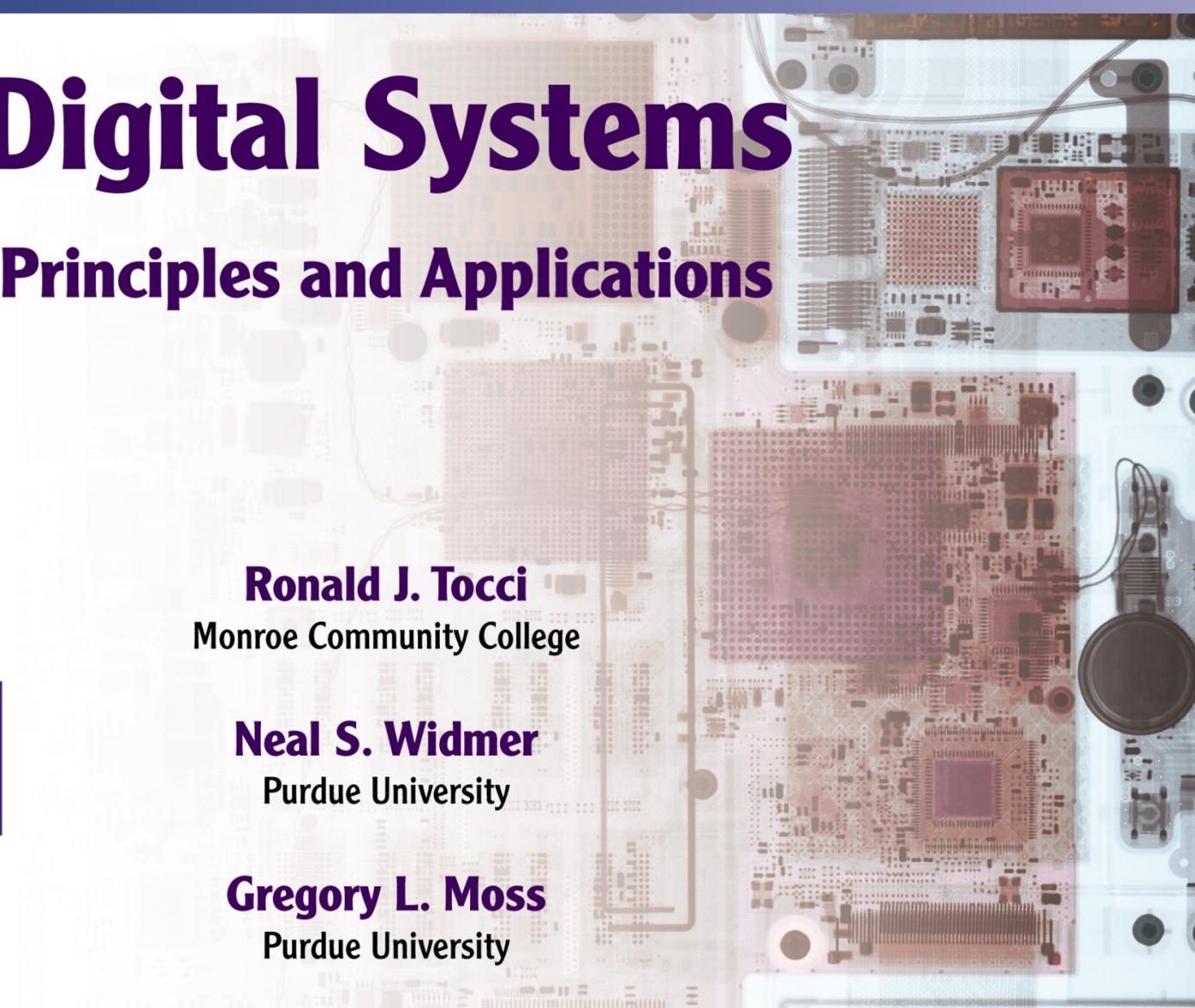


Digital Systems

Principles and Applications



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- *Selected areas covered in this chapter:*
 - Converting between number systems.
 - Decimal, binary, hexadecimal.
 - Advantages of the hexadecimal number system.
 - Counting in hexadecimal.
 - Representing decimal numbers using the BCD code.
 - Pros and cons of using BCD.
 - Differences between BCD and straight binary.
 - Purpose of alphanumeric codes such as ASCII code.
 - Parity method for error detection.
 - Determine the parity bit to be attached to a digital data string.

2-1 Binary to Decimal Conversion

- Convert binary to decimal by summing the positions that contain a 1:

1 1 0 1 1_2

$$\begin{aligned}2^4 + 2^3 + 0 + 2^1 + 2^0 &= 16 + 8 + 2 + 1 \\&= 27_{10}\end{aligned}$$

- An example with a greater number of bits:

1 0 1 1 0 1 0 $1_2 =$

$$2^7 + 0 + 2^5 + 2^4 + 0 + 2^2 + 0 + 2^0 = 181_{10}$$

2-1 Binary to Decimal Conversion

- The double-dabble method avoids addition of large numbers:
 - Write down the left-most 1 in the binary number.
 - Double it and add the next bit to the right.
 - Write down the result under the next bit.
 - Continue with steps 2 and 3 until finished with the binary number.

