

input → ? → output

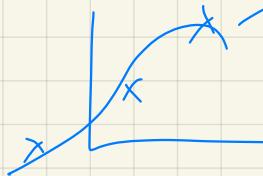
how to know abt the process

give it input

find the output

plot the graph

Gaussian elimination



parabola

$$p(x) = x_0 + x_1 x + x_2 x^2$$

3 unknowns

use Gaussian

give input and give → test case

→ output given

3 rows 3 rows

↓

4 rows 3 rows

→ 3 rows 3 rows

check answer

check

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & -2 & | & 0 \\ -2 & -3 & 4 & | & 3 \\ 4 & 3 & 2 & | & 3 \end{pmatrix}$$

2 2 -2 0
-2 -3 4 3
4 3 -2 3

$$\begin{pmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & -3 \\ 0 & 0 & 1 & | & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 & | & 0 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & 6 & | & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & | & 3 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 4 & | & 0 \end{pmatrix}$$

check answer

0 = 0 → many solutions

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & -2 & | & 0 \\ -2 & 3 & 4 & | & 3 \\ 4 & 3 & -2 & | & 4 \end{pmatrix}$$

$$0 = 1 \text{ falsch}$$

$$\begin{pmatrix} 1 & -\frac{1}{5} & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 5 & 2 & | & 3 \\ 0 & -1 & 2 & | & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & +1 \\ 0 & 1 & -\frac{2}{5} \\ 0 & 0 & \frac{5}{5} \end{pmatrix} \begin{pmatrix} 1 & 0 & -\frac{7}{5} & | & -\frac{3}{5} \\ 0 & 1 & \frac{2}{5} & | & \frac{3}{5} \\ 0 & 0 & \frac{7}{5} & | & \frac{7}{5} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & | & \frac{5}{5} \\ 0 & 1 & 0 & | & \frac{1}{5} \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

Wegnehmen
/ 5

$$Ax = b$$

define $x_s \quad x_n$

$$Ax_s = b$$

$$Ax_n = 0$$

$$x = x_s + Bx_n$$

$$Ax = A(x_s + Bx_n) = b$$

$$Ax_s + ABx_n = b$$

$$b + B0 = b$$

วิธี

$$\left(\begin{array}{ccc|c} 2 & 2 & -2 & 0 \\ -2 & -3 & 4 & 3 \\ 4 & 3 & -2 & 4 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 2 & 6 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{cases} 2x_0 + 0 + 2x_2 = 6 \\ -x_1 + 2x_2 = 3 \end{cases} \text{ let } x_5$$

$$\begin{cases} 2x_0 + 2x_2 = 0 \\ -x_1 + 2x_2 = 0 \end{cases} \text{ let } x_h$$

อันนี้ถ้าจับค่า 0

กำหนด free variable

เปลี่ยนตัวแปรที่รู้ค่าได้

let x_5 กำหนด x_2 เป็น free var

set $x_2 = 0$

$$\begin{cases} 2x_0 + 2(0) = 6 \\ -x_1 + 2(0) = 3 \end{cases} \Rightarrow \begin{cases} x_0 = 3 \\ x_1 = -3 \\ x_2 = 0 \end{cases}$$

$$x_5 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}$$

let x_h
กำหนด x_2 เป็น fv \rightarrow set $x_2 = 1$

$$\begin{aligned} -x_1 + 2(1) &= 0 \\ x_1 &= 2 \end{aligned}$$

$$x_h = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

กำหนดตัวแปรที่รู้ค่าได้ whatever

$$x = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} + B \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

