คุณสมบัติของ cfl

Positive Properties of Context-Free languages

Union

Context-free languages
are closed under: **Union**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 \cup L_2$ is context-free

Example

Language | **Grammar**

ยกภาษาขึ้นมา 2 ก้อน

$$L_1 = \{a^n b^n\}$$

$$S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\}$$

$$S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Union

ไปทางไหนก็ได้

$$L = \{a^n b^n\} \cup \{ww^R\}$$

$$S \rightarrow S_1 \mid S_2$$

ถ้าใช้การ union เป็นคุณสมบัติปิดของ cfl

In general:

For context-free languages $L_1, \; L_2$
with context-free grammars $G_1, \; G_2$
and start variables $S_1, \; S_2$

The grammar of the **union** $L_1 \cup L_2$
has new start variable S
and additional production $S \to S_1 \mid S_2$

② Concatenation

Context-free languages
are closed under: **Concatenation**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 L_2$ is context-free

Example

Language | **Grammar**

$$L_1 = \{a^n b^n\} \qquad S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\} \qquad S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Concatenation

$$L = \{a^n b^n\} \circ \{ww^R\} \qquad S \rightarrow S_1 S_2$$

เอามาเชื่อมกัน

ctf อยู่ดี

In general:

For context-free languages $L_1, \; L_2$

with context-free grammars $G_1, \; G_2$

and start variables $S_1, \; S_2$

The grammar of the **concatenation** $L_1 L_2$

has new start variable S

and additional production $S \rightarrow S_1 S_2$

3 Star Operation

Context-free languages
are closed under: **Star-operation**

L is context free $\Rightarrow$ L^* is context-free

Example

Language

Grammar

$$L = \{a^n b^n\}$$

$$S \rightarrow aSb \mid \lambda$$

Star Operation

แค่เขียน Grammar เพิ่มไป

$$L = \{a^n b^n\}^*$$

$$S_1 \rightarrow SS_1 \mid \lambda$$

In general:

For context-free language L
with context-free grammar G
and start variable S

The grammar of the **star operation** L^*
has new start variable S_1
and additional production $S_1 \to SS_1 \mid \lambda$

คุณสมบัติแง่ลบ

Negative Properties of Context-Free Languages

Intersection

Context-free languages
are **not** closed under: **intersection**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 \cap L_2$

not necessarily
context-free

Example

$$L_1 = \{a^n b^n c^m\}$$

$\cap$

ได้ $a^n b^n c^n$

จาก previous

this language is not cfl

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \rightarrow AC$$

$$A \rightarrow aAb \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

Context-free:

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow bBc \mid \lambda$$

Intersection

$$L_1 \cap L_2 = \{a^n b^n c^n\}$$ **NOT** context-free

Complement

Context-free languages
are **not** closed under: **complement**

L is context free $\Rightarrow$ $\overline{L}$ **not** necessarily context-free

Example

$L_1 = \{a^n b^n c^m\}$

Context-free:

$S \to AC$

$A \to aAb \mid \lambda$

$C \to cC \mid \lambda$

$L_2 = \{a^n b^m c^m\}$

Context-free:

$S \to AB$

$A \to aA \mid \lambda$

$B \to bBc \mid \lambda$

Complement

$$\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \{a^n b^n c^n\}$$

NOT context-free

Intersection of Context-free languages and Regular Languages

The intersection of

a context-free language and

a regular language

is a context-free language

L_1 context free

L_2 regular

$\Rightarrow$

ป้อบ ก้วน

$L_1 \cap L_2$

context-free

Example:

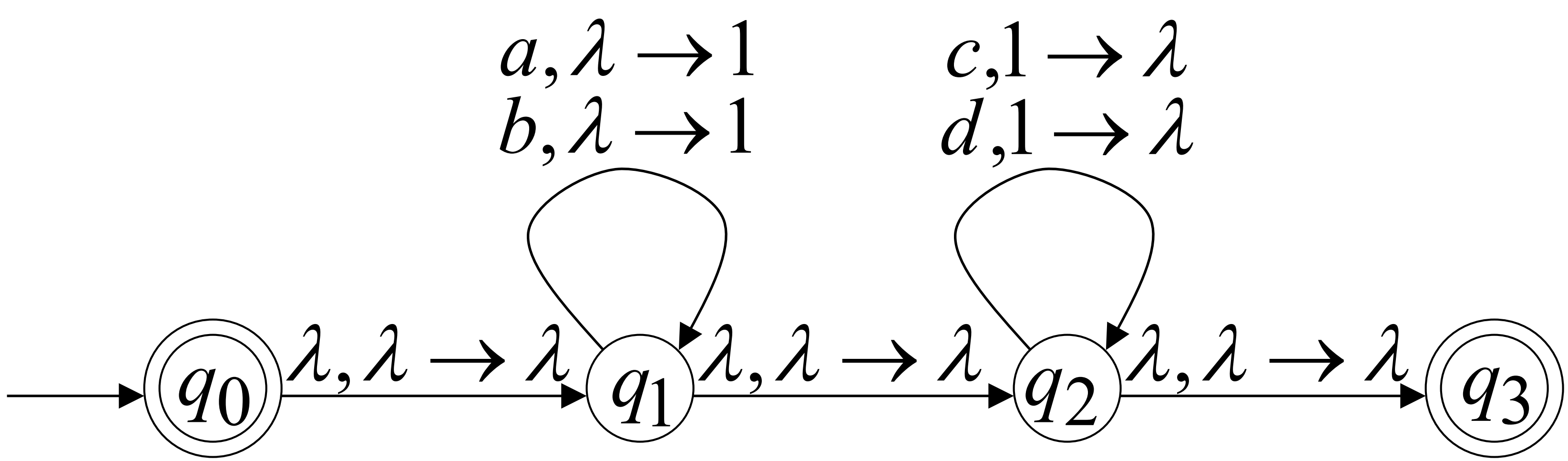
context-free

$$L_1 = \{w_1 w_2 : |w_1| = |w_2|, w_1 \in \{a,b\}^*, w_2 \in \{c,d\}^*\}$$

$\{a,b\}^* \{c,d\}^*$

prove → context with create NPDA for M_1

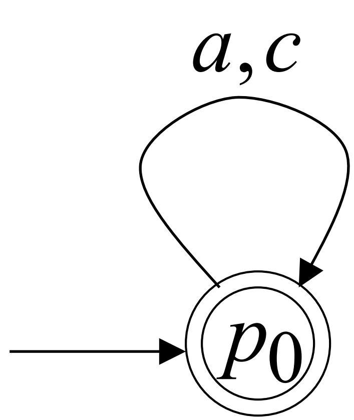
NPDA M_1



regular

$$L_2 = \{a, c\}^*$$

DFA M_2



context-free

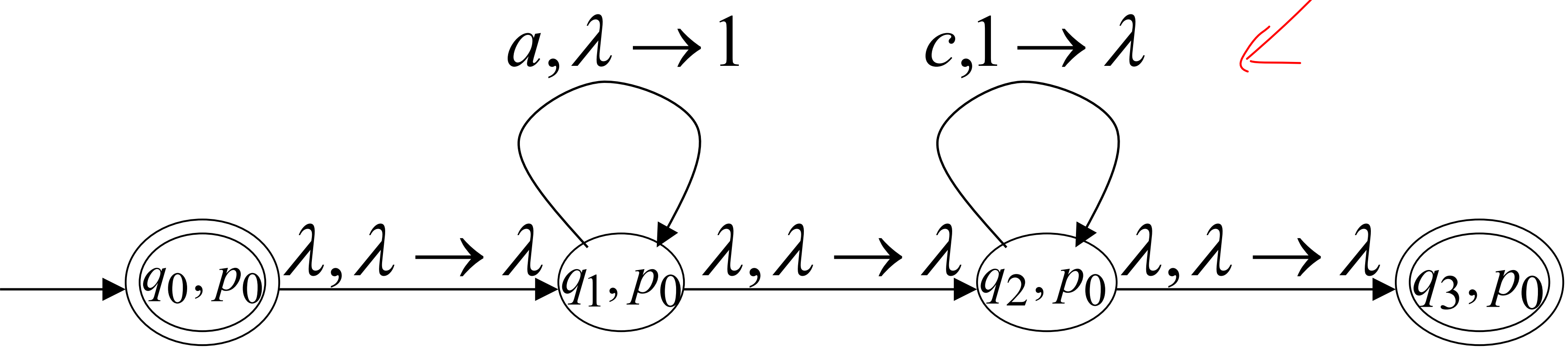
contex f~

Automaton for: $L_1 \cap L_2 = \{a^n c^n : n \geq 0\}$

ถ้าเอามาintersect

จะได้ก้อนนี้

NPDA M



Applications
of
Regular Closure

การใช้งานคุณสมบัติปิด

The intersection of

a context-free language and

a regular language

is a context-free language

L_1 context free

L_2 regular

Regular Closure

$L_1 \cap L_2$

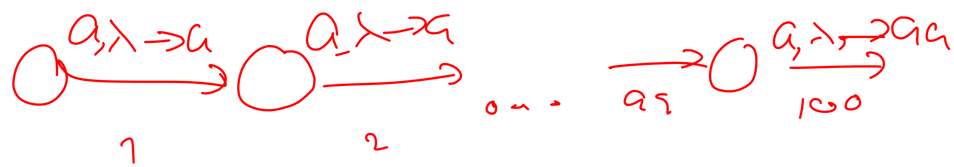
context-free

An Application of Regular Closure

Prove that: $L = \{a^n b^n : \; n \neq 100, n \geq 0\}$

is context-free

นึกภาพสร้าง NPDA ดู



เป็นๆ เลยตาย

We know:

$\{a^n b^n : n \geq 0\}$ is context-free

We also know:

$$L_1 = \{a^{100}b^{100}\}$$ is regular

finite state is always regular

complement = ดูนอกของที่ปิด

$$\overline{L_1} = \{(a+b)^*\} - \{a^{100}b^{100}\}$$ is regular

$\{a^n b^n\}$ context-free

$\overline{L_1} = \{(a+b)^*\} - \{a^{100} b^{100}\}$ regular

(regular closure) $\{a^n b^n\} \cap \overline{L_1}$ context-free

เหมือนเอามา ∩ กัน

$$\{a^n b^n\} \cap \overline{L_1} = \{a^n b^n : \quad n \neq 100, n \geq 0\} = L$$

รูปทั่วไป

prove

is context-free

Another Application of Regular Closure

Prove that: $L = \{w: \;\; n_a = n_b = n_c\}$

is **not** context-free

If $L = \{w: \ n_a = n_b = n_c\}$ is context-free

(regular closure)

$a^{n_a} b^{n_a} c^{n_c}$

Then $L \cap \{a^*b^*c^*\} = \{a^n b^n c^n\}$

context-free regular context-free

this bm cfl ∩ res = cfl

Impossible!!!

Therefore, L is **not** context free