

# Artificial Intelligence

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# Lecture 5

## Logical Agents

- Knowledge-based agents
- Propositional logic
- Semantic of propositional logic
- Proof by inference
- Proof by resolution

เป็น agent ที่ฉลาด

↓  
ใช้การค้นหา (search)

↓  
ใช้การอนุมาน (inference) & การแก้ปัญห

# Knowledge-based agents

→ แปล text file ที่ได้ → text มาทำเป็น rule เอาไว้ (pre program)  
↳ fact เก็บข้อมูลที่เราสนใจประมวลผล

- Knowledge-based agent is the agent that has the **reasoning process** that operates on internal knowledge.  
↳ มีการประมวลผลภายใน
- The central component of a knowledge-based agent is its **knowledge base**, or **KB**, which is a set of **sentences**.  
↳ ตัวของ knowledge base มาจาก → sentence (fact)
- Each **sentence** is expressed in a special form and represents some **fact** about the world.  
↳ sentence เป็น fact ของโลกจริง  
today is hot ⇒ fact  
is(today, hot) ⇒ sentence
- Knowledge-based agent can operate by being **told** or **learning** new knowledge about the environment and they can adapt to changes in the environment by **updating** the relevant knowledge.  
↳ agent ได้รับ (sense) percept จาก env  
update knowledge  
↑  
ได้ข้อมูลมา  
↑  
ประมวลผล  
agent เก็บความรู้ knowledge base  
method  
↑  
เก็บความรู้  
↑  
ใช้รูปแบบการเก็บความรู้แบบที่เรามาบอก
- TELL** is a way to add new sentences to the knowledge base.  
↑  
method  
↑  
เก็บความรู้  
↑  
ใช้รูปแบบการเก็บความรู้แบบที่เรามาบอก
- ASK** is a way to query what is known.  
↑  
method  
↑  
เก็บความรู้  
↑  
ใช้รูปแบบการเก็บความรู้แบบที่เรามาบอก

↑  
update knowledge  
↑  
ได้ข้อมูลมา  
↑  
ประมวลผล  
agent เก็บความรู้ knowledge base  
↑  
method  
↑  
เก็บความรู้  
↑  
ใช้รูปแบบการเก็บความรู้แบบที่เรามาบอก

② เสนอความรู้ โดย tell → ข้อ fact ว่า  $9 > 6$

Fact: 9 is greater than 6

TELL

ตัวความรู้ใช้เงื่อนไขเงื่อนไข

KB

① เสนอความรู้ใช้เงื่อนไข

Rule: If  $(> x y)$  and  $(> y z)$  then  $(> x z)$

③ แปล fact → sentence

Sentence:  $(> 9 6)$

Sentence:  $(> 6 3)$

⑤ แปล

Reasoning (Inference)

New sentence:  $(> 9 3)$

④ ไปได้ไหม

ตรวจสอบ

ได้ไหม  
 $? = 3, 6$

$A \Rightarrow B$   
true  $\Rightarrow$  true

④ โดย tell ว่า

Fact: 6 is greater than 3

TELL

⑥ รับความรู้จากแหล่งอื่น  
 $(> 9 ?)$

ASK

ตัวความรู้

↓ sentence

fact: จากข้อ 5 รับมา

ตัวความรู้แบบเป็นแบบตัวเลข

agent ต้องตรวจสอบความรู้ก่อน

↓  
ตรวจสอบ

$if (> 9 6) \text{ and } (> 6 3)$

↓ สอดคล้องกับ  $9 > 3$

↓ แปลเป็น sentence  $(> 9 3)$

subcode

பெரிய பழைய

រតនា ឥសានី

**function** KB-AGENT(*percept*) **returns** an *action*

**persistent:**  $KB$ , a knowledge base

$t$ , a counter, initially 0, indicating time

Constructs a sentence asserting that the agent perceived the given percept at the given time.

① TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))

Is sentence in percept an fact

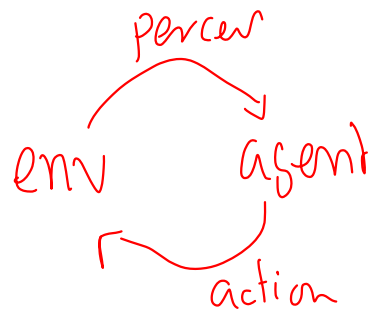
$$action \leftarrow \text{ASK}(KB, \text{MAKE-ACTION-QUERY}(t)) \quad (9)$$

TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))

Constructs a sentence that asks what action should be done at the current time.

$$t \leftarrow t + 1$$
**return** *action*

Constructs a sentence asserting that the chosen action was executed.



action might change env or percept



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 OK <b>A</b> OK	2,1 OK	3,1	4,1

(a)

**A** = Agent  
**B** = Breeze  
**G** = Glitter, Gold  
**OK** = Safe square  
**P** = Pit  
**S** = Stench  
**V** = Visited  
**W** = Wumpus

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 <b>A</b> B OK	3,1 P?	4,1

(b)

**Figure 7.3** The first step taken by the agent in the wumpus world. (a) The initial situation, after percept  $[None, None, None, None, None]$ . (b) After one move, with percept  $[None, Breeze, None, None, None]$ .

The prudent agent will turn around, go back to  $[1,1]$ , and then proceed to  $[1,2]$

Assume that the agent turns and moves to [2,3].

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 <b>A</b> S OK	2,2  OK	3,2	4,2
1,1  V OK	2,1 B  V OK	3,1 P!	4,1

(a)

**A** = Agent  
**B** = Breeze  
**G** = Glitter, Gold  
**OK** = Safe square  
**P** = Pit  
**S** = Stench  
**V** = Visited  
**W** = Wumpus

1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 <b>A</b> S G B	3,3 P?	4,3
1,2 S  V OK	2,2  V OK	3,2	4,2
1,1  V OK	2,1 B  V OK	3,1 P!	4,1

(b)

**Figure 7.4** Two later stages in the progress of the agent. (a) After the third move, with percept [*Stench, None, None, None, None*]. (b) After the fifth move, with percept [*Stench, Breeze, Glitter, None, None*].



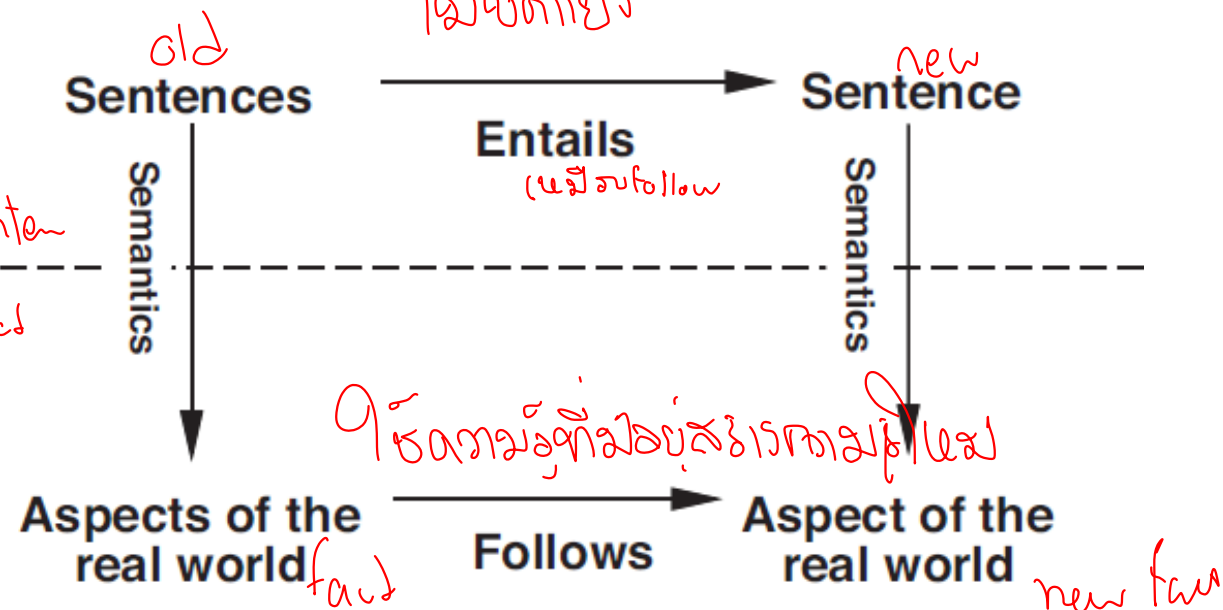
การที่ฉันจะคิดได้แบบนี้ → เราต้องนำหลักการ logic ลงไป

## Reasoning

“การอนุมาน” หรือ “inference”

- Reasoning is the process of constructing new physical configurations (sentences) from old ones.   
↳ การสร้าง sentence ใหม่จาก sentence เดิมที่มีอยู่
- Proper reasoning should ensure that the new configuration represents facts that actually **follow** from the fact that the old configurations represent. In other words, new sentence **entails** the old sentences.

