 1

$\frac{6}{36}$ $\frac{6}{36}$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 6/36 + 6/36 - 1/36$$
$$= 11/36$$

A Special Case

When two events A and B are **mutually exclusive,** $P(A\cap B) = 0$ and $\mathbf{P(A\cup B) = P(A) + P(B)}$.

A: male with brown hair

$P(A) = 20/120$

B: female with brown hair

$P(B) = 30/120$

	Brown	Not Brown
Male	20	40
Female	30	30

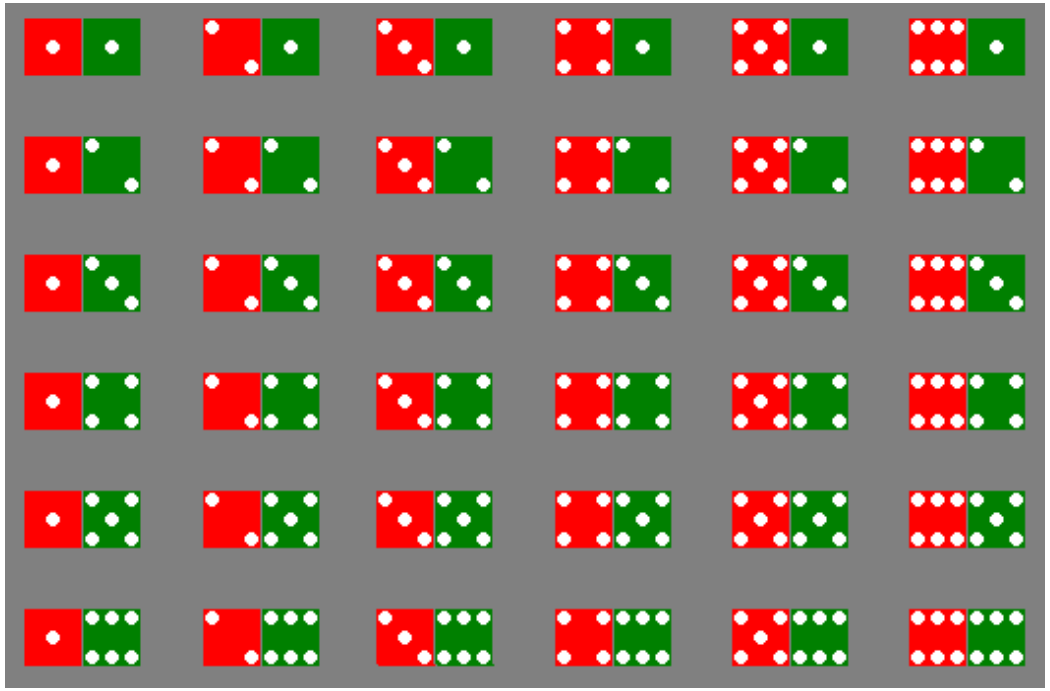
A and B are mutually exclusive, so that

$P(A\cup B) = P(A) + P(B)$
$= 20/120 + 30/120$
$= 50/120$

Example: Two Dice

A: dice add to 3

B: dice add to 6



A and B are mutually exclusive, so that

$P(A \cup B) = P(A) + P(B)$

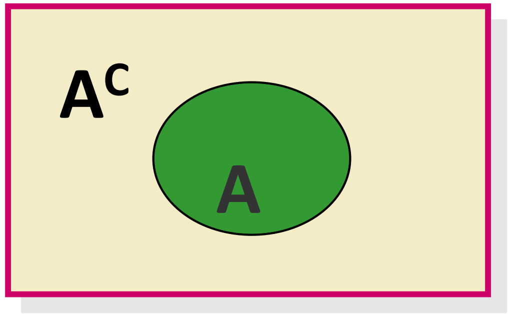
$= 2/36 + 5/36$

$= 7/36$

Calculating Probabilities for Complements

- We know that for any event **A:**
  - **$P(A \cap A^C) = 0$**
- Since either **A** or **A^C** must occur,

  $P(A \cup A^C) = 1$
- so that **$P(A \cup A^C) = P(A) + P(A^C) = 1$**



$$\boxed{P(A^C) = 1 - P(A)}$$

Example

Select a student at random from the classroom. Define:

A: male

P(A) = 60/120

B: female

P(B) = ?