For example

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & -3 \\ -1 & 0 & 1 & 2 \end{array} \right) \left(\begin{array}{ccc|c} 2 & -4 & -2 & 4 \\ -2 & 4 & 1 & -3 \\ 2 & -4 & 0 & 2 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array} \right) \left(\begin{array}{ccc|c} 2 & -4 & -2 & 4 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 2 & -2 \end{array} \right)$$

$$2x_0 - 4x_1 - 2x_2 = 4$$

$$-1x_2 = 1$$

$$2x_2 = -2$$

$$x_2 = -1$$

$$X_s \text{ define } x_1 = 0$$

$$2x_0 - 4x_1 - 2x_2 = 4$$

$$x_2 = -1$$

$$2x_0 - 4x_1 = 2 \quad \left| \quad x_2 = -1 \right.$$

$$2x_0 - 4(0) = 2$$

$$x_0 = 1$$

$$X_s = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$X_n \text{ define } x_1 = 1$$

$$2x_0 - 4(1) = 2$$

$$2x_0 = 6 \quad \left| \quad x_2 = 1 \right.$$

$$x_0 = 3$$

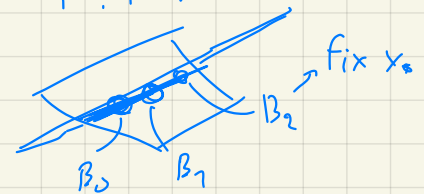
$$X_n = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + B \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$2x_0 - 2x_2 = 4$$

$$x_2 = -1$$

จุดที่พิกัดของ B คือ หน่วยในแนวตั้ง



free var \rightarrow complete solution

1 param 3 angles

$$Ax_s = 6$$

$$Ax_{n0} = 0$$

$$Ax_{n1} = 0$$

$$x_0 - 2x_1 + 4x_2 = -1 \quad \text{in } x_s$$

$$x_0 - 2x_1 + 4x_2 = 0 \quad \text{in } x_n$$

$$X = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + B_0 \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix} + B_1 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

จัดแนวกับ unit set

$$X = x_s + B_0 x_{n0} + B_1 x_{n1}$$

in x_s

\hookrightarrow define $x_1 = 0, x_2 = 0$

$$x_0 = -1$$

$$X_s = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

in x_{n0}

\hookrightarrow define $x_1 = 0, x_2 = 1$

$$x_0 + 2(0) + 4(1) = 0$$

$$x_0 = -4$$

$$\begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$$

in x_{n1}

\hookrightarrow define $x_1 = 1, x_2 = 0$

$$x_0 + 2(1) + 4(0) = 0$$

$$x_0 = -2$$

$$\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

3x4 4x1 4x1

Gauß-Verfahren

L) VOR, 2. NB

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 & 2 & | & 1 \\ 2 & 6 & 4 & 8 & | & 3 \\ 0 & 0 & 2 & 4 & | & 1 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_0 + 3x_1 + x_2 + 2x_3 = 1$$

$$2x_2 + 4x_3 = 1$$

$$x_0 + 3x_1 + x_2 + 2x_3 = 0$$

$$2x_2 + 4x_3 = 0$$

$$\begin{pmatrix} 1 & 3 & 1 & 2 & | & 1 \\ 0 & 0 & 2 & 4 & | & 1 \\ 0 & 0 & 2 & 4 & | & 1 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

an x_3 nehmen $x_1 = 0, x_3 = 0$

$$x_0 + x_2 = 1$$

$$2x_2 = 1$$

$$x_0 + \frac{1}{2} = 1$$

$$x_0 = -\frac{1}{2}$$

$$\begin{pmatrix} -\frac{1}{2} \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

4. Schritt 2. Schritt

an x_{n0} nehmen $x_1 = 0, x_3 = 1$

$$x_0 + x_2 + 2 = 0$$

$$2x_2 + 4 = 0$$

$$x_2 = -2$$

$$x_0 = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

an x_{n1} nehmen $x_1 = 1, x_3 = 0$

$$x_0 + 3 + x_2 = 0$$

$$2x_2 = 0$$

$$x_2 = 0$$

$$x_0 = -3$$

$$\begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

$$X = \begin{pmatrix} -\frac{1}{2} \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \beta_0 \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix} + \beta_1 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Freiervariablen