2-1 Binary to Decimal Conversion

- Binary numbers verify the double-dabble method:

Given:

$$1 \quad 1 \quad 0 \quad 1 \quad 1,$$

Results:

$$1 \times 2 = 2$$

+ 1

$$\begin{array}{r} 3 \times 2 = 6 \\ \underline{-} \\ + 0 \end{array}$$

$$6 \times \underline{2} = 12$$

+1

$$13 \times 2 = 26$$

$$\frac{+1}{27_{10}}$$

1 1 0 1 1
2 3
6 6
12 13
26
27

~~144~~ 154

~~18~~

~~128 + 16 + 8 + 2~~

ပုဂ္ဂနိုင်

$$1001 \overline{)1010} = (154)$$

2+0 4+0 8+1 18+1 38 76+1 154

သီတေသနရေးဝန်ကြီးခွဲ

| | 0 0 1 0 0 |

2-1 Binary to Decimal Conversion

- Reverse process described in 2-1.
 - Note that all positions must be accounted for.

$$45_{10} = 32 + 8 + 4 + 1 = 2^5 + 0 + 2^3 + 2^2 + 0 + 2^0$$

$= 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1_2$

$\cancel{2^6} + \cancel{2^4}$

- Another example:

$$76_{10} = 64 + 8 + 4 = 2^6 + 0 + 0 + 2^3 + 2^2 + 0 + 0$$

$= 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0_2$

2-1 Binary to Decimal Conversion

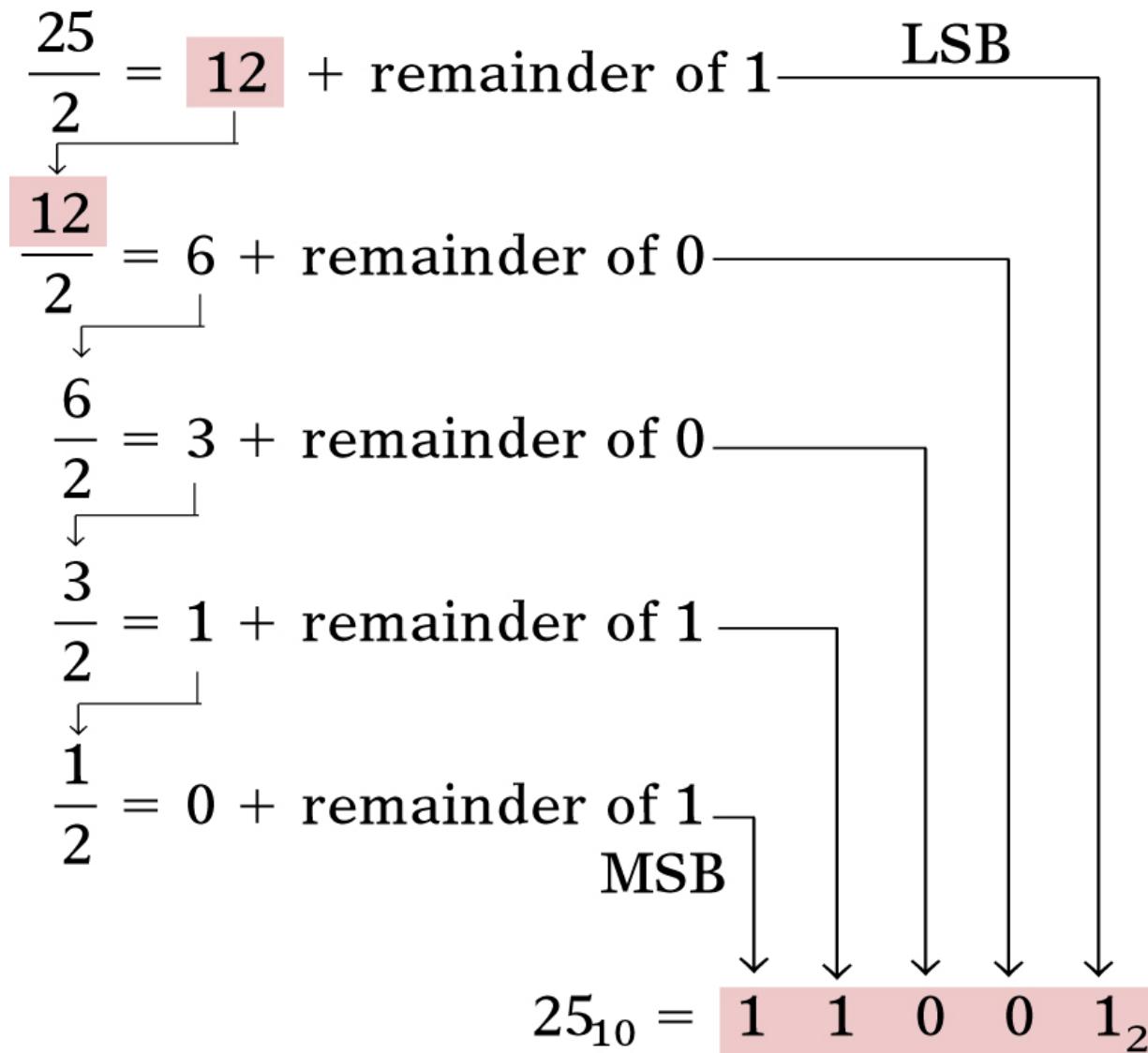
Repeated Division

Divide the decimal number by 2.

Write the remainder after each division until a quotient of zero is obtained.