	Brown	Not Brown
Male	20	40
Female	30	30

A and B are complementary, so that

P(B) = 1- P(A)

= 1- 60/120 = 60/120

Calculating Probabilities for Intersections

In the previous example, we found $P(A \cap B)$ directly from the table. Sometimes this is impractical or impossible. The rule for calculating $P(A \cap B)$ depends on the idea of **independent and dependent events.**

Two events, **A** and **B**, are said to be **independent** if the occurrence or nonoccurrence of one of the events does not change the probability of the occurrence of the other event.

Questions

- You draw at random five cards from a standard deck of 52 cards. What is the probability that there is an ace among the five cards and a king or queen?

Question

- The probability that a visit to a particular car dealer results in neither buying a second-hand car nor a Japanese car is 55%. Of those coming to the dealer, 25% buy a second-hand car and 30% buy a Japanese car. What is the probability that a visit leads to buying a second-hand Japanese car?

Conditional Probabilities

The probability that A occurs, given that event B has occurred is called the **conditional probability** of A given B and is defined as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0$$

“given”

เมื่อเกิด B ก่อนแล้ว A ตาม

เหตุการณ์นี้ต้องเกิดขึ้นก่อน

มี 2 สูตร

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$P(A\cap B) = P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

จากเหตุการณ์ย้อนกลับไปได้

"Bayes' Theorem"

Example 1

Toss a fair coin twice. Define
- A: head on second toss
- B: head on first toss

ครั้งแรกหัว
ครั้ง2 หัว

$P(A) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

HH	1/4
HT	1/4
TH	1/4
TT	1/4

เมื่อครั้งแรกเป็นหัวแล้วครั้งที่2 เป็นหัว

โอกาสเกิดครึ่งหนึ่งนะ

P(A|B) = ½

P(A|not B) = ½

P(A) does not change, whether B happens or not…

เมื่อลดเหตุการณ์ก็ไม่ได้เปลี่ยนแต่ตัวหาร

A and B are independent!

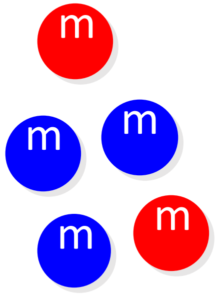
แสดงว่าทั้ง2 เหตุการณ์ไม่มีความสัมพันธ์กัน

ค่าเท่าเดิม

Example 2

A bowl contains five M&Ms®, two red and three blue. Randomly select two candies, and define

- A: second candy is red.
- B: first candy is blue.



P(A|B) =P(2nd red|1st blue)= 2/4 = 1/2

P(A|not B) = P(2nd red|1st red) = 1/4

เหตุการณ์ทั้ง 2 มีความ
เชื่อมโยงกัน

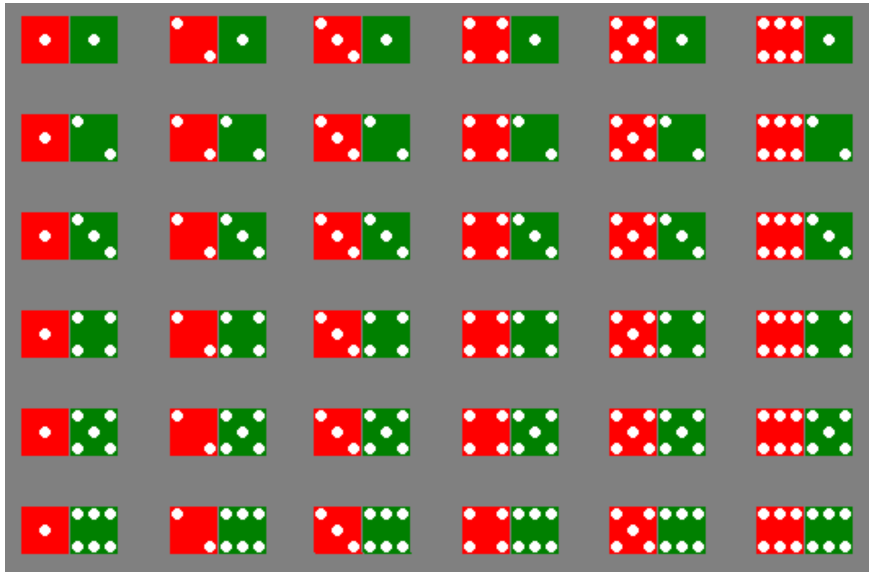
P(A) does change, depending on whether B happens or not…

A and B are dependent!