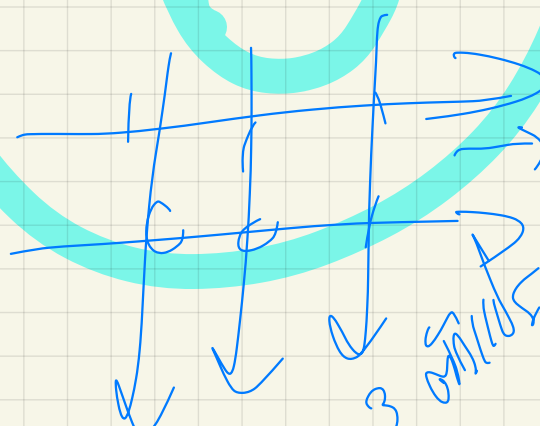
β_0, β_1 Parameter

↓

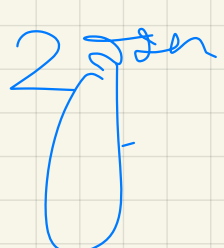
$$\beta_0 x_{n0} + \beta_1 x_{n1}$$

$$\begin{pmatrix} 1 & 4 & 0 \\ 0 & 5 & 5x4 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \textcircled{X} & X & 0 & 0 \\ 0 & 0 & \textcircled{X} & X \rightarrow \text{row 2} \\ 0 & 0 & X & X \\ 0 & 0 & X & X \end{pmatrix}$$

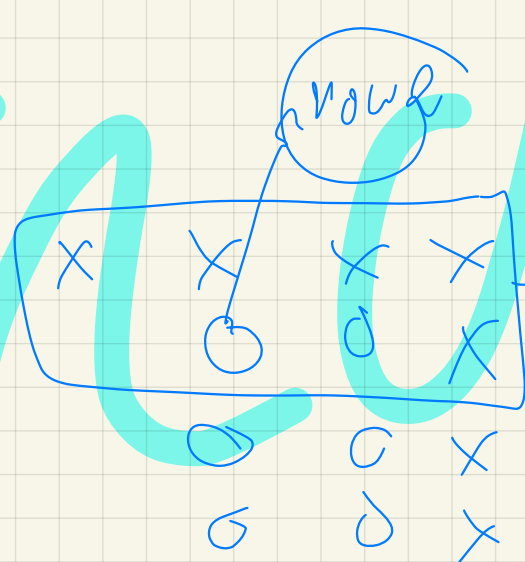


3 rows 2 columns

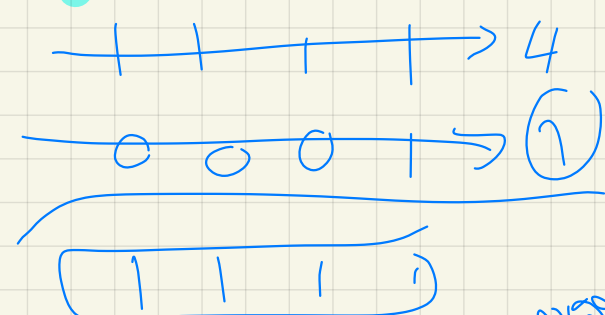


$$2x + 3y = 5$$

$$3x + 4y = 10$$



Fix variable = 3

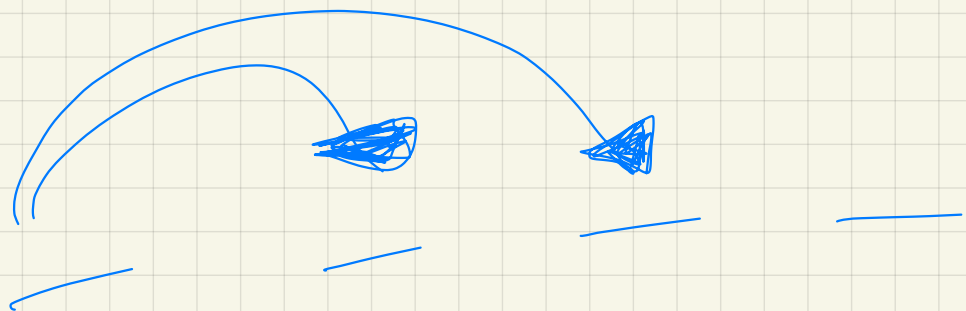


erasedum = 2x2x2

$$\begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

3×4 4×1

Gauß-Verfahren



ein von 90 auf 100 %

(90 % auf 100 %)

100 %