การจะอนุมาน sentence ใหม่ต้อง  
↳ ใช้ข้อเท็จจริง  
กับ fact  
เดิมที่มีอยู่แล้ว



# Propositional Logic: A Very Simple Logic

**Syntax:** The **syntax** of propositional logic defines the allowable sentences.

- The **atomic sentences** consist of a single proposition symbol.
  - **Symbol** represents proposition (fact) that can be true or false; e.g.,  $W_{1,3}$  to stand for the proposition that the wumpus is in [1,3]. Symbol is atomic, i.e., W, 1, and 3 are not meaningful parts of the symbol.
  - There are **two proposition symbols** with fixed meanings: **True** is the always-true proposition and **False** is the always-false proposition

↓  
นี่คือการเขียน logic ใหม่

↗ atomic sentence အစိတ်အပိုင်း

- **Complex sentences** are constructed from simpler sentences, using parentheses and logical connectives.

There are five logical connectives in common use:

- **¬ (not)** A sentence such as  $\neg W_{1,3}$  is called the **negation** of  $W_{1,3}$ .
- **∧ (and)** A sentence whose main connective is  $\wedge$ , such as  $W_{1,3} \wedge P_{3,1}$ , is called a **conjunction**.
- **∨ (or)** A sentence using  $\vee$ , such as  $(W_{1,3} \wedge P_{3,1}) \vee W_{2,2}$ , is a **disjunction** of the disjuncts  $(W_{1,3} \wedge P_{3,1})$  and  $W_{2,2}$ .
- **⇒ (implies)** A sentence such as  $(W_{1,3} \wedge P_{3,1}) \Rightarrow \neg W_{2,2}$  is called an implication (or conditional). Its **premise** or **antecedent** is  $(W_{1,3} \wedge P_{3,1})$ , and its **conclusion** or **consequent** is  $\neg W_{2,2}$ .
- **⇔ (if and only if)** The sentence  $W_{1,3} \Leftrightarrow \neg W_{2,2}$  is a **biconditional**. Some other books write this as  $\equiv$ .

<i>Sentence</i>	$\rightarrow$	<i>AtomicSentence</i>   <i>ComplexSentence</i>	
<i>AtomicSentence</i>	$\rightarrow$	<i>True</i>   <i>False</i>   <i>P</i>   <i>Q</i>   <i>R</i>   ...	
<i>ComplexSentence</i>	$\rightarrow$	( <i>Sentence</i> )   [ <i>Sentence</i> ]	
		$\neg$ <i>Sentence</i>	
		<i>Sentence</i> $\wedge$ <i>Sentence</i>	
		<i>Sentence</i> $\vee$ <i>Sentence</i>	
		<i>Sentence</i> $\Rightarrow$ <i>Sentence</i>	
		<i>Sentence</i> $\Leftrightarrow$ <i>Sentence</i>	
<u>OPERATOR PRECEDENCE</u>	:	$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$	

ក្រុងប្រទេស

ក្រុង

← →

**Figure 7.7** A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Note that we can use parenthesis in propositional logic along with other operators.

# Semantics

Propositional logic  $\xrightarrow{\text{syntax}} (A \wedge B \Rightarrow A)$   
 $\xrightarrow{\text{semantic}} \text{models}$

- The semantics defines the rules for determining the truth of a sentence with respect to a particular model.
- The truth of a sentence can be evaluated recursively like a **truth table**.