Example 3: Two Dice

Toss a pair of fair dice. Define

- A: red die show 1
- B: green die show 1



P(A|B) = P(A and B)/P(B)

=1/36/1/6=1/6=P(A)

P(A) does not change, whether B happens or not…

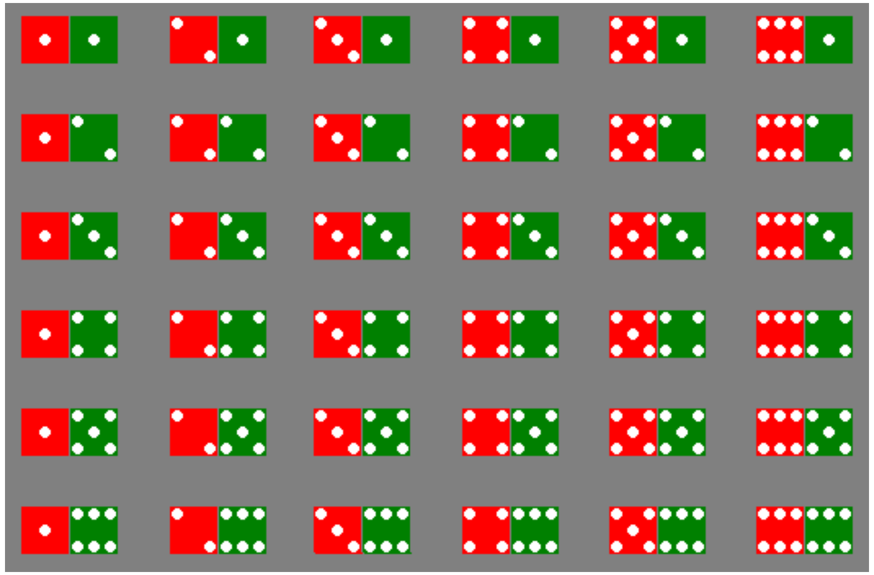
→

A and B are independent!

Example 3: Two Dice

Toss a pair of fair dice. Define

- A: add to 3
- B: add to 6



P(A|B) = P(A and B)/P(B)

=0/36/5/6=0

P(A) does change when B happens

A and B are dependent! In fact, when B happens, A can’t

Defining Independence

- We can redefine independence in terms of conditional probabilities:

Two events A and B are **independent** if and only if

↳ ไม่ขึ้นอยู่ต่อกัน

$$P(A|B) = P(A) \qquad \text{or} \qquad P(B|A) = P(B)$$

Otherwise, they are **dependent**.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

The Multiplicative Rule for Intersections

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A\cap B) = P(B)\,P(A|B)$$

P(A ∩ B) = P(A) P(B given that A occurred)
= P(A)P(B|A)

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

เมื่อ A ไม่เกี่ยวข้องกับ B

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

P(A ∩ B) = P(A) P(B)

ถ้าเหตุการณ์ทั้ง 2 ไม่มีความเกี่ยวข้องกัน

เอาความน่าจะเป็นมาคูณแล้ว

เรื่องที่ยากคือ
เราจะกำหนดอันไหนเป็นอะไร

Example 1

ใช้เสี่ยงหาตัวหารไม่ได้ หาตัวตั้งไม่ได้ แต่เขาให้ % ให้

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

$= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1)= 3(.1)(.9)^2 = .243$

10ๆ. มีใครสักคนเป็น heart attack เลือกมา 3 อาชีพจากประชากร โอกาสจะไม่รอด

1/3 คนที่ขอรับผล=เป็นคนที่มีอาการ heart attack

Example 2

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, $P(F) = .49$ and $P(H|F) = .08$.

