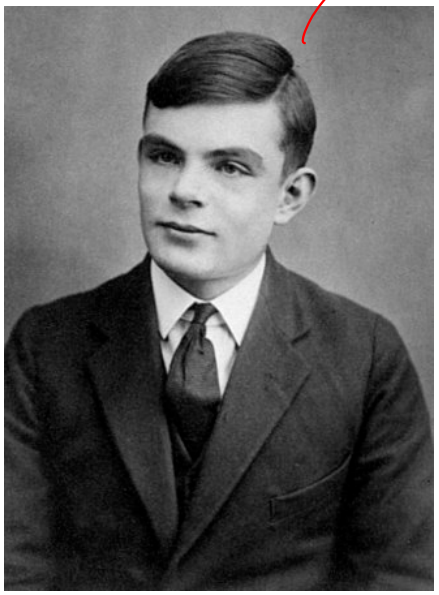
model เริ่มต้นของความซับซ้อน

เป็น automata ชนิดสุดท้าย

Turing Machines

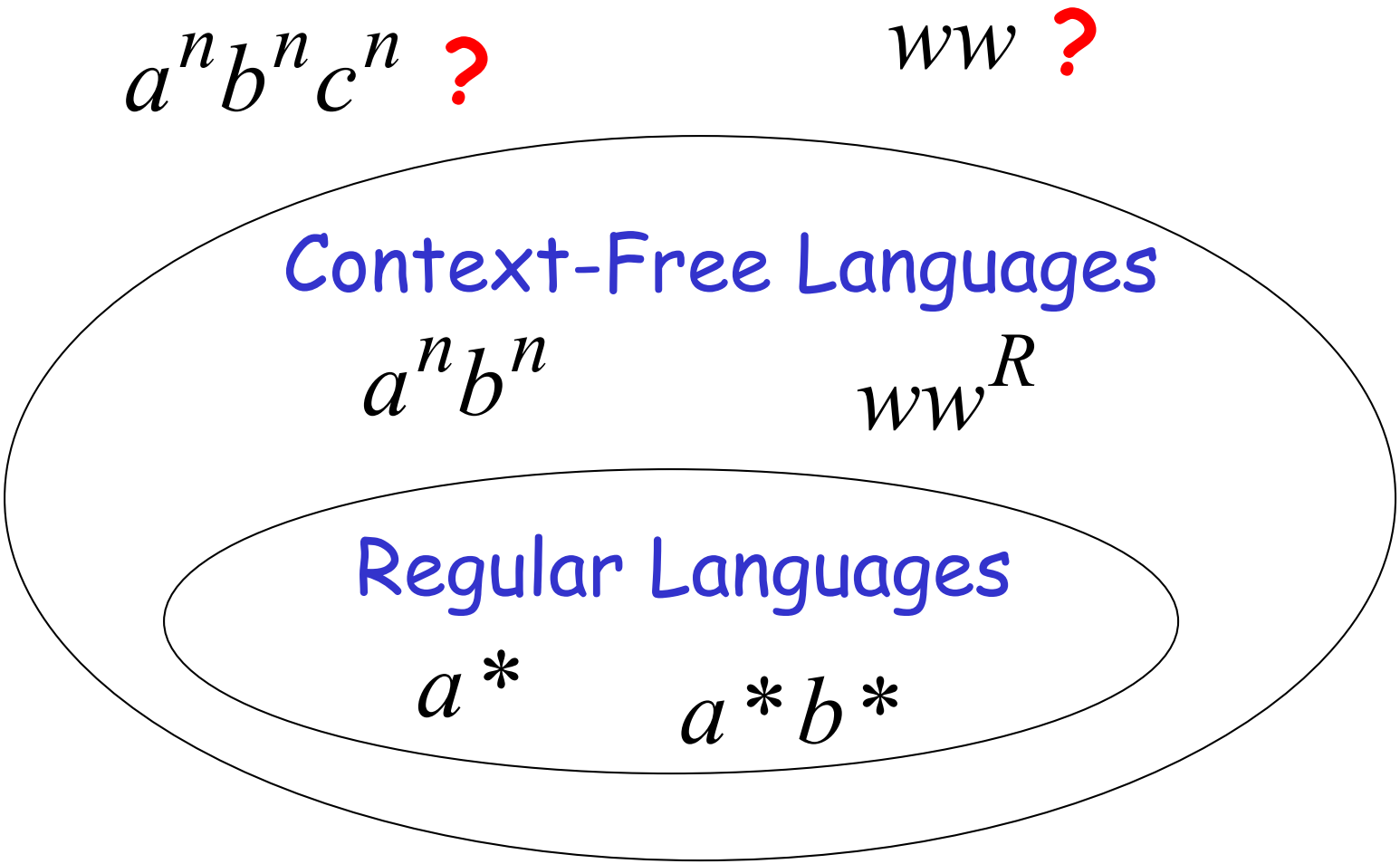


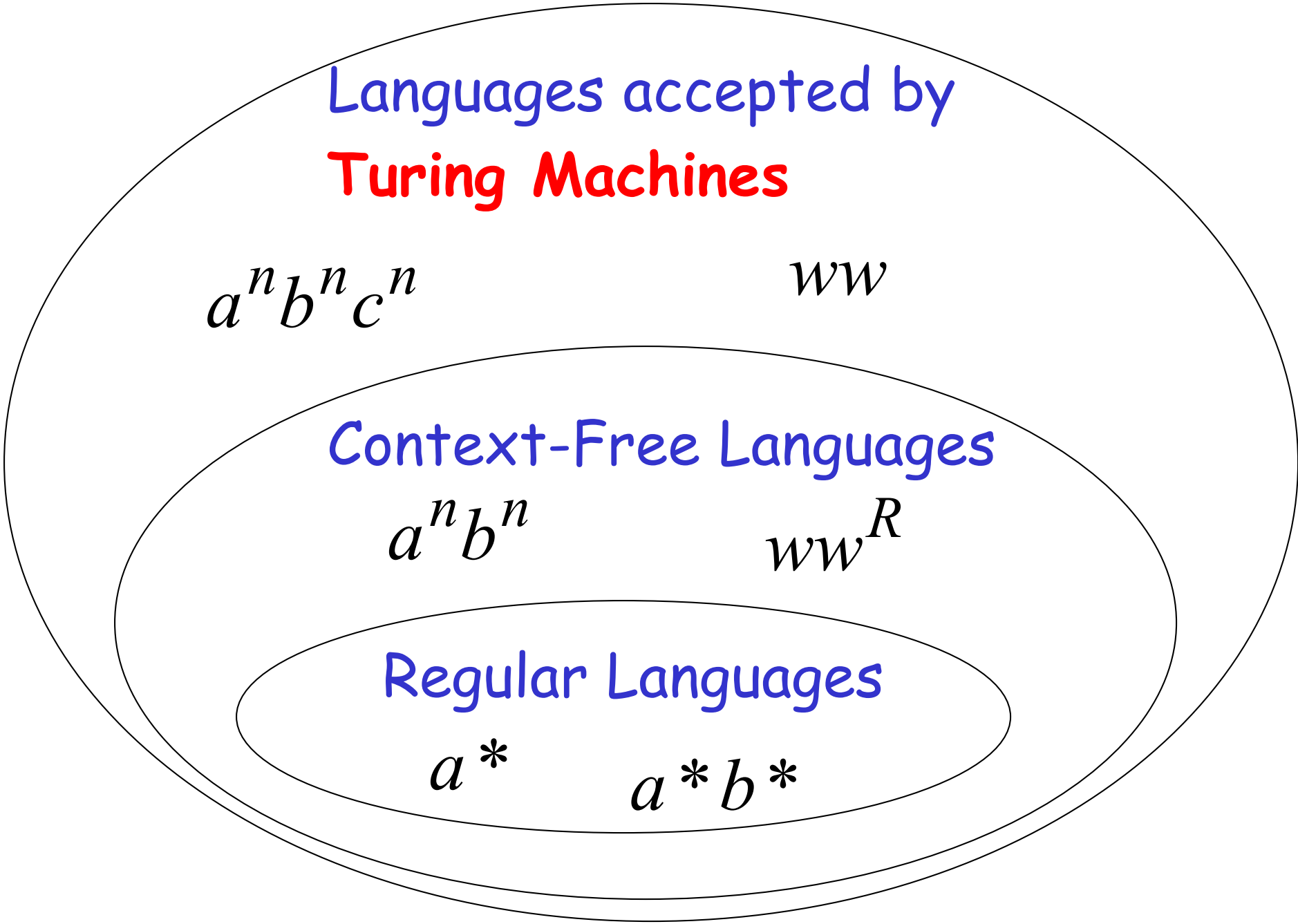
Alan M. Turing (1912-1954)

- English mathematician, computer scientist
- providing the concepts of algorithm and computation with the Turing machine, which can be considered a model of a computer.

- During the 2nd World War, Turing devised a technique for breaking German ciphers machine (Enigma).
- Turing was prosecuted in 1952 for homosexual acts.
- He accepted chemical treatment, as an alternative to prison. However, it was unsuccessful.
- In 1954, he committed suicide by cyanide poisoning.
- In 2017, UK retroactively apologize for an outlawed homosexual acts.

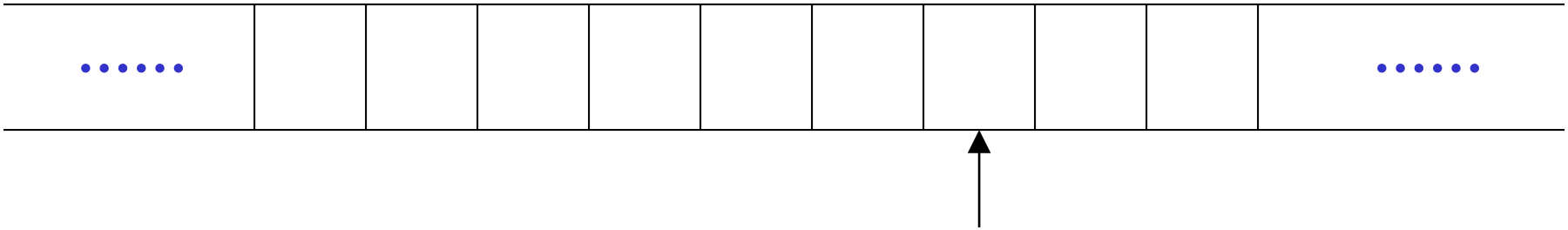
The Language Hierarchy




Languages accepted by
Turing Machines
$a^n b^n c^n$
ww
Context-Free Languages
$a^n b^n$
ww^R
Regular Languages
$a*$
$a*b*$

A Turing Machine

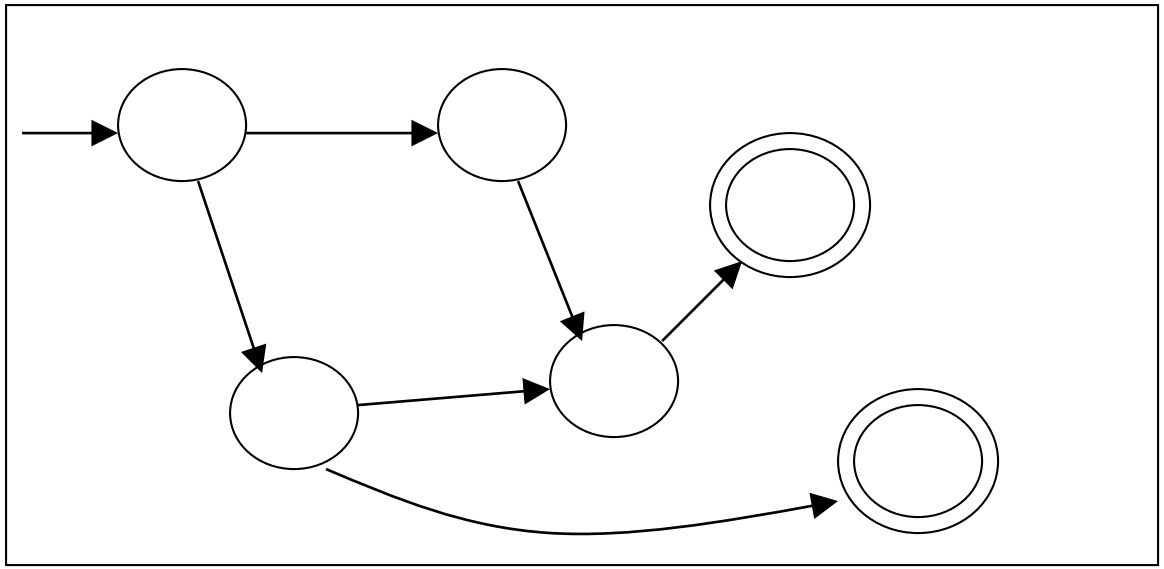
Tape



Read-Write head

ตัวpointer อ่านเขียน

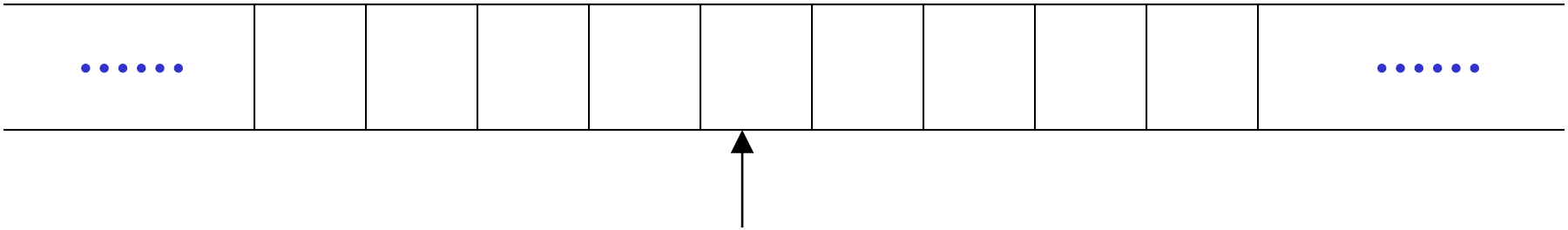
Control Unit



The Tape

ยาวไม่จำกัด

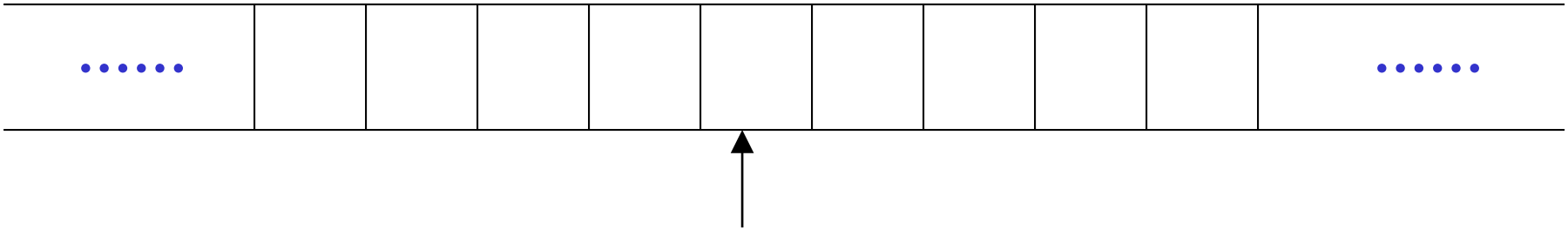
No boundaries -- infinite length



Read-Write head

อ่านหรือเขียนก็ได้

The head moves Left or Right



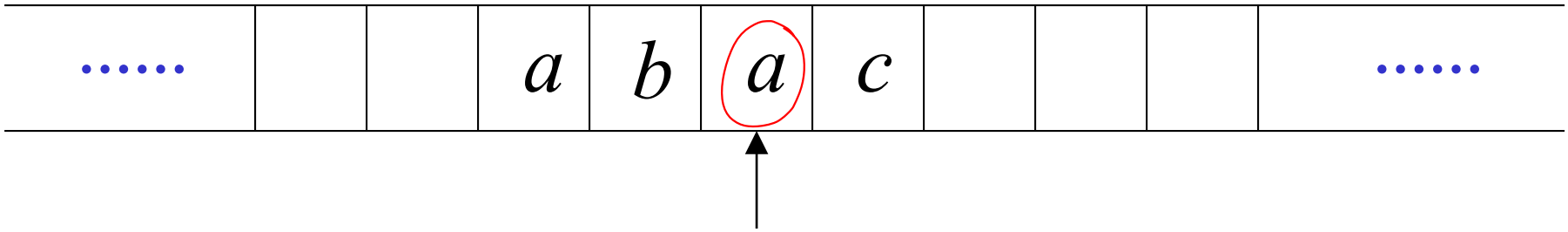
Read-Write head

The head at each time step:

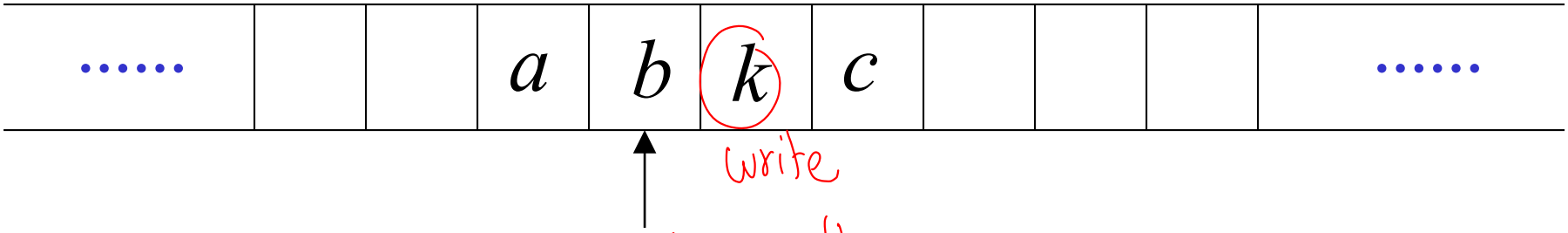
1. Reads a symbol
2. Writes a symbol
3. Moves Left or Right

Example:

Time 0

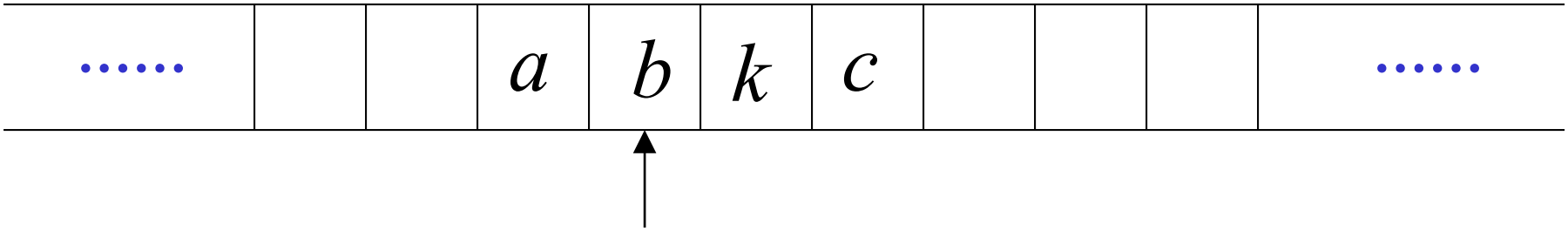


Time 1

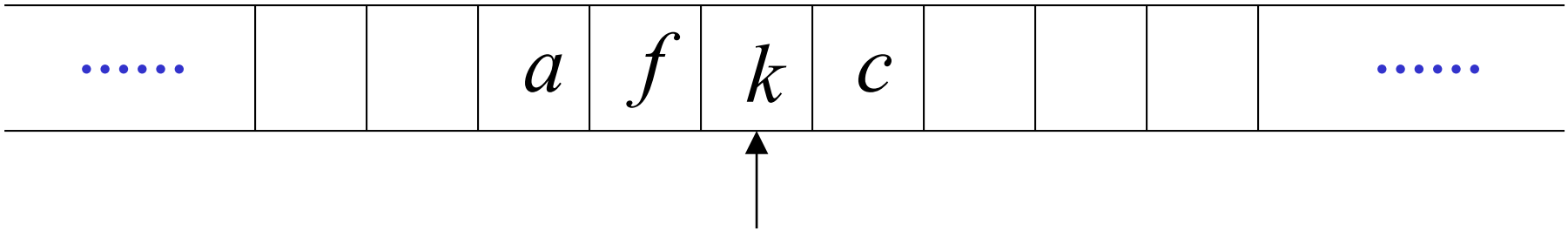


1. Reads a
2. Writes k
3. Moves Left

Time 1

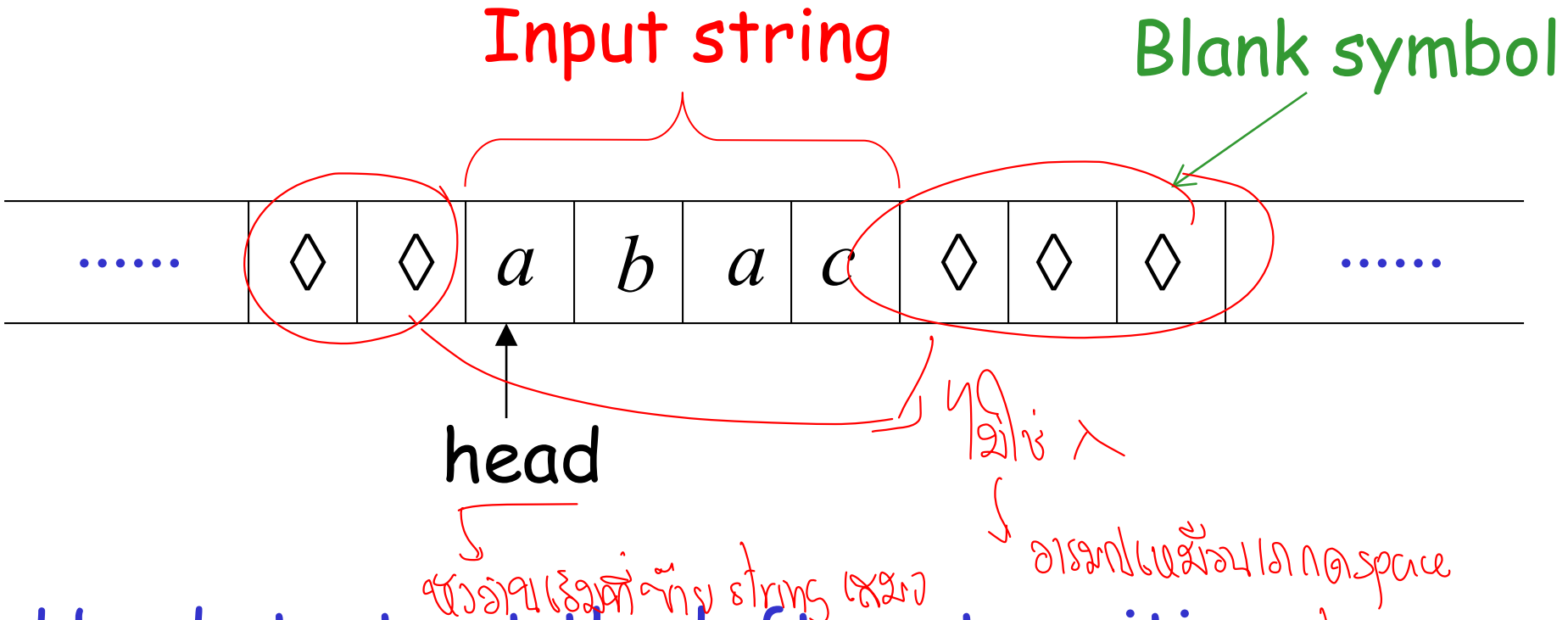


Time 2

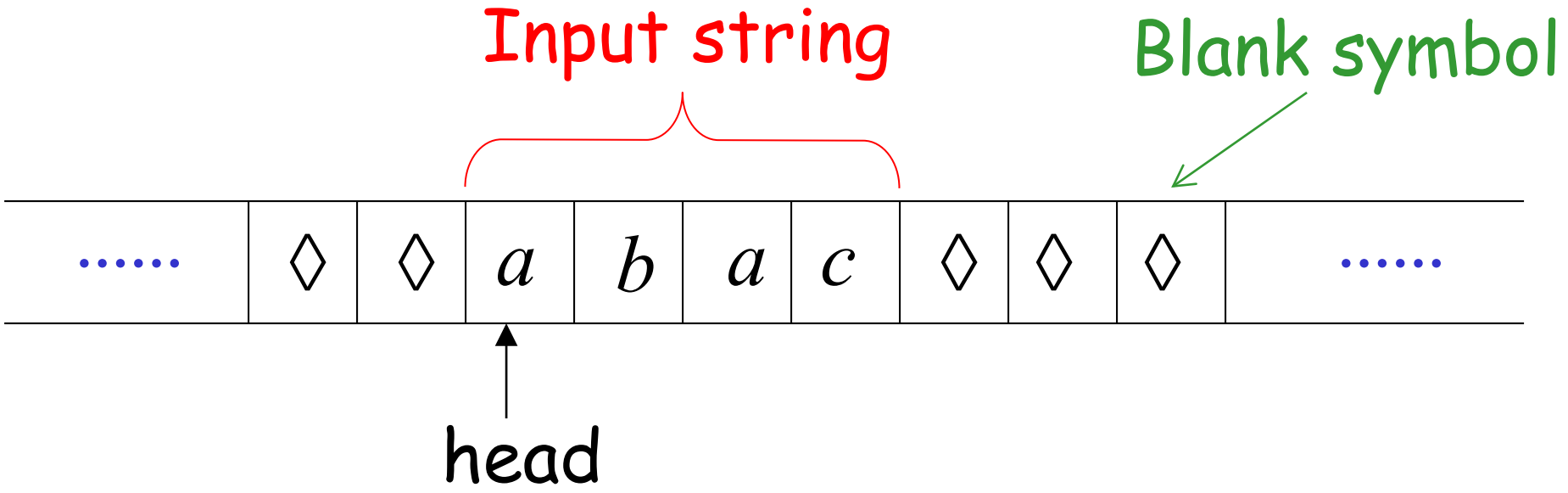


1. Reads b
2. Writes f
3. Moves Right

The Input String

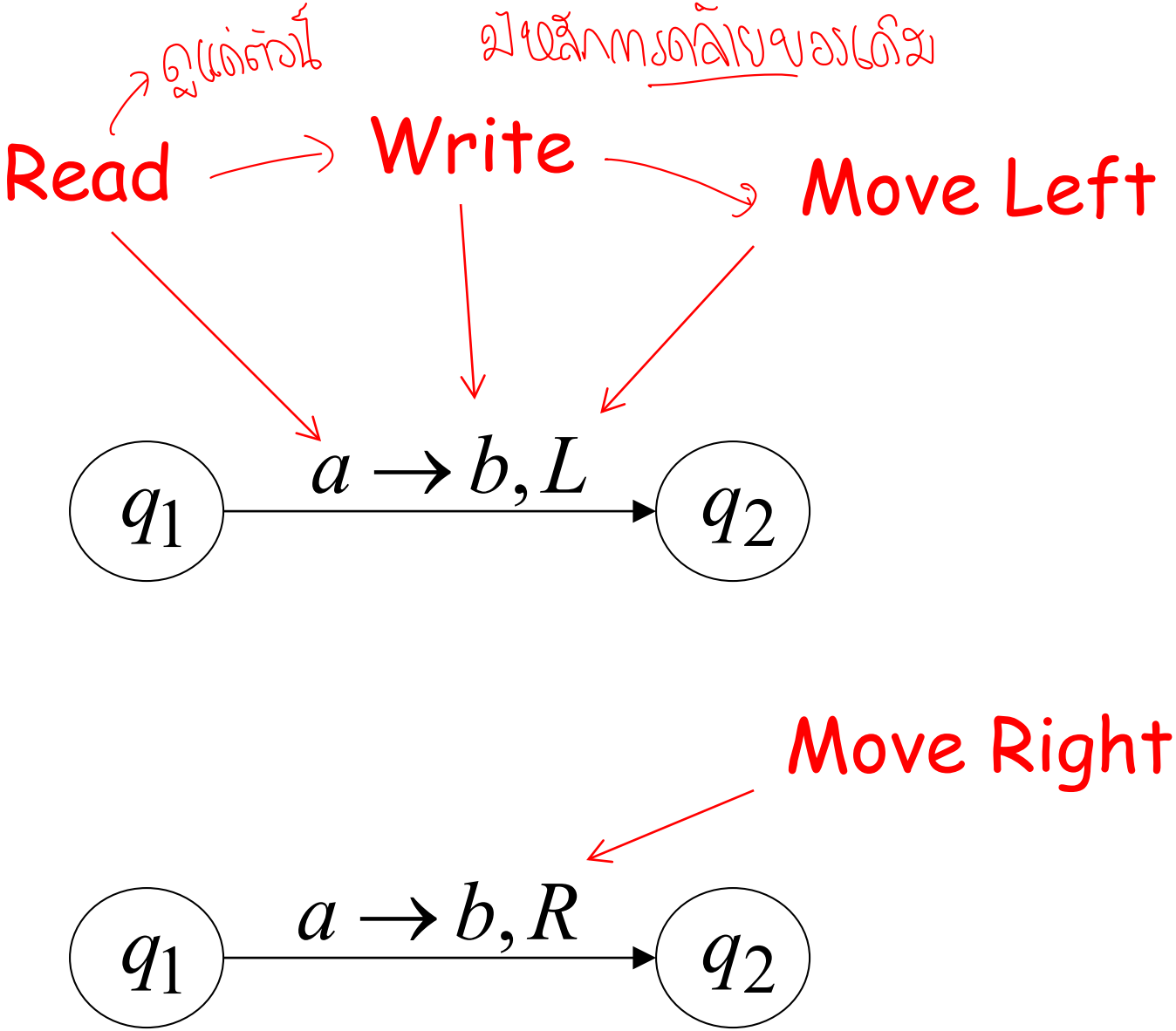


Head starts at the leftmost position of the input string



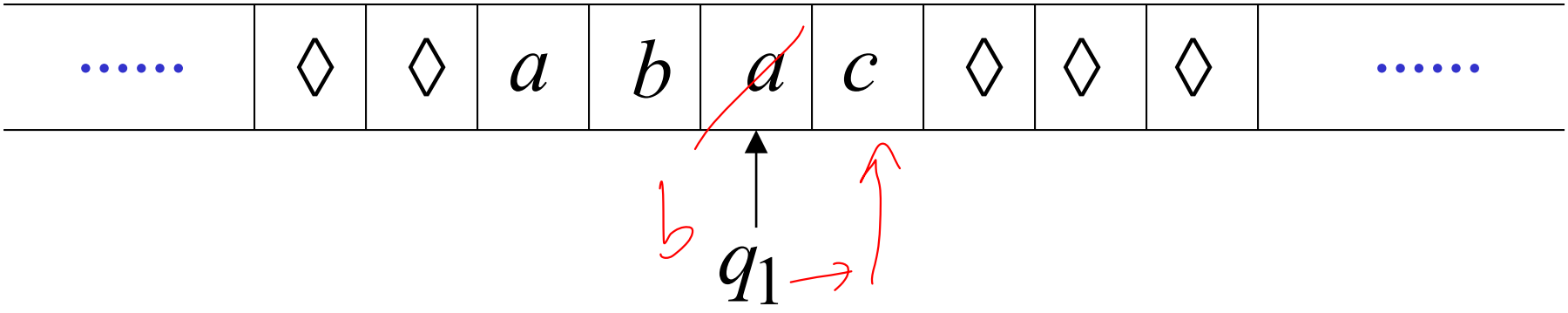
Remark: the input string is never empty

States & Transitions

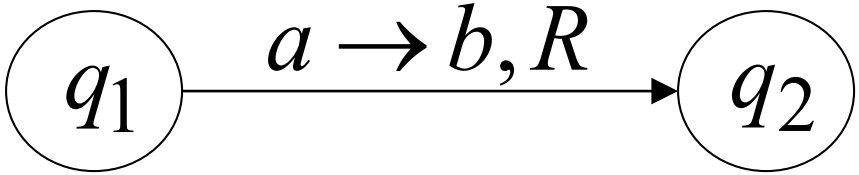


Example:

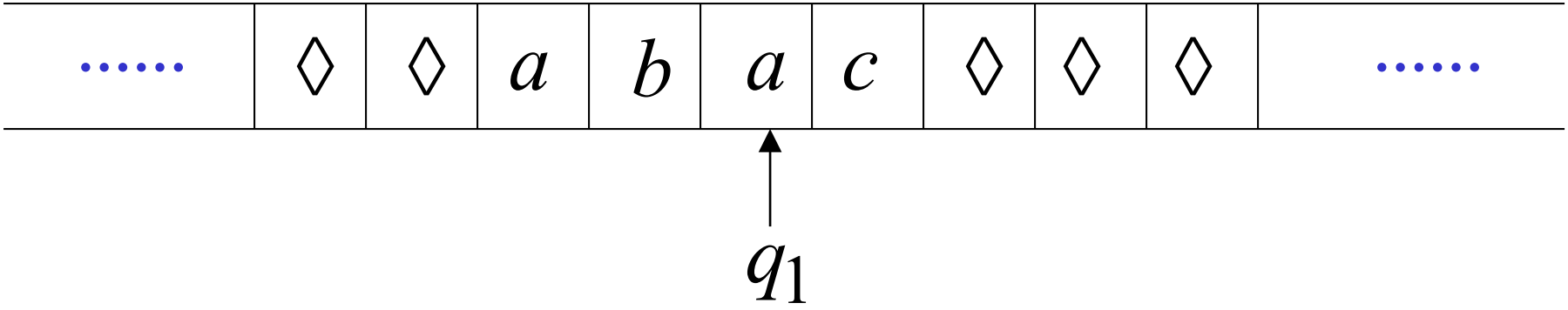
Time 1



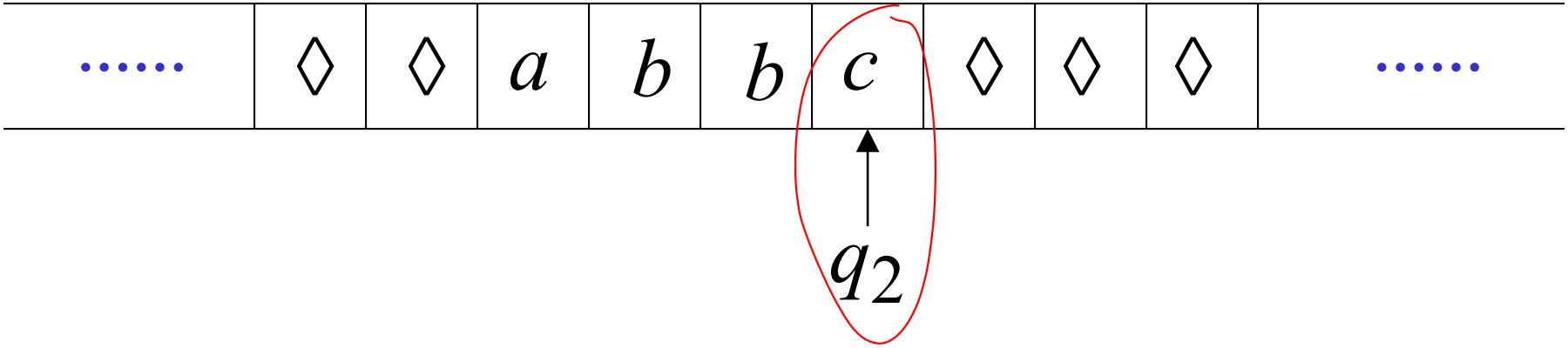
current state

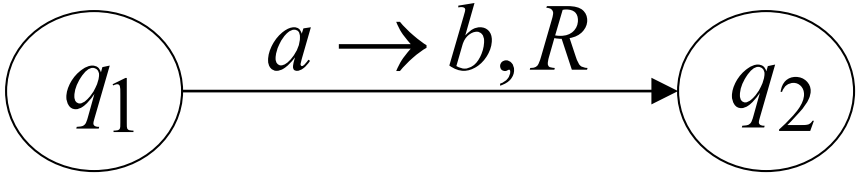


Time 1



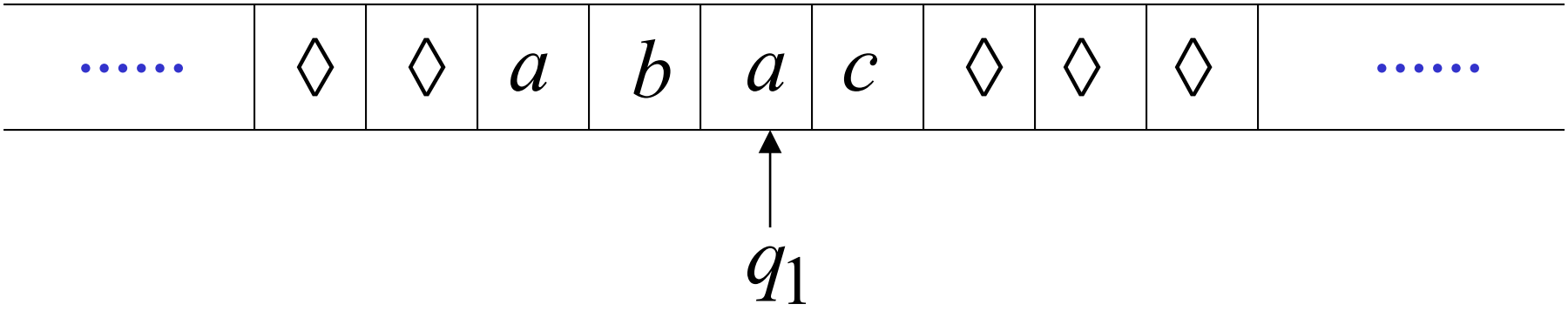
Time 2



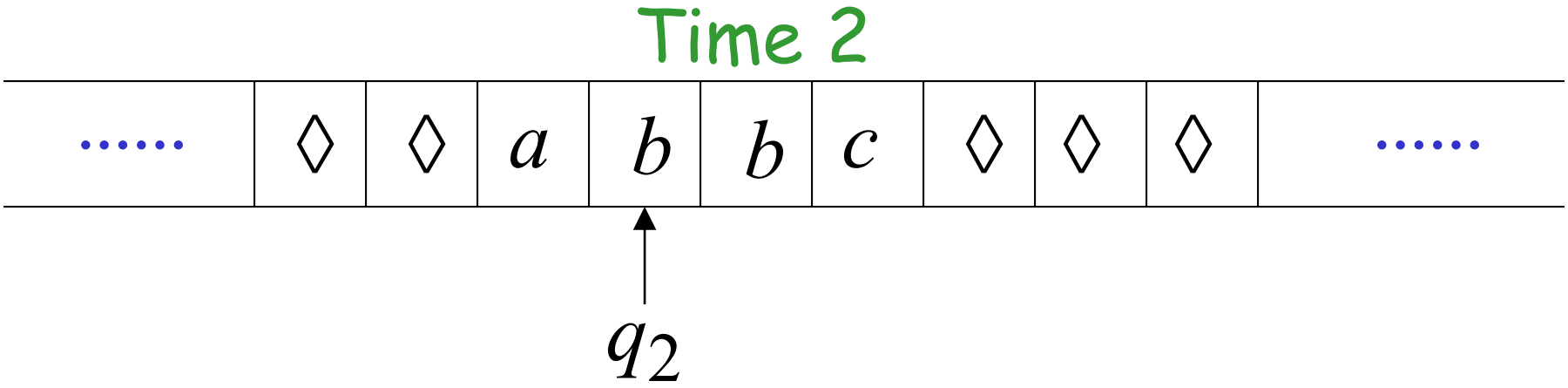


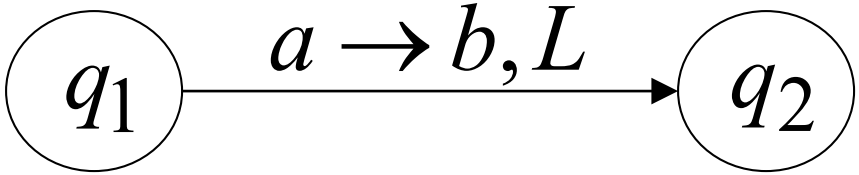
Example:

Time 1



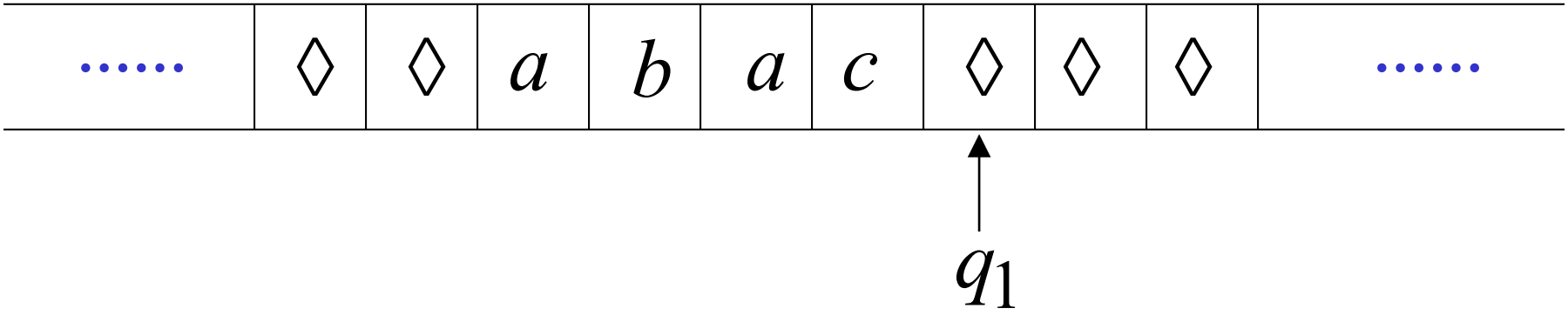
Time 2



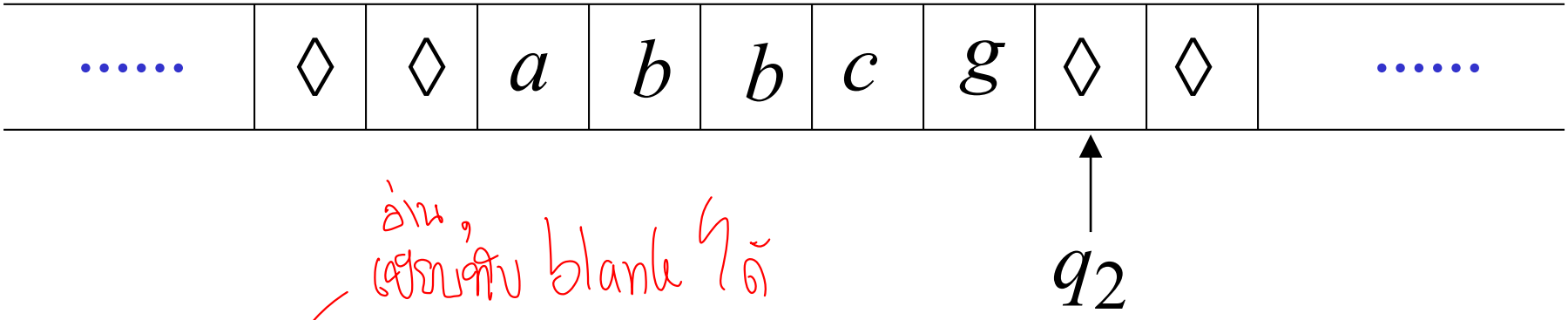


Example:

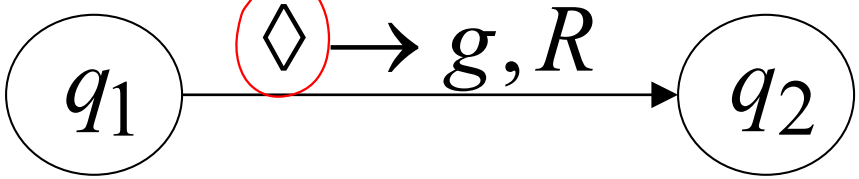
Time 1



Time 2



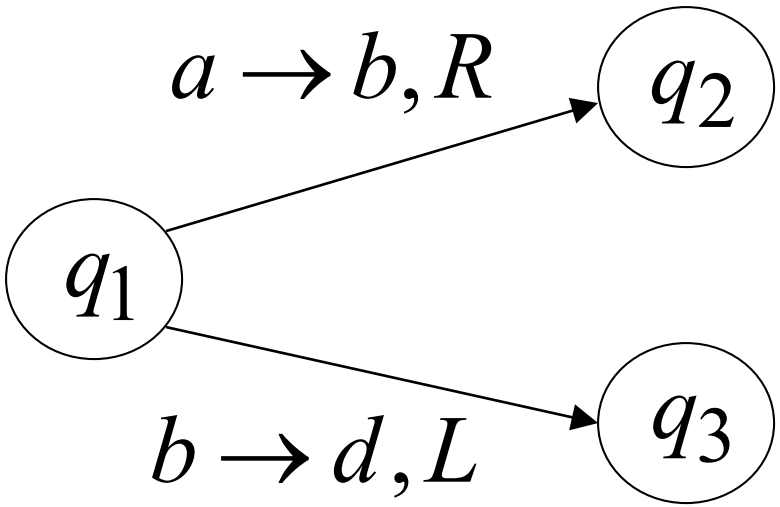
อ่าน, เขียนทับ blank ได้



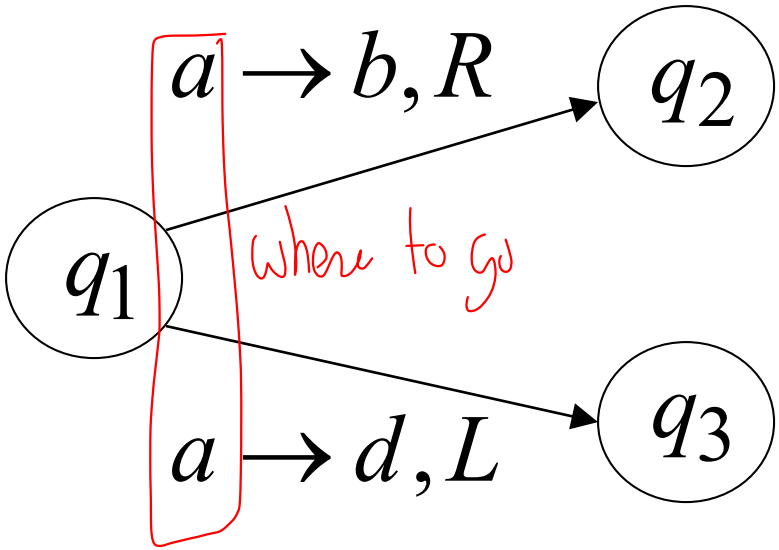
Determinism

Turing Machines are deterministic

Allowed



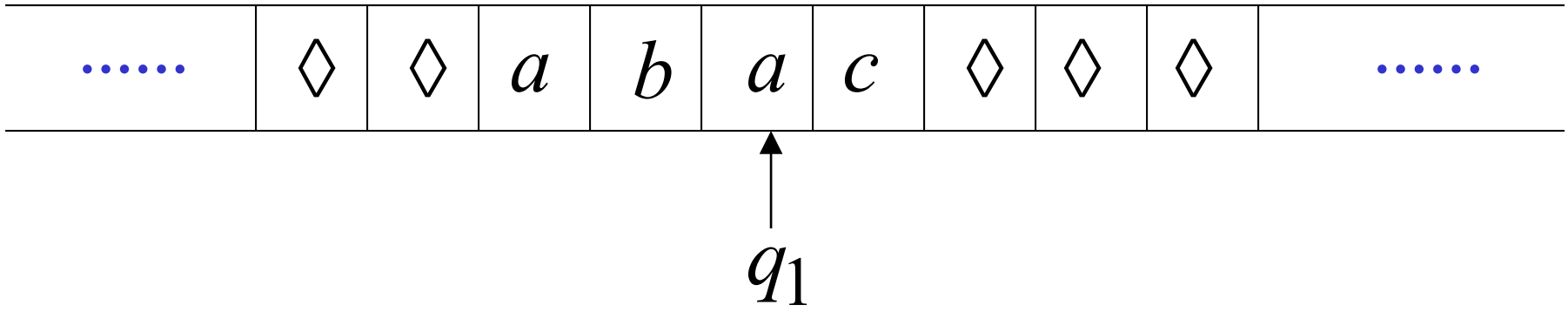
Not Allowed

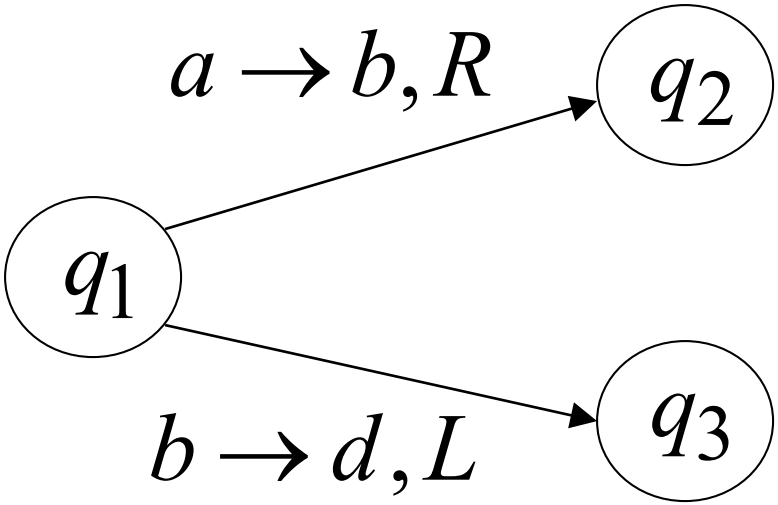


No lambda transitions allowed

Partial Transition Function

Example:





Allowed:

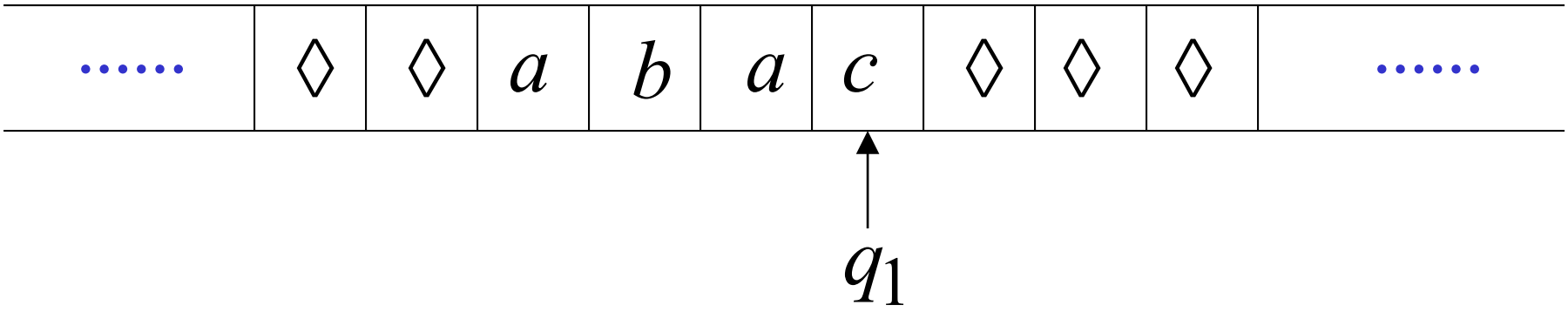
No transition for input symbol c

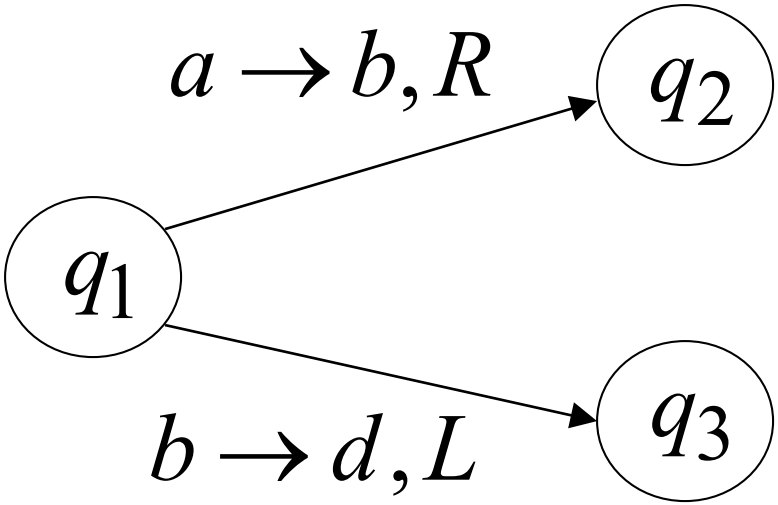
Halting

หยุดเมื่อการ halt = หยุด

The machine ***halts*** if there are no possible transitions to follow

Example:

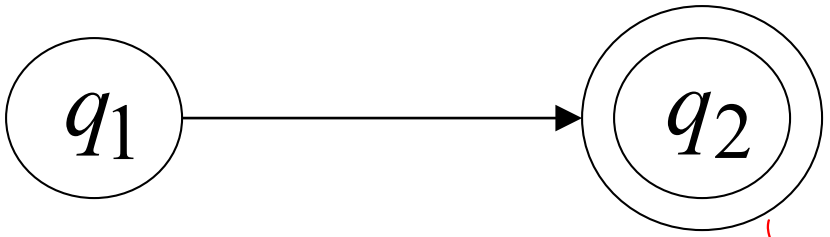




No possible transition

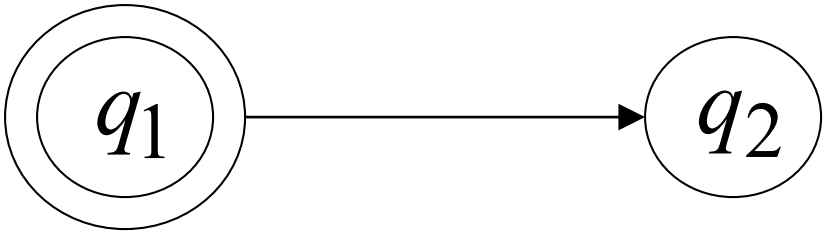
HALT!!!

Final States



Allowed

เข้าได้อย่างเดียว ห้ามออก

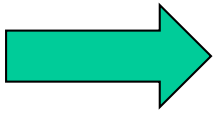


Not Allowed

- Final states have no outgoing transitions
- In a final state the machine halts

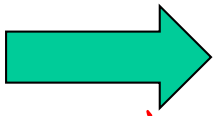
Acceptance

Accept Input



If machine halts
in a final state

Reject Input



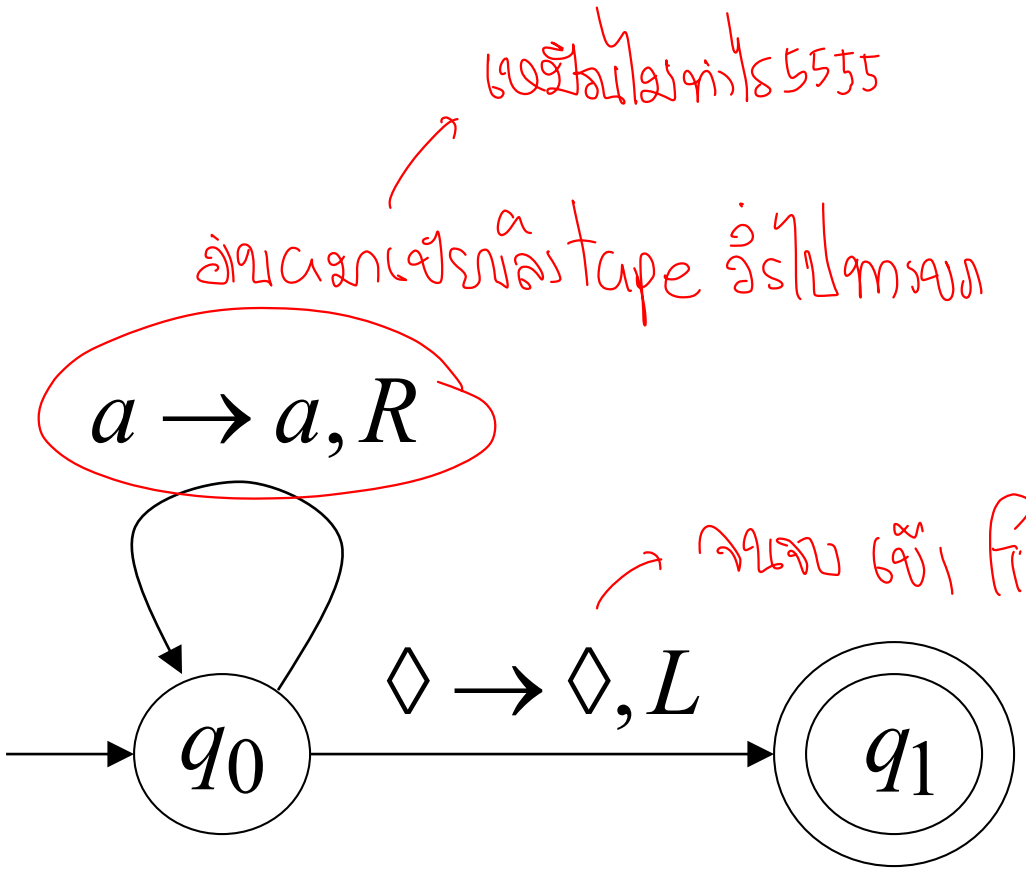
If machine halts
in a non-final state

or

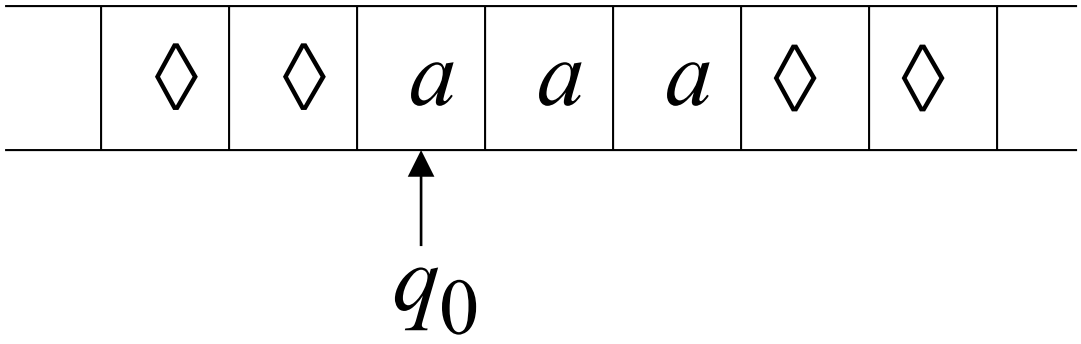
If machine enters
an *infinite loop*

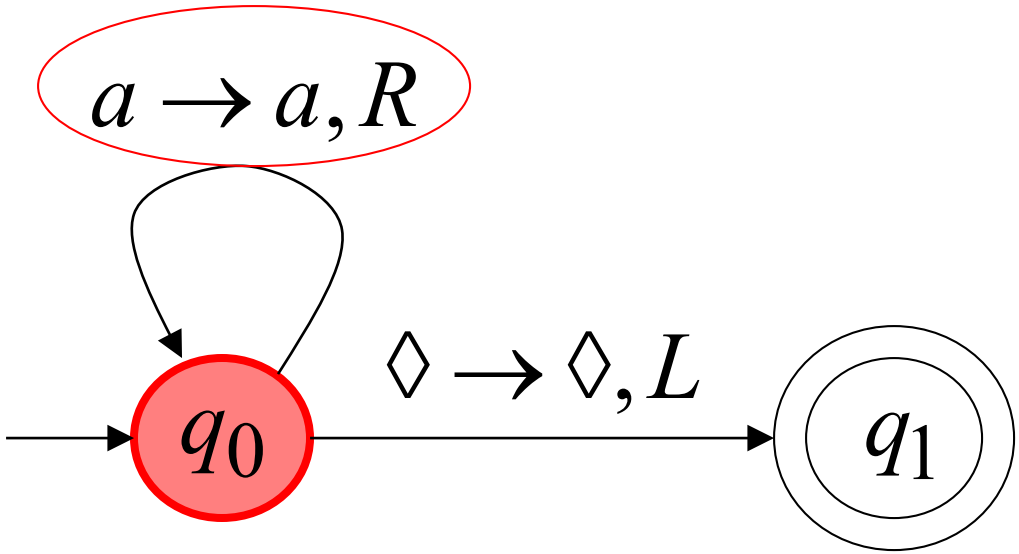
Turing Machine Example

A Turing machine that accepts the language:

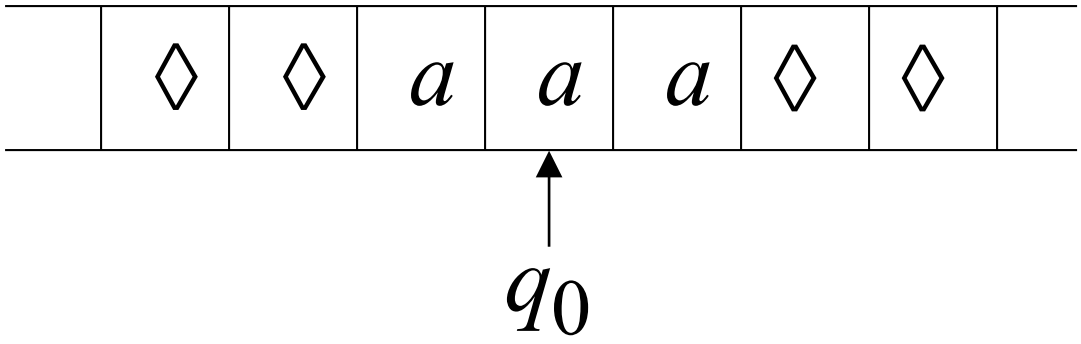


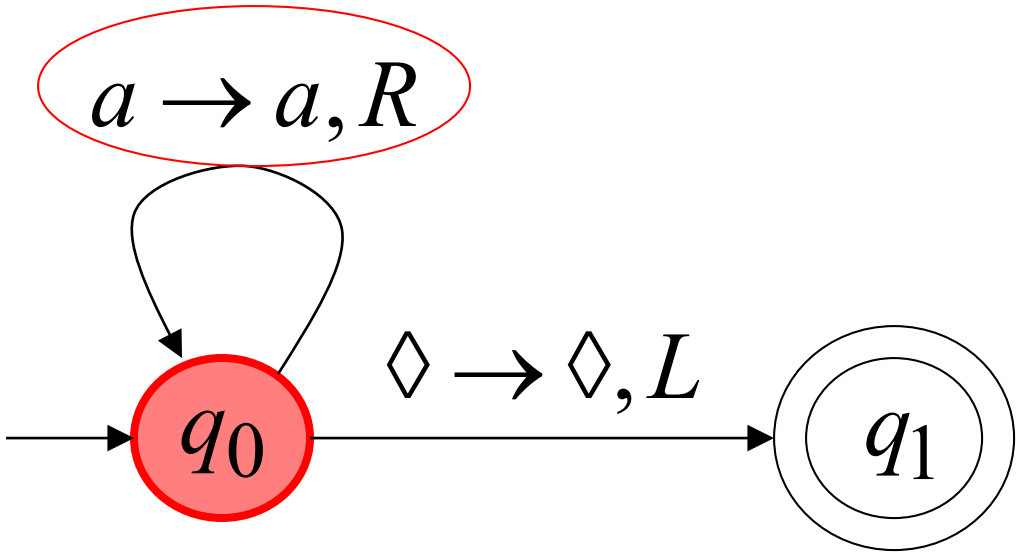
Time 0



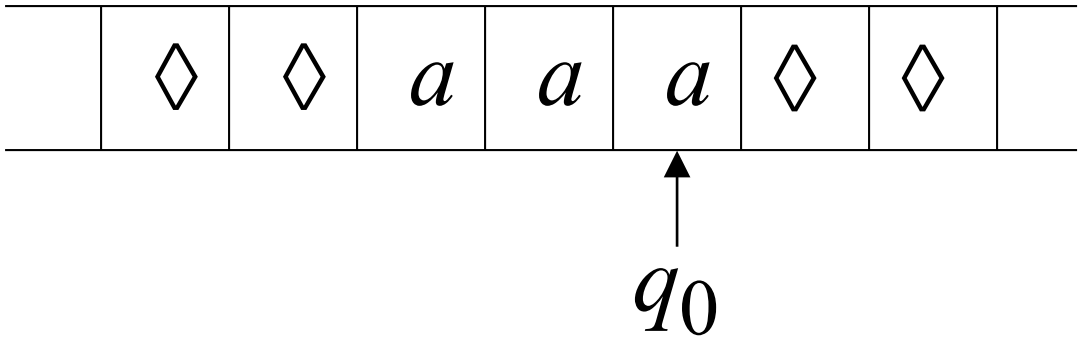


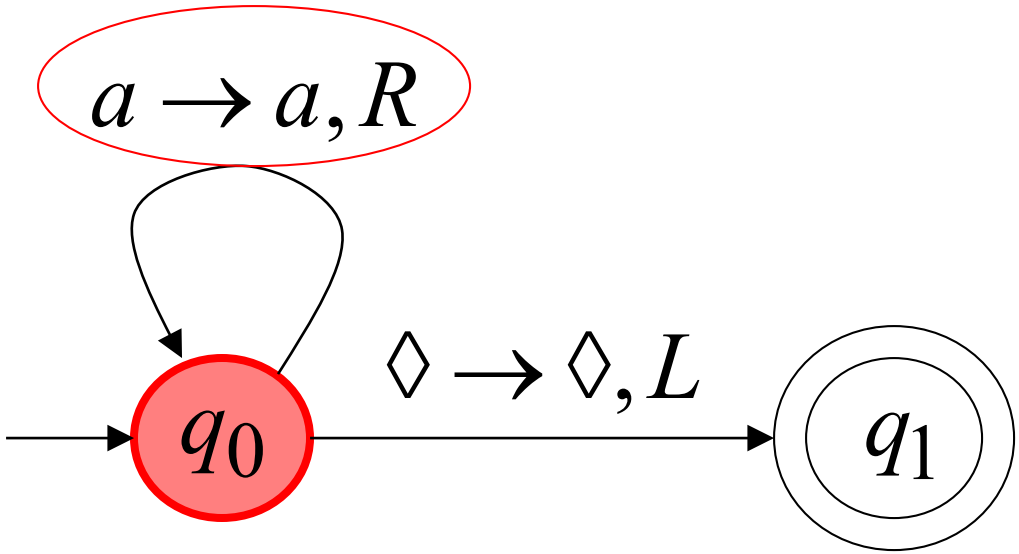
Time 1



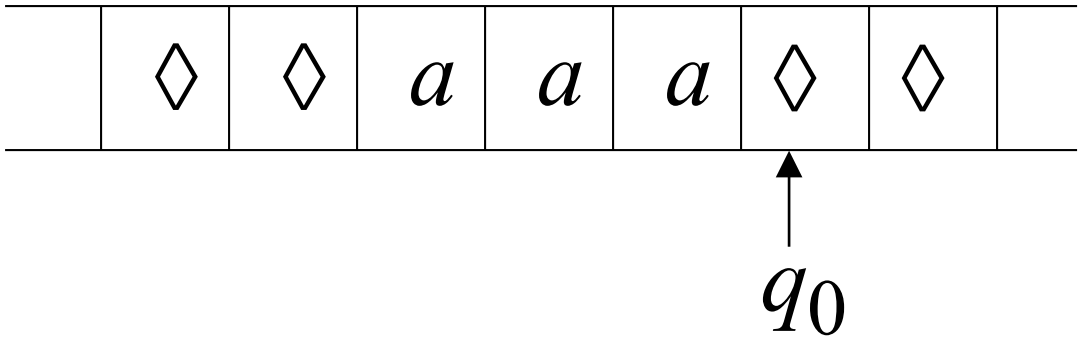


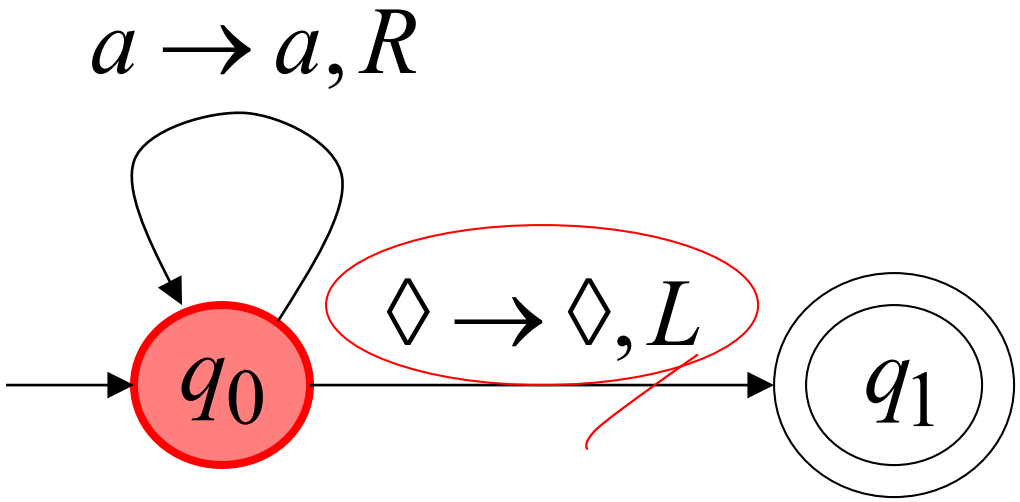
Time 2



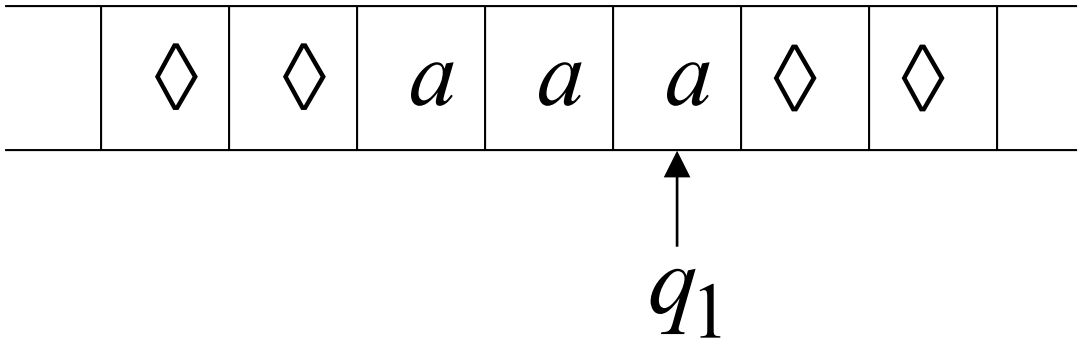


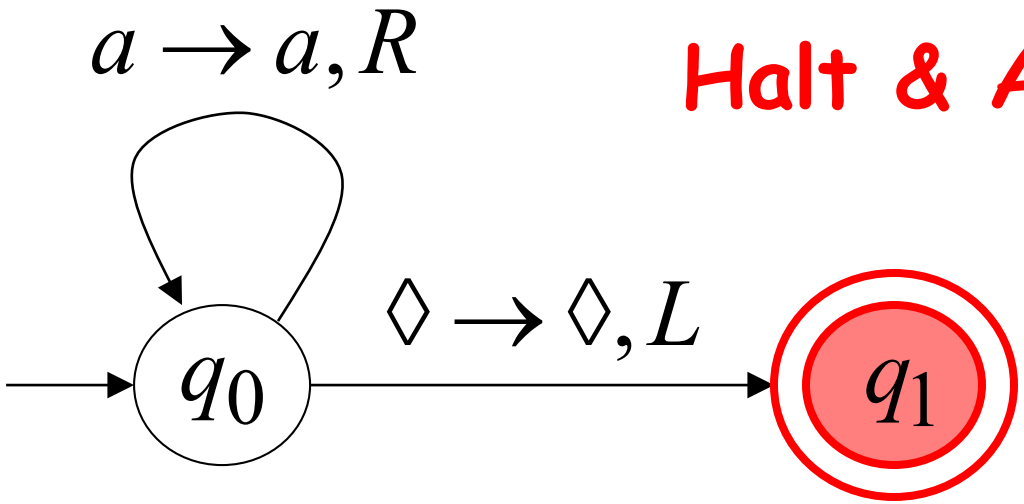
Time 3





Time 4

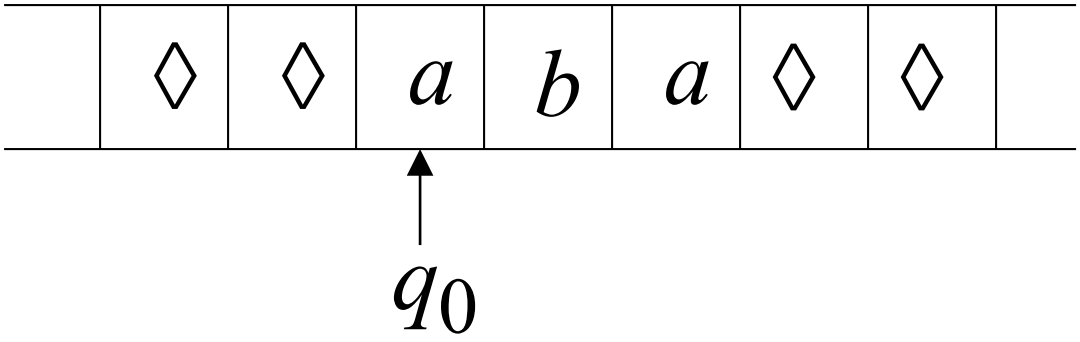


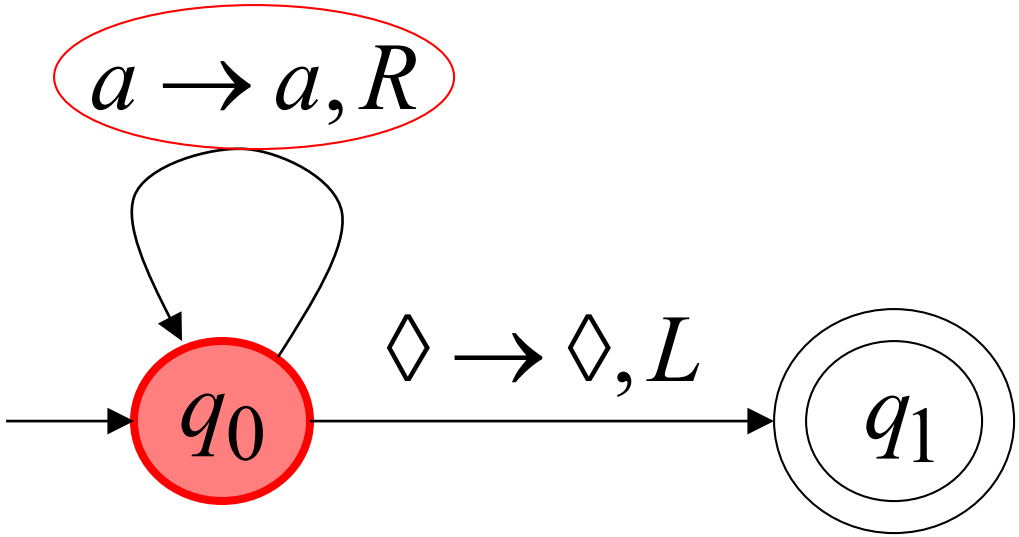


Halt & Accept

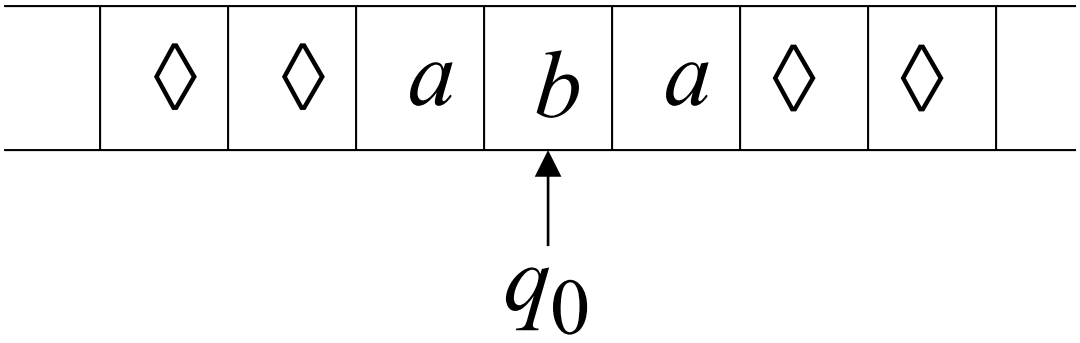
Rejection Example ตรวจสอบการ reject

Time 0



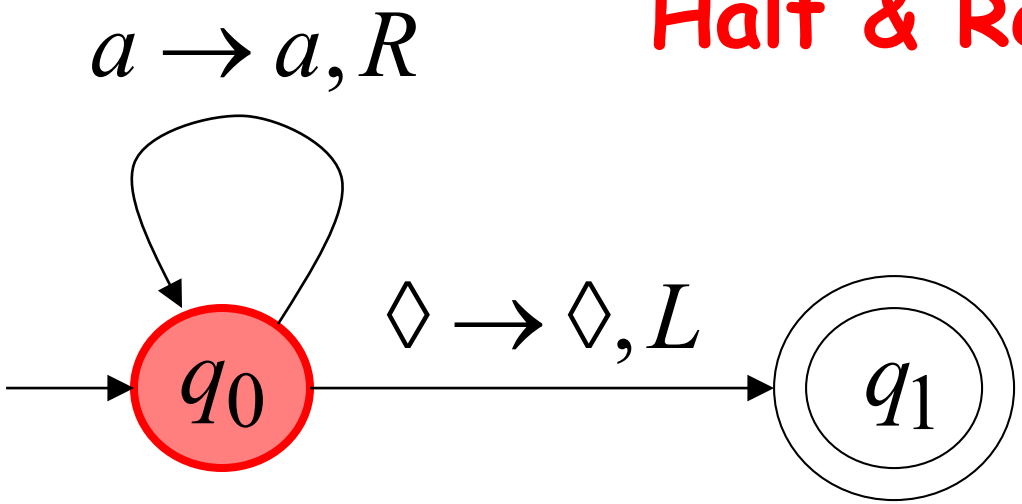


Time 1

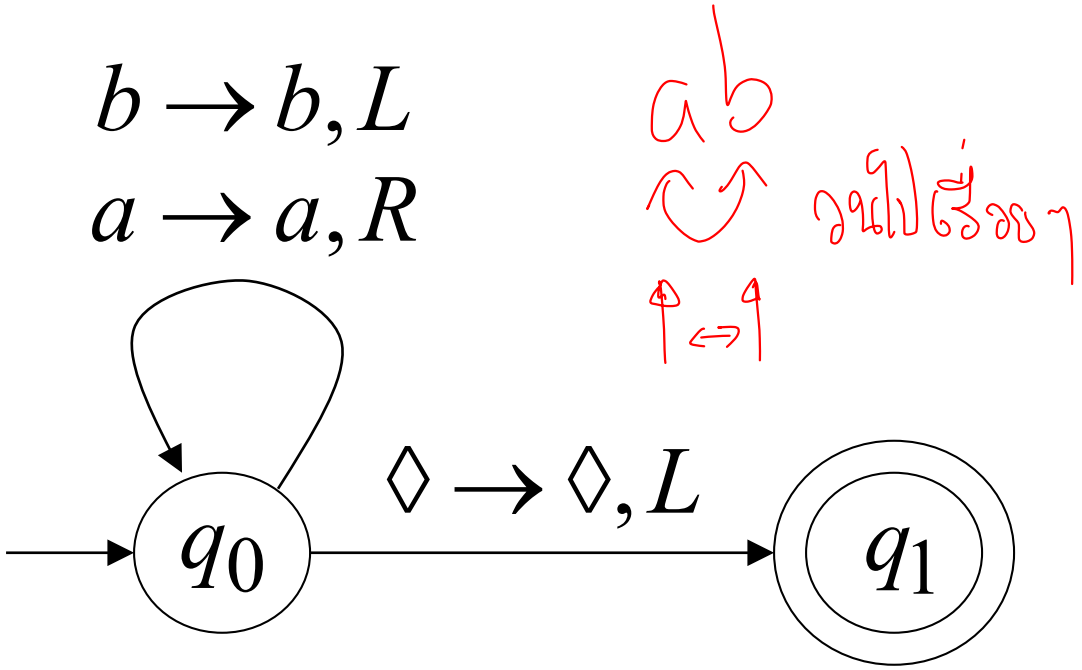


No possible Transition

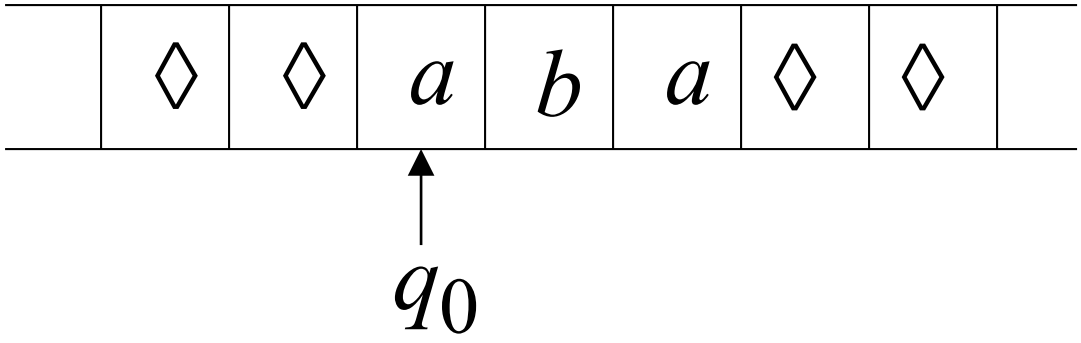