$P$	$Q$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

- For example, the sentence  $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1})$   
when  $P_{1,2} = \text{false}$ ,  $P_{2,2} = \text{false}$ ,  $P_{3,1} = \text{true}$ , gives  
 $\text{true} \wedge (\text{false} \vee \text{true}) = \text{true} \wedge \text{true} = \text{true}.$

กฎการอนุมานเชิงประพจน์

หลักการทั่วไป

Premise  
Conclusion

ข้อสมมติ  
ข้อสรุป

# Seven inference rules for propositional logic

- Modus Ponens

$$\frac{\alpha \rightarrow \beta, \alpha}{\beta}$$

เป็นจริง (จริง)

truth table ที่จริงได้  
 $T \rightarrow X \approx T$

- And-Elimination

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n}{\alpha_i} ; 1 \leq i \leq n$$

ถ้าเป็นจริงโดยที่ก็เป็นจริง

- And-Introduction

$$\frac{\alpha_1, \alpha_2, \dots, \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n}$$

ถ้าเป็นจริง  
ถ้าเป็นจริงก็จริง

- Or-Introduction

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \dots \vee \alpha_n} ; 1 \leq i \leq n$$

ถ้าเป็นจริง  
or กับ true  $\approx T$

- Double-negation Elimination

$$\frac{\neg \neg \alpha}{\alpha}$$

- Unit Resolution

$$\frac{\alpha \vee \beta, \neg \beta}{\alpha}$$

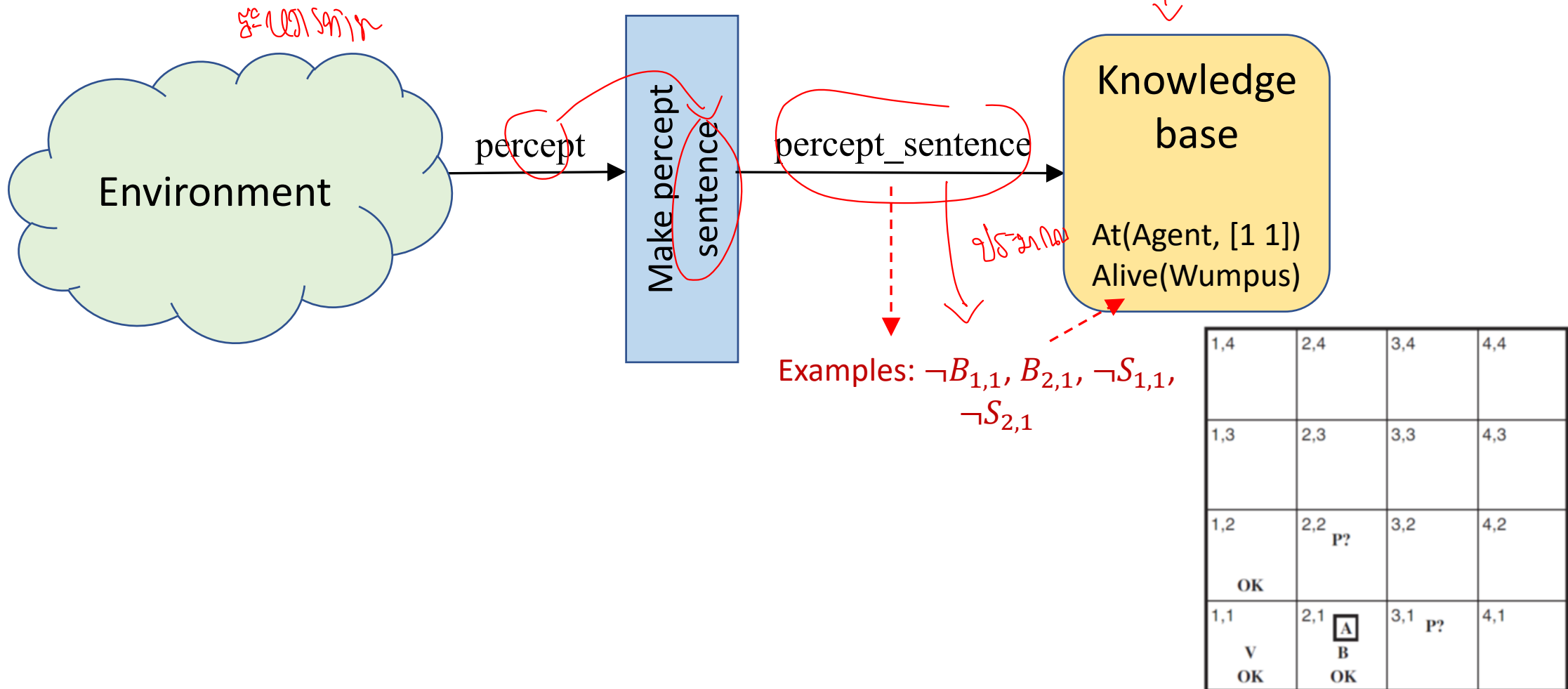
$$\frac{T \vee F}{T}$$

- Resolution

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

$$\frac{\neg \alpha \rightarrow \beta, \beta \rightarrow \gamma}{\neg \alpha \rightarrow \gamma} \approx \alpha \vee \gamma$$