Use the Multiplicative Rule:

$$P(\text{high risk female}) = P(H \cap F)$$

$$= P(F)P(H|F) = .49(.08) = .0392$$

Example : Use Conditional Probability

- \- Five people are sitting at a table in a restaurant. Two of them order coffee and the other three order tea. The waiter forgot who ordered what and puts the drinks in a random order for the five persons. Specify an appropriate sample space and determine the probability that each person gets the correct drink

Answer from basic probability operation : (2! X 3!) / 5! = 0.1

Let's consider the waiter first serves the two cups of coffee

A1 = first coffee is given to a person having order

A2 = second coffee is given to a person having order

[illegible]

Answer = $P(A1 \cap A2) = P(A1) \times P(A2|A1) = 2/5 \times 1/4 = 0.1$

Example : Use Conditional Probability

- - Somebody is looking for a top-floor apartment. She hears about two vacant apartments in a building with 7 floors x 8 apartments per floor. What is the probability that there is a vacant apartment on the top floor?

- Answer from basic probability operation : $(8C1 \times 48C1 + 8C2) / 56C2 = 384+28 / 1540 = 0.2675$
- Consider : A1 = The first vacancy is not on the top floor , A2 = The second vacancy is not on the top floor
- Find : $1 - P(A1 \cap A2) = 1 - P(A1) \times P(A2|A1) = 1 - 48/56 * 47/55 = 0.2675$

Question : Use Conditional Probability

- \- You and two of your friends are in a group of 10 people. The group is randomly split up into two groups of 5 people each. Specify an appropriate sample space and determine the probability that you and your two friends are in the same group.

- Answer from basic probability operation : (7C2 + 7C5) / 10C5 = 1/6
- Consider : A1 = … , A2 = …
- Find : ….

Question : Use Conditional Probability

- \- You are dealt a hand of four cards from a well-shuffled deck of 52 cards. Specify an appropriate sample space and determine the probability that you receive the four cards J, Q, K, A in any order, with suit irrelevant.

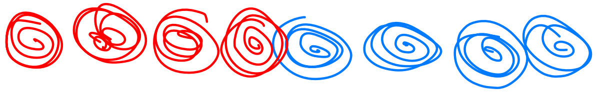
- Answer from basic probability operation :

4C1 x 4C1 * 4C1 * 4C1 / 52C4 = 9.46 x 10^(-4)

- Consider : A1 = … , A2 = … , A3 = …. , A4 = ….
- Find : …

Question : Use Conditional Probability

- \- A bowl contains four red and four blue balls. As part of drawing lots,you choose four times two balls at random from the bowl without replacement. What is the probability that one red and one blue ball are chosen each time?



หยิบออกมา แล้วไม่ต้องใส่คืน 4 ครั้ง

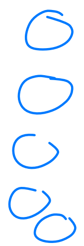
$$\frac{\binom{4}{1}\binom{4}{1}}{\binom{8}{2}} \quad \frac{\binom{3}{1}\binom{3}{1}}{\binom{6}{2}}$$

$$= \frac{8}{35}$$

Question : Use Conditional Probability

- \- There are three English teams among the eight teams that have reached the quarter-finals of the Champions League soccer. What is the probability that the three English teams will avoid each other in the draw if the teams are paired randomly?