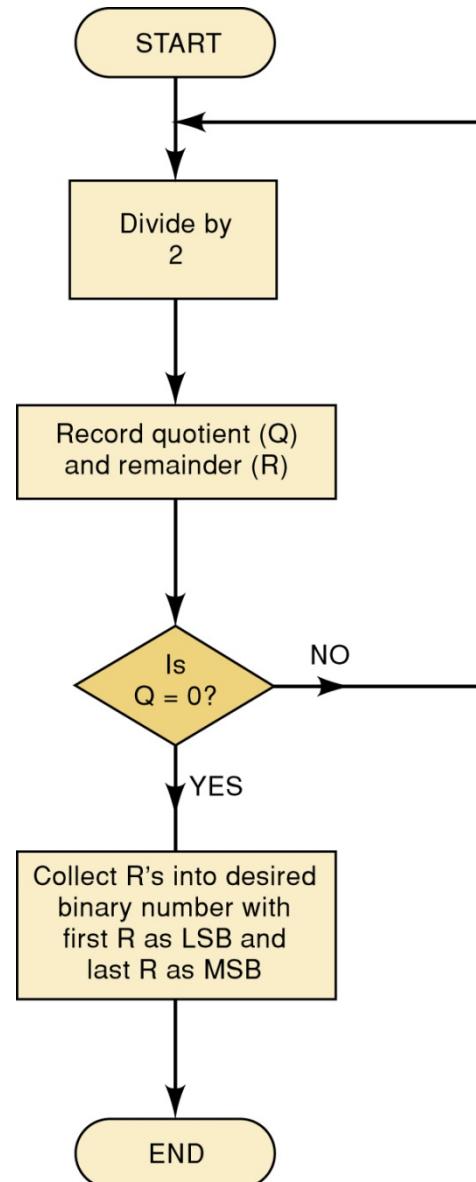
The first remainder is the LSB.

The last is the MSB.



Repeated Division

This flowchart describes the process and can be used to convert from decimal to any other number system.



2-3 Hexadecimal Number System

- Convert 37_{10} to binary:

$$\frac{37}{2} = 18.5 \longrightarrow \text{remainder of 1 (LSB)}$$

$$\frac{18}{2} = 9.0 \longrightarrow 0$$

$$\frac{9}{2} = 4.5 \longrightarrow 1$$

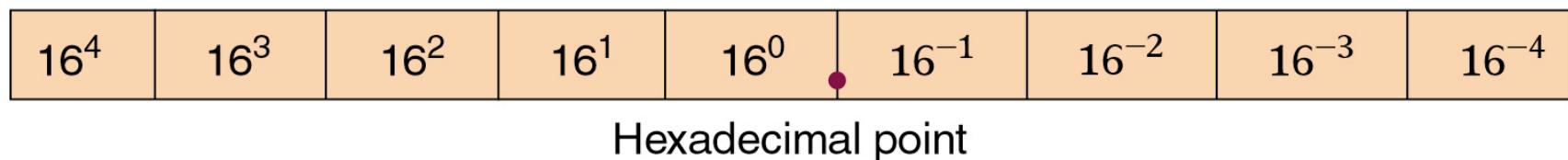
$$\frac{4}{2} = 2.0 \longrightarrow 0$$

$$\frac{2}{2} = 1.0 \longrightarrow 0$$

$$\frac{1}{2} = 0.5 \longrightarrow 1 \text{ (MSB)}$$

2-3 Hexadecimal Number System

- Hexadecimal allows convenient handling of long binary strings, using **groups of 4 bits**—Base 16
 - 16 possible symbols: 0-9 and A-F *(numerals)*



2-3 Hexadecimal Number System

Relationships between hexadecimal, decimal, and binary numbers.

Hexadecimal	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

2-3 Hexadecimal Number System – Hex to Decimal

- Convert from hex to decimal by multiplying each hex digit by its positional weight.

$$\begin{aligned}356_{16} &= 3 \times 16^2 + 5 \times 16^1 + 6 \times 16^0 \\&= 768 + 80 + 6 \\&= 854_{10}\end{aligned}$$

- In a 2nd example, the value 10 was substituted for A and 15 substituted for F.

$$\begin{aligned}2AF_{16} &= 2 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 \\&= 512 + 160 + 15 \\&= 687_{10}\end{aligned}$$

ស្ថាបនុលំមាត្រាករណវត្ថុទាំងនេះ

For practice, verify that $1BC2_{16}$ is equal to 7106_{10}

2-3 Hexadecimal Number System – Decimal to Hex

- Convert from decimal to hex by using the repeated division method used for decimal to binary conversion.
- Divide the decimal number by 16
 - The first remainder is the LSB—the last is the MSB.

2-3 Hexadecimal Number System – Decimal to Hex

- Convert 423_{10} to hex:

$$\begin{array}{r} 423 \\ \hline 16 \\ \downarrow \\ 26 \\ \hline 16 \\ \downarrow \\ 1 \\ \hline 16 \\ \downarrow \\ 1 \\ \hline \end{array} = 26 + \text{remainder of } 7 \quad \boxed{26} + \text{remainder of } 10 \quad \boxed{1} + \text{remainder of } 1$$

$423_{10} = \boxed{1A7}_{16}$

2-3 Hexadecimal Number System – Decimal to Hex

- Convert 214_{10} to hex:

$$\begin{array}{r} 214 \\ \hline 16 \\ \downarrow \\ 13 \end{array} = 13 + \text{remainder of } 6$$
$$\begin{array}{r} 13 \\ \hline 16 \\ \downarrow \\ 0 \end{array} = 0 + \text{remainder of } 13$$
$$214_{10} = \boxed{\text{D6}}_{16}$$

2-3 Hexadecimal Number System – Hex to Binary

- Leading zeros can be added to the left of the MSB to fill out the last group.