- The agent must start out with some knowledge of environment. For example, agent's position is [1,1], wumpus is alive.
- Knowledge base is told new percept sentences or inferred sentences at each time step.



## Example

- Let us see how these inference rules and equivalences can be used in the wumpus world.
- We start with the knowledge base containing R1 through R5 and show how to prove  $\neg P_{1,2}$ , that is, there is **no pit in [1,2]**.

$R_1 : \neg P_{1,1} .$  (นิเสธของ)  
 $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}) .$  นิเสธของ B ขึ้นอยู่กับ P  
 $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1}) .$  นิเสธของ B ขึ้นอยู่กับ P  
 $R_4 : \neg B_{1,1} .$   
 $R_5 : B_{2,1} .$

predefine กฎของระบบ  
↓  
เราต้องทำแบบตารางให้ได้อะไรบางอย่าง  
↓  
ตารางนี้แสดงถึงสิ่งที่ได้กับของเดิม  
↓  
จัดให้ agent ง่าย ๆ

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1



Proof by inference  $\neg P_{1,2}$ : There is no pit in [1,2].

- First, we apply biconditional elimination to R2 to obtain:

$$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}) .$$

- Then we apply And-Elimination to R6 to obtain:

$$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}) .$$

- Logical equivalence for contrapositives gives:

$$R_8 : (\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})) . \text{ This is called "inference".}$$

- Now we can apply Modus Ponens with R8 and the percept R4 (i.e.,  $\neg B_{1,1}$ ), to obtain

$$R_9 : \neg(P_{1,2} \vee P_{2,1}) .$$

- Finally, we apply De Morgan's rule, giving the conclusion

$$R_{10} : \neg P_{1,2} \wedge \neg P_{2,1} .$$

$$a \leftrightarrow b \\ a \rightarrow b \wedge b \rightarrow a$$

$$a \rightarrow b \quad \neg b \rightarrow \neg a$$

assume  $a$  is true, then  $b$  is true

if  $a$  is true, then  $b$  is true

if  $a$  is true, then  $b$  is true  $\rightarrow$  if  $a$  is false, then  $b$  is false

<sup>2</sup>  
Proof by resolution :  $P_{3,1}$  : There is a pit at [3,1]

Let us consider the steps which the agent returns from [2,1] to [1,1] and then goes to [1,2], where it perceives a stench, but no breeze. We add the following facts to the KB:

$R_{11} : \neg B_{1,2} .$  → เกิดตานั้น  
 $R_{12} : B_{1,2} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3}) .$  → เจ็บจนกลับมามีความ 1, 2 ของ B  
define กฎใหม่

By the same process that led to R10 earlier, we can derive

$R_{13} : \neg P_{2,2} .$   
 $R_{14} : \neg P_{1,3} .$

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 <span style="border: 1px solid black; padding: 2px;">A</span> S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

$B_{1,2} \rightarrow (P_{1,1} \vee P_{2,2} \vee P_{1,3}) \wedge (P_{1,1} \vee P_{2,2} \vee P_{1,3}) \rightarrow B_{1,2}$   
← spill  
 $\neg B_{1,2} \rightarrow \neg(P_{1,1} \vee P_{2,2} \vee P_{1,3})$   
 $\neg P_{1,1} \wedge \neg P_{2,2} \wedge \neg P_{1,3}$   
~ มีอยู่แล้ว

We can also apply biconditional elimination to R3, followed by Modus Ponens with R5, to obtain R15

$R_{15} : P_{1,1} \vee P_{2,2} \vee P_{3,1} .$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$\frac{B_{2,1}}{\neg} \rightarrow (\square) \wedge (\square) \rightarrow B_{2,1}$

$(\square)$  unt resolution นี้จริง or

Now using the resolution rule: the literal  $\neg P_{2,2}$  in R13 resolves with the literal  $P_{2,2}$  in R15 to give the resolvent.