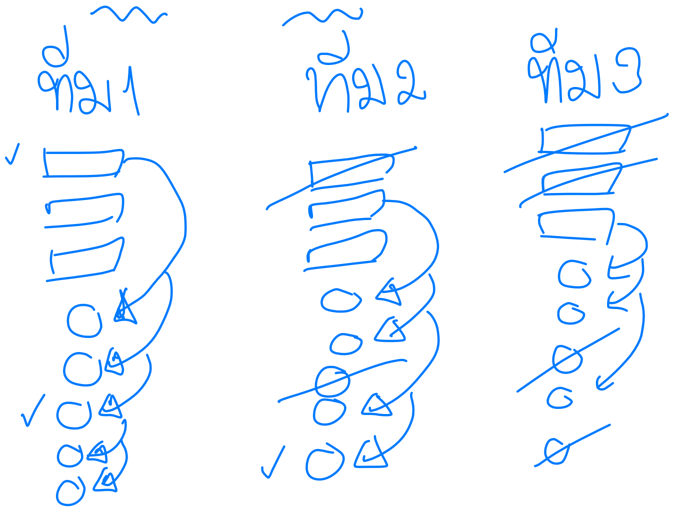
liver
Man U
Braca



โอกาสที่จับคู่แล้วไม่เจอ 3 ทีม

ผลคูณโอกาสที่แต่ละทีมจะเจอทีมที่
ตัวเองไม่ได้อยู่ใน 3ทีม

$$\frac{5}{7} \quad \frac{4}{5} \quad \frac{3}{4} = \frac{4}{7}$$



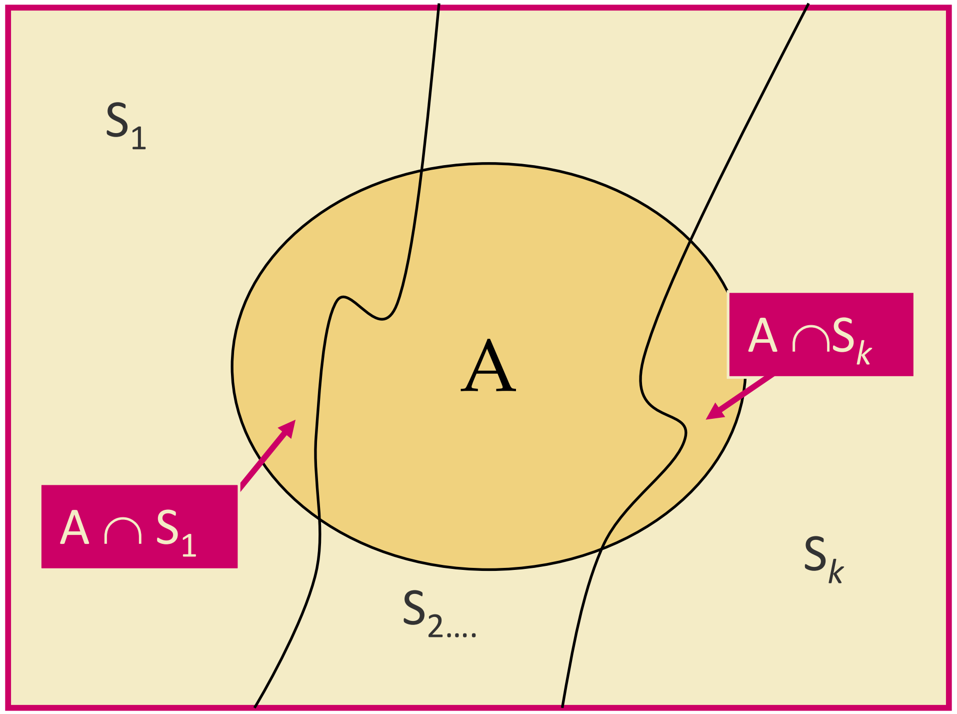
The Law of Total Probability

Let S_1 , S_2 , S_3 ,..., S_k be mutually exclusive and exhaustive events (that is, one and only one must happen). Then the probability of any event A can be written as

$$\mathbf{P(A) = P(A \cap S_1) + P(A \cap S_2) + \ldots + P(A \cap S_k)}$$

$$\mathbf{= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)}$$

The Law of Total Probability



$$P(A) = P(A \cap S_1) + P(A \cap S_2) + \ldots + P(A \cap S_k)$$

$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)$$

Bayes' Rule

เมื่อเรารู้ P(L|E) และสามารถหา P(E|L) ได้

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(B \cap (A) = P(B|A)P(A)$

$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

สัลบูณ์ศาเต็ม ตัวหาร

คือตัวแรก

Let S_1 , S_2 , S_3 ,..., S_k be mutually exclusive and exhaustive events with prior probabilities $P(S_1)$, $P(S_2)$,...,$P(S_k)$. If an event A occurs, the posterior probability of S_i, given that A occurred is

$$P(S_i \mid A) = \frac{P(S_i)P(A \mid S_i)}{\sum P(S_i)P(A \mid S_i)} \quad \text{for } i = 1, 2, \ldots k$$

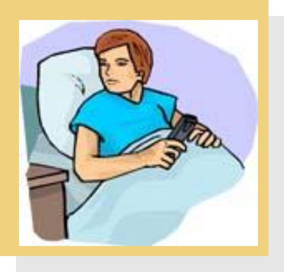
P(A) ไง

Proof

$$P(A \mid S_i) = \frac{P(AS_i)}{P(S_i)} \longrightarrow P(AS_i) = P(S_i)P(A \mid S_i)$$

$$P(S_i \mid A) = \frac{P(AS_i)}{P(A)} = \frac{P(S_i)P(A \mid S_i)}{\sum P(S_i)P(A \mid S_i)}$$