16 bits 12 位二進制 2 bits

$$\begin{aligned} 9F2_{16} &= \begin{array}{ccccccc} 9 & & & F & & & 2 \\ \downarrow & & & \downarrow & & & \downarrow \\ = 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ = 100111110010_2 \end{array} \end{aligned}$$

For practice, verify that $BA6_{16} = 101110100110_2$

1011 1010 0110

2-3 Hexadecimal Number System – Binary to Hex

- Convert from binary to hex by grouping bits in four starting with the LSB.
 - Each group is then converted to the hex equivalent
- The binary number is grouped into groups of four bits & each is converted to its equivalent hex digit.

$$\begin{aligned}1110100110_2 &= \underbrace{0\ 0}_{3} \underbrace{1\ 1}_{A} \underbrace{1\ 0\ 1\ 0}_{6} \underbrace{0\ 0\ 1\ 1\ 0}_{6} \\&= 3A6_{16}\end{aligned}$$

For practice, verify that $101011111_2 = 15F_{16}$

2-3 Hexadecimal Number System – Decimal to Hex to Binary

- Convert decimal 378 to a 16-bit binary number by first converting to hexadecimal.

$$\begin{array}{r} 378 \\ \hline 16 \\ \downarrow \\ 23 \\ \hline 16 \\ \downarrow \\ 1 \\ \hline 16 \\ \end{array} = 23 + \text{remainder of } 10_{10} = A_{16}$$
$$\begin{array}{r} 23 \\ \hline 16 \\ \downarrow \\ 1 \\ \hline 16 \\ \end{array} = 1 + \text{remainder of } 7$$
$$\begin{array}{r} 1 \\ \hline 16 \\ \end{array} = 0 + \text{remainder of } 1$$

2-3 Hexadecimal Number System

16 → 2

Hex	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

To perform conversions between hex & binary, it is necessary to know the four-bit binary numbers (0000 - 1111), and their equivalent hex digits.

2-3 Hexadecimal Number System – Counting in Hex

- When counting in hex, each digit position can be incremented (increased by 1) from 0 to F.
 - On reaching value F, it is reset to 0, and the next digit position is incremented.

Example:

38 , 39 , 3A , 3B , 3C , 3D , 3E , 3F , 40 , 41 , 42

When there is a 9 in a digit position, it becomes an A when it is incremented.

With three hex digits, we can count from 000_{16} to FFF_{16} which is 0_{10} to 4095_{10} — a total of $4096 = 16^3$ values.

2-4 BCD Code

- Binary Coded Decimal (BCD) is a widely used way to present decimal numbers in binary form.
 - Combines features of both decimal and binary systems.
 - Each digit is converted to a binary equivalent.
- BCD is *not* a number system.
 - It is a decimal number with each digit encoded to its binary equivalent.
- A BCD number is *not* the same as a straight binary number.
 - The primary advantage of BCD is the relative ease of converting to and from decimal.

Digit 10 9 8 7 6 5 4 3 2

2-4 BCD Code

- Convert the number 874_{10} to BCD:
 - Each decimal digit is represented using 4 bits.
 - Each 4-bit group can never be greater than 9.

8	7	4	(decimal)
↓	↓	↓	
1000	0111	0100	(BCD)

→ අභ්‍යන්තරික binary

- Reverse the process to convert BCD to decimal.

9	4	3	(decimal)
↓	↓	↓	
1001	0100	0011	(BCD)

2-4 BCD Code

- Convert 0110100000111001 (BCD) to its decimal equivalent.

0110 1000 0011 1001
6 8 3 9

Divide the BCD number into four-bit groups and convert each to decimal.

2-4 BCD Code

- Convert 0110100000111001 (BCD) to its decimal equivalent.

0110 1000 0011 1001
6 8 3 9

Divide the BCD number into four-bit groups and convert each to decimal.

2-4 BCD Code

- Convert BCD 011111000001 to its decimal equivalent.

0111 1100 0001
7 ↓ 1

The forbidden group represents
an error in the BCD number.

2-5 The Gray Code

- The Gray code is used in applications where numbers change rapidly.
 - Only one bit changes from each value to the next.