$$R_{16} : P_{1,1} \vee P_{3,1} .$$

F

Similarly, the literal  $\neg P_{1,1}$  in R1 resolves with the literal  $P_{1,1}$  in R16 to give

$$R_{17} : P_{3,1} .$$

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 <div style="border: 1px solid black; display: inline-block; padding: 2px;">A</div> S OK	2,2  OK	3,2	4,2
1,1  V OK	2,1 B V OK	3,1 P!	4,1

Proof by resolution :  $W_{1,3}$  : There is the Wumpus at [1,3]

$$R_1 : \neg S_{1,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$$

$$R_2 : \neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$$

$$R_3 : \neg S_{1,2} \Rightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$$

$$R_4 : S_{1,2} \Rightarrow W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee W_{1,1}$$

*rule*

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 <div style="border: 1px solid black; padding: 2px; display: inline-block;">A</div> S OK	2,2  OK	3,2	4,2
1,1  V OK	2,1 B  V OK	3,1 P!	4,1

Finding the Wumpus:

1. Agent starts out at [1,1] and gets the percept  $\neg S_{1,1}$ . By using R1 and Modus Ponens, we obtain

$$\neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1} \quad \checkmark \text{ and elimination}$$

සාධනයක්  $W, S$  තවදුරටත් පරීක්ෂා

2. Applying And-Elimination:

$$\neg W_{1,1} \quad \neg W_{1,2} \quad \neg W_{2,1}$$

ආරම්භකයෙන් 3 කියවීමේදී වේ

3. Agent moves to [2,1] and gets percept  $\neg S_{2,1}$ .

By using R2, Modus Ponens, And-Elimination, we obtain

$$\neg W_{2,2} \quad \neg W_{2,1} \quad \neg W_{3,1} \quad \neg W_{1,1}$$

4. Agent moves to [1,1] then [1,2] and get  $S_{1,2}$ .

By using R4 and Modus Ponens, we obtain

$$W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee \cancel{W_{1,1}} \rightarrow F$$