Example



From a previous example, we know that 49% of the population are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. A single person is selected at random and found to be high risk. What is the probability that it is a male? Define H: high risk F: female M: male

We know:

P(F) = .49

P(M) = .51

P(H|F) = .08

P(H|M) = .12

$$P(M|H) = \frac{P(M)P(H|M)}{P(M)P(H|M) + P(F)P(H|F)}$$

$$= \frac{.51(.12)}{.51(.12) + .49(.08)} = .61$$

Example

Suppose a rare disease infects one out of every 1000 people in a population. And suppose that there is a good, but not perfect, test for this disease: if a person has the disease, the test comes back positive 99% of the time. On the other hand, the test also produces some false positives: 2% of uninfected people are also test positive. And someone just tested positive. What are his chances of having this disease?

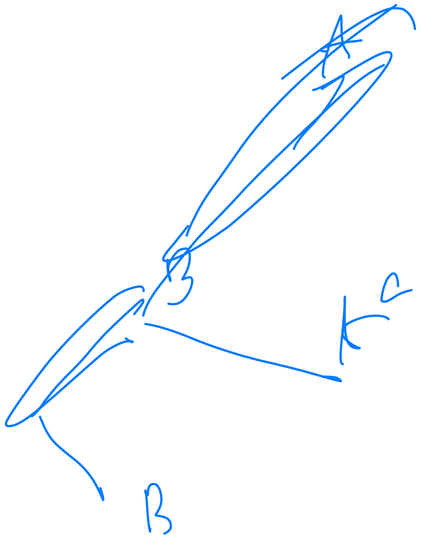
Example

Define A: has the disease   B: test positive

We know:

$P(A) = .001$ $P(A^c) = .999$

$P(B|A) = .99$ $P(B|A^c) = .02$



We want to know $P(A|B)=?$

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)}$$

[illegible]

$$= \frac{.001 \times .99}{.001 \times .99 + .999 \times .02} = .0472$$

Question

- In a binary transmission channel, a 1 is transmitted with probability 0.8 and a 0 with probability 0.2. The conditional probability of receiving a 1 given that a 1 was sent is 0.95, the conditional probability of receiving a 0 when a 0 was sent is 0.99. What is the probability that a 1 was sent when receiving a 1?

Question

- An oil explorer performs a seismic test to determine whether oil is likely to be found in a certain area. The probability that the test indicates the presence of oil is 90% if oil is indeed present in the test area and the probability of a false positive is 15% if no oil is present in the test area. Before the test is done, the explorer believes that the probability of presence of oil in the test area is 40%. Use Bayes’ rule to revise the value of the probability of oil being present in the test area given that the test gives a positive signal.

Question

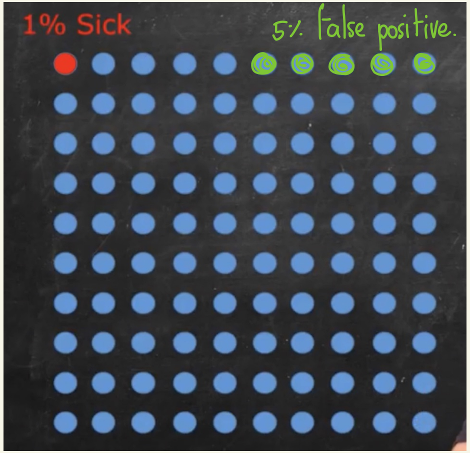
- A family is chosen at random from all three-child families. What is the probability that the chosen family has one boy and two girls if the family has a boy among the three children?