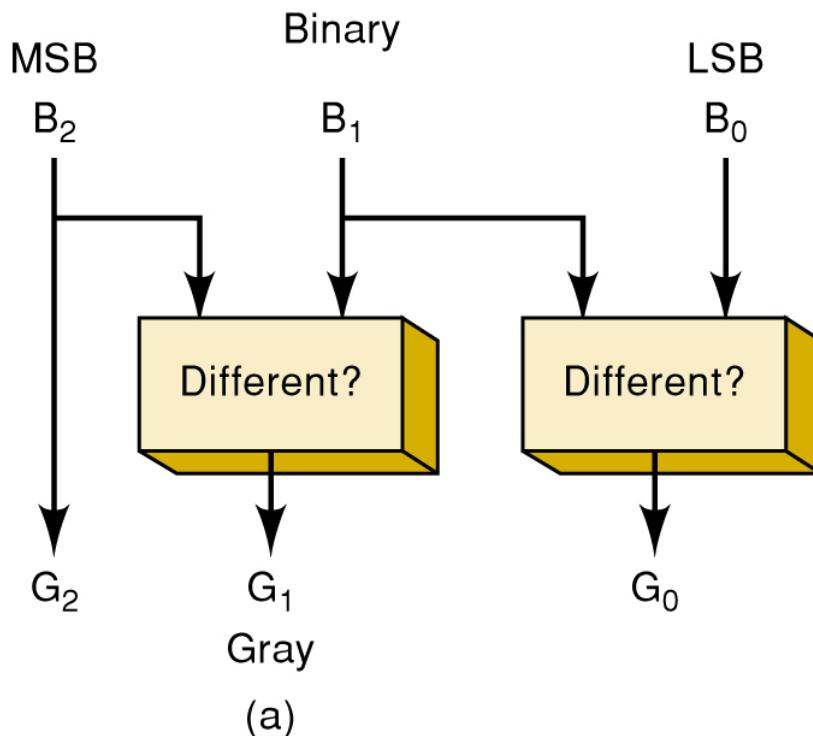
Gray
Assume encode
↓
to 3 encoder

Three bit binary
and Gray code
equivalents.

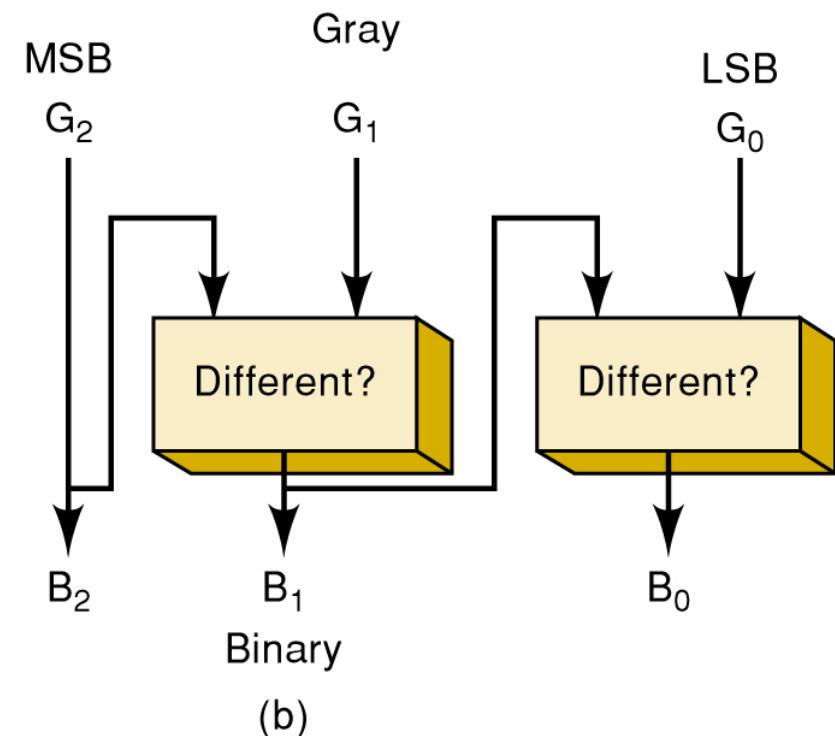
B ₂	B ₁	B ₀	G ₂	G ₁	G ₀
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	1	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