5. Applying the unit resolution, where  $\alpha$  is  $W_{1,3} \vee W_{1,2} \vee W_{2,2}$  and  $\beta$  is  $W_{1,1}$ .

$$W_{1,3} \vee W_{1,2} \vee \cancel{W_{2,2}} \rightarrow F$$

6. Applying the unit resolution again, where  $\alpha$  is  $W_{1,3} \vee W_{1,2}$  and  $\beta$  is  $W_{2,2}$ .

$$W_{1,3} \vee \cancel{W_{1,2}} \rightarrow F$$

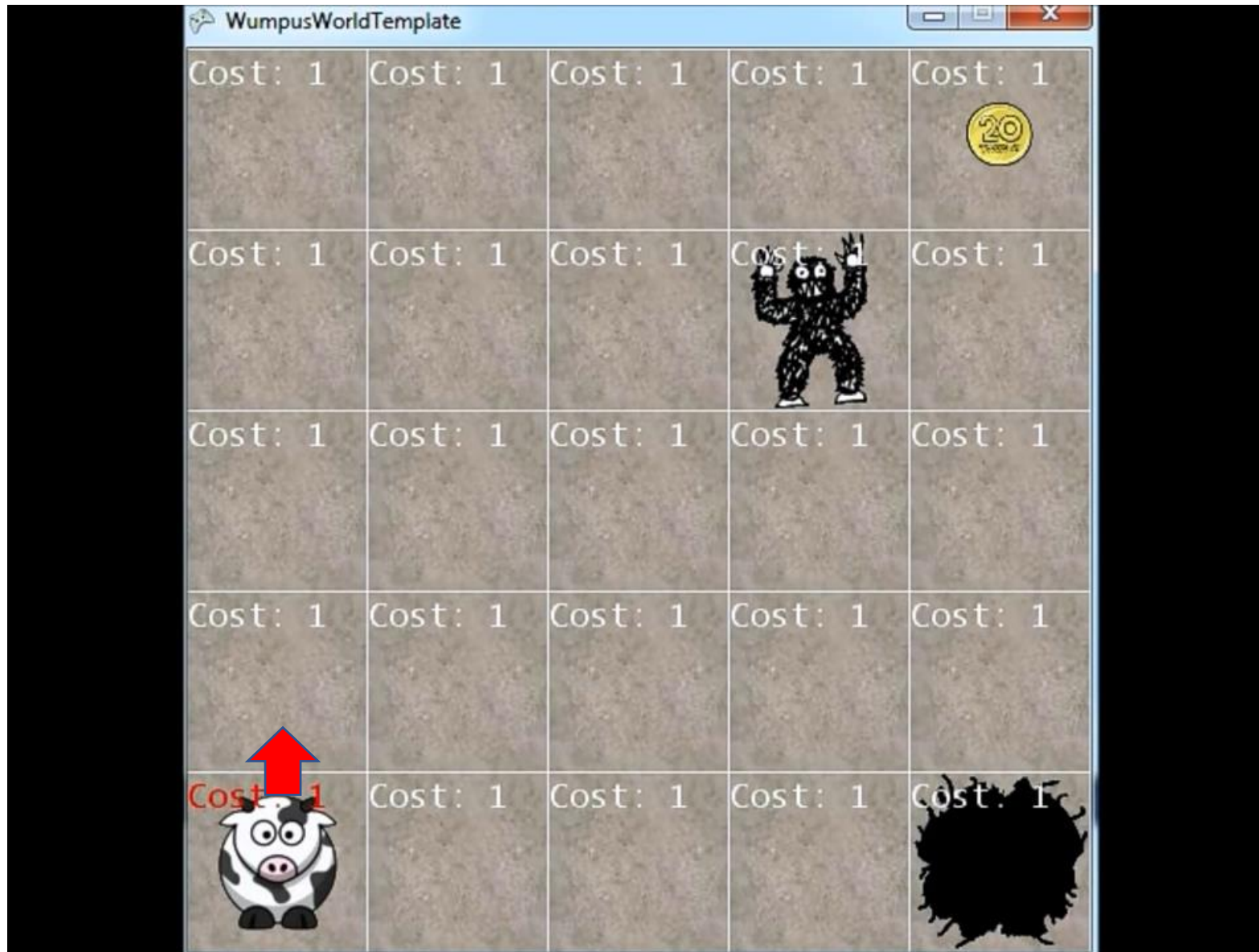
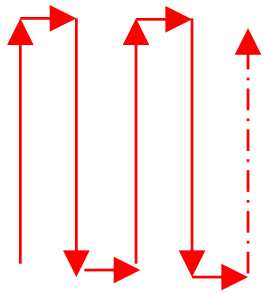
7. One more resolution where  $\alpha$  is  $W_{1,3}$  and  $\beta$  is  $W_{1,3}$ .

$$\cancel{W_{1,3}}$$

ආරම්භකයෙන්  $\rightarrow$  ව්‍යුත්පන්නයක්  $\rightarrow$  ව්‍යුත්පන්නයක්  $\rightarrow$  ව්‍යුත්පන්නයක්  $\rightarrow$  ✓

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

## Exercise : Find a set of positions where agent visits.



## Translating knowledge into action

We need additional rules that relate the current state of the world to the actions the agent should take.

For example,

$$A_{1,1} \wedge East_A \wedge \neg W_{2,1} \Rightarrow Forward$$

$$A_{1,2} \wedge North_A \wedge W_{1,3} \Rightarrow TurnRight$$

$$A_{1,2} \wedge North_A \wedge W_{1,3} \wedge Alive(W) \Rightarrow Shoot$$

ឯកតាអាចដកវាចេញ W, B, S ចំពោះស្ថានភាព

ស្ថានភាពអាចប្រែប្រួលបាន, ក៏ដូចជា

ក្នុងការប្រើ action

## Problems of the propositional agent

1. There are too many propositions to handle.

For example, The simple rule “Don’t go forward if the Wumpus is in front of the agent” can be stated in propositional logic by 64 rules.

2. Since the world can change configuration at each time step, we must specify **time** in the inference rules.

(Suppose that, when agent wants to turn, it always turns left.)

$$A_{1,1}^0 \wedge East_A^0 \wedge \neg W_{2,1} \Rightarrow Forward^0$$

$$A_{1,1}^6 \wedge East_A^6 \wedge \neg W_{2,1} \Rightarrow TurnLeft^6$$

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
OK			
1,1	2,1	3,1	4,1
OK A →	OK		