มี false negative ด้วย

Bayes' Theorem - False Positive

↳ ความผิดพลาด เช่น ตรวจโรค มีโอกาส 5% ที่จะผิดพลาด → เราเสือกตรวจได้ผลผิด ↗ จริงๆ เราไม่ได้เป็น

95% ที่จะถูก



จะมีคนป่วย 1% จาก 100 คน

อีก 99 คนจะไปตรวจโรค

↓ อีก 5% จะเจอ false positive

ใช้ Baye's Theorem

$$\underbrace{P(A|B)}_{\text{want}} = \frac{\overbrace{P(B|A)\,P(A)}^{\text{have}}}{P(B)}$$

A เป็นโรค

B ตรวจเจอโรค

B เกิดได้ 2 กรณี
เป็นโรค → ตรวจเจอโรค (B)
ไม่เป็นโรค → ตรวจเจอโรค (B) # false positive

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + \boxed{P(B|\sim A)\,P(\sim A)}}$$

ไม่มีโรคแต่เสือกตรวจเจอ

A คือ เป็นโรค

B คือ ตรวจเจอโรค

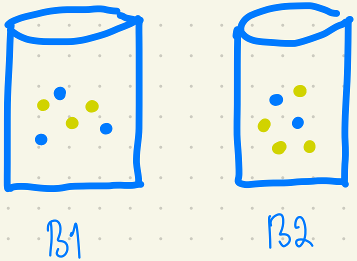
↳ ผิด, ถูก ว่ากันอีกที

$$= \frac{(0.9)(0.01)}{(0.9)(0.01) + (0.05)(0.99)}$$

false positive

$$= 0.154$$

หรือ 15.4 %



โจทย์ ถ้าสุ่มหยิบ ลูกสีน้ำเงิน โอกาสเท่าไหร่ที่จะเป็นถังแรก

เสริมรายละเอียด: สุ่มเลือกกล่องด้วย สุ่มลูกด้วยนะ

กำหนด A : เลือกลูกสีน้ำเงิน

$$P(A|B_1) = \frac{1}{2}$$

$$P(A|B_2) = \frac{1}{3}$$

} โอกาสเจอสีน้ำเงิน

$P(B_1) = P(B_2) = \frac{1}{2}$ # ยืนยัน 2 กล่องตรง

$$P(A) = ?$$

$$= P(A\cap B_1) + P(A\cap B_2)$$

→ ไปได้สองทาง

$$P(A|B_1) = \frac{P(A\cap B_1)}{P(B_1)} \qquad P(A|B_2) = \frac{P(A\cap B_2)}{P(B_2)}$$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$

$$= \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{6}$$

$$= \frac{10}{24} = \frac{5}{12}$$

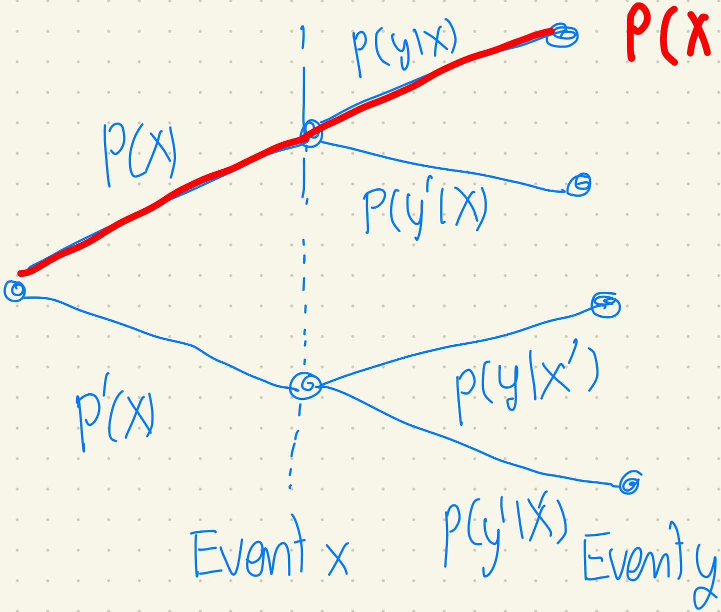
$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{5}{12}}$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{12}^{3}}{5}$$

$$= \frac{3}{5}$$

} Bayes' Theorem -disjoint union.



$$P(x \text{ and } y) = \underline{P(X)}\,\underline{P(y|X)}$$

เป็นหลักของ $P(y|X) = \dfrac{P(x \text{ and } y)}{P(X)}$

ทำให้เกิด $P(y|X) = \dfrac{P(X|y)P(y)}{P(X)}$

ลองมองเป็น tree diagram