Halt & Reject

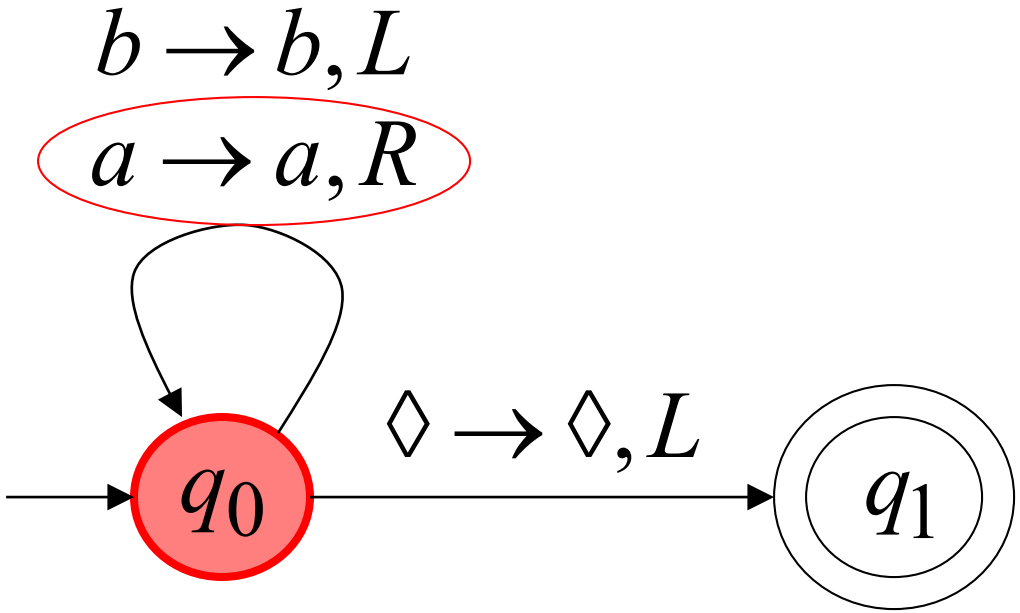


Infinite Loop Example

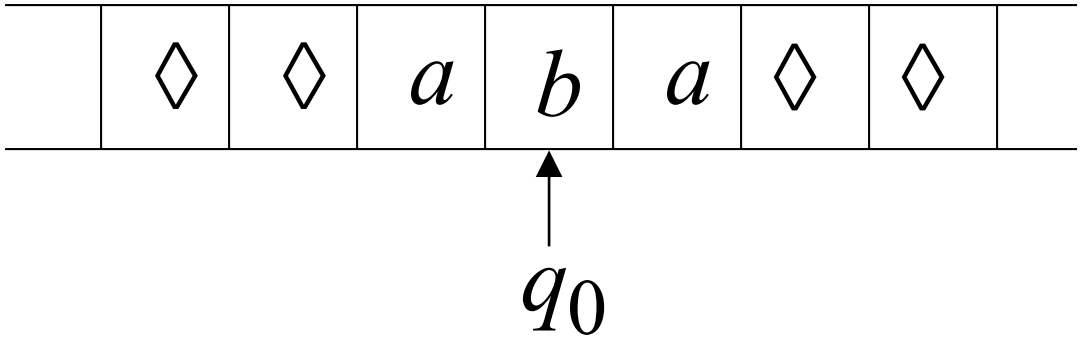


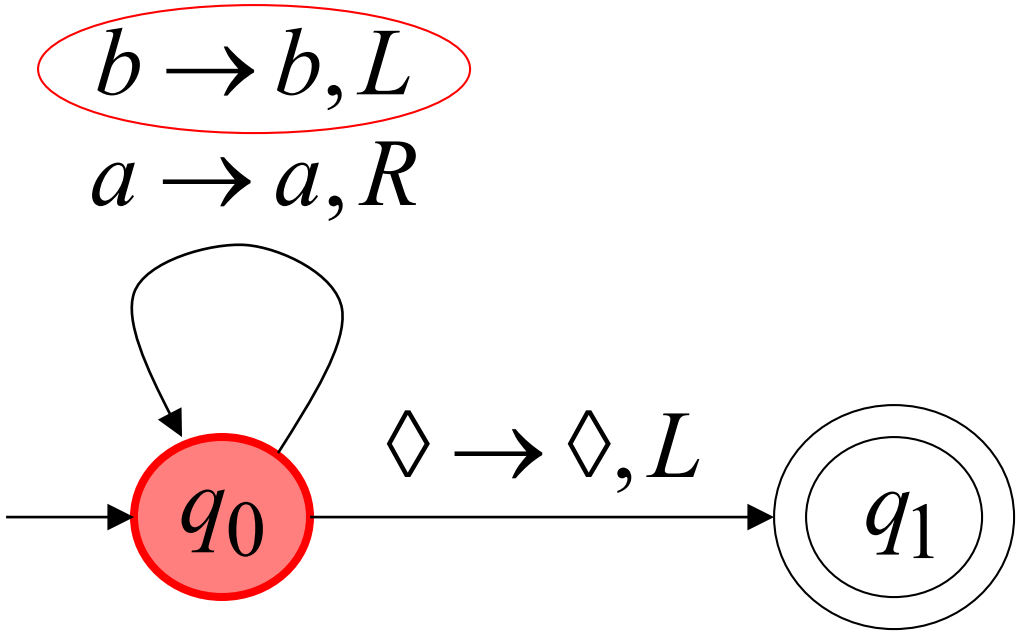
Time 0



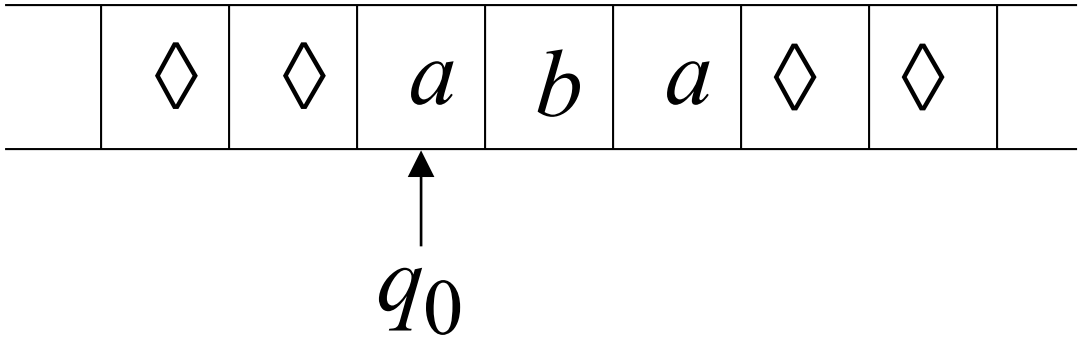


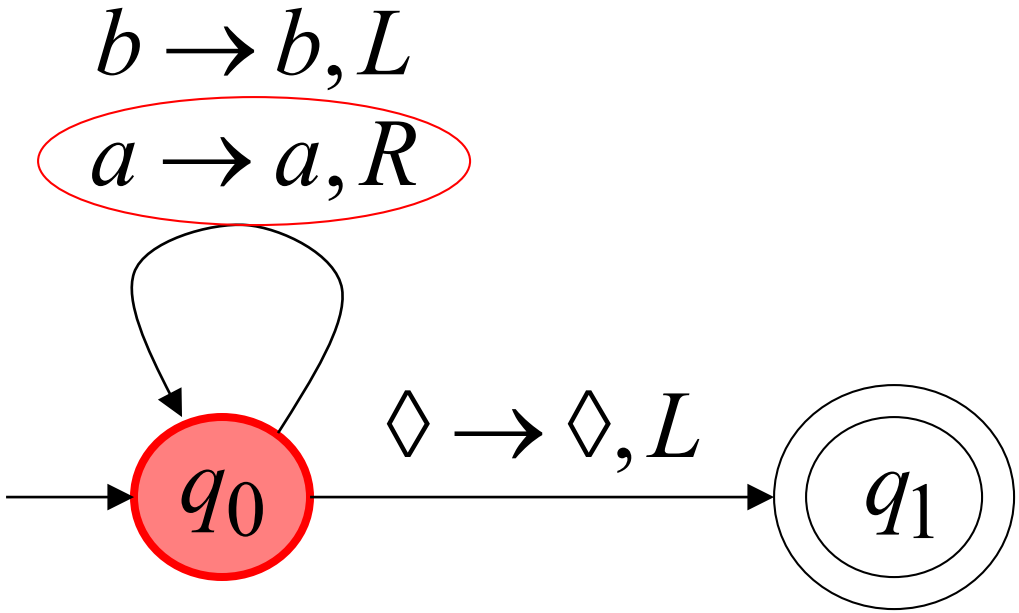
Time 1



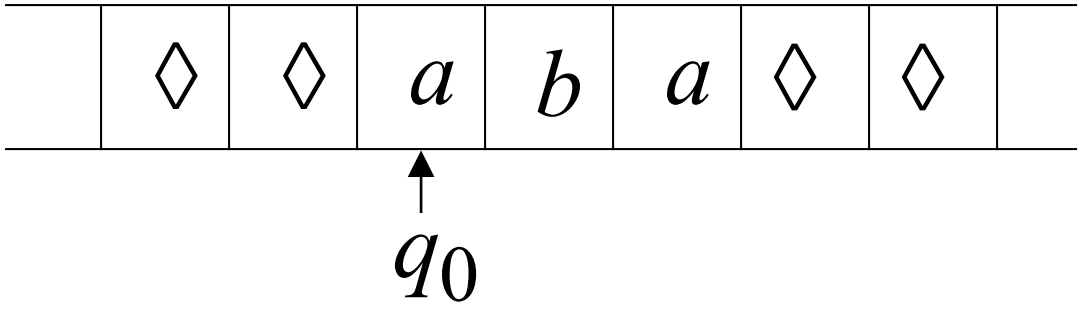


Time 2

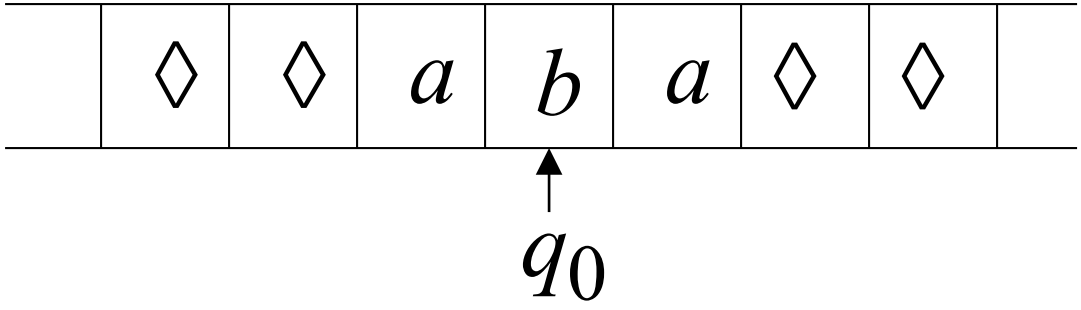




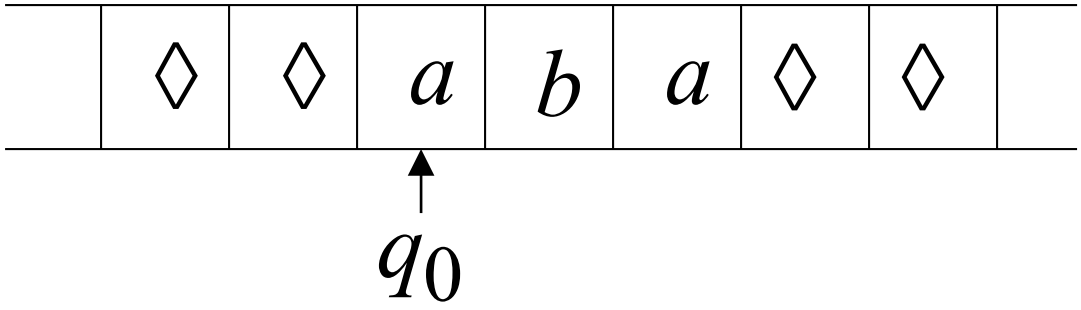
Time 2



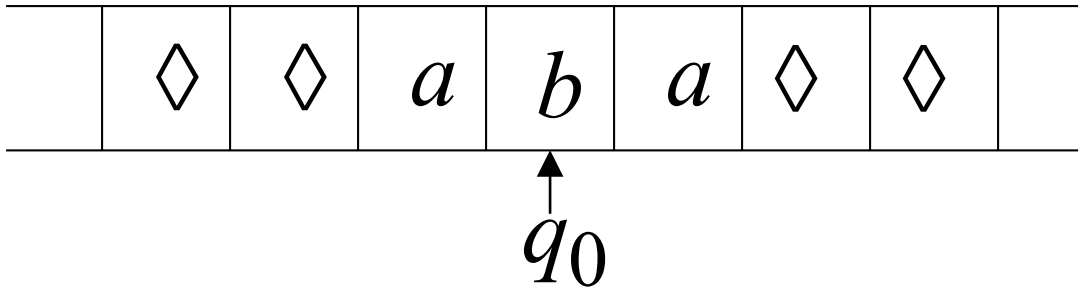
Time 3



Time 4



Time 5



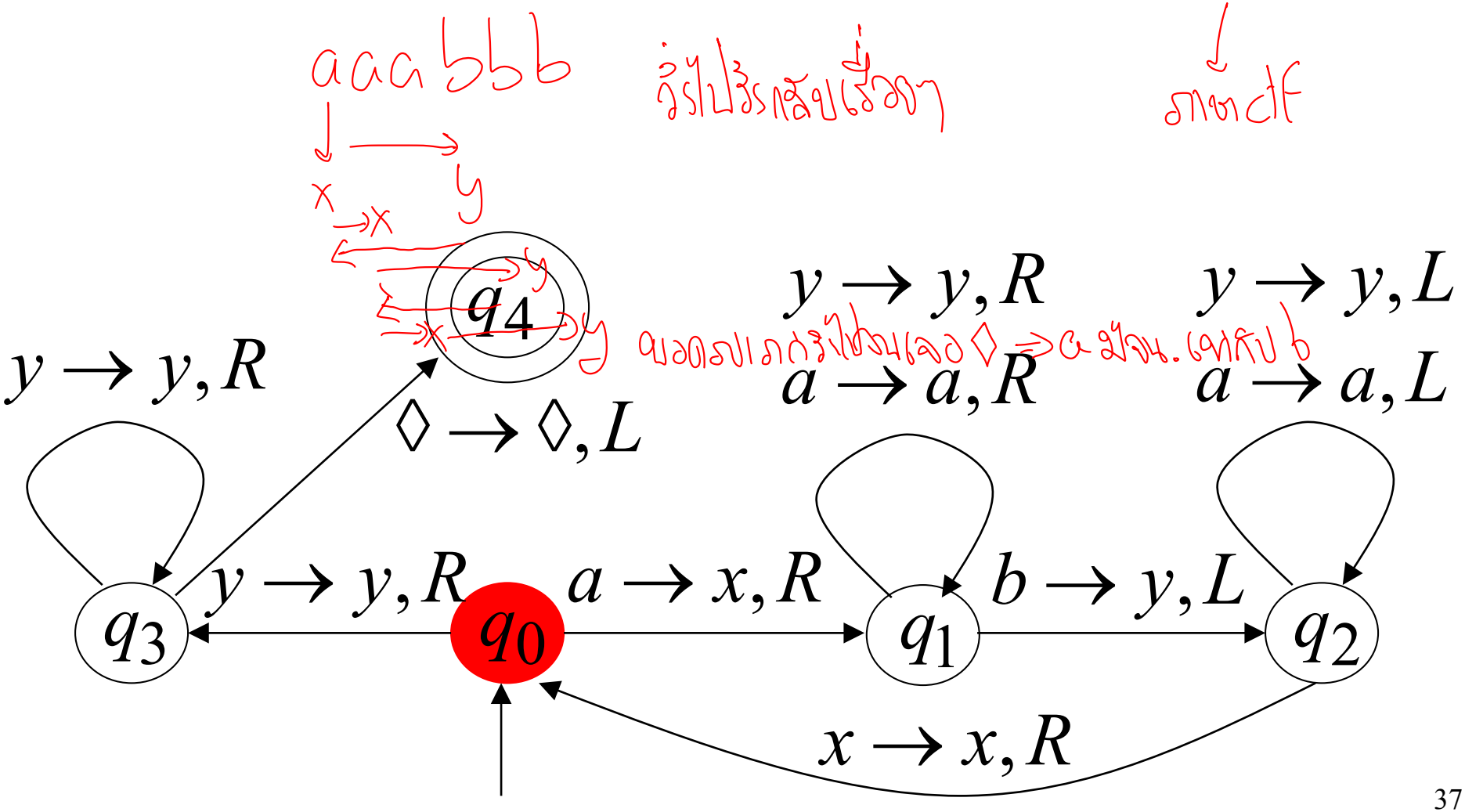
Infinite loop

Because of the infinite loop:

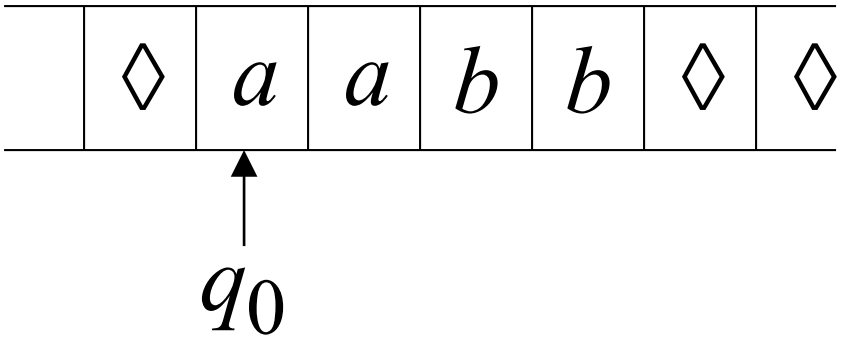
- The final state cannot be reached
- The machine never halts
- The input is not accepted

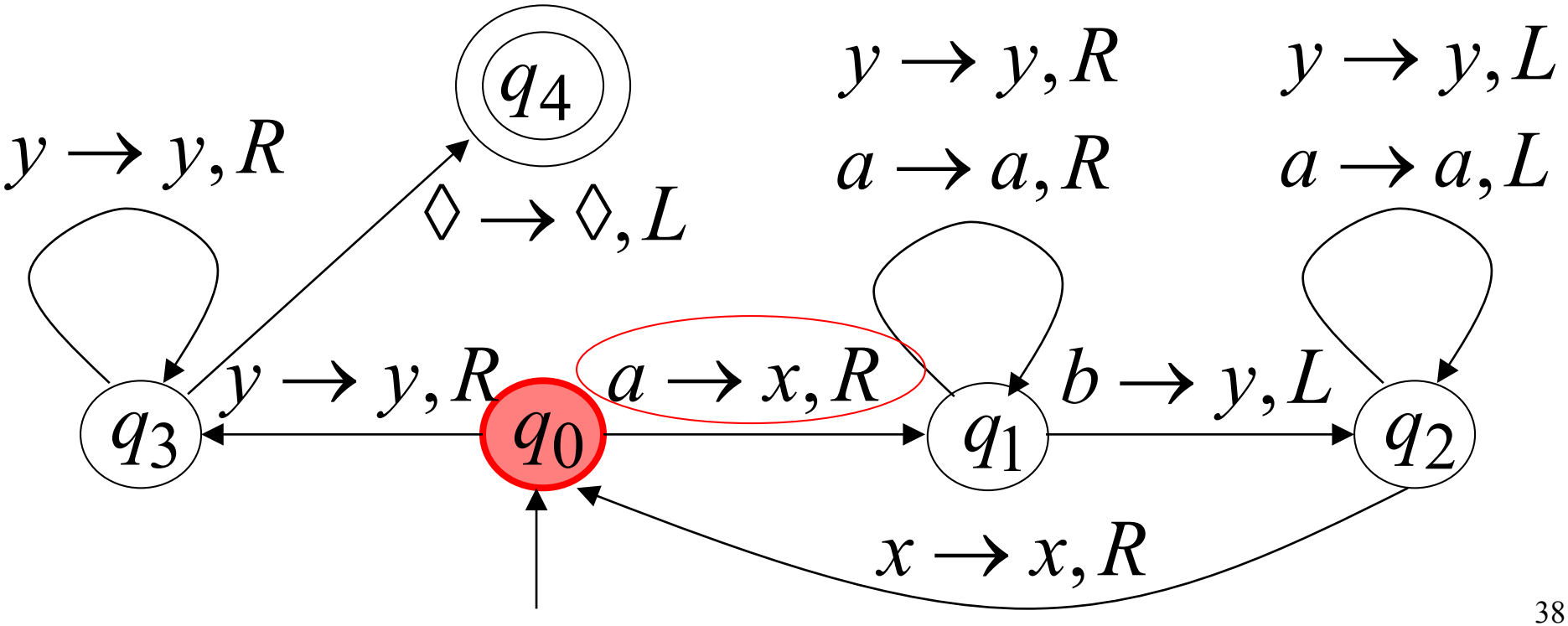
Another Turing Machine Example

Turing machine for the language $\{a^n b^n\}$

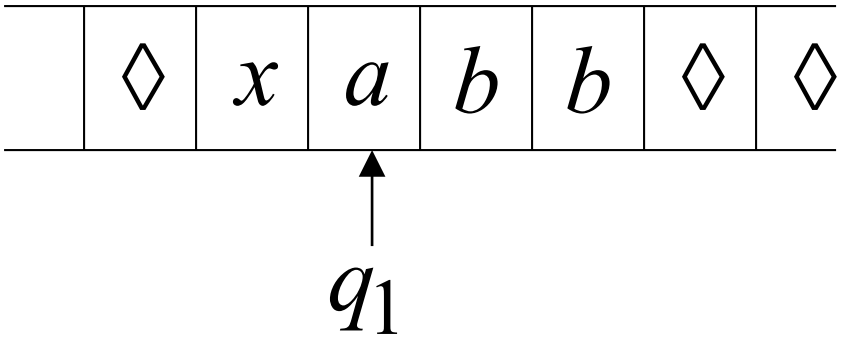


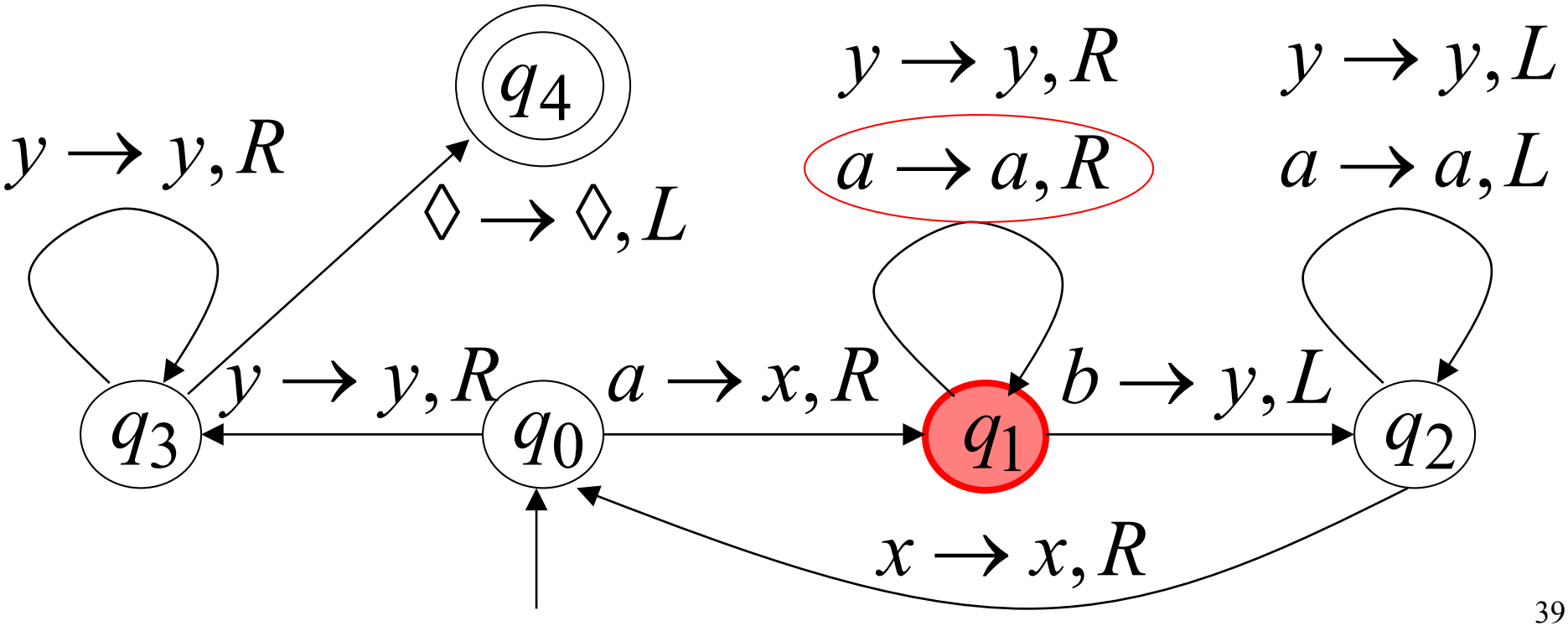
Time 0


◊ a a b b ◊ ◊
q_0

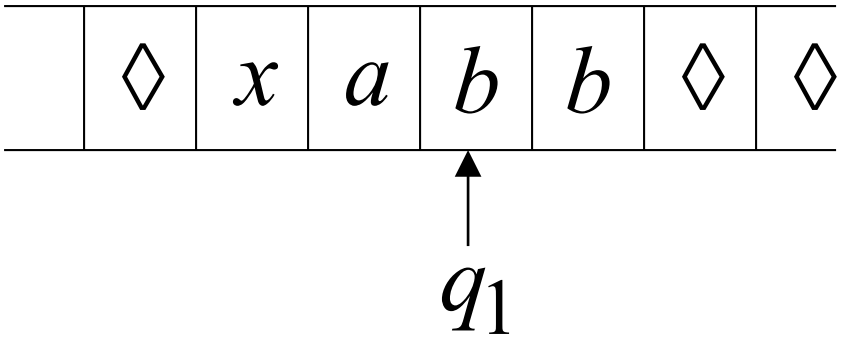

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

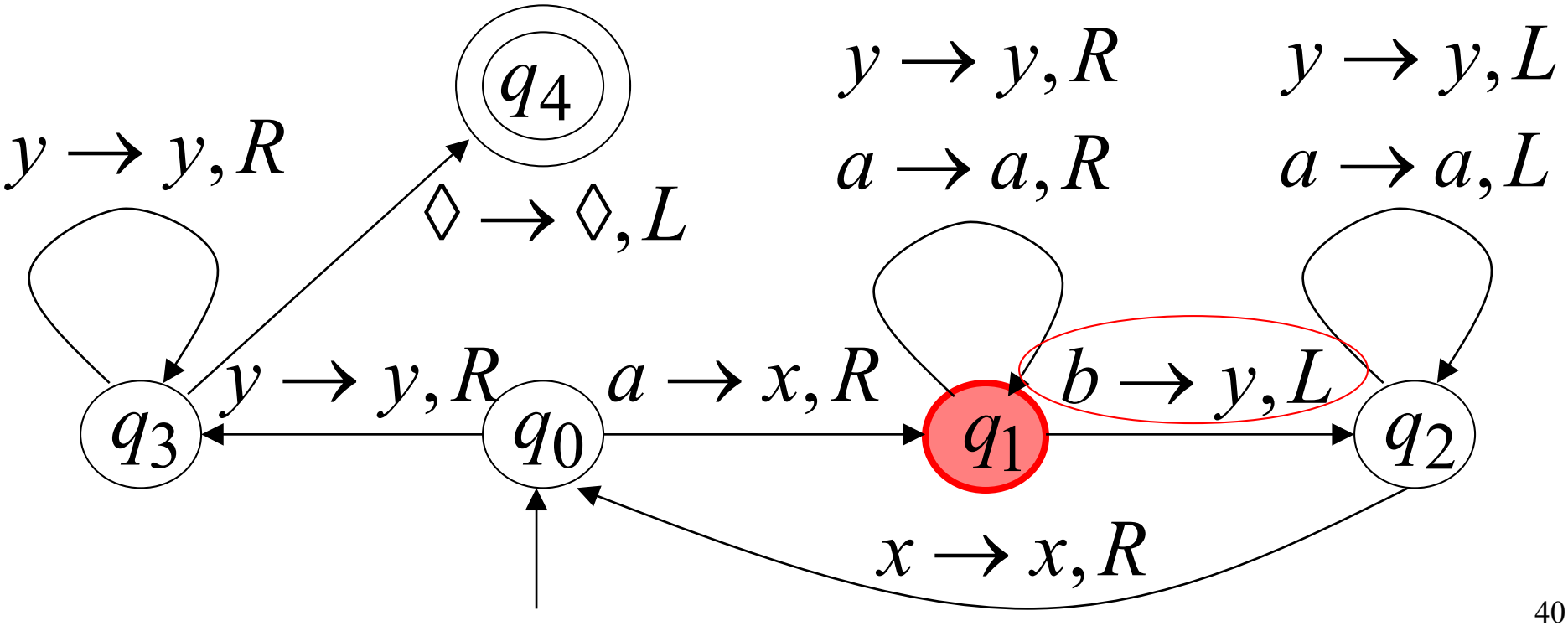
Time 1


◊ x a b b ◊ ◊
q_1

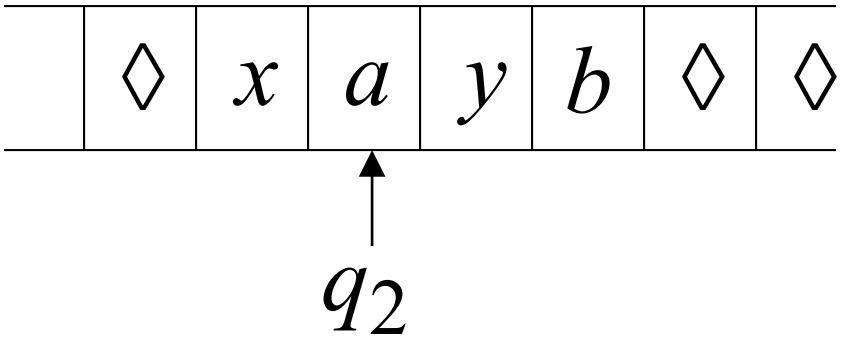

q_4
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
q_3
$y \to y, R$
q_0
$a \to x, R$
q_1
$b \to y, L$
q_2
$x \to x, R$

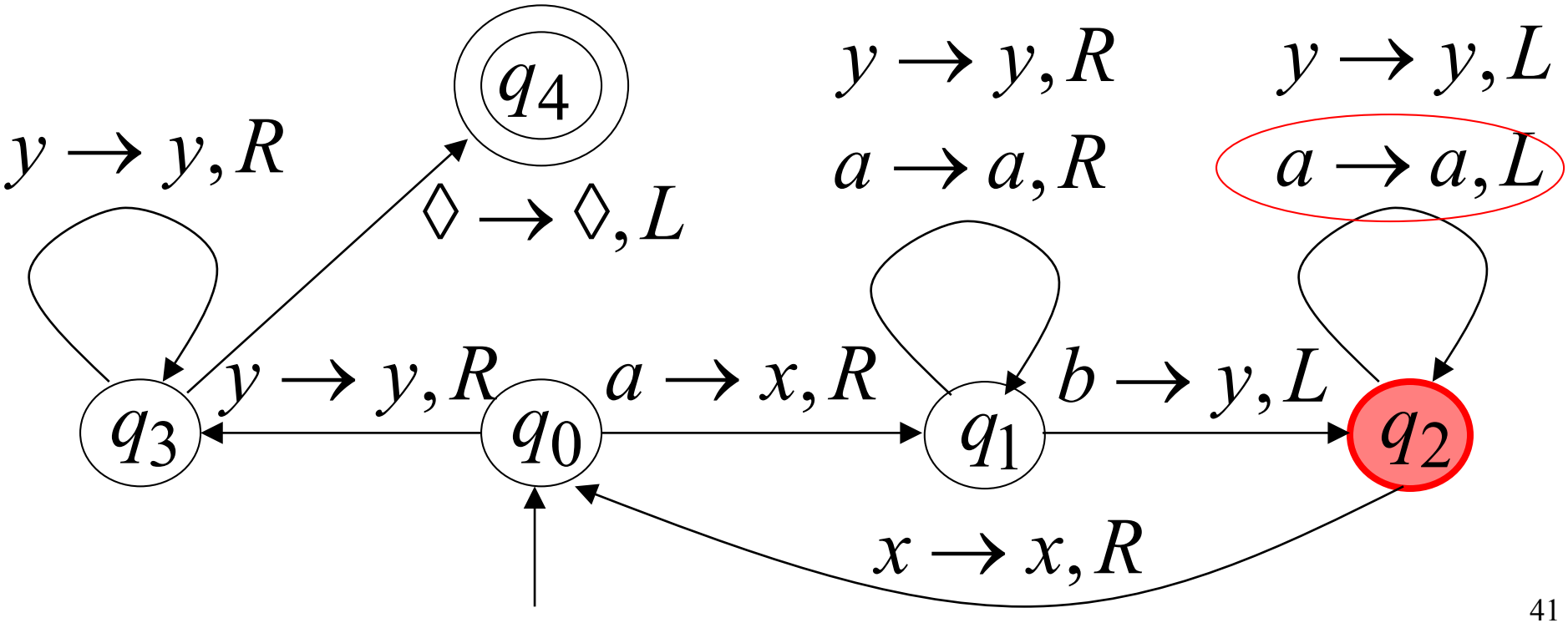
Time 2


◊ x a b b ◊ ◊
q_1

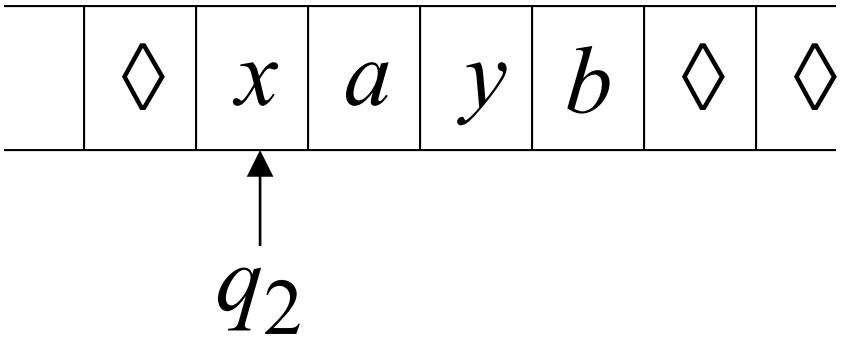

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

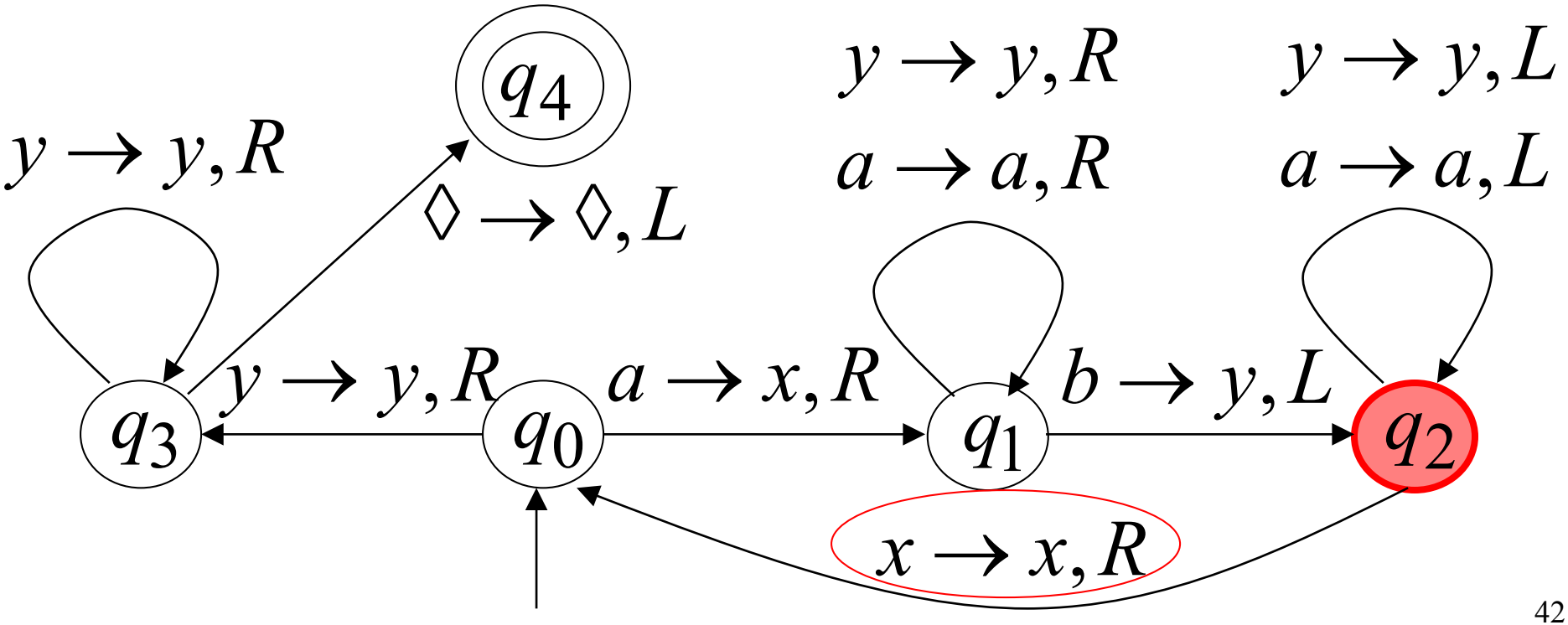
Time 3


◊ x a y b ◊ ◊
q_2

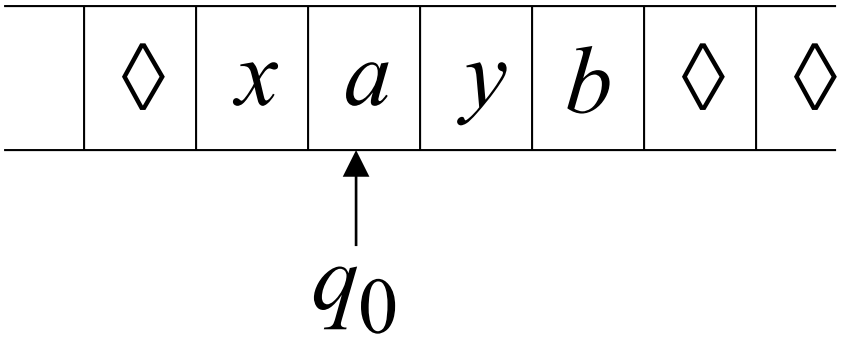

q_4
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
q_3
$y \to y, R$
q_0
$a \to x, R$
q_1
$b \to y, L$
q_2
$x \to x, R$

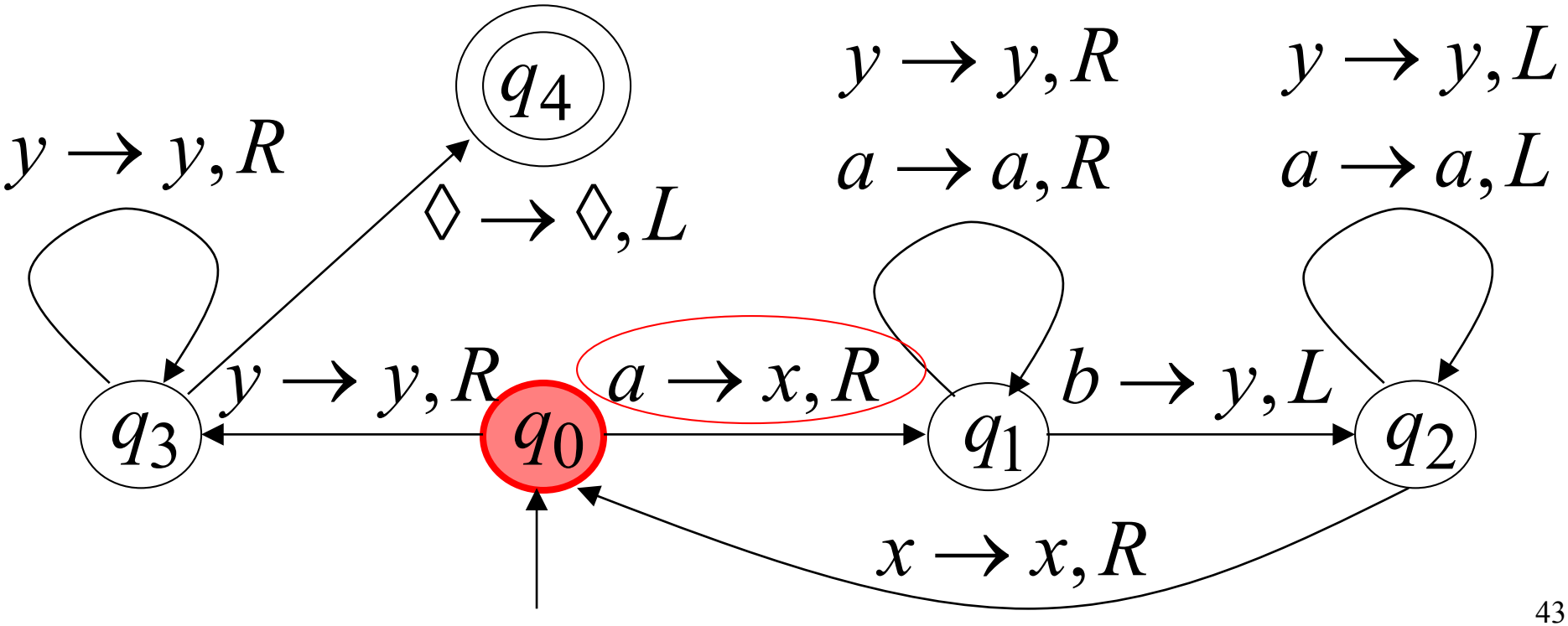
Time 4


◊ x a y b ◊ ◊
q_2

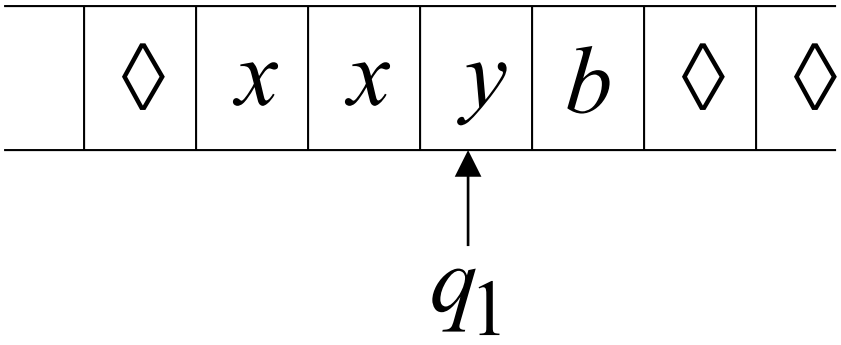

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

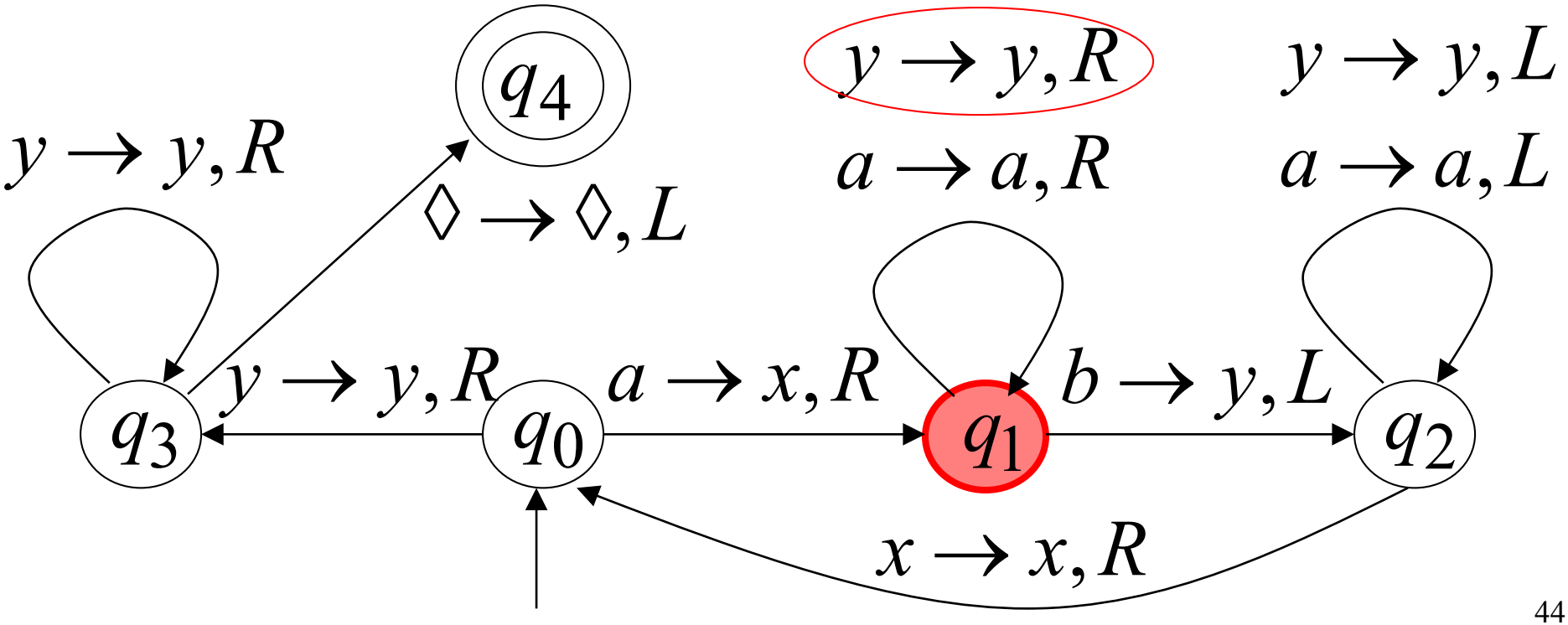
Time 5


◊ x a y b ◊ ◊
q_0

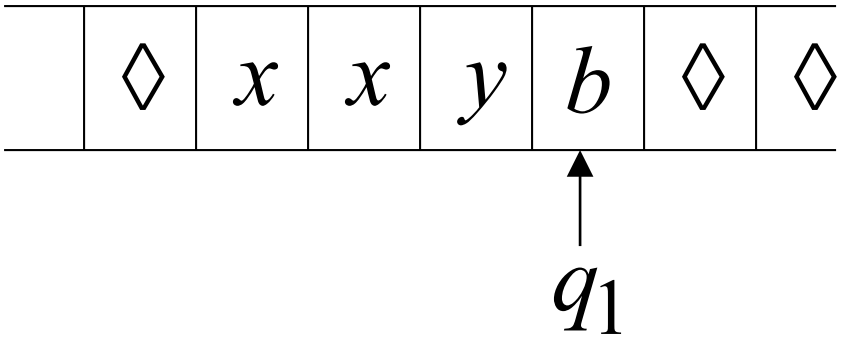

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

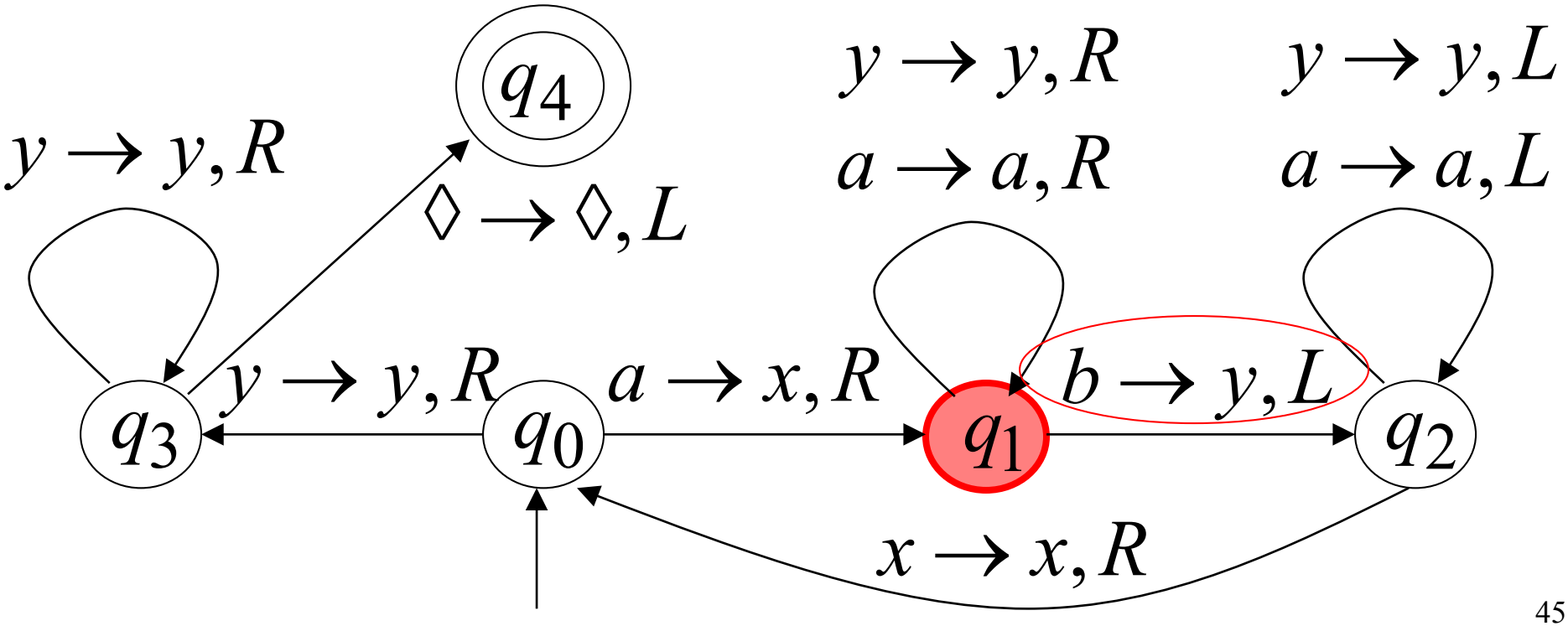
Time 6


◊ x x y b ◊ ◊
q_1

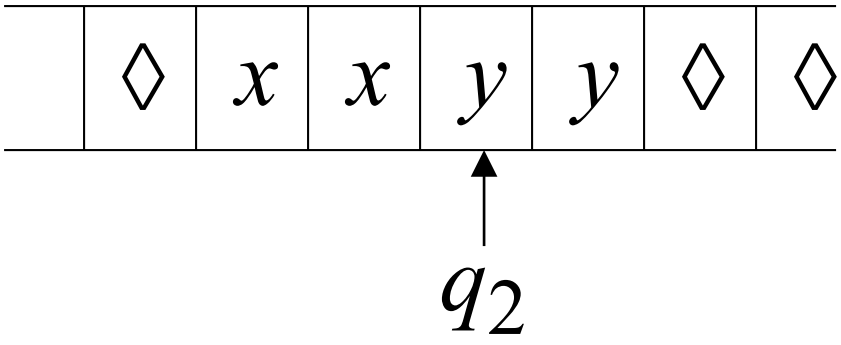

q_4
$y \to y, R$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

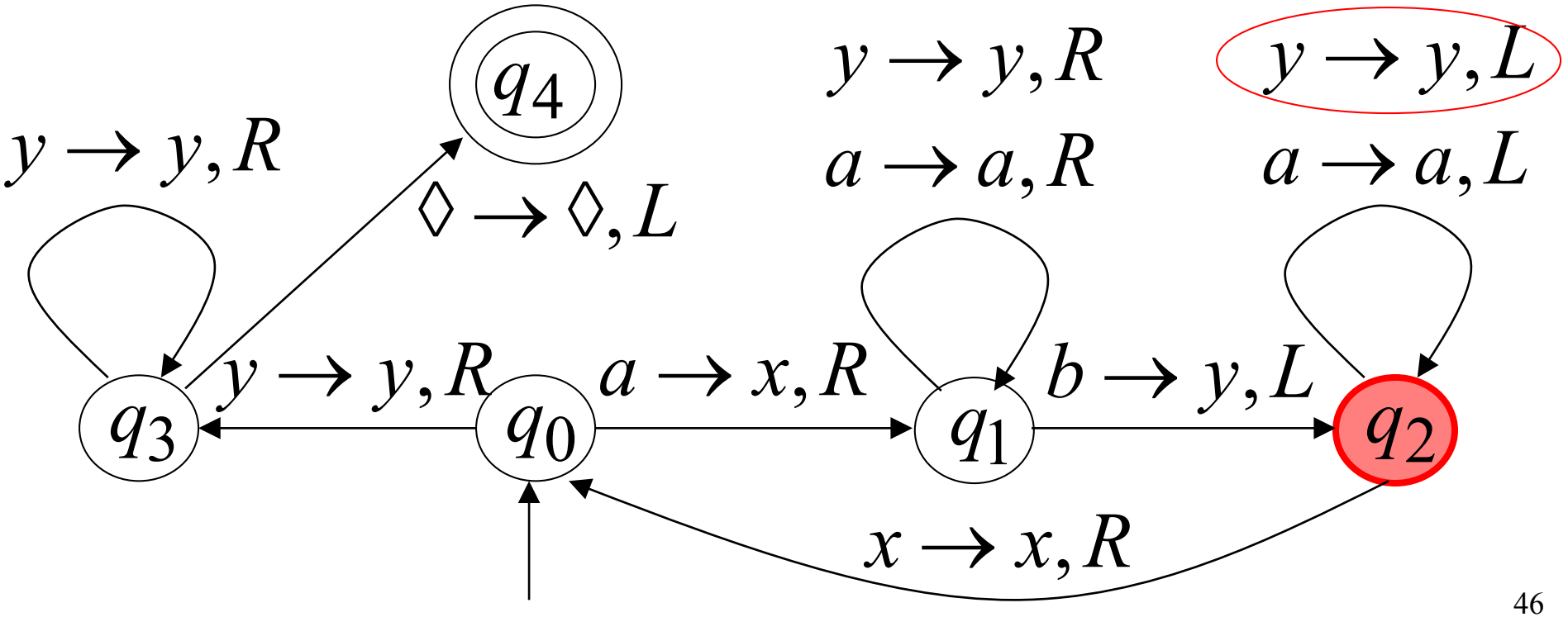
Time 7


◊ x x y b ◊ ◊
q_1

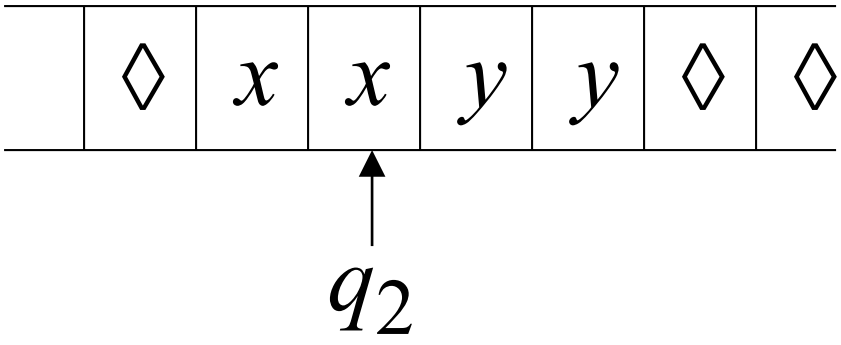

q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

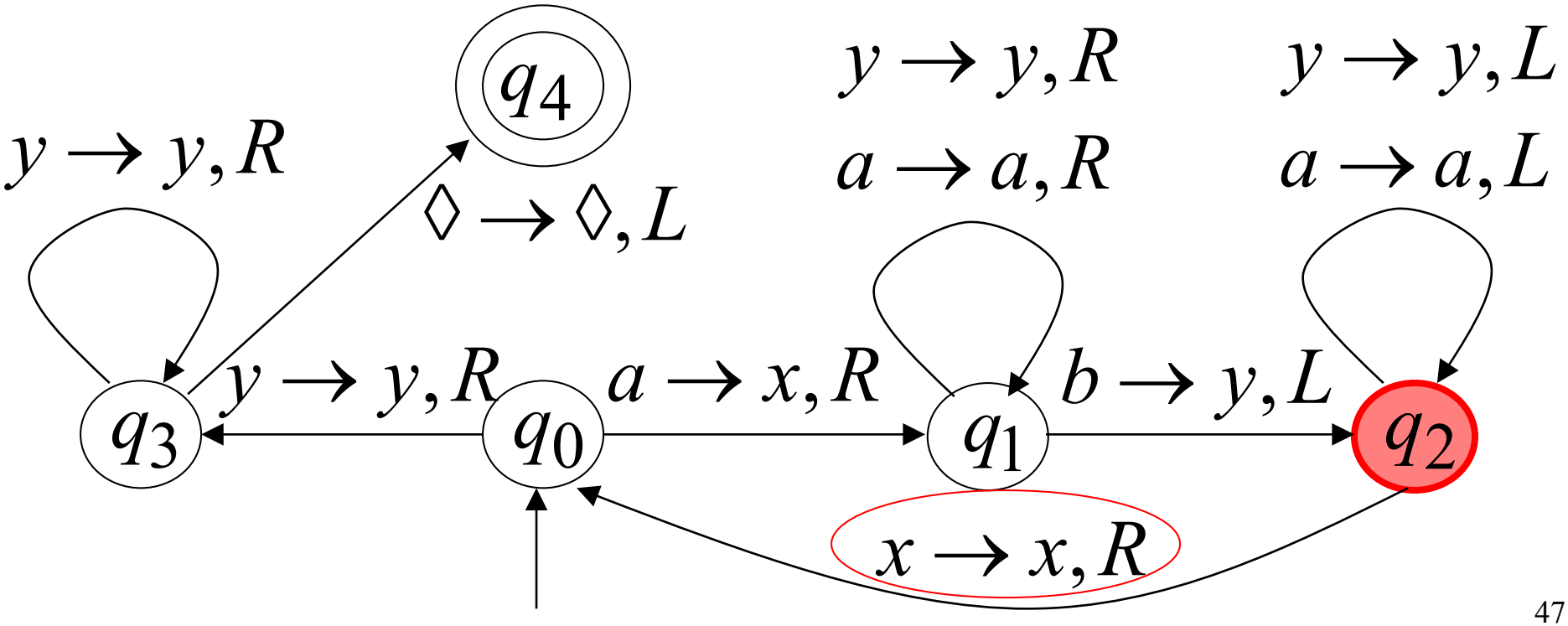
Time 8


◊ x x y y ◊ ◊
q_2

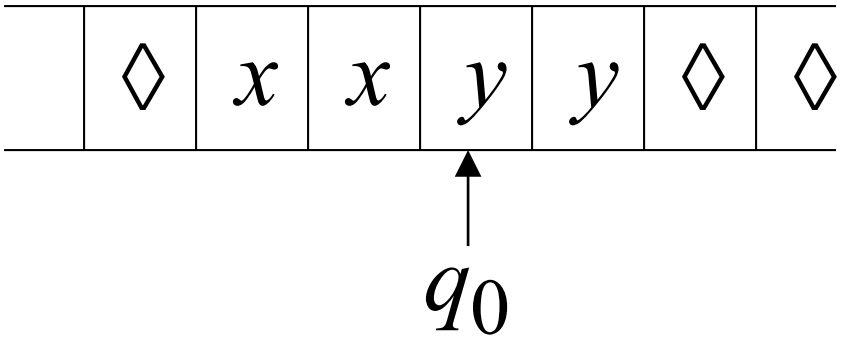

q_4
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
q_3
$y \to y, R$
q_0
$a \to x, R$
q_1
$b \to y, L$
q_2
$x \to x, R$

Time 9

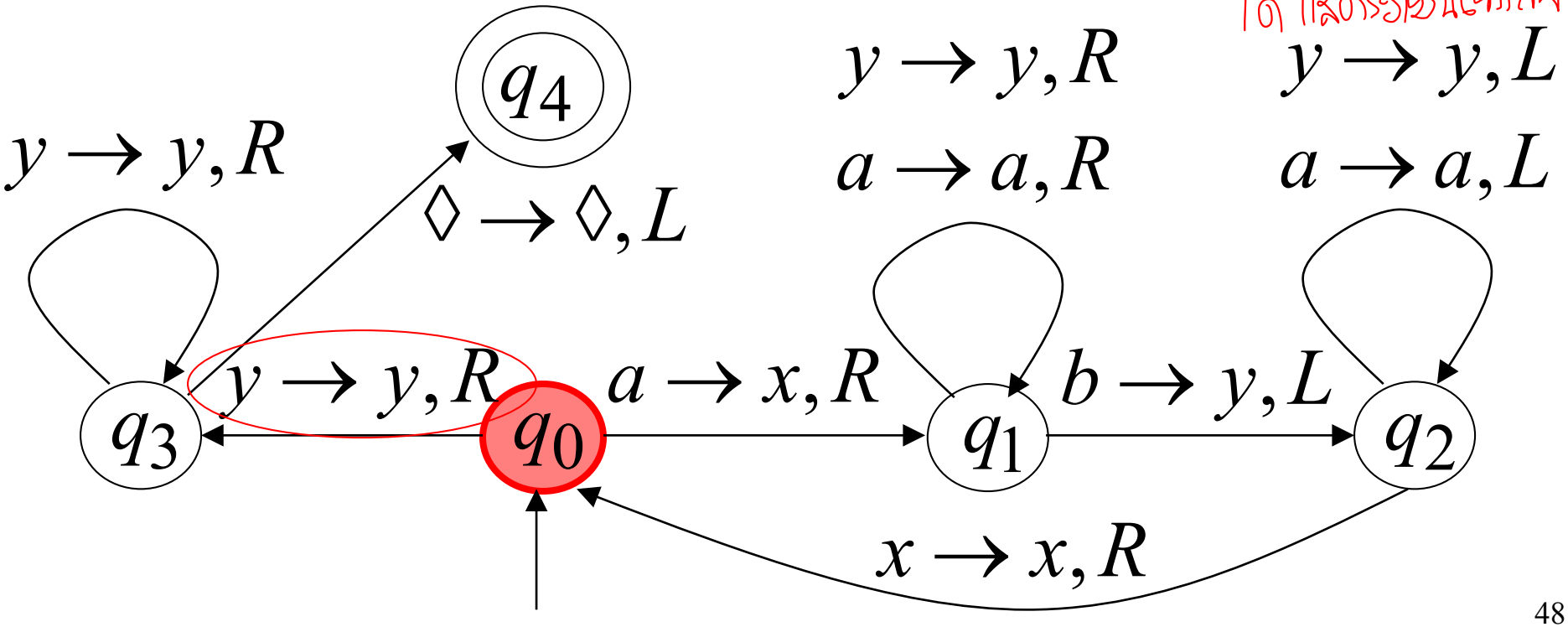

◊ x x y y ◊ ◊
q_2


q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
$y \to y, R$
$a \to x, R$
$b \to y, L$
q_3
q_0
q_1
q_2
$x \to x, R$

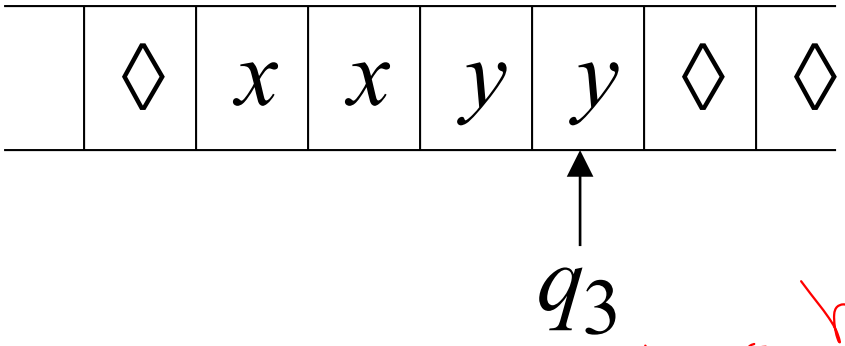
Time 10

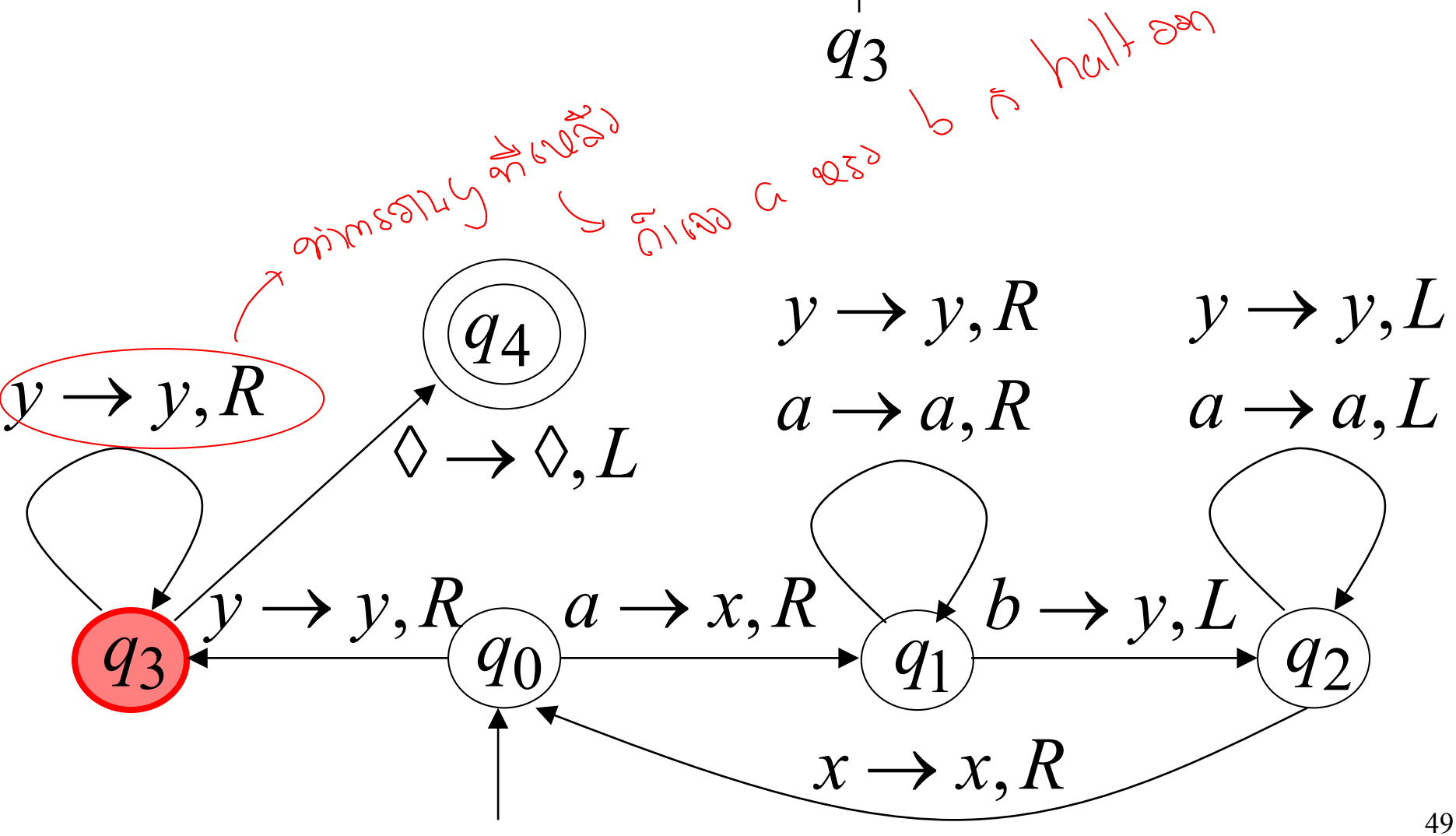


เมื่อชนขอบเทปถือสุดท้ายที่ไหนไป ๆ ได้ แสดงว่าชนเท่านั้น

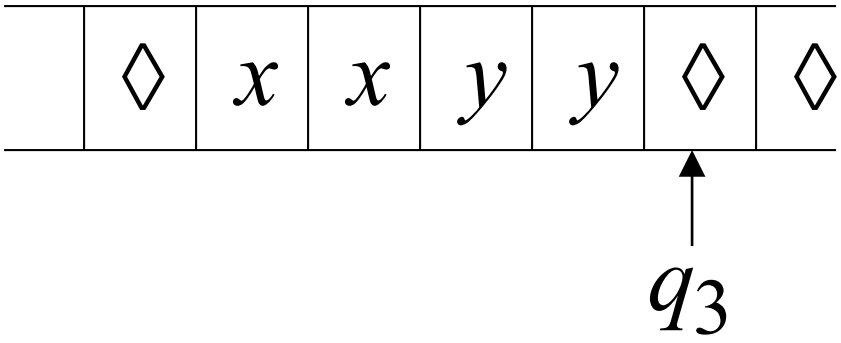


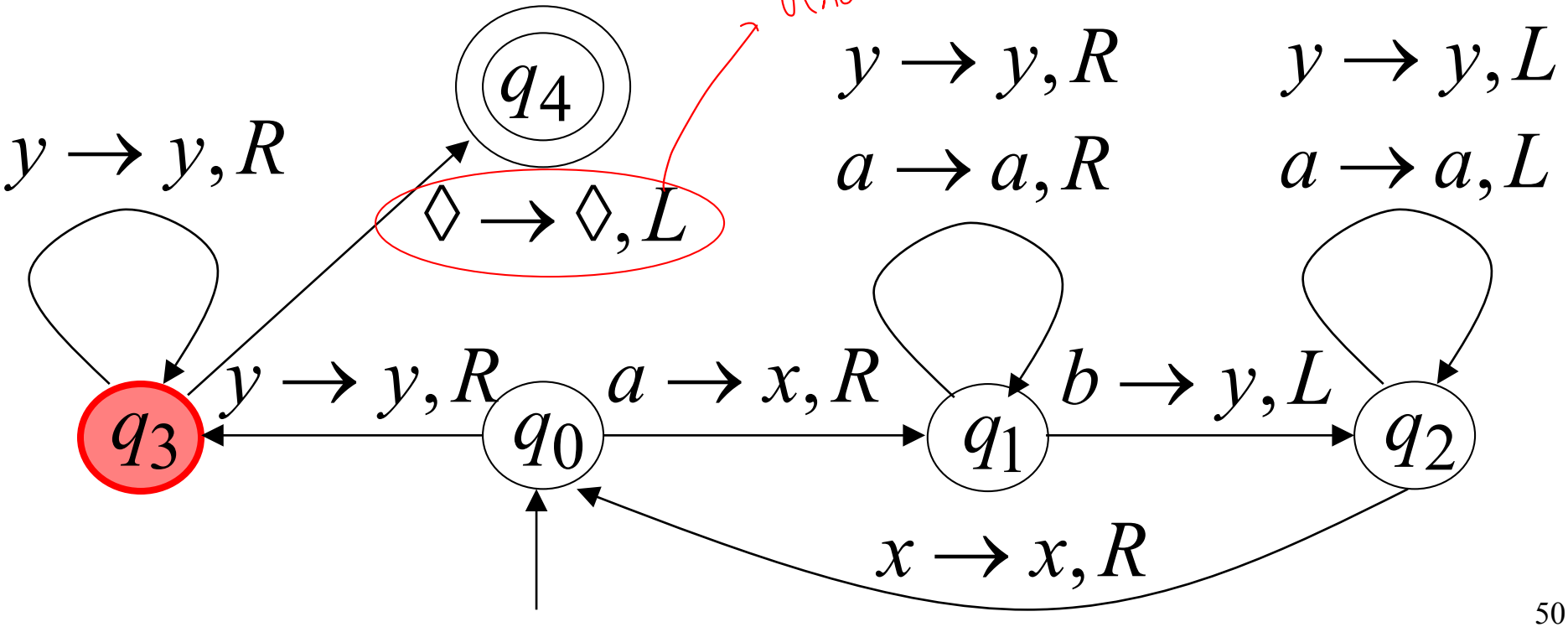
Time 11


◊ x x y y ◊ ◊
q_3

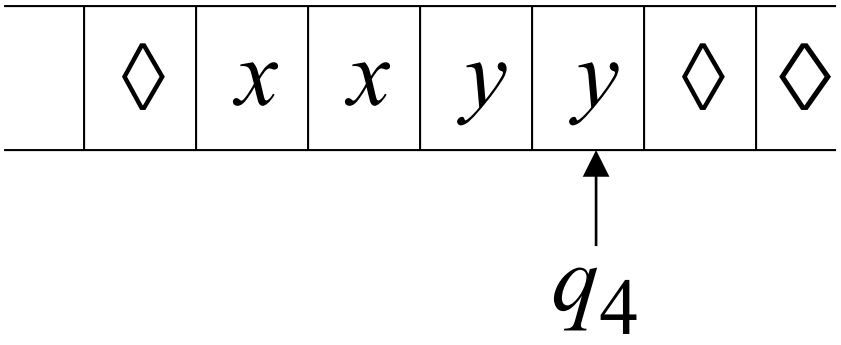

$y \to y, R$
q_4
$\diamond \to \diamond, L$
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
q_3
$y \to y, R$
q_0
$a \to x, R$
q_1
$b \to y, L$
q_2
$x \to x, R$

Time 12

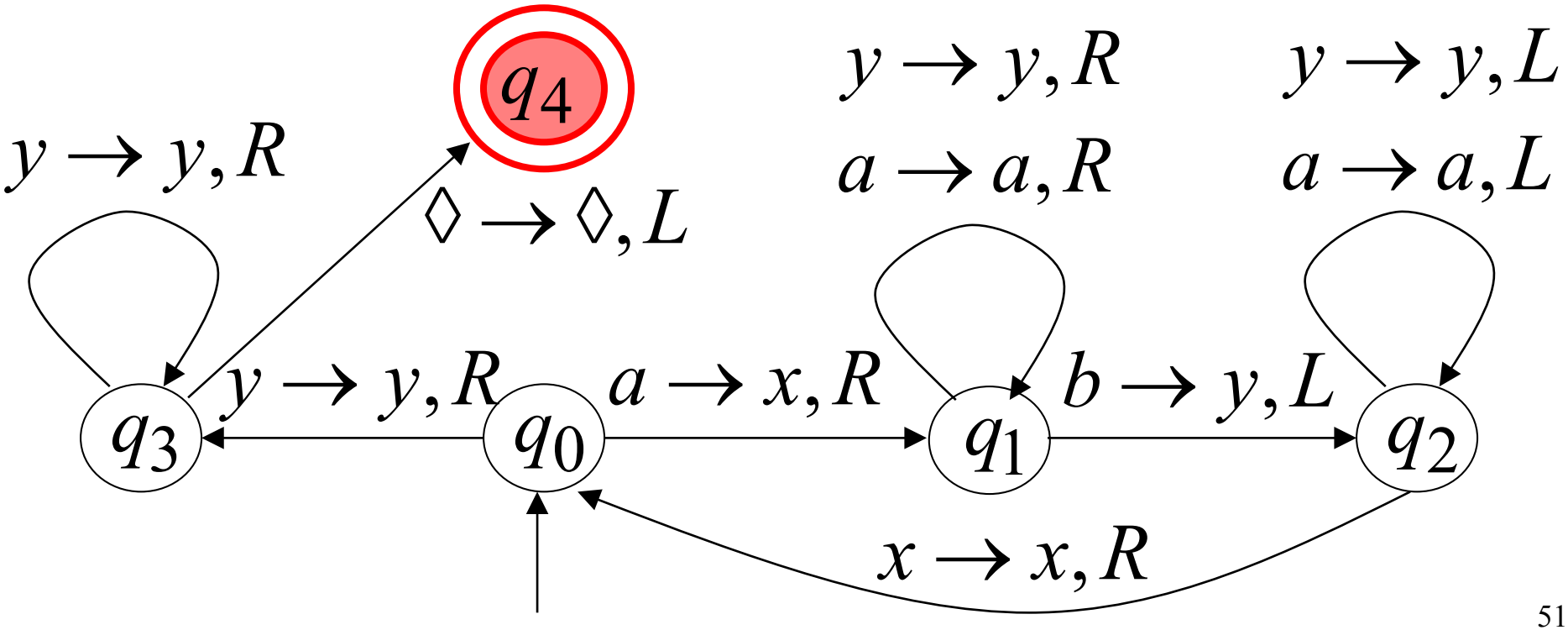

◊ x x y y ◊ ◊
q_3


q_4
$y \to y, R$
$a \to a, R$
$y \to y, L$
$a \to a, L$
$y \to y, R$
$\Diamond \to \Diamond, L$
q_3
$y \to y, R$
q_0
$a \to x, R$
q_1
$b \to y, L$
q_2
$x \to x, R$

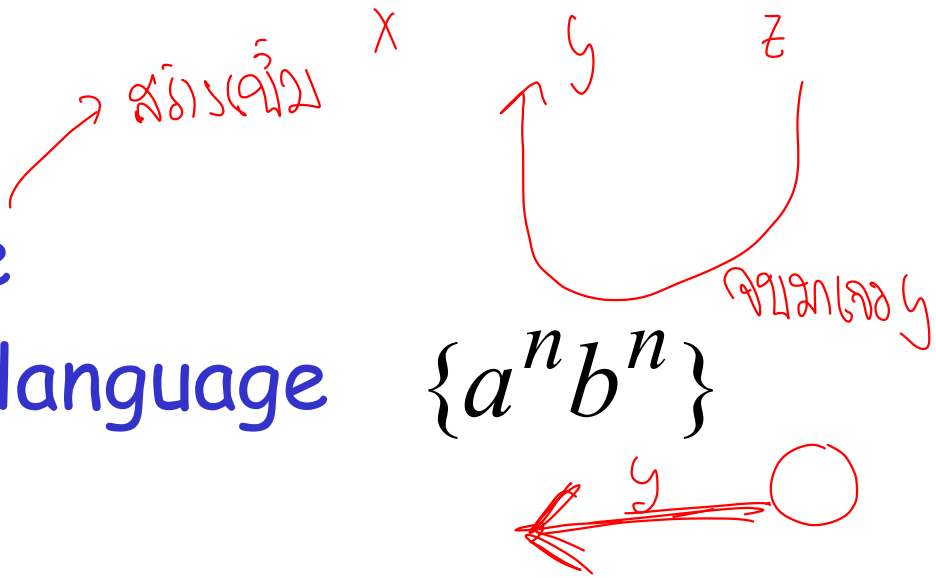
Time 13



Halt & Accept



Observation:



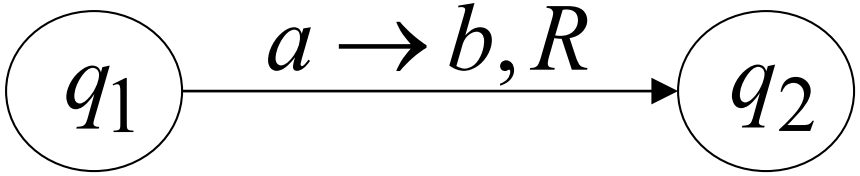
If we modify the
machine for the language $\{a^n b^n\}$

we can easily construct
a machine for the language $\{a^n b^n c^n\}$

Formal Definitions for Turing Machines

Transition Function

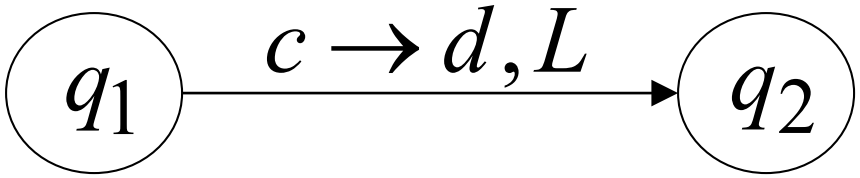
DFA $q(q_1, a) = q_2$ DPDA $\delta(q_1, a, b) = q_2$



ฟังก์ชันเปลี่ยนสถานะ

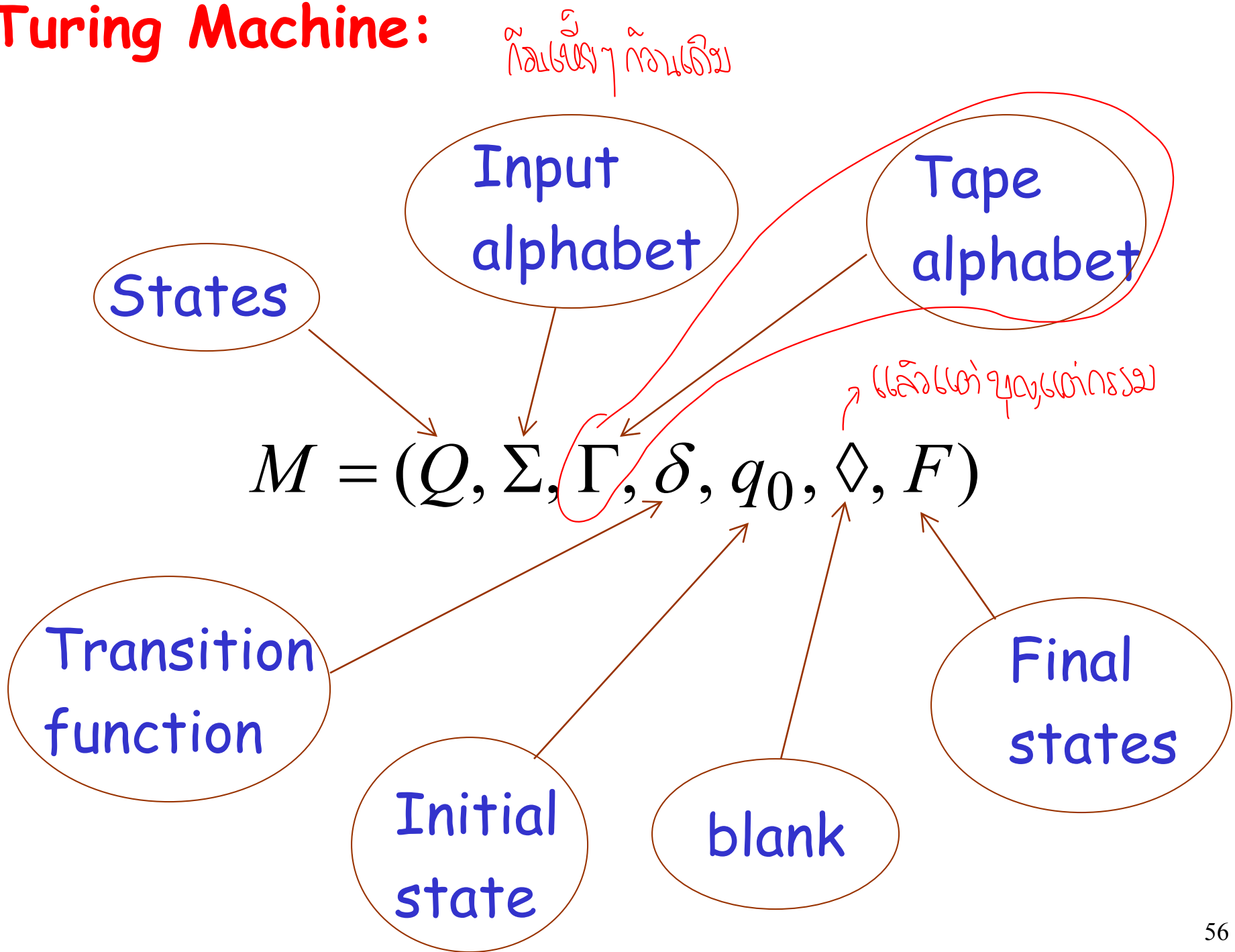
$$\delta(q_1, a) = (q_2, b, R)$$

Transition Function

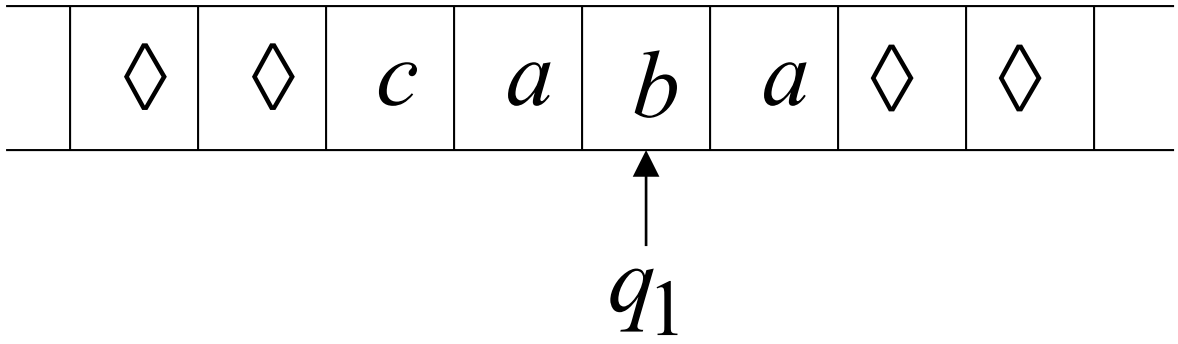


$$\delta(q_1, c) = (q_2, d, L)$$

Turing Machine:



Configuration



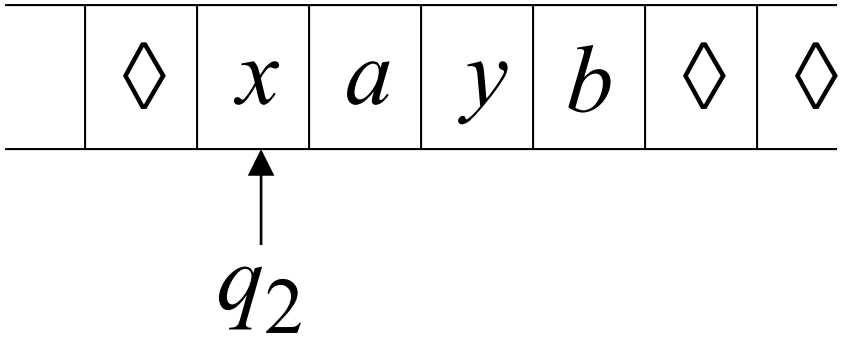
แทน state ณ ขณะนั้น

Instantaneous description: $ca\,q_1\,ba$

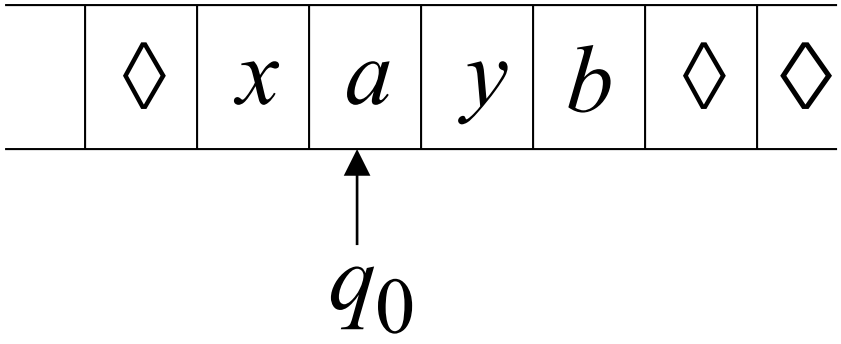
ที่เหลือ

ปัจจุบัน q_1

Time 4

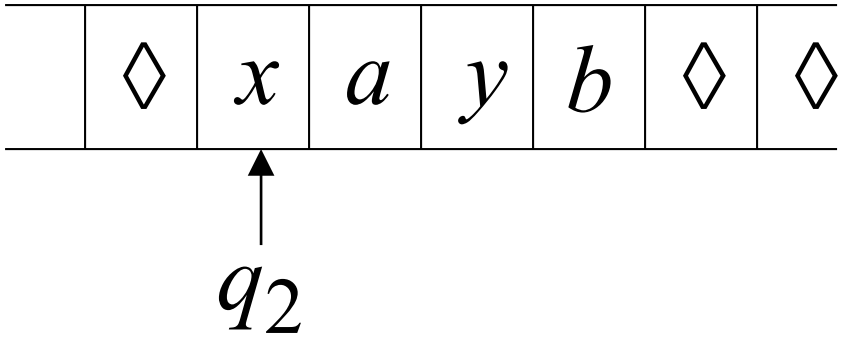


Time 5

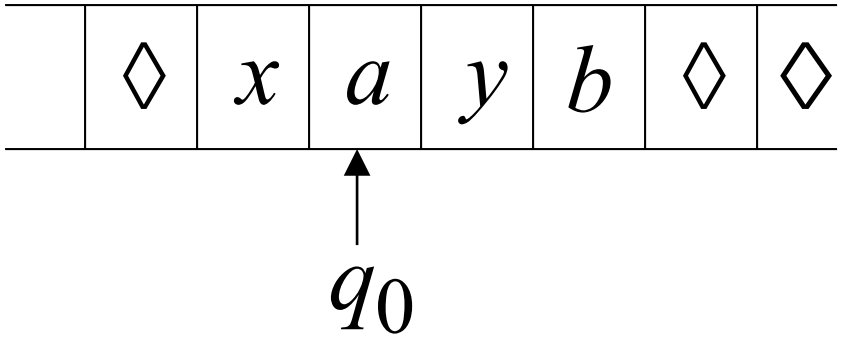


A Move: $q_2\ xayb \succ x\ q_0\ ayb$

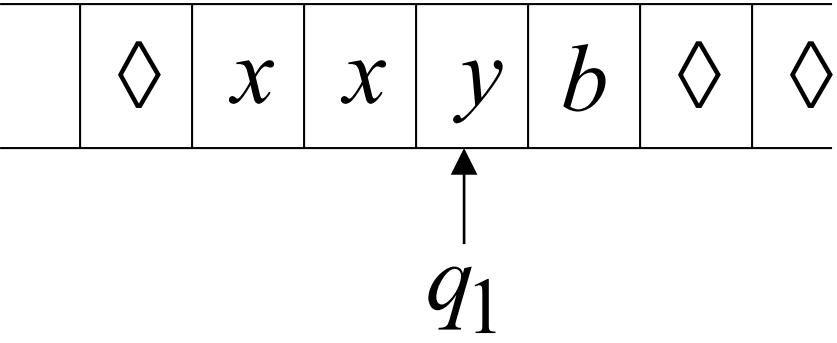
Time 4



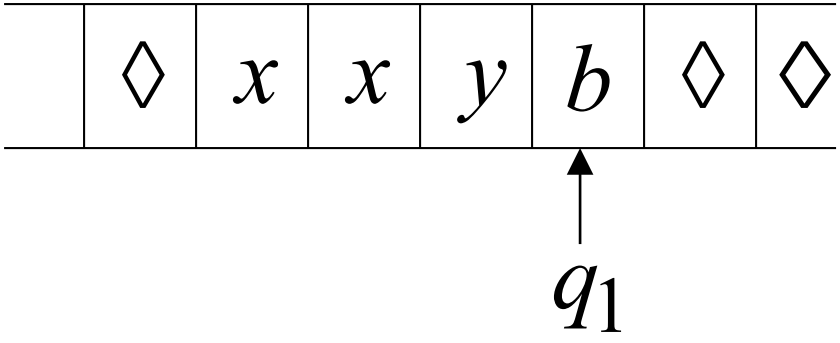
Time 5



Time 6



Time 7



$$q_2\ xayb \succ x\ q_0\ ayb \succ xx\ q_1\ yb \succ xxy\ q_1\ b$$

$$q_2\ xayb \succ x\ q_0\ ayb \succ xx\ q_1\ yb \succ xxy\ q_1\ b$$

เขียนแบบนี้ได้