เพื่อต้นแบบทางการณ์

2-5 The Gray Code

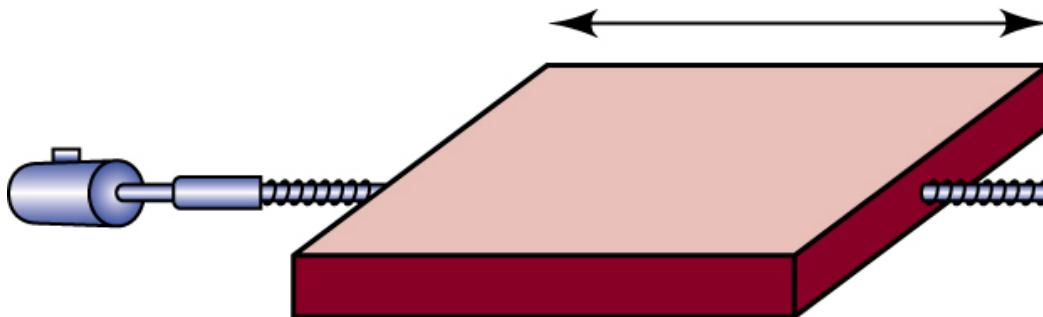


Binary to Gray

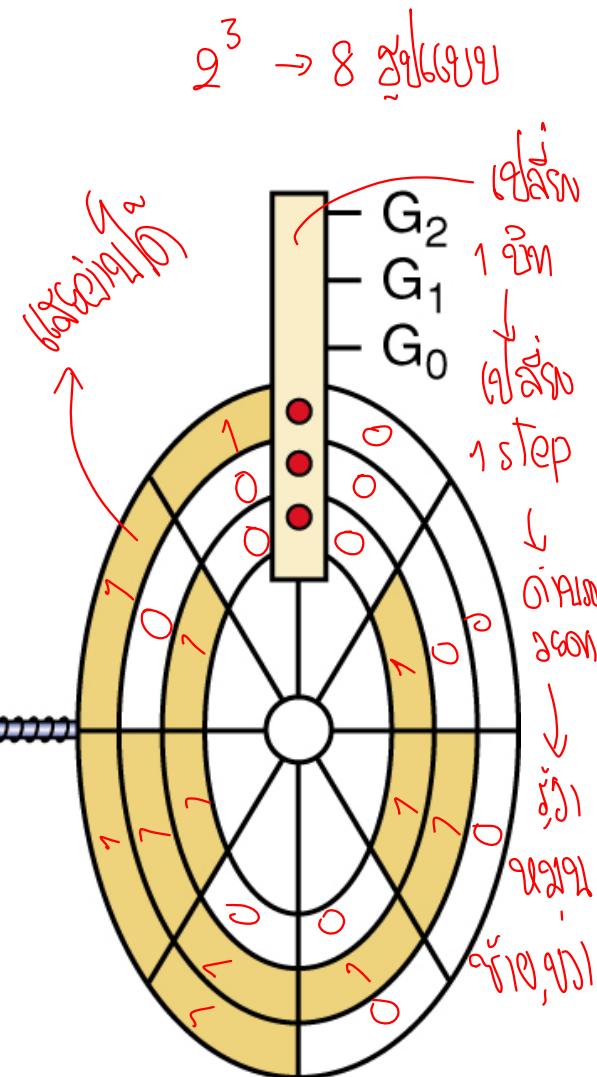


Gray to Binary

An eight-position, three-bit shaft encoder



These devices produce a binary value that represents the position of a rotating mechanical shaft.



2-6 Putting It All Together

0-7 ② ខ្លួន 8 លើ 3 ចិត្ត នៅ bit ខាង 3 bit សែត្រ (តិច)

Decimal numbers 1 – 15 in binary, hex, BCD, Gray

Decimal	Binary	Hexadecimal	BCD	GRAY
0	0	0	0000	0000
1	1	1	0001	0001
2	10	2	0010	0011
3	11	3	0011	0010
4	100	4	0100	0110
5	101	5	0101	0111
6	110	6	0110	0101
7	111	7	0111	0100
8	1000	8	1000	1100
9	1001	9	1001	1101
10	1010	A	0001 0000	1111
11	1011	B	0001 0001	1110
12	1100	C	0001 0010	1010
13	1101	D	0001 0011	1011
14	1110	E	0001 0100	1001
15	1111	F	0001 0101	1000

2-7 The Byte, Nibble, and Word

1 bit = 1 digit
8 bits = 1 byte

- Most microcomputers handle and store binary data and information in groups of eight bits.
 - 8 bits = 1 byte.
 - A byte can represent numerous types of data/information.
- Binary numbers are often broken into groups of four bits.
 - Because a group of four bits is half as big as a byte, it was named a **nibble**.
- A **word** is a group of bits that represents a certain unit of information.
 - **Word size** can be defined as the number of bits in the binary word a digital system operates on.
 - PC word size is eight bytes (64 bits).

2-8 Alphanumeric Codes

- Represents characters and functions found on a computer keyboard.
 - 26 lowercase & 26 uppercase letters, 10 digits, 7 punctuation marks, 20 to 40 other characters.
- ASCII – American Standard Code for Information Interchange.
 - Seven bit code: $2^7 = 128$ possible code groups
 - Examples of use: transfer information between computers; computers & printers; internal storage.

ເປົ້າສະບັບໂດຍອ່ານວ່າ ກົດ character ສຳເນົາ ໂດຍໄລຍະ

81 8 bit

)

ຊື່ Unicode ພົມ

ASCII – American Standard Code for Information Interchange

Character	HEX	Decimal	Character	HEX	Decimal	Character	HEX	Decimal	Character	HEX	Decimal
NUL (null)	0	0	Space	20	32	@	40	64	.	60	96
Start Heading	1	1	!	21	33	A	41	65	a	61	97
Start Text	2	2	"	22	34	B	42	66	b	62	98
End Text	3	3	#	23	35	C	43	67	c	63	99
End Transmit.	4	4	\$	24	36	D	44	68	d	64	100
Enquiry	5	5	%	25	37	E	45	69	e	65	101
Acknowledge	6	6	&	26	38	F	46	70	f	66	102
Bell	7	7	'	27	39	G	47	71	g	67	103
Backspace	8	8	(28	40	H	48	72	h	68	104
Horiz. Tab	9	9)	29	41	I	49	73	i	69	105
Line Feed	A	10	*	2A	42	J	4A	74	j	6A	106
Vert. Tab	B	11	+	2B	43	K	4B	75	k	6B	107
Form Feed	C	12	,	2C	44	L	4C	76	l	6C	108
Carriage Return	D	13	-	2D	45	M	4D	77	m	6D	109
Shift Out	E	14	.	2E	46	N	4E	78	n	6E	110

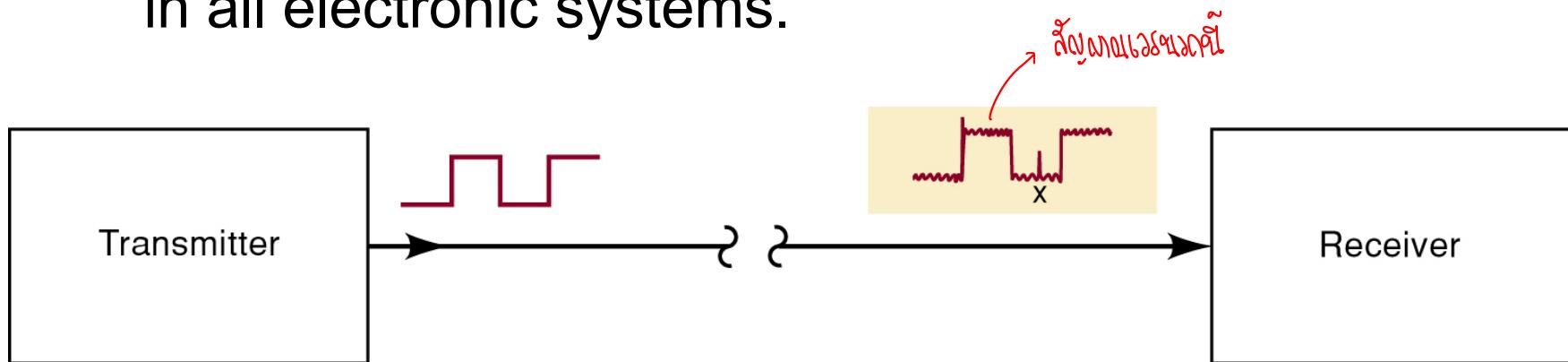
See the entire table on page 49 of your textbook.

2-9 Parity Method for Error Detection

- Binary data and codes are frequently moved between locations:
 - Digitized voice over a microwave link.
 - Storage/retrieval of data from magnetic/optical disks.
 - Communication between computer systems over telephone lines, using a modem.

2-9 Parity Method for Error Detection

- Electrical **noise** can cause errors during transmission.
 - Spurious fluctuations in voltage or current present in all electronic systems.



- Many digital systems employ methods for error detection—and sometimes correction.
 - One of the simplest and most widely used schemes for error detection is the **parity method**.

សមតាមរបៀប
បញ្ជាគុណការកំណត់កំណត់កំណត់

2-9 Parity Method for Error Detection

- The parity method of error detection requires the addition of an extra bit to a code group.
 - Called the parity bit, it can be either a 0 or 1, depending on the number of 1s in the code group.
- There are two parity methods, even and odd.
 - The transmitter and receiver must “agree” on the type of parity checking used.
 - Even seems to be used more often.

2-9 Parity Method for Error Detection

- Even parity method—the total number of bits in a group *including* the parity bit must add up to an even number.
 - The binary group **1 0 1 1** would require the addition of a parity bit **1**, making the group **1 1 0 1 1**.
 - The parity bit may be added at either end of a group.

↑
1 1 0 0 0 0 1 1
added parity bit*

↑ ស្មើរៀបចំព័ត៌មាន

1 0 0 0 0 1 0 } នៅក្នុង parity នេះ

មិនត្រូវការផ្តល់ទូទាត់ 1 bit ឡើង ឬមិន

parity ត្រូវ add នៅ ឬ (ជីវិត្រូវ 1)

នៅលើក្រុងការផ្តល់ទូទាត់

កំណត់ទូទាត់មិនអាច

2-9 Parity Method for Error Detection

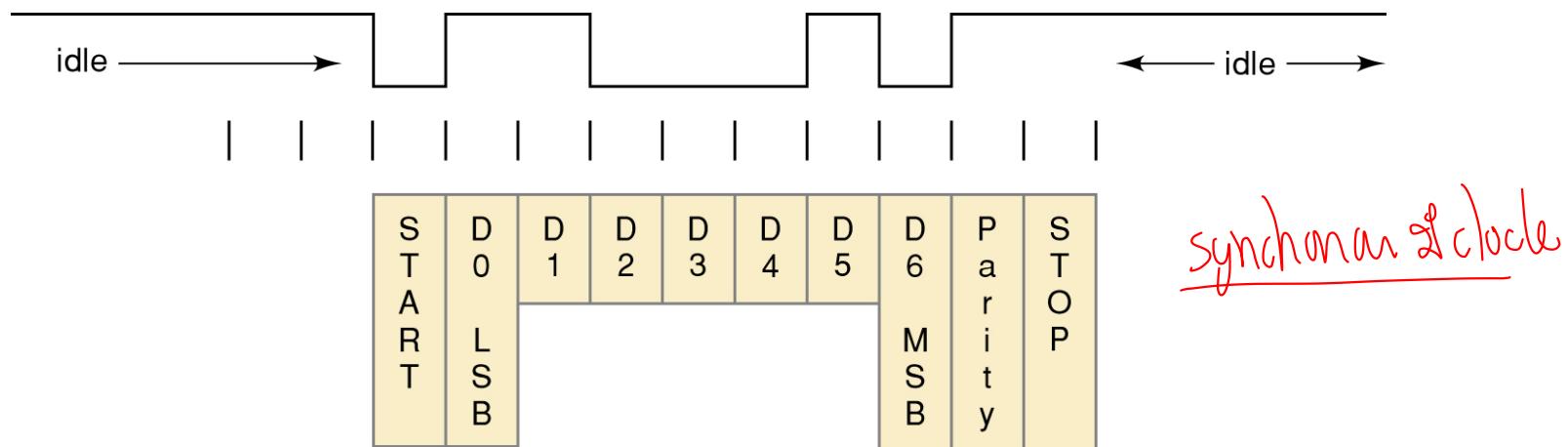
- Odd parity method—the total number of bits in a group *including* the parity bit must add up to an odd number.
 - The binary group **1 1 1 1** would require the addition of a parity bit **1**, making the group **1 1 1 1 1**.

The parity bit becomes a part of the code word.

**Adding a parity bit to the seven-bit ASCII
code produces an eight-bit code.**

- When ASCII characters are transmitted there must be a way to tell the receiver a new character is coming.
 - There is often a need to detect errors in the transmission as well.
- The method of transfer is called asynchronous data communication.

- An ASCII character must be “framed” so the receiver knows where the data begins and ends.
 - The first bit must always be a start bit (logic 0).
- ASCII code is sent LSB first and MSB last.
 - After the MSB, a parity bit is appended to check for transmission errors.
 - Transmission is ended by sending a stop bit (logic 1).



END

ELEVENTH EDITION

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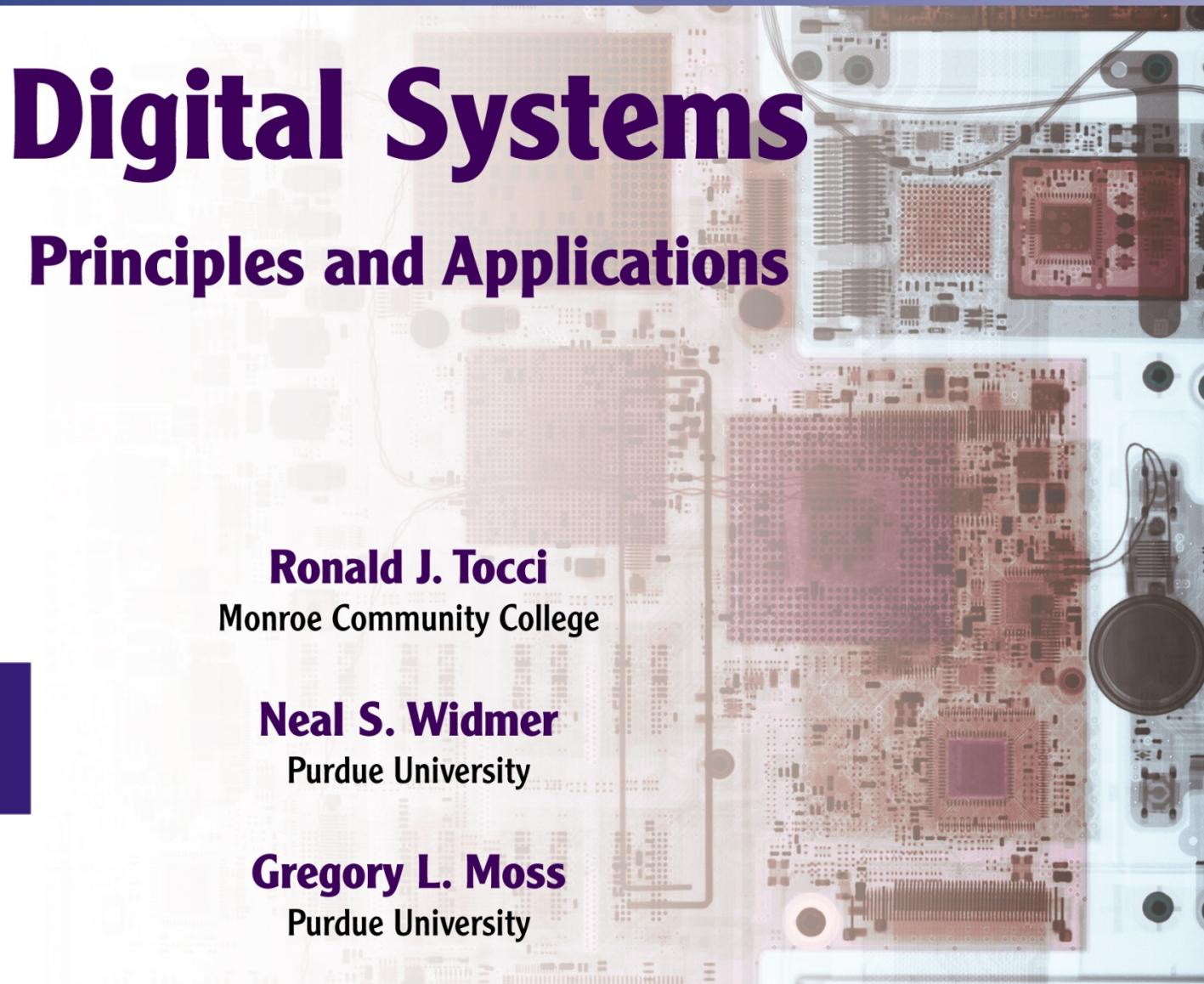
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$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$2^{10} = 1024$$

(key point) $2^{10} = 1024$

$$2^{10} = 1024 = 1K$$

$$2^{20} = 1024 \times 1024 = 1M$$

$$2^{30} = 1024 \times 1024 \times 1024 = 1G$$

$$2^{40} = 1024 \times 1024 \times 1024 \times 1024 = 1T$$

$$\text{Ex: } 2^{35} = 2^{30} \times 2^5 = 32 \times G \\ = 32G$$

$$2^{41} = 2 \times 2^{40} = 2T$$

giga

$$2^{27} = 2^7 \times 2^{20} = 128M$$

$$2^{49} = 2^9 \times 2^{40} = 4T$$