Theory of Computation

Exercise 13&14: (Pumping Lemma for NonCFL)

Prove by P.L. that the following languages are not CFL.

1. L1 $=\{a^n b^{n+1} c^{n+2} : n \geq 0\}$

<u>Proof</u>: Assume that L1 is CFL. There is PDA accepts L1 with m number of states

1) We pick a string w = $a^m b^{m+1} c^{m+2}$, |w| >= m

2) Opponent decomposes w into uvxyz, |vxy|<=m, |vy|>=1.

2.1) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

|--u--vxy-|---------z------------|

Let ~~vxy~~ vy = a^k; k>=1 condition

We pick i=2, $a^{m+k} b^{m+1} c^{m+2} \notin$ L1 since k>=1.

2.2) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

`|----u-----|---vxy--|------z----|`

2.3) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

`|-----------u----------|-vxy--z-|`

Similar analysis to case 2.1.

2.4) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

`|--u-|-v-|x|-y-|-------z-------|`

Let |v|=k1, |y|=k2 , k1+k2 >=1

We pick i=0, $a^{m-k1}\ b^{m+1-k2}\ c^{m+2} \notin$ L1 since k1+k2 >=1.

2.5) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

`|--u----|--v--|xy|------z------|`

We pick i=2, $a^{m}\ b^{k2}\ a^{k1}\ b^{k2+m+1}\ c^{m+2} \notin$ L1.

2.6) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m+1}\ \overbrace{cc \ldots c}^{m+2}$

`|-uvx|---y----|-------z---------|`

Similar analysis to case 2.5.

2.7) – 2.9) vxy overlaps b^{m+1} and c^{m+2} which have similar analysis to 2.4) – 2.6).

3) We can reach contradiction with P.L. in all cases. Therefore, L1 is not CFL.

2. L2 ={ $w \in \{a, b, c\}^*$: $n_a(w) = \min(\, n_b(w), n_c(w)\,)$ }

Proof: Short version (You suppose to show a full version in the exam).

$w = a^m b^m c^m$

Case 1) $\overbrace{aa \ldots a}^{m}\ \overbrace{bb \ldots b}^{m}\ \overbrace{cc \ldots c}^{m}$ i = 0, 2, 3, …

|---vxy---|

Case 2) |---vxy---| i = 0

Case 3) |--vxy--| i = 0

Case 4) vxy overlaps $a^m b^m$, $v=a^{k1}$, $y=b^{k2}$

If k1!=0 AND k2!=0, use i=2, 3,…
If k1==0 AND k2!=0, use i=0
If k1!=0 AND k2==0, use i=0, 2, 3,…

Case 5) vxy overlaps $a^m b^m$, v overlaps $a^m b^m$, $y=b^k$ i = 2, 3, …

Case 6) vxy overlaps $a^m b^m$, $v = a^k$, y overlaps $a^m b^m$ i = 2, 3, …

Case 7) vxy overlaps $b^m c^m$, $v=b^{k1}$, $y=c^{k2}$ Similar analysis to Case 4.

Case 8) vxy overlaps $b^m c^m$, v overlaps $b^m c^m$, $y=c^k$ i = 0, 2, 3, …

Case 9) vxy overlaps $b^m c^m$, $v=b^k$, y overlaps $b^m c^m$ i = 0, 2, 3, …