



# Linear Regression

*การคำนวณแบบ predictive*

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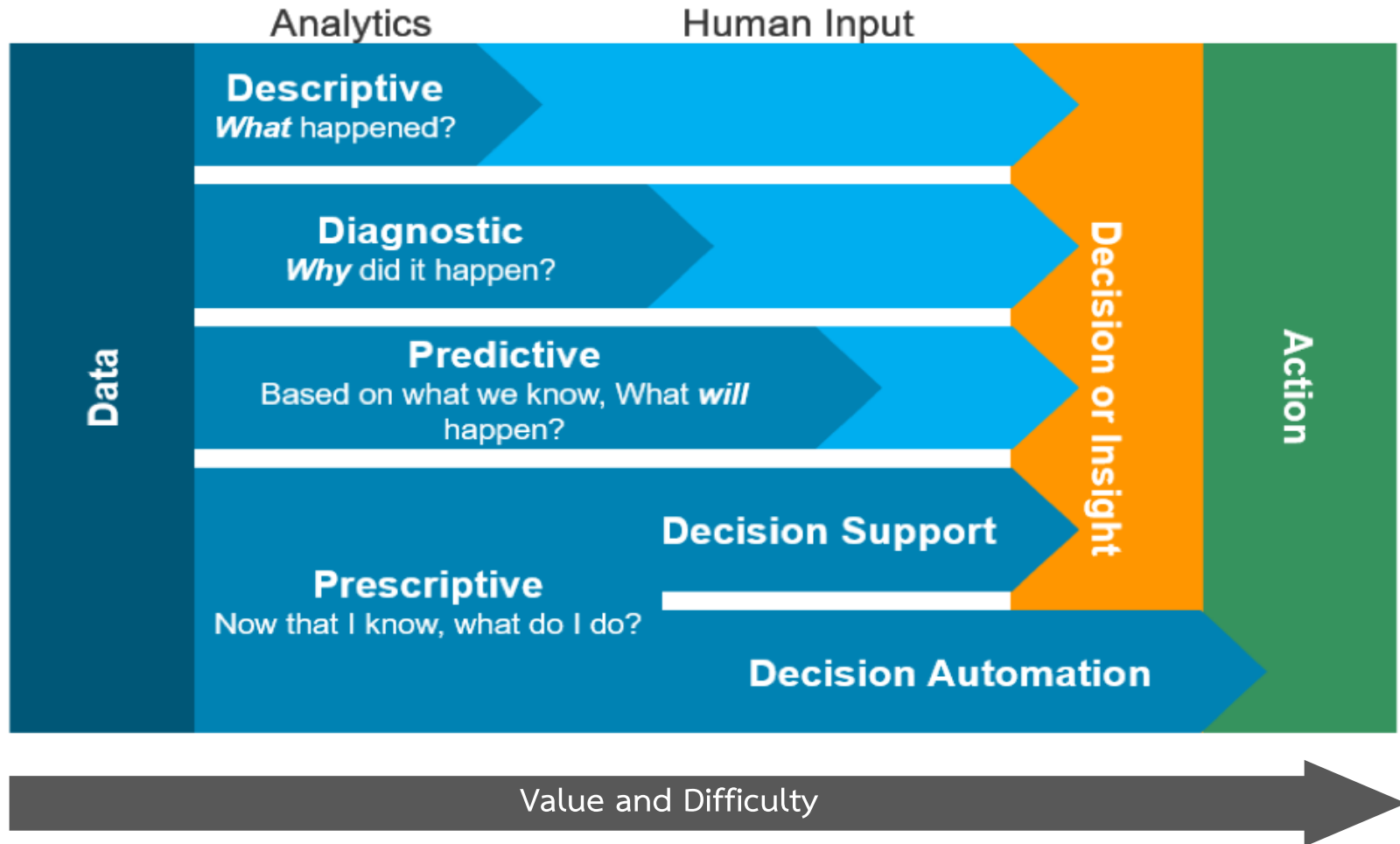
Faculty of Engineering

King Mongkut's Institute of Technology Ladkrabang

# Agenda

- Linear Regression
- Evaluation Methods
- Feature Extraction
- Cross Validation
- Feature Scaling

# Data Analytics



# Machine Learning

## Supervised Learning

Develop predictive model based on both input and output data

### Regression

- Linear Regression
- Polynomial Regression

### Classification

- Decision Tree
- Logistic Regression
- Neural Network
- etc.

## Unsupervised Learning

Develop predictive model based on both input and output data

### Clustering

- K-Means
- DB-SCAN
- etc.

# Linear Regression



# Your Salary?

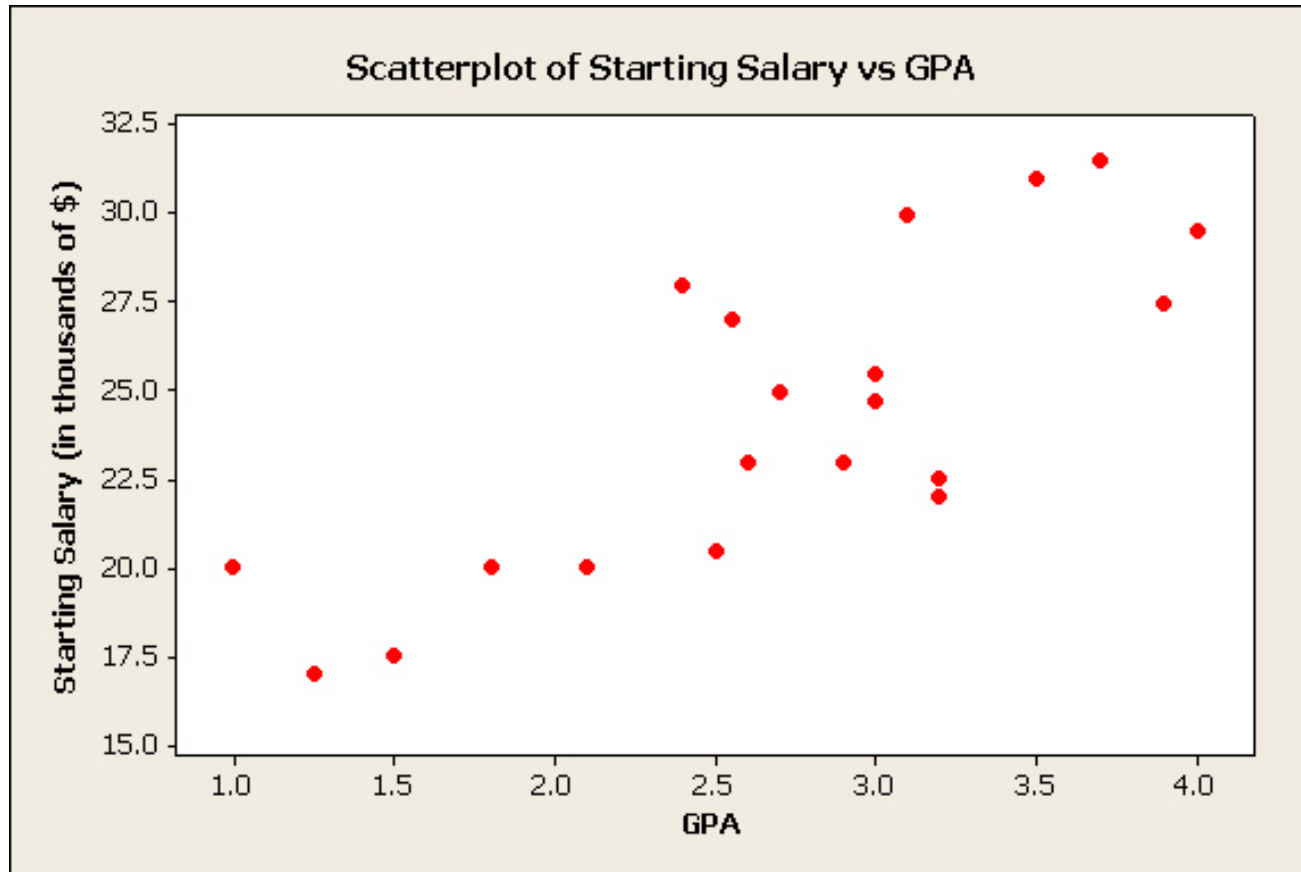
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GPA	Salary
1.07	15,400
2.32	20,500
2.75	23,300
3.02	28,200
3.25	27,900
3.63	32,100
3.98	30,200

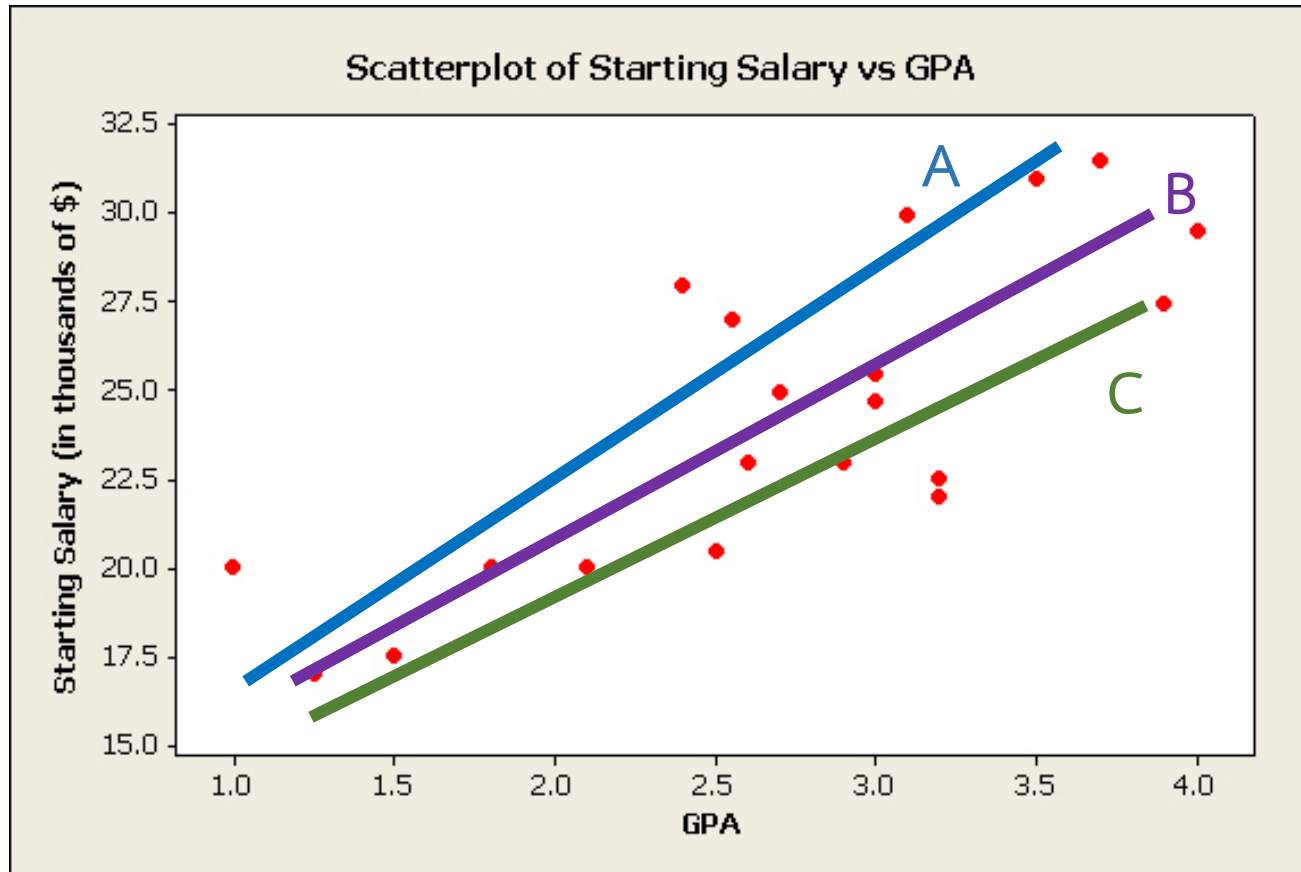
(assumed data)

# Prediction !

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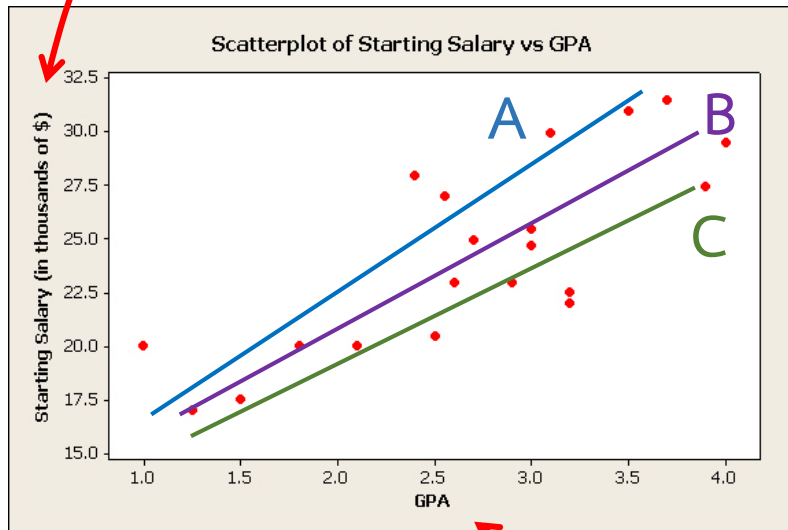


# Prediction !





# Linear Equation



เขียนเป็น  
สมการเส้นตรง

$$y = ax + c$$

?

?

# Simple Linear Regression

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$$\hat{Y}_i = b_0 + b_1 X_i$$

# Simple Linear Regression

---

$$\hat{Y}_i = b_0 + b_1 X_i$$

Salary
15,400
20,500
23,300
26,200
27,900
32,100
30,200

GPA
1.07
2.32
2.75
3.02
3.25
3.63
3.98

# Your Salary?

---

GPA	U. Ranking	Salary
1.07	6	15,400
2.32	7	20,500
2.75	8	23,300
3.02	9	28,200
3.25	7	27,900
3.63	10	32,100
3.98	5	30,200

(assumed data)

# Multiple Linear Regression

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$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \dots$$

# Multiple Linear Regression

---

$$\hat{Y}_i = b_0 + b_1X_{1i} + b_2X_{2i} + \dots$$

Salary
15,400
20,500
23,300
28,200
27,900
32,100
30,200

GPA
1.07
2.32
2.75
3.02
3.25
3.63
3.98

U. Ranking
6
7
8
9
7
10
5

# Coefficients

---

$$\hat{Y}_i = 4006 + 5655 * GPA + 698 * URank$$

# Python

---

1. `from sklearn import linear_model`
2. `X = df[“GPA”, “Urank”]`
3. `y = df[“Salary”]`
4. `lm = linear_model.LinearRegression()`
5. `lm.fit (X,y)`
6. `lm.coef_`  
`array([ 406, 565, 698])`

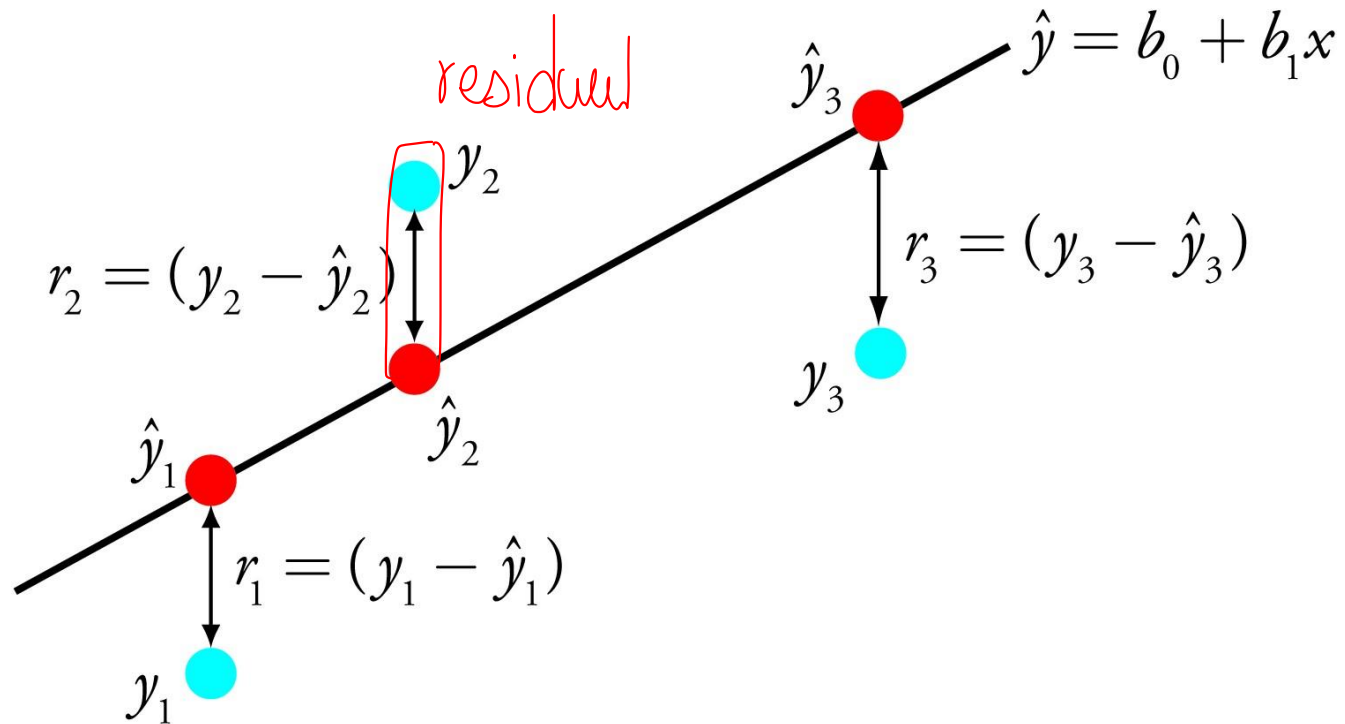


monitoring and evaluation?

# Evaluation Methods



# Residual



# Root Mean Squared Error (RMSE)

---

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (\hat{y}_i - y_i)^2}{N}}$$

Diagram illustrating the components of the RMSE formula:

- $\hat{y}_i$ : predicted
- $y_i$ : actual
- $N$ : number of samples

# RMSE

$$\hat{Y}_i = ? + ?? * GPA + ??? * URank$$

GPA	U. Ranking	Actual Salary
1.07	6	15,400
2.32	7	20,500
2.75	8	23,300
3.02	9	28,200
3.25	7	27,900
3.63	10	32,100
3.98	5	30,200

# RMSE

---

$$\hat{Y}_i = 4006 + 5655 * GPA + 698 * URank$$

GPA	U. Ranking	Actual Salary
1.07	6	15,400
2.32	7	20,500
2.75	8	23,300
3.02	9	28,200
3.25	7	27,900
3.63	10	32,100
3.98	5	30,200

# RMSE

---

$$\hat{Y}_i = 4006 + 5655 * GPA + 698 * URank$$

GPA	U. Ranking	Actual Salary	Predicted Salary
1.07	6	15,400	16,350
2.32	7	20,500	23,100
2.75	8	23,300	25,750
3.02	9	28,200	27,600
3.25	7	27,900	27,750
3.63	10	32,100	31,150
3.98	5	30,200	30,400

# RMSE

$$\hat{Y}_i = 4006 + 5655 * GPA + 698 * URank$$

GPA	U. Ranking	Actual Salary	Predicted Salary	$\hat{y}_i - y_i$	$(\hat{y}_i - y_i)^2$
1.07	6	15,400	16,350	1,155.15	1,334,371.52
2.32	7	20,500	23,100	-1,511.60	2,284,934.56
2.75	8	23,300	25,750	-1,841.25	3,390,201.56
3.02	9	28,200	27,600	833.90	695,389.21
3.25	7	27,900	27,750	629.25	395,955.56
3.63	10	32,100	31,150	586.35	343,806.32
3.98	5	30,200	30,400	197.10	38,848.41
				SUM	8,483,507.15
				MEAN	1,211,929.59
				RMSE	1,100.88

# Other Evaluation Methods

ตัววัดอื่น ๆ

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Mean squared error

$$\text{MSE} = \frac{1}{n} \sum_{t=1}^n e_t^2$$

---

Root mean squared error

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^n e_t^2}$$

---

Mean absolute error

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^n |e_t|$$

---

Mean absolute percentage error

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^n \left| \frac{e_t}{y_t} \right|$$

---



# Python

---

- `from sklearn import linear_model`
- `from sklearn.metrics import mean_squared_error`
- `from math import sqrt`
- `X = df[“GPA”, “Urank”]`
- `y = df[“Salary”]`
- `lm = linear_model.LinearRegression()`
- `lm.fit (X,y)`
- `y_predicted = lm.predict(X)`
- `rmse = sqrt(mean_squared_error(y, y_predicted))`

# Learning



# Solve by Matrix

SOLVE

$$\begin{cases} 3x + 5y = -1 \\ x - 2y = 4 \end{cases} \quad A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$
$$\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$
$$A^{-1} = \frac{1}{3(-2) - 1(5)} \begin{pmatrix} -2 & -5 \\ -1 & 3 \end{pmatrix}$$

# Solve by .....



SOLVE  $\begin{cases} 3x + 5y = -1 \\ x - 2y = 4 \end{cases} \quad A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$
$$A^{-1} = \frac{1}{3(-2) - 1(5)} \begin{pmatrix} -2 & -5 \\ -1 & 3 \end{pmatrix}$$



New algorithm ( $n \geq 1$ ):

Repeat {

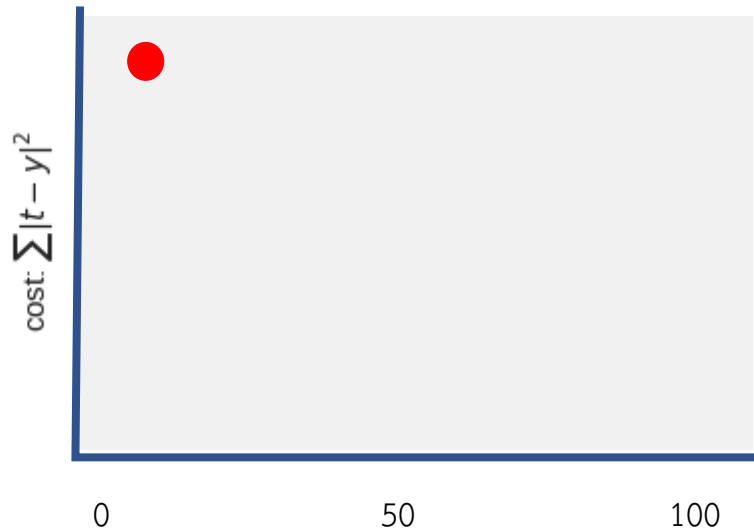
$$\downarrow \frac{\partial}{\partial \theta_j} J(\theta)$$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

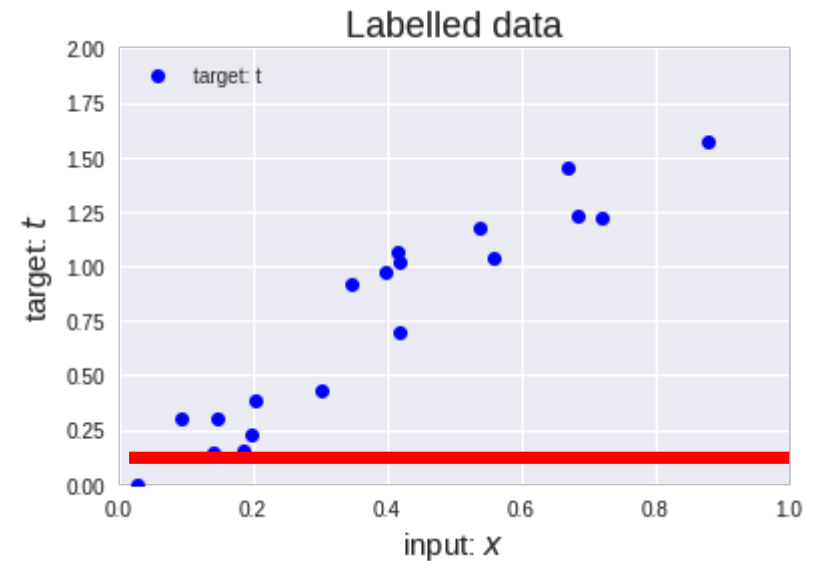
(simultaneously update  $\theta_j$  for  $j = 0, \dots, n$ ) }

# Learning based on Gradient Descent

$$y = \textcolor{red}{a}x + c$$

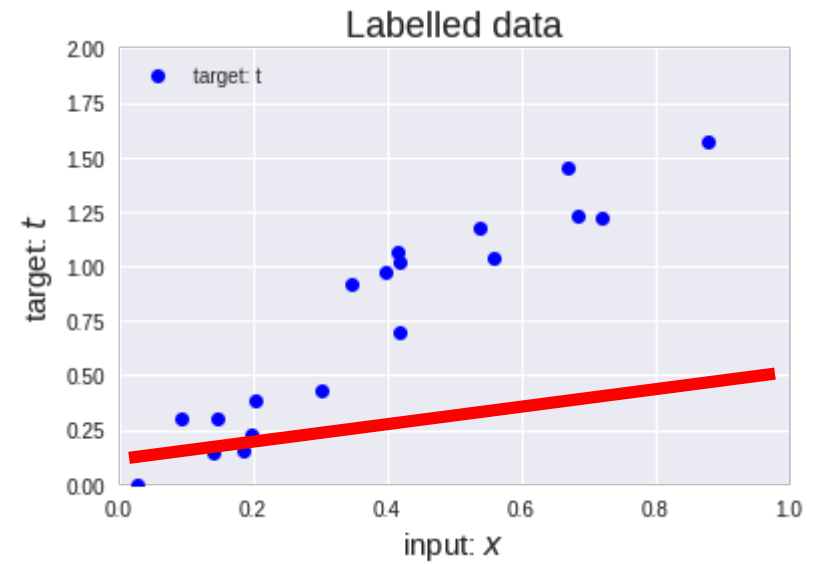
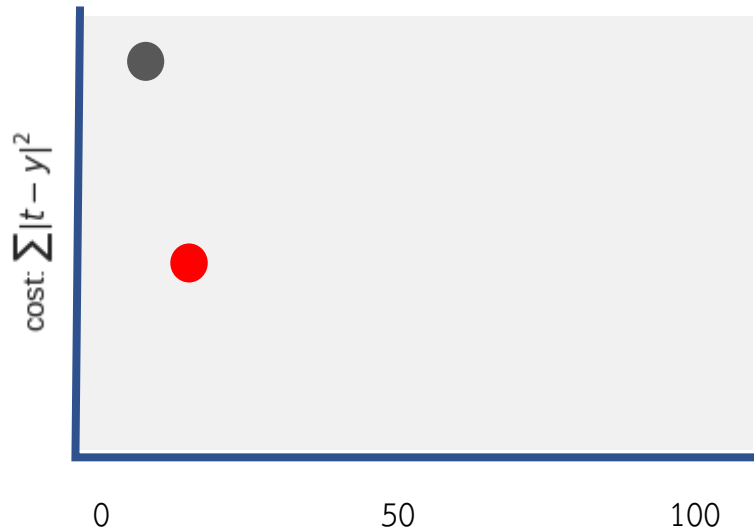


$\textcolor{red}{a}$

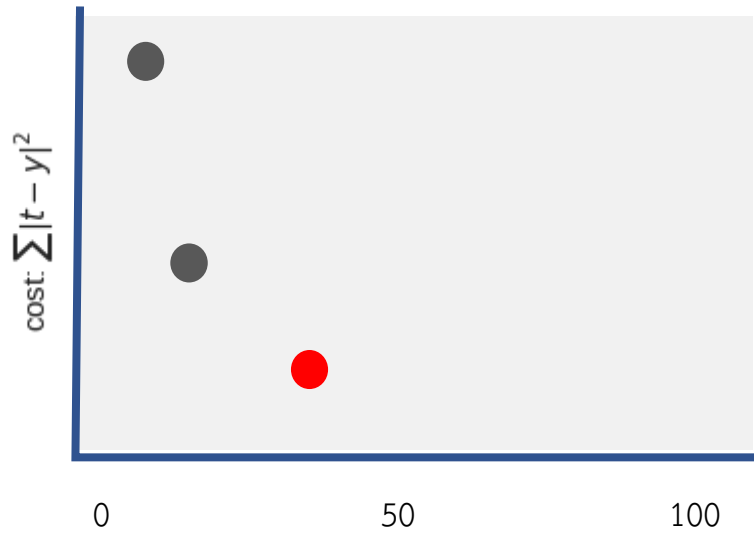


# Learning

$$y = \textcolor{red}{a}x + c$$

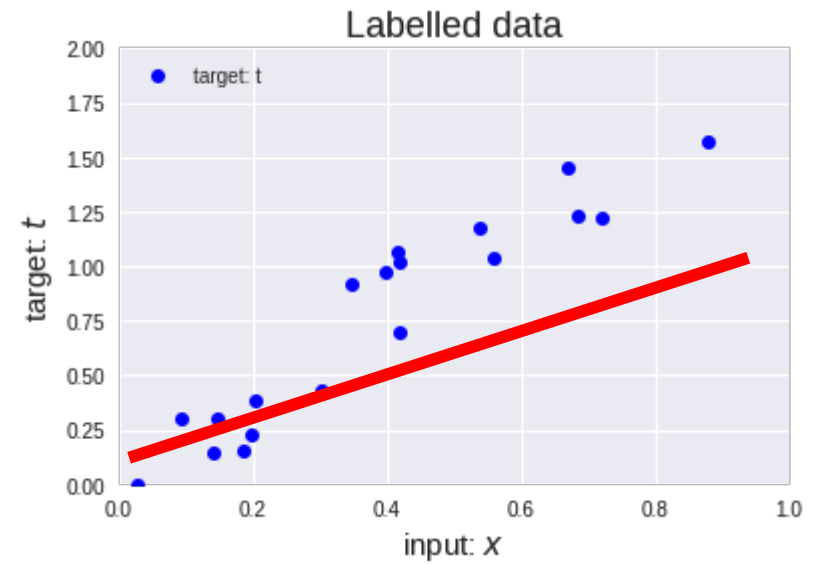


# Learning

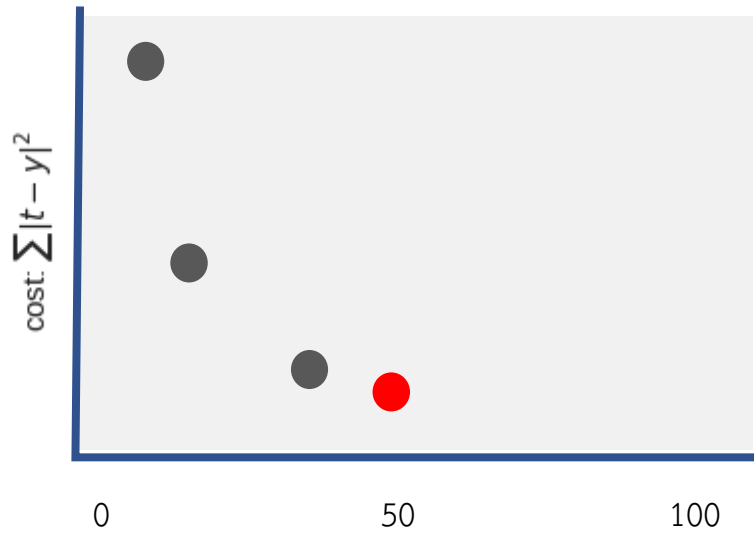


$a$

$$y = \textcolor{red}{a}x + c$$

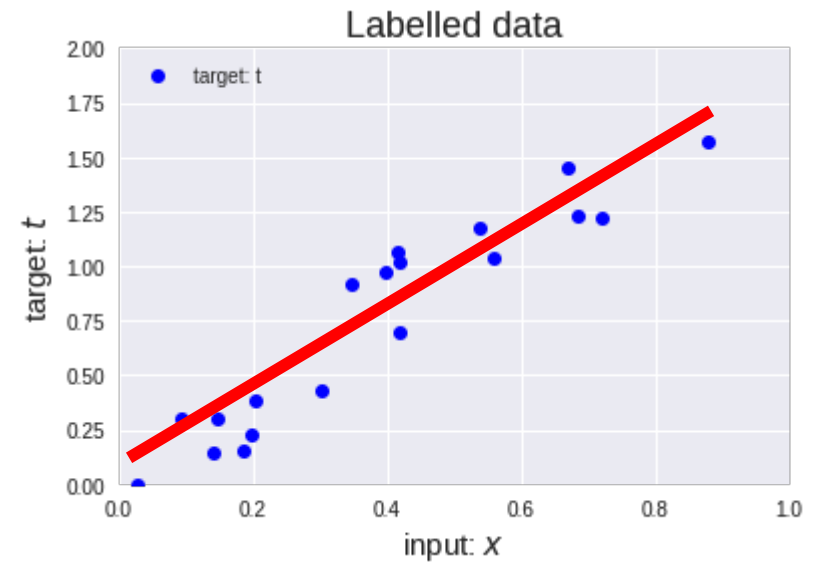


# Learning



$a$

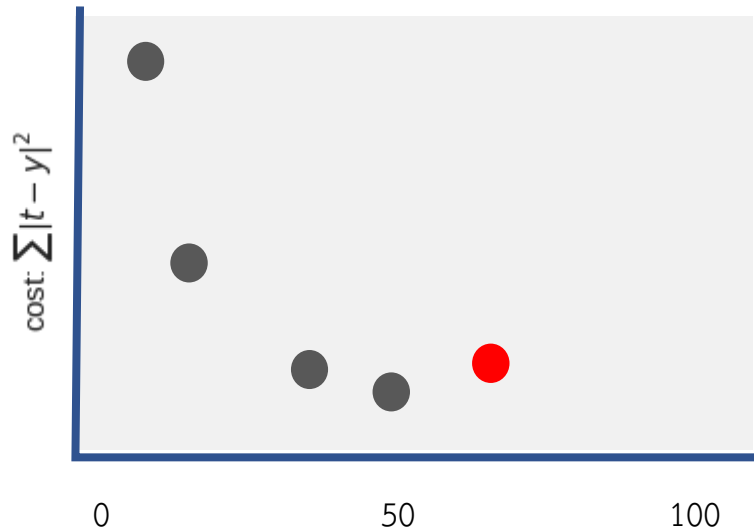
$$y = \textcolor{red}{a}x + c$$



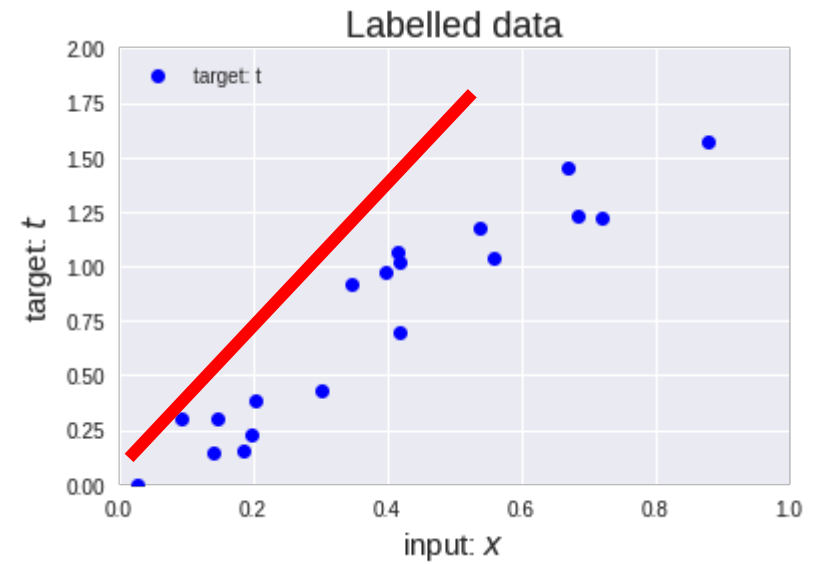


# Learning

$$y = \textcolor{red}{a}x + c$$

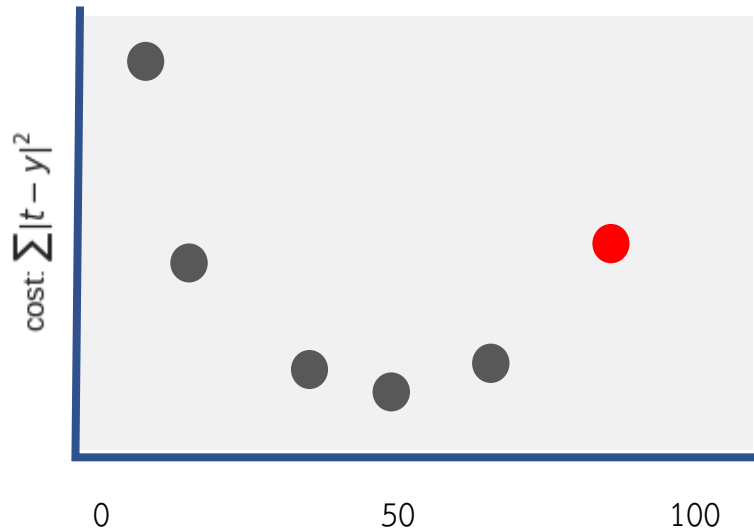


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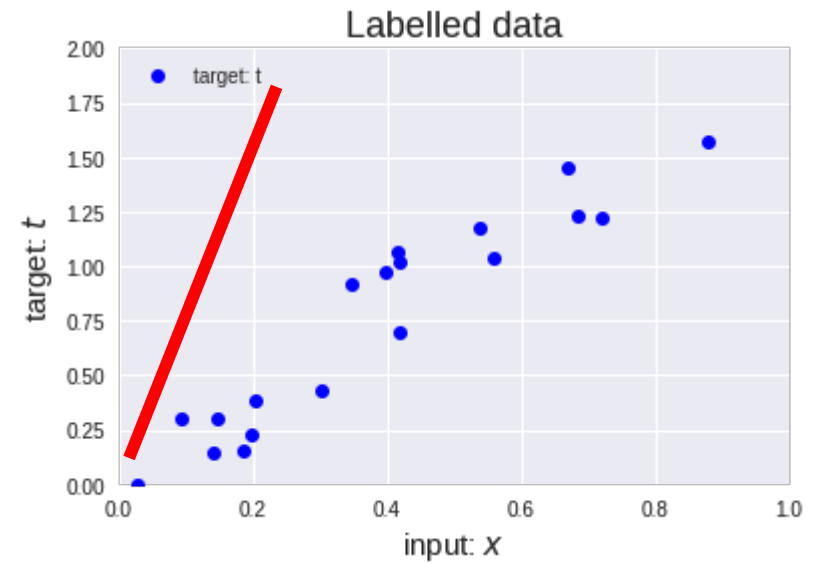


# Learning

$$y = \textcolor{red}{a}x + c$$

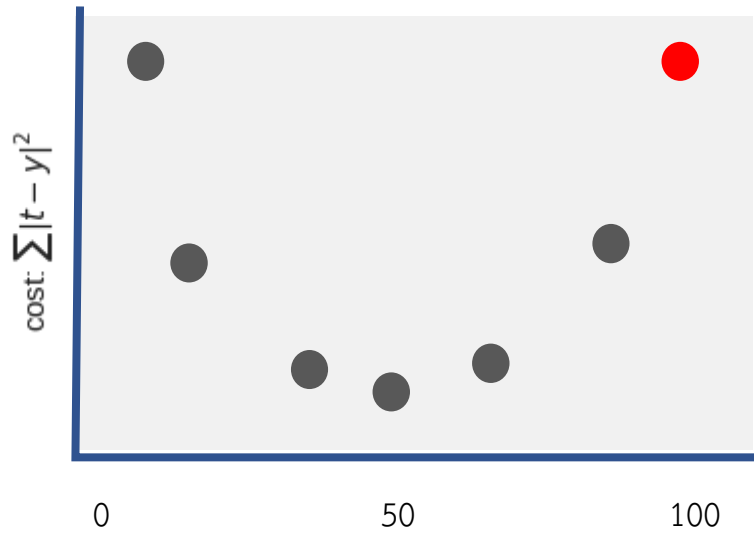


$a$

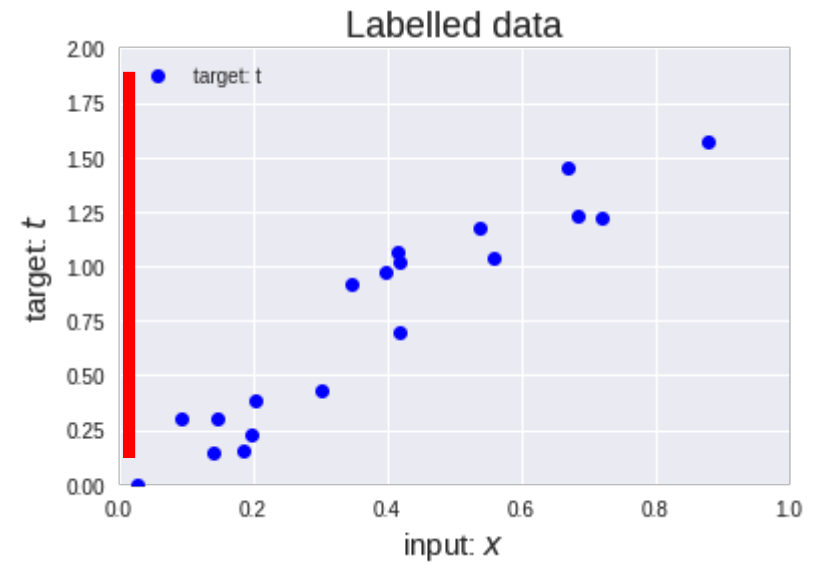


# Learning

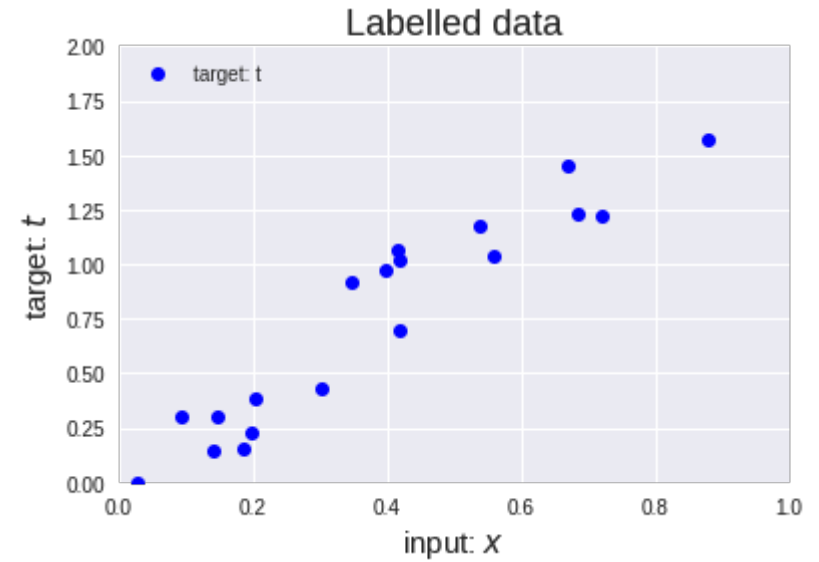
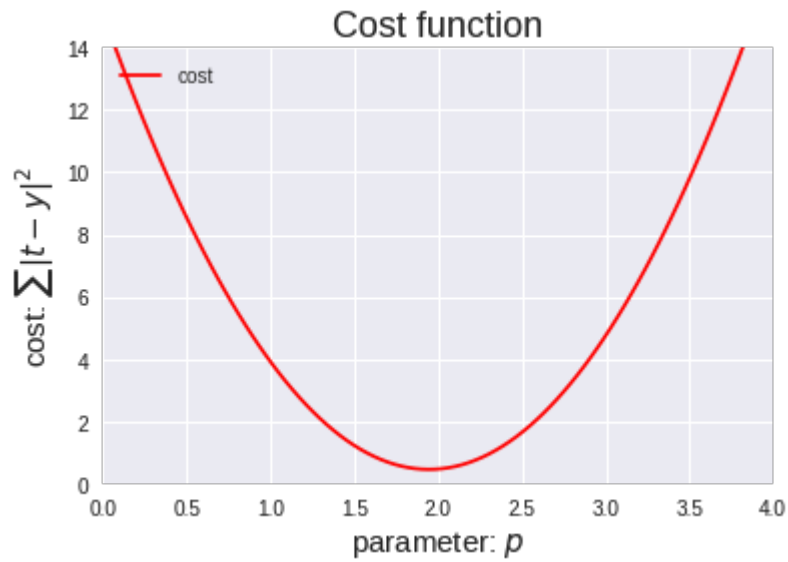
$$y = \textcolor{red}{a}x + c$$



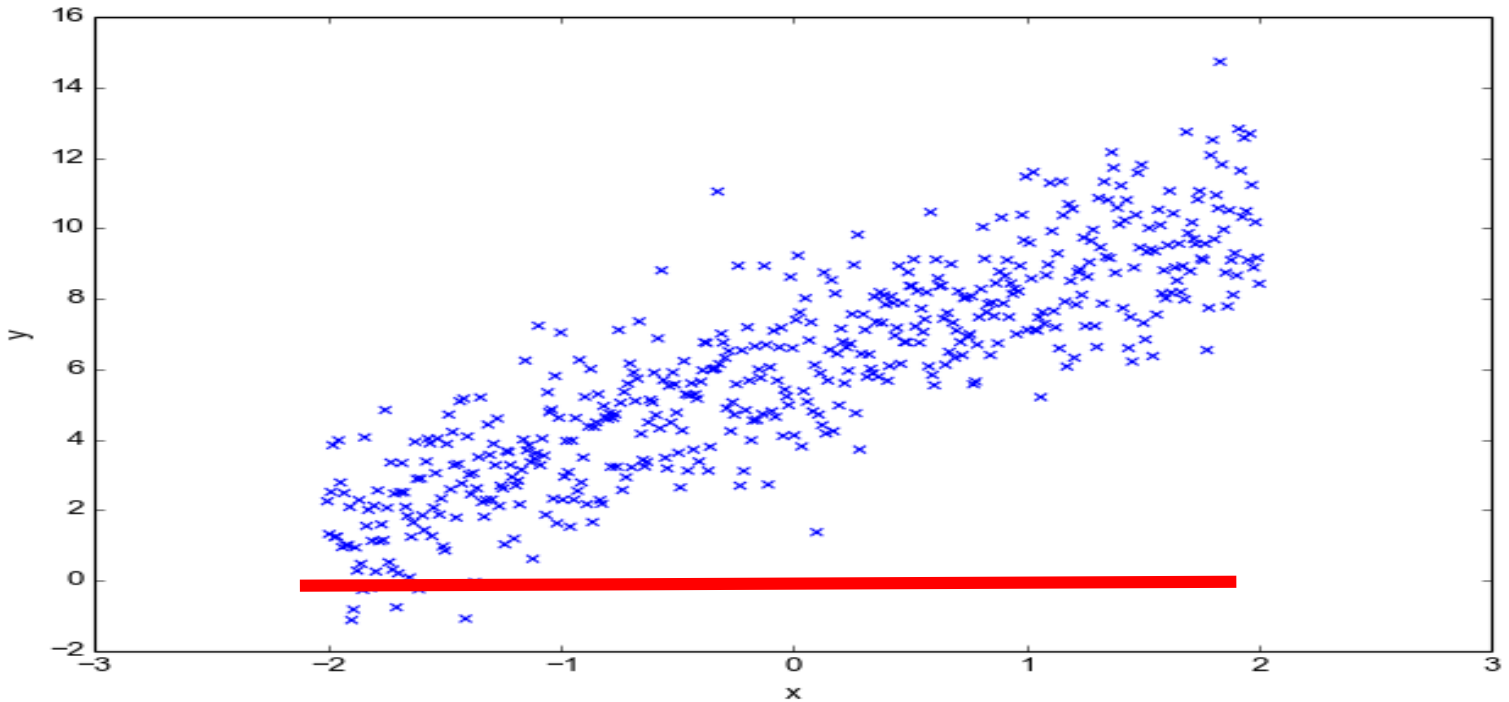
$a$



# Learning



# Learning

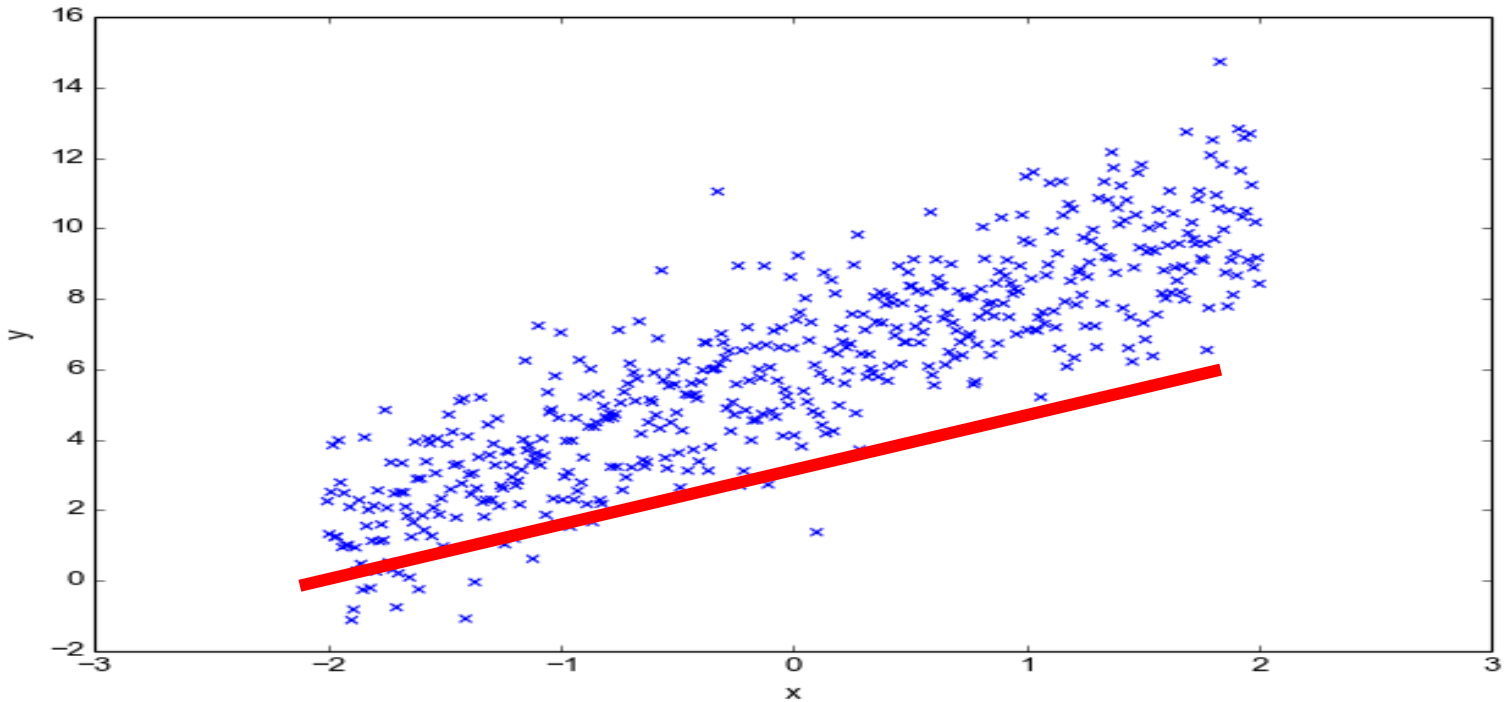


cost:  $\sum |t - y|^2$

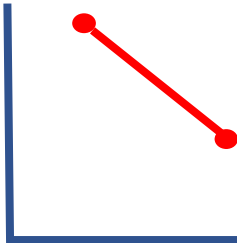


*n*-th round

# Learning

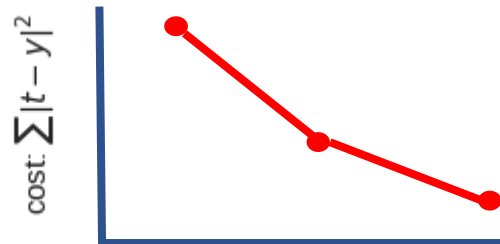
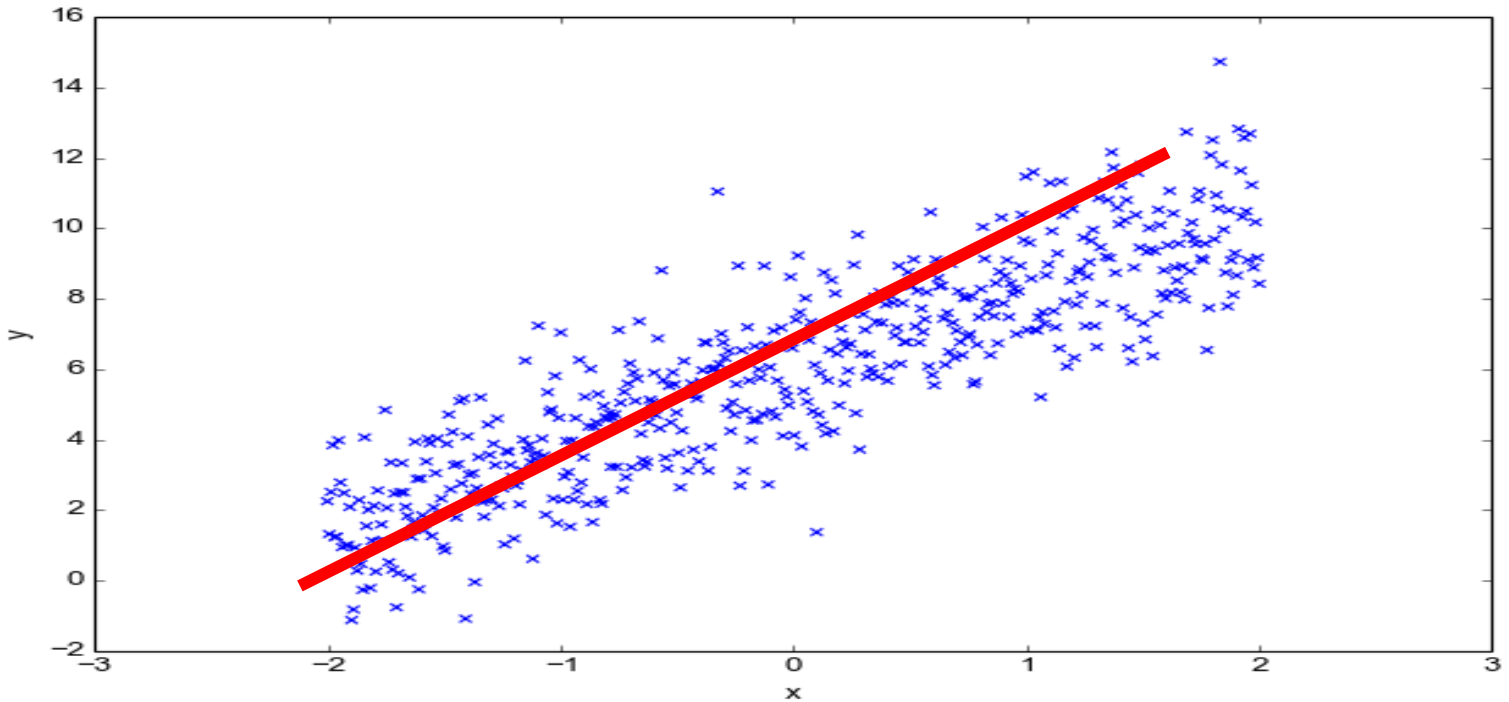


cost:  $\sum |t - y|^2$



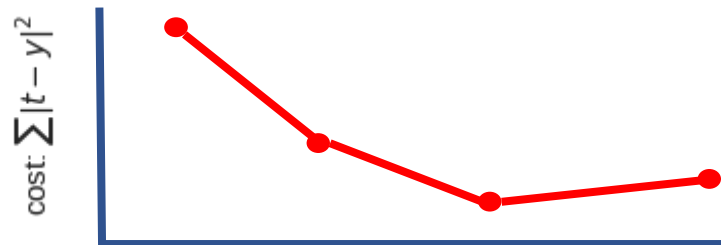
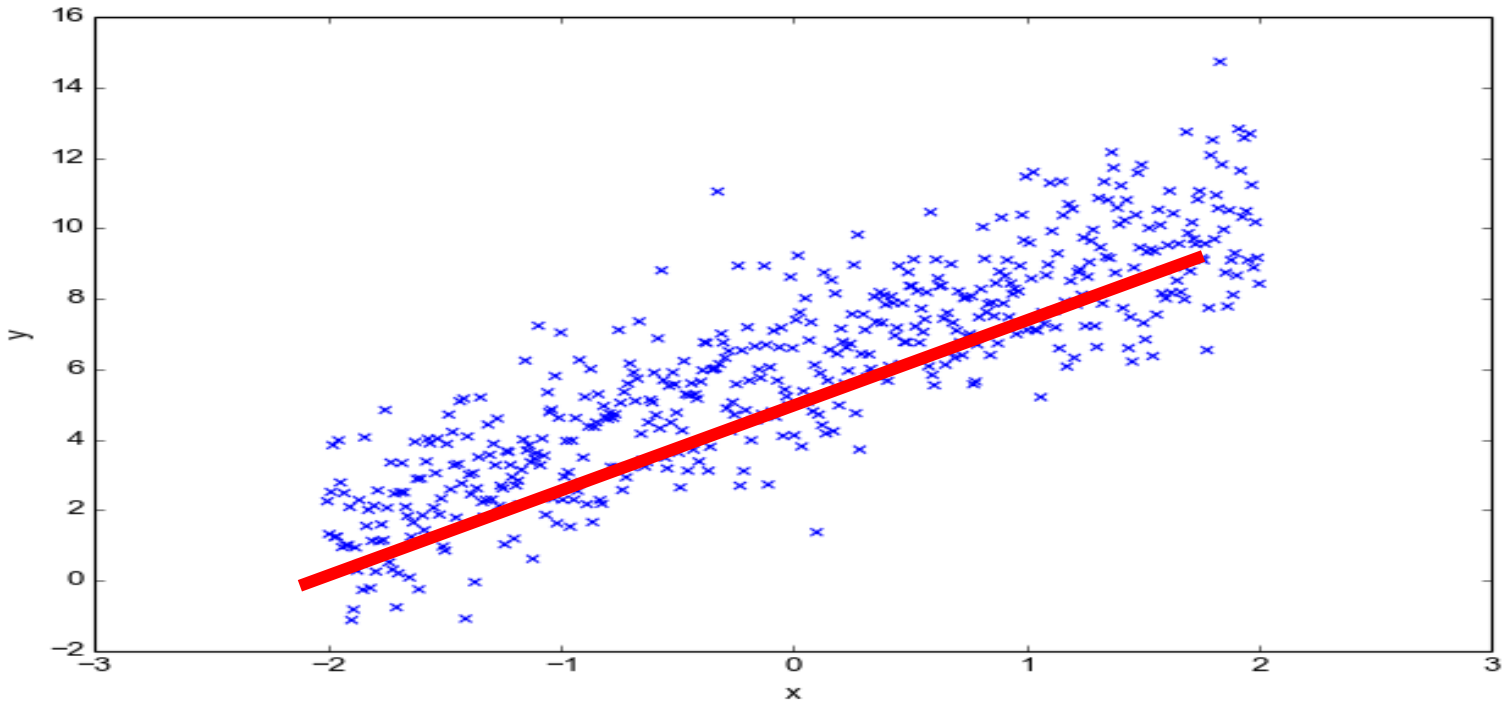
*n*-th round

# Learning



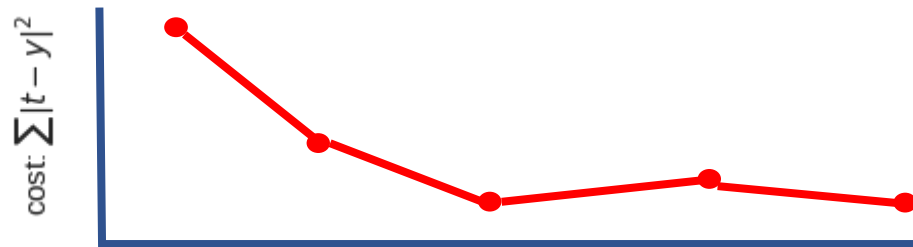
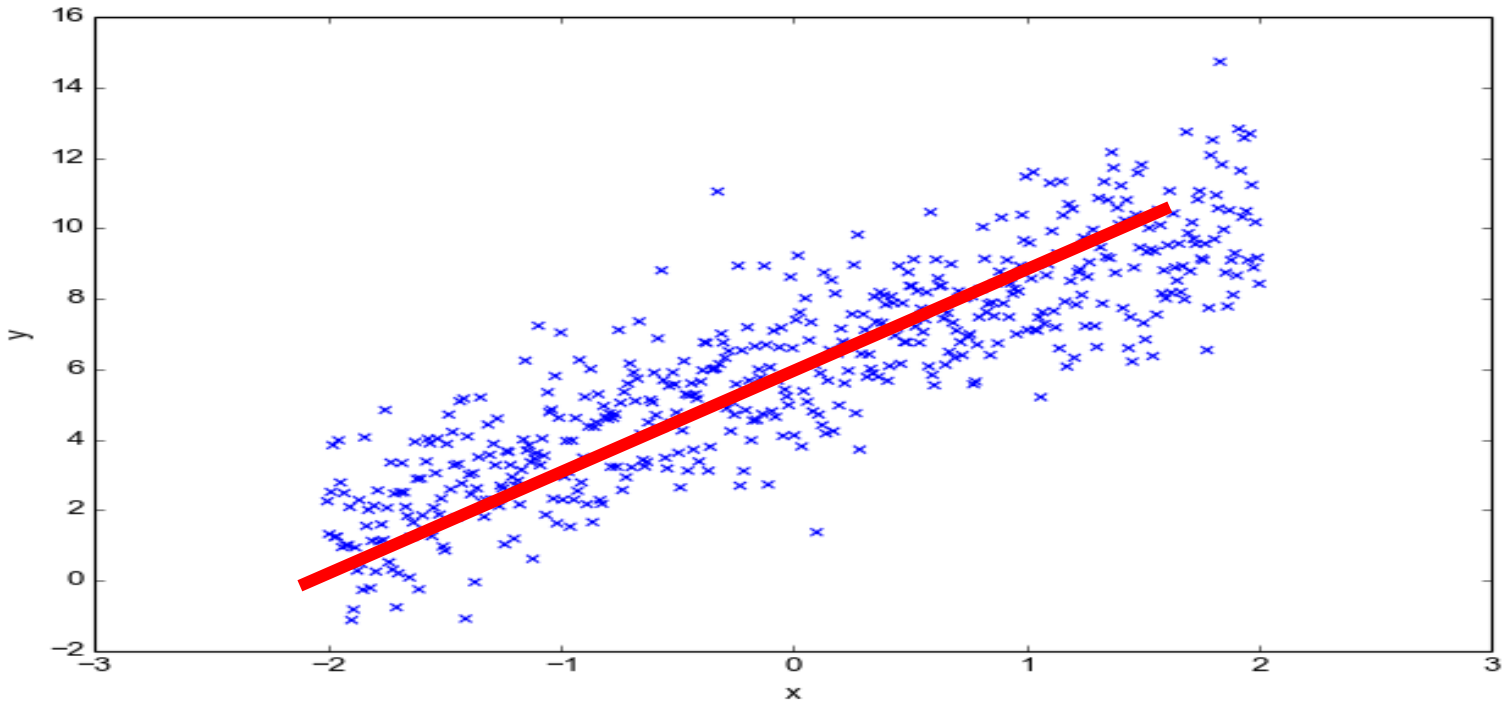
*n*-th round

# Learning



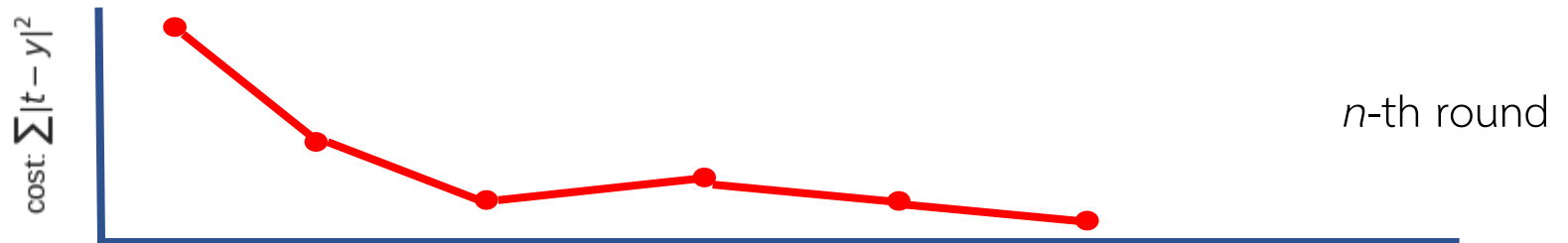
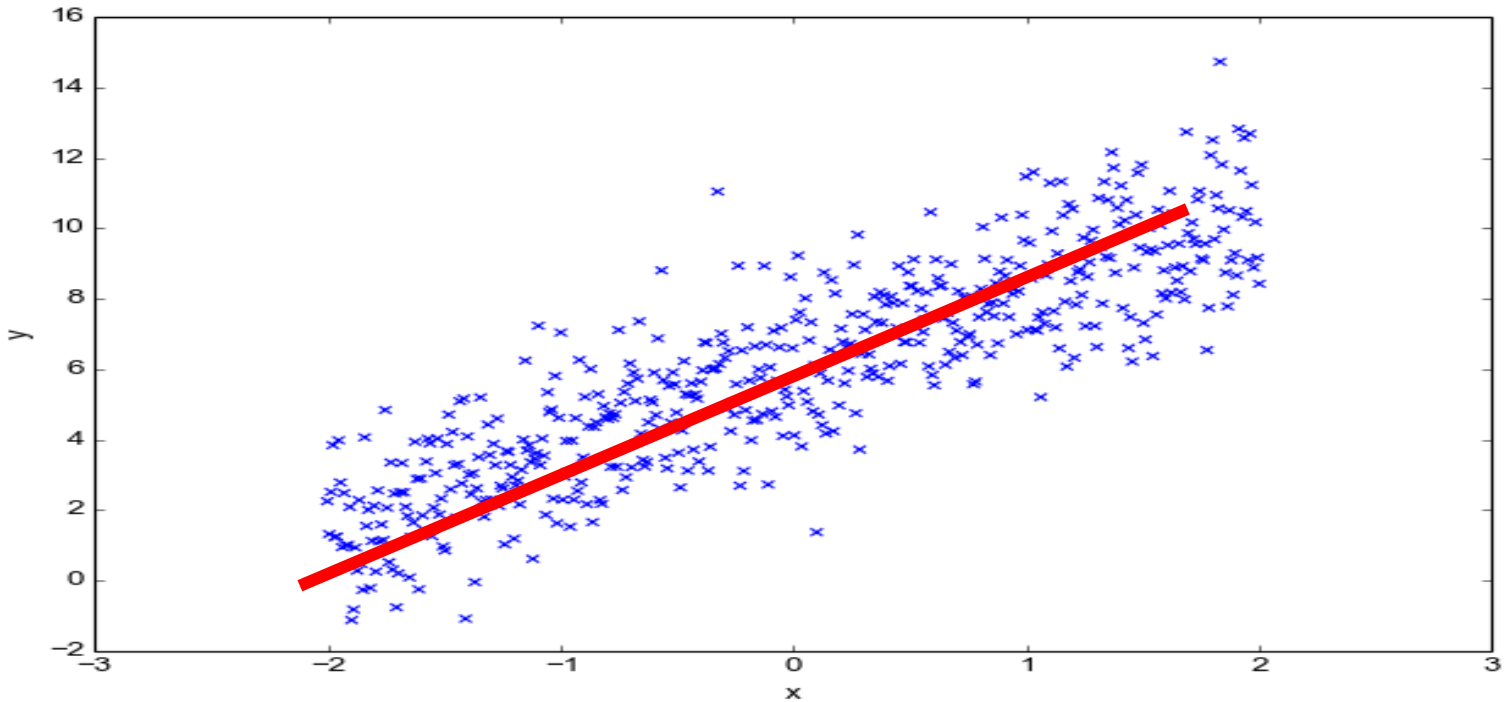


# Learning

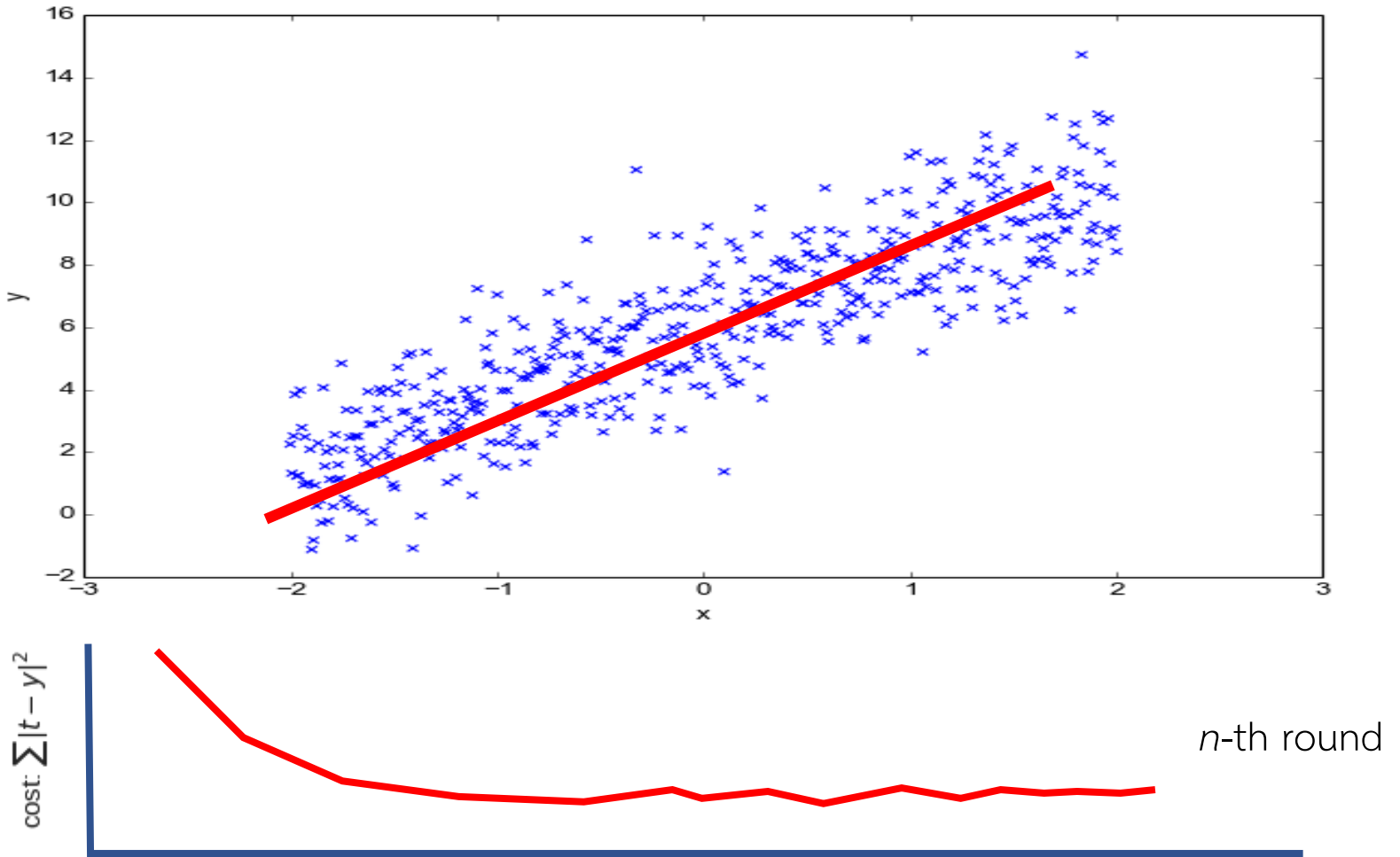


$n$ -th round

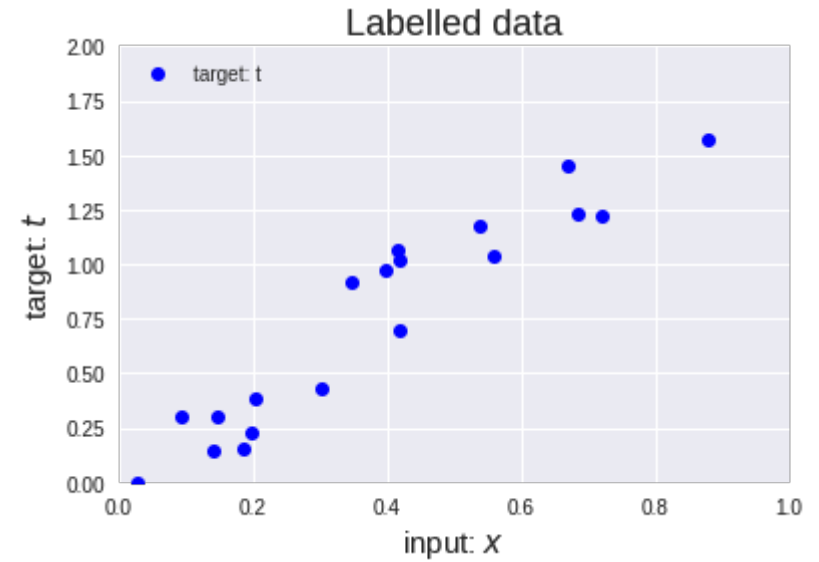
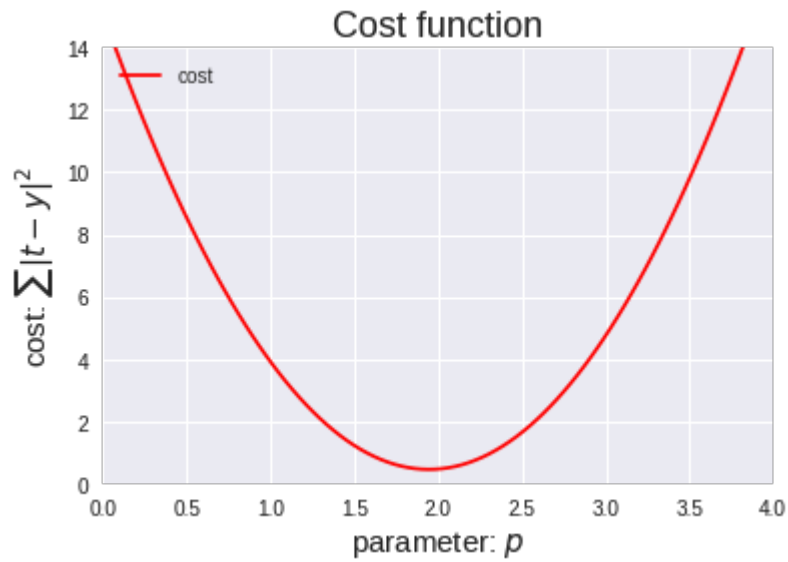
# Learning



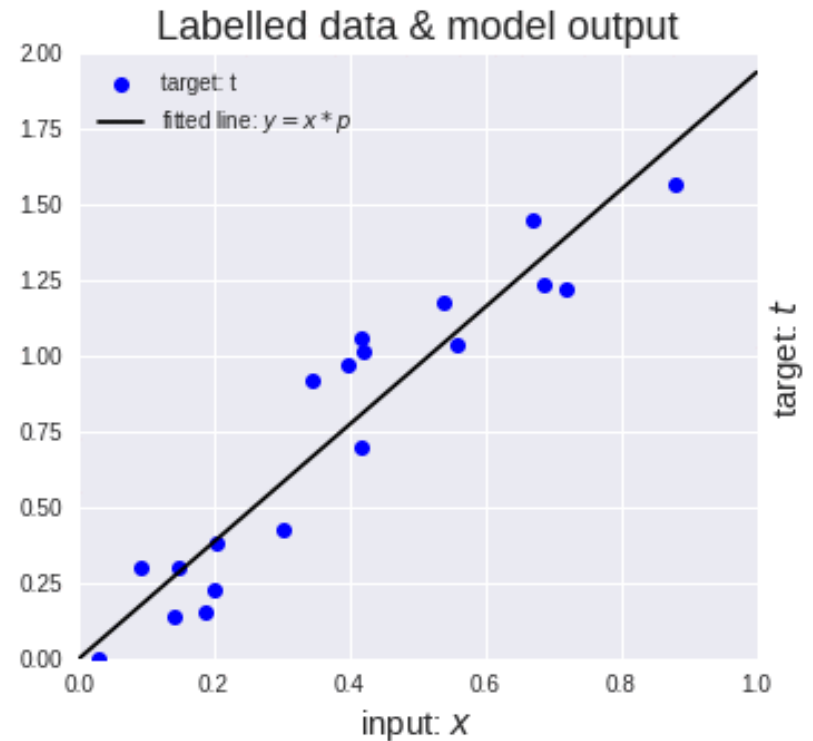
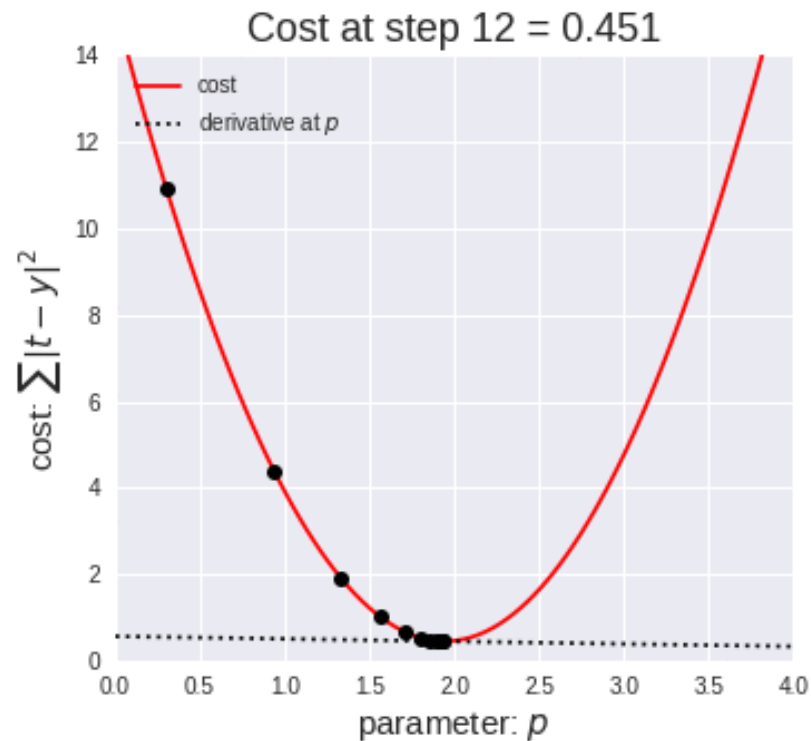
# Learning



# Learning



# Learning



# Gradient Descent for Linear Regression

(minimization: subtract gradient term because we move towards local minima)

$$\mathbf{b} = \mathbf{a} - \gamma \nabla f(\mathbf{a})$$

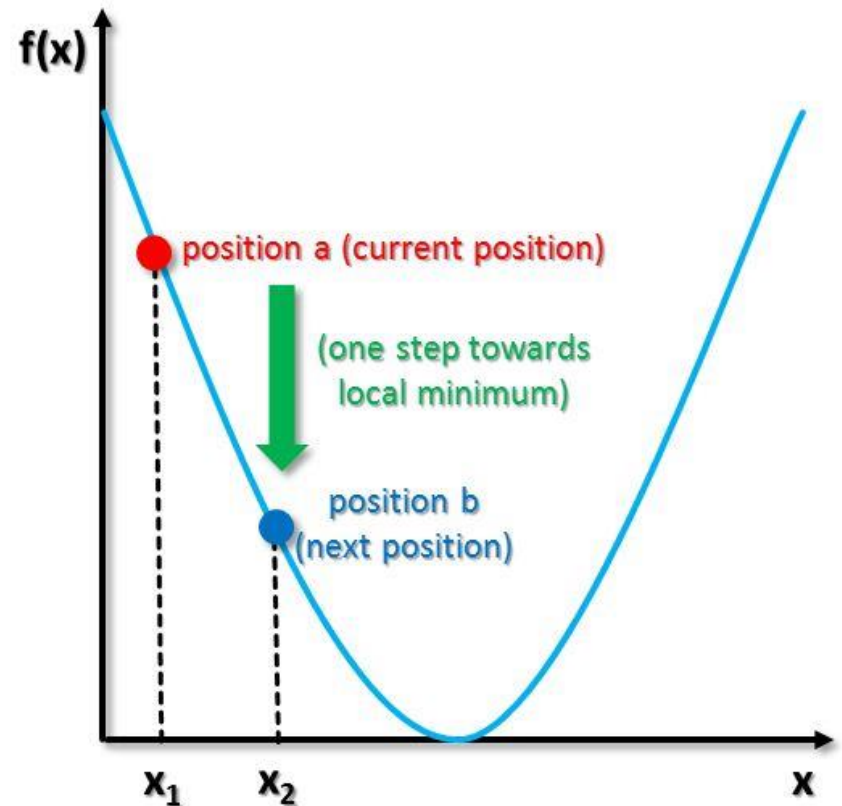
(new position after the step)

(old position before the step)

(weighting factor known as step-size, can change at every iteration, also called learning rate)

(the derivative of  $f$  with respect to  $\mathbf{a}$ )

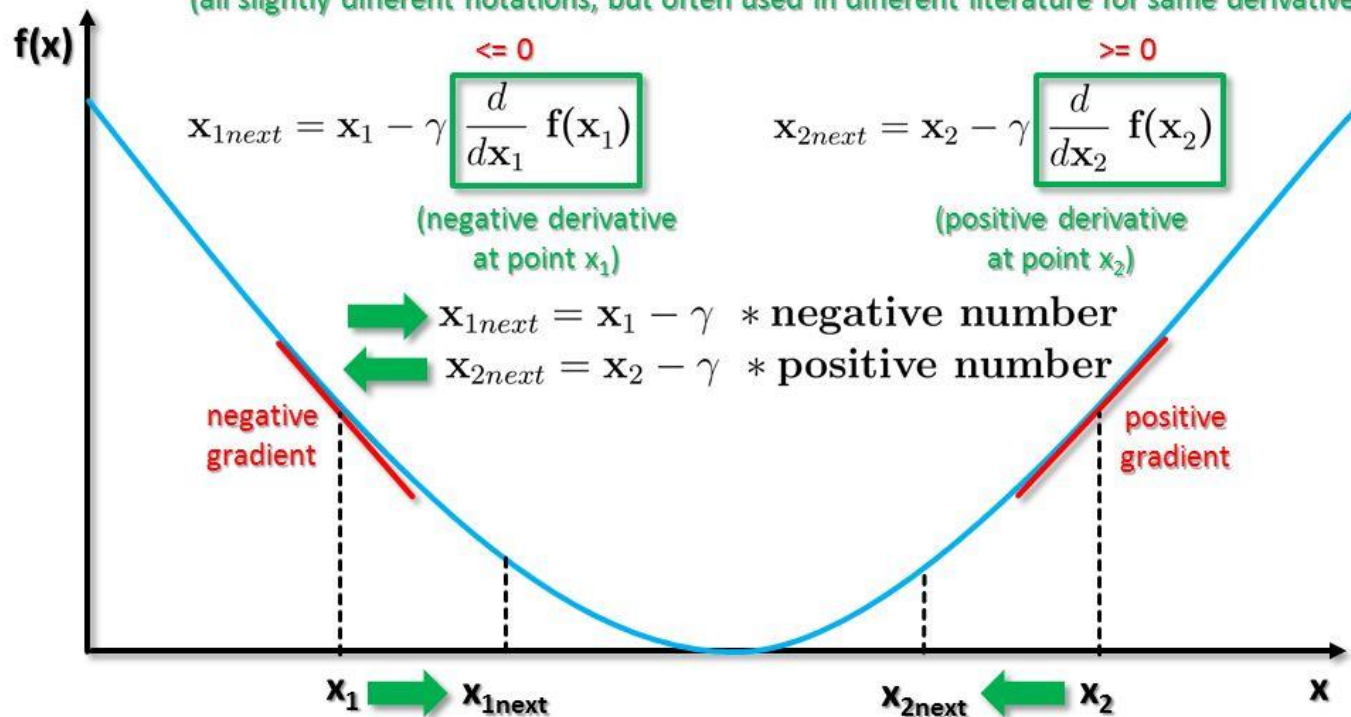
(gradient term is steepest ascent)



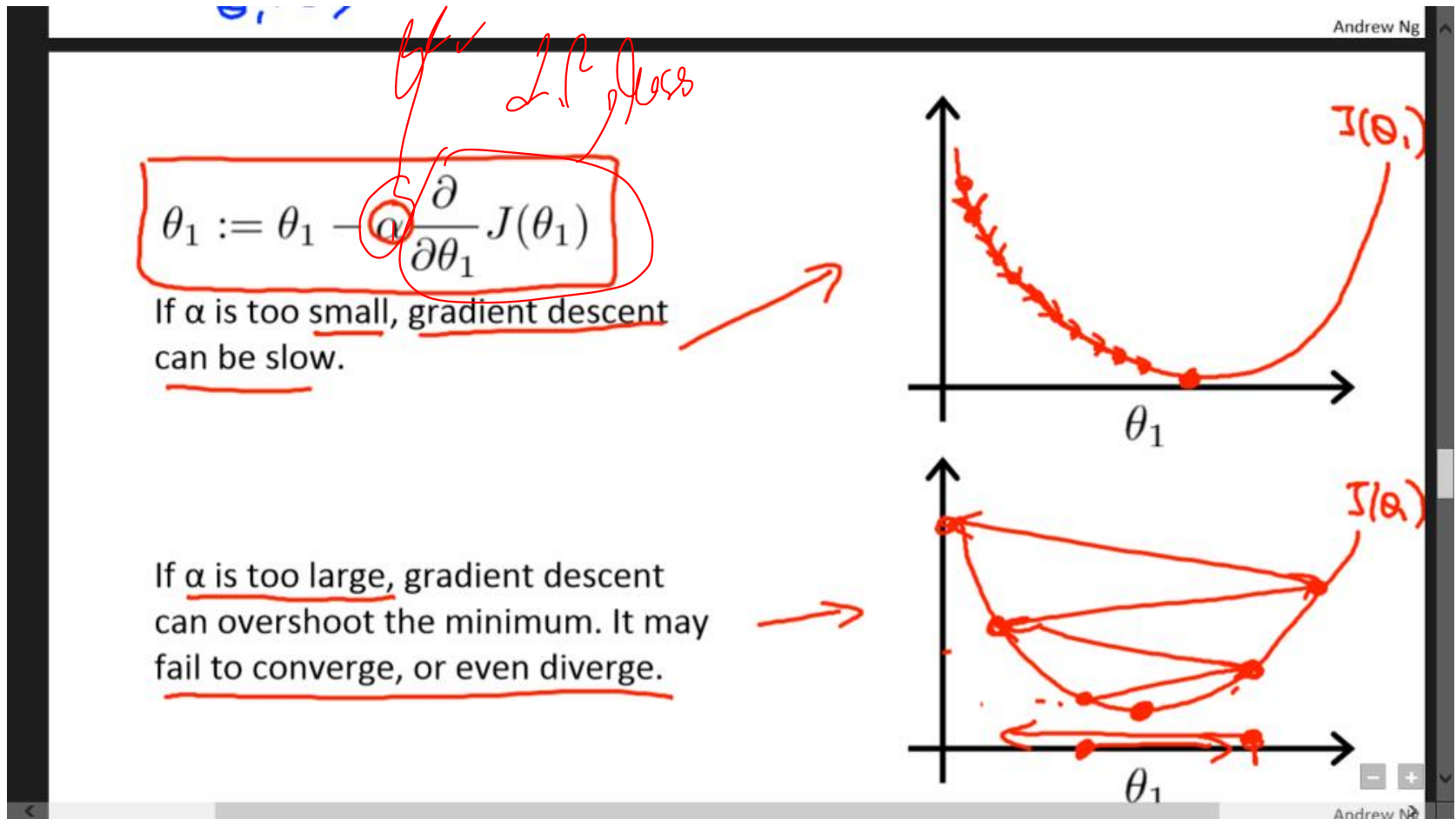
# Gradient Descent for Linear Regression

$$\mathbf{b} = \mathbf{a} - \gamma \nabla f(\mathbf{a}) \quad \mathbf{b} = \mathbf{a} - \gamma \frac{\partial}{\partial \mathbf{a}} f(\mathbf{a}) \quad \mathbf{b} = \mathbf{a} - \gamma \frac{d}{d\mathbf{a}} f(\mathbf{a})$$

(all slightly different notations, but often used in different literature for same derivative term)



# Gradient Descent for Linear Regression





# Learning

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$$y = \theta_0 + \theta_1 x_1$$

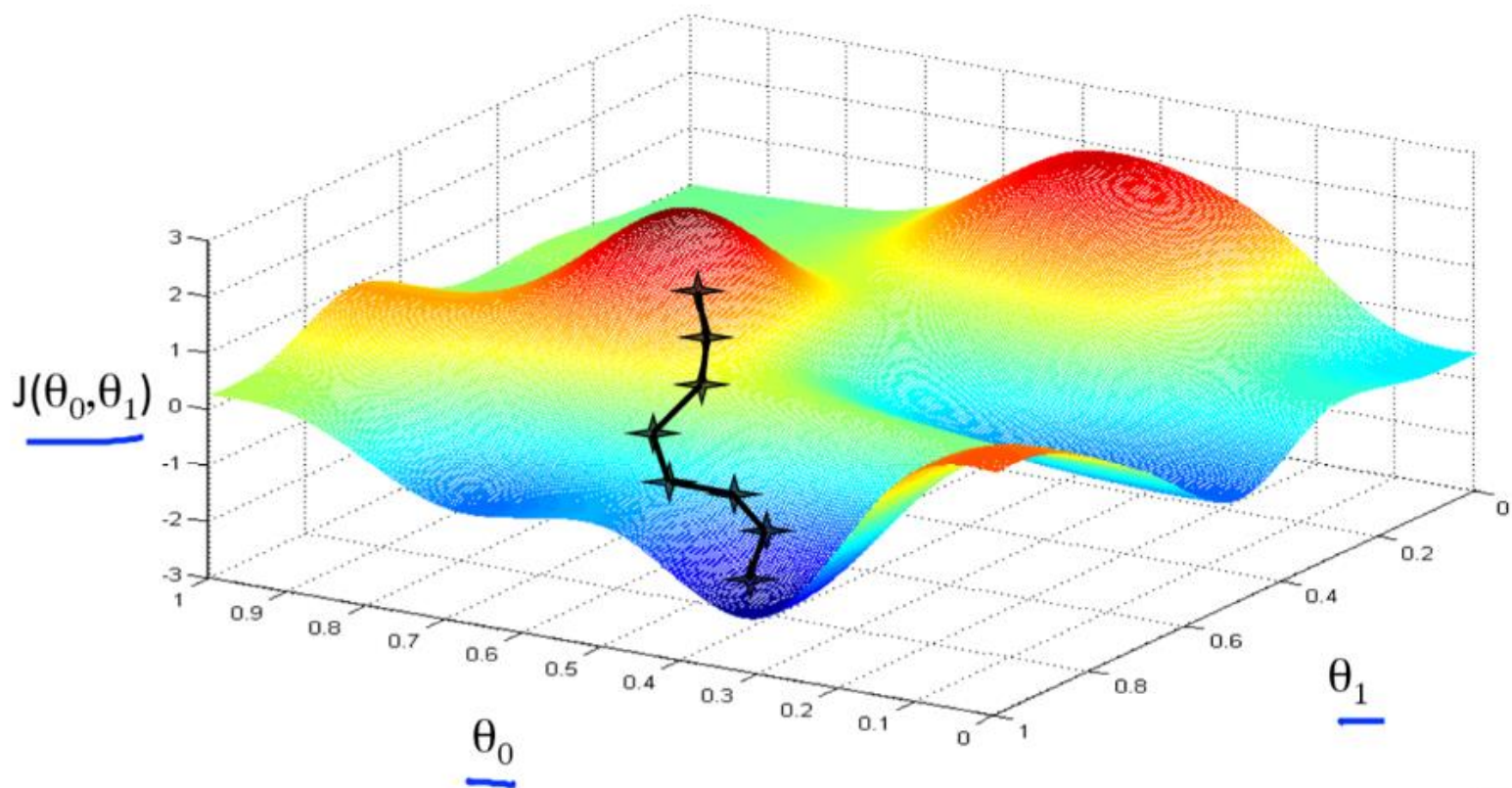
repeat until convergence: {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m ((h_{\theta}(x_i) - y_i) x_i)$$

}

# Gradient Descent for Linear Regression



วันนี้สำคัญกว่าทุกอย่างที่เราจะเรียน

# Feature Extraction

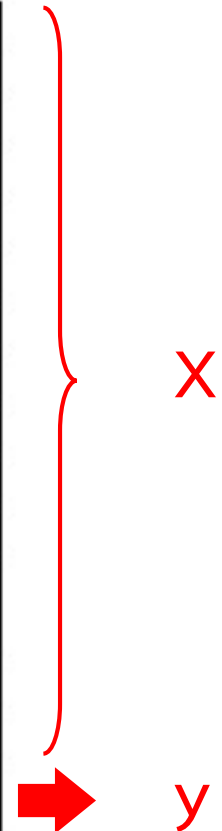


# Boston Housing

CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV	CAT. MEDV
0.00632	18	2.31	0	0.538	6.575	65.2	4.09	1	296	15.3	396.9	4.98	24	0
0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.9	9.14	21.6	0
0.02729	0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7	1
0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4	1
0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.9	5.33	36.2	1
0.02985	0	2.18	0	0.458	6.43	58.7	6.0622	3	222	18.7	394.12	5.21	28.7	0
0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5	311	15.2	395.6	12.43	22.9	0
0.14455	12.5	7.87	0	0.524	6.172	96.1	5.9505	5	311	15.2	396.9	19.15	27.1	0
0.21124	12.5	7.87	0	0.524	5.631	100	6.0821	5	311	15.2	386.63	29.93	16.5	0
0.17004	12.5	7.87	0	0.524	6.004	85.9	6.5921	5	311	15.2	386.71	17.1	18.9	0
0.22489	12.5	7.87	0	0.524	6.377	94.3	6.3467	5	311	15.2	392.52	20.45	15	0
0.11747	12.5	7.87	0	0.524	6.009	82.9	6.2267	5	311	15.2	396.9	13.27	18.9	0
0.09378	12.5	7.87	0	0.524	5.889	39	5.4509	5	311	15.2	390.5	15.71	21.7	0
0.62976	0	8.14	0	0.538	5.949	61.8	4.7075	4	307	21	396.9	8.26	20.4	0
0.63796	0	8.14	0	0.538	6.096	84.5	4.4619	4	307	21	380.02	10.26	18.2	0

# Boston Housing

CRIM	Per capita crime rate by town
ZN	Proportion of residential land zoned for lots over 25,000 sq.ft.
INDUS	Proportion of non-retail business acres per town
CHAS	Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
NOX	Nitric oxides concentration (parts per 10 million)
RM	Average number of rooms per dwelling
AGE	Proportion of owner-occupied units built prior to 1940
DIS	Weighted distances to five Boston employment centers
RAD	Index of accessibility to radial highways
TAX	Full-value property-tax rate per \$10,000
PTRATIO	Pupil-teacher ratio by town
B	$1000(B_k - 0.63)^2$ where $B_k$ is the proportion of African-Americans by town
LSTAT	% Lower status of the population
MEDV	Median value of owner-occupied homes in \$1000's
CAT.MEDV	Binary variable that indicates based on the MEDV variable. If $MEDV > 30$ , $CAT.MEDV = 1$



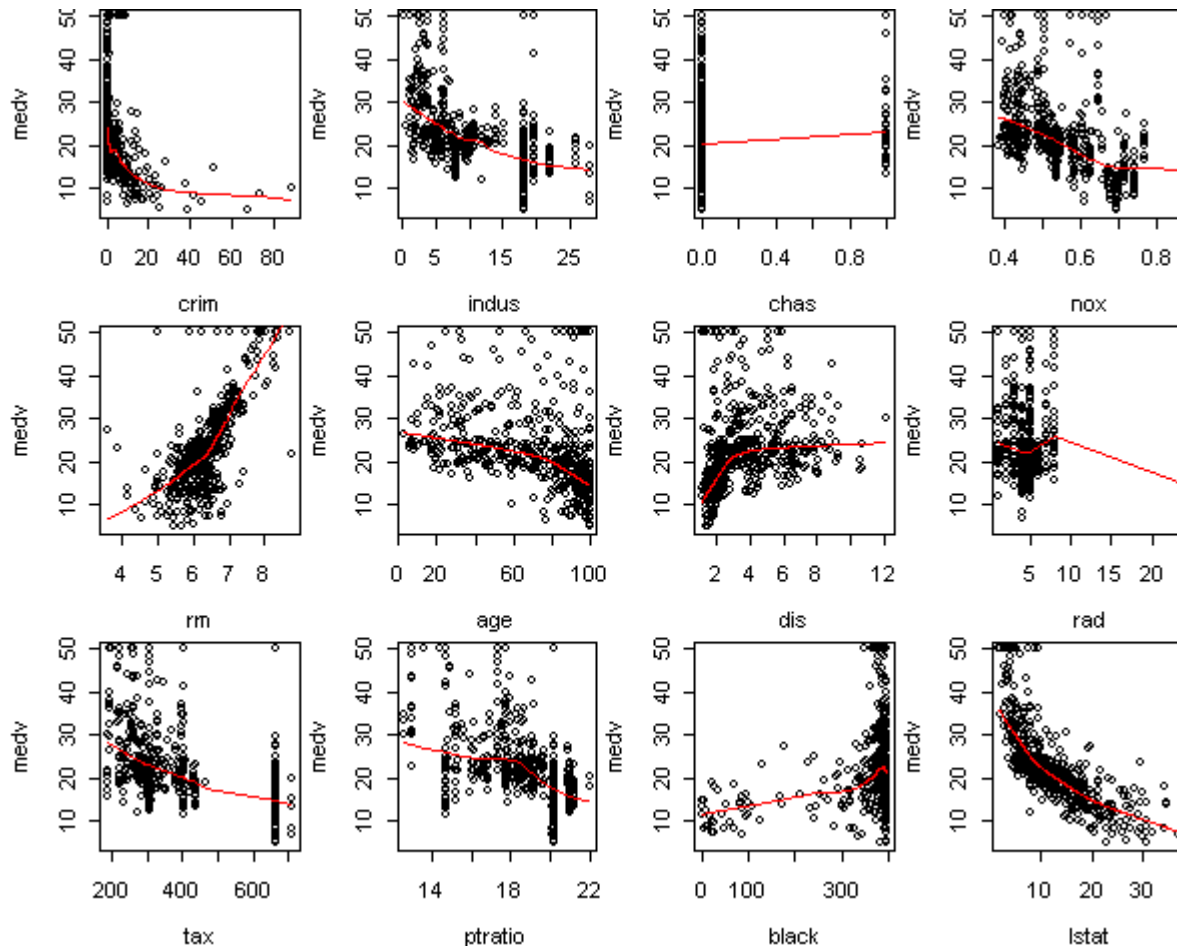
x

y

# Correlations

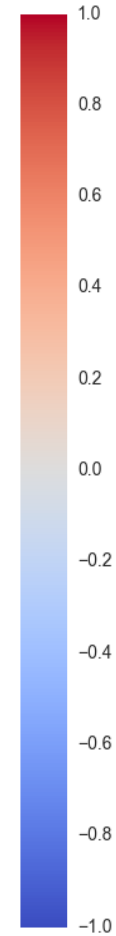
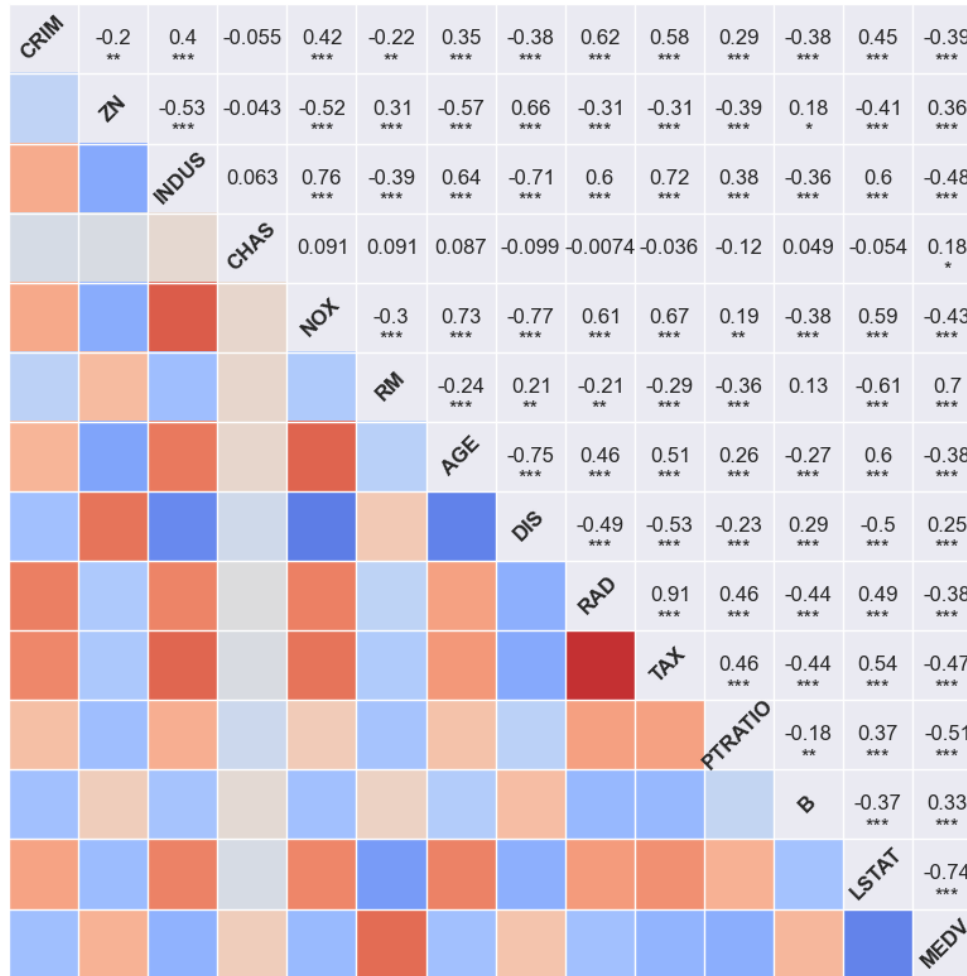
lineary correlation plot

- Content



# Correlations

การกระจายของตัวแปร X, Y



regression  
↓  
95  
pearson correlation  
คือค่า

ค่าเฉลี่ย  
≈  $|X| > 0.7$  (สำคัญ)

# Cross Validation

↓  
กรณี





# Rotation Estimation

① train\_test\_split

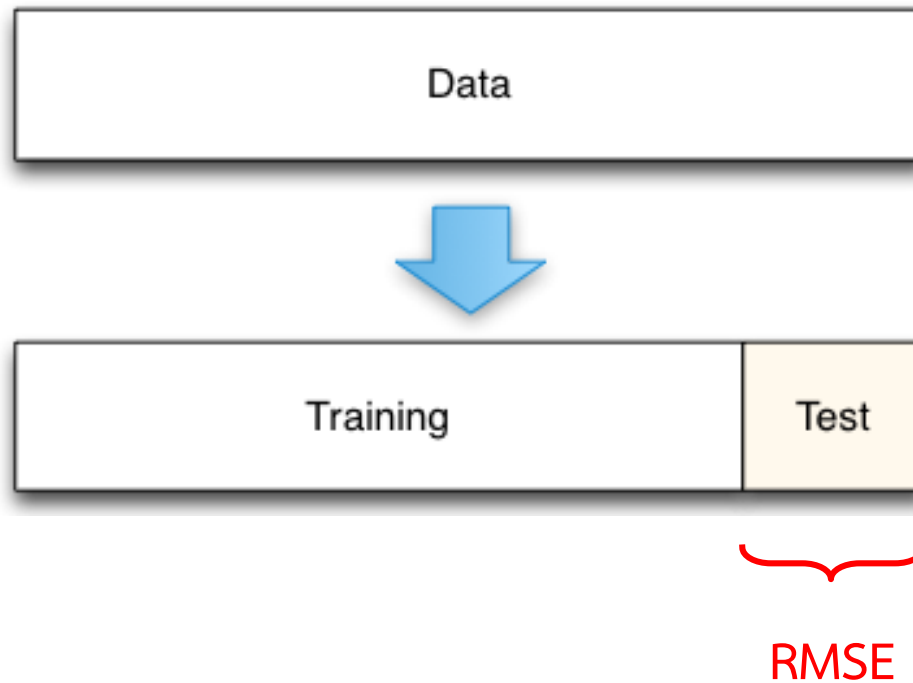
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## How accurately a predictive model will perform in practice?

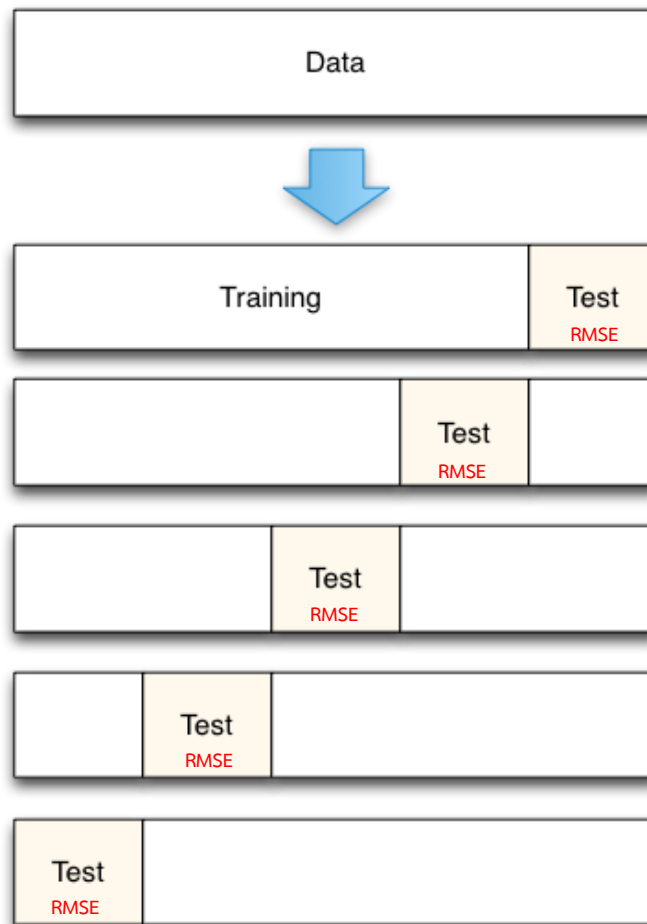
- Cross-validation is a technique to evaluate predictive models by partitioning the original sample into a training set to train the model, and a test set to evaluate it.
- In k-fold cross-validation, the original sample is randomly partitioned into k equal size subsamples. Of the k subsamples, a single subsample is retained as the validation data for testing the model, and the remaining k-1 subsamples are used as training data. The cross-validation process is then repeated k times (the folds), with each of the k subsamples used exactly once as the validation data. The k results from the folds can then be averaged (or otherwise combined) to produce a single estimation. The advantage of this method is that all observations are used for both training and validation, and each observation is used for validation exactly once.

# Cross-Validation

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# K-Fold Cross-Validation



ทำหลายๆ ครั้ง

หาค่าเฉลี่ย

Average  
RMSE

หาค่าเฉลี่ยแล้ว → เปรียบ

# Python Code

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1. `from sklearn import neighbors, datasets, preprocessing`
2. `from sklearn.model_selection import train_test_split`
3. `iris = datasets.load_iris()`
4. `X, y = iris.data[:, :2], iris.target`
5. `X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=33)`
6. `lm = linear_model.LinearRegression()`
7. `lm.fit(X_train, y_train)`
8. `y_predicted = lm.predict(X_test)`
9. `rmse = sqrt(mean_squared_error(y_test, y_predicted))`

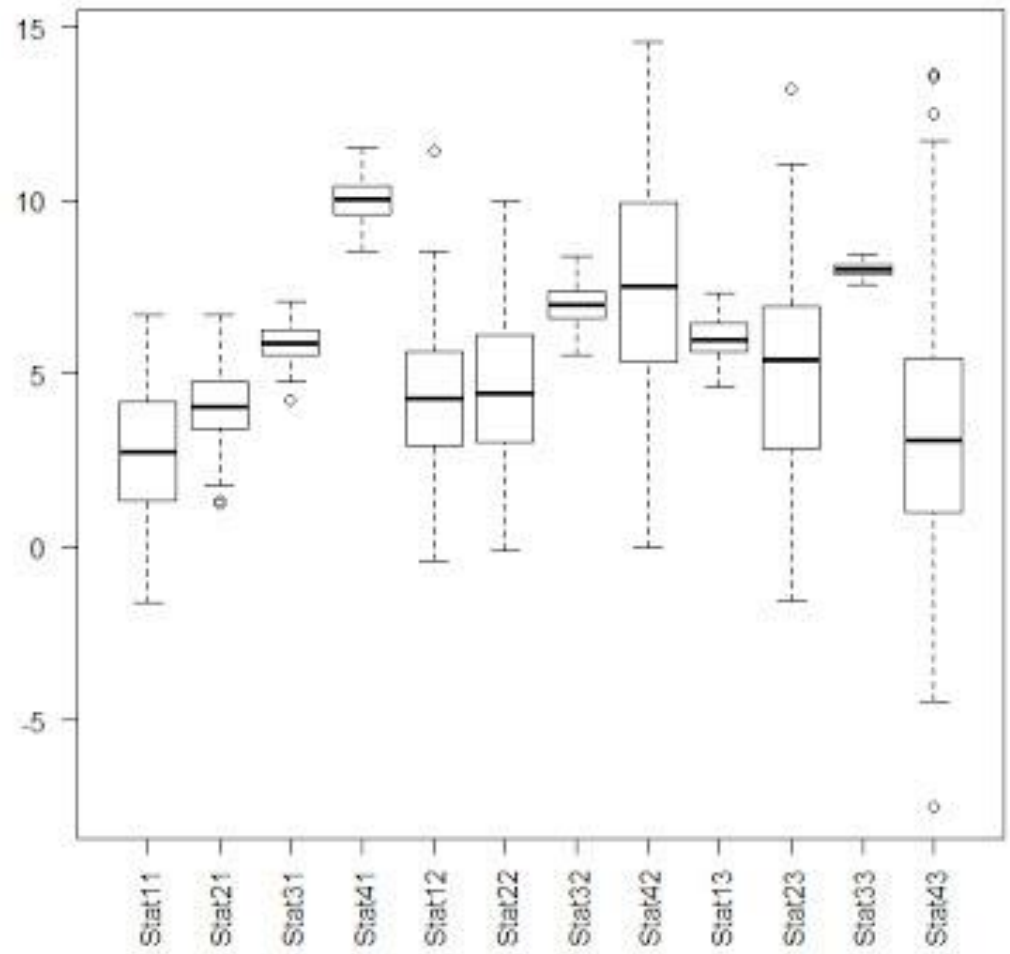
# Feature Scaling

↓  
space ที่ใช้  
↓  
ขนาด , size  
↓  
memory space ใช้



# Feature Scaling

- Feature scaling is a method used to standardize the range of independent variables or features of data. In data processing, it is also known as data normalization and is generally performed during the data preprocessing step.



# Rescaling

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- The simplest method is rescaling the range of features to scale the range in  $[0, 1]$  or  $[-1, 1]$ . Selecting the target range depends on the nature of the data. The general formula is given as:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

normalization

- where  $x$  is an original value  $x'$  is the normalized value. For example, suppose that we have the students' weight data, and the students' weights span . To rescale this data, we first subtract 160 from each student's weight and divide the result by 40 (the difference between the maximum and minimum weights).

# Mean Normalization

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$$x' = \frac{x - \text{mean}(x)}{\max(x) - \min(x)}$$

- where  $x$  is an original value,  $x'$  is the normalized value.



# Standardization

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- In machine learning, we can handle various types of data, e.g. audio signals and pixel values for image data, and this data can include multiple dimensions. Feature standardization makes the values of each feature in the data have zero-mean (when subtracting the mean in the numerator) and unit-variance.

$$x' = \frac{x - \bar{x}}{\sigma}$$

- Where  $x$  is the original feature vector,  $\bar{x}$  is the mean of that feature vector, and  $\sigma$  is its standard deviation.

# Scaling to unit length

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- Another option that is widely used in machine-learning is to scale the components of a feature vector such that the complete vector has length one. This usually means dividing each component by the Euclidean length of the vector:

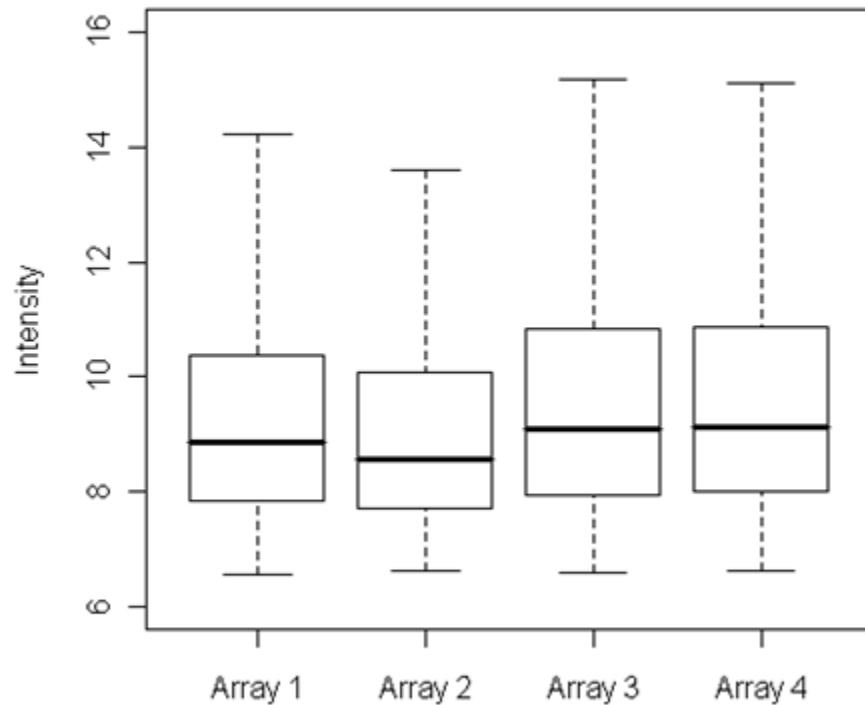
$$x' = \frac{x}{||x||}$$

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විභාජනය

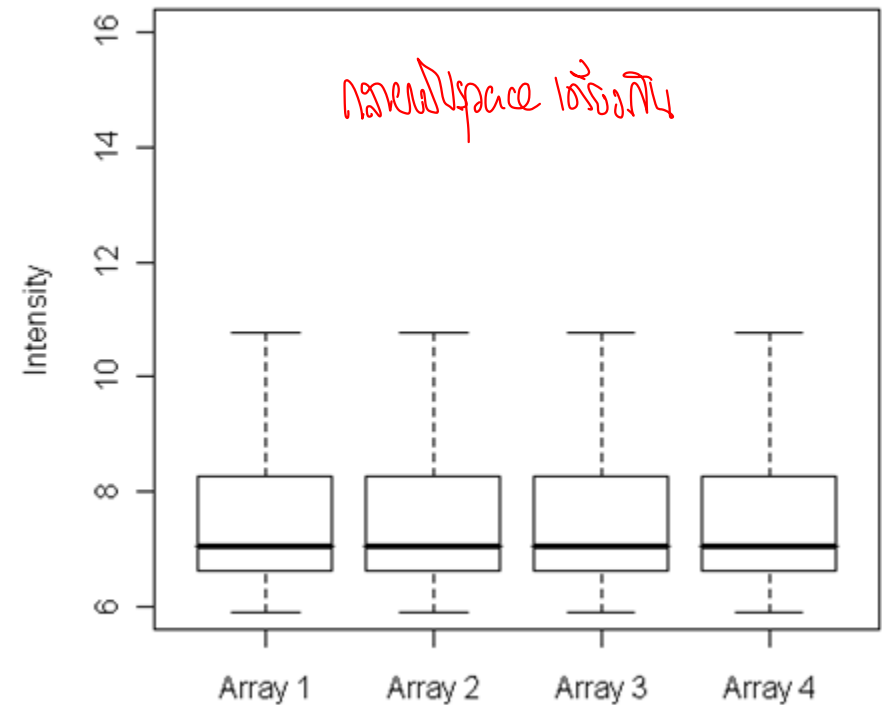
- In some applications (e.g. Histogram features) it can be more practical to use the L1 norm (i.e. Manhattan Distance, City-Block Length or Taxicab Geometry) of the feature vector. This is especially important if in the following learning steps the Scalar Metric is used as a distance measure.

# Box Plot

**Before normalization**



**After normalization**



# Python Code

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1. `from sklearn import neighbors, datasets, preprocessing`
2. `from sklearn.model_selection import train_test_split`
3. `from sklearn.metrics import accuracy_score`
4. `iris = datasets.load_iris()`
5. `X, y = iris.data[:, :2], iris.target`
6. `X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=33)`
7. `scaler = preprocessing.StandardScaler().fit( X_train )`
8. `X_train = scaler.transform(X_train)`
9. `X_test = scaler.transform(X_test)`
10. `# Do Prediction`

↓  
ต้องใส่ X\_train ให้ scaler

“

Data is a precious thing and will last  
longer than the systems themselves.

”

Tim Berners-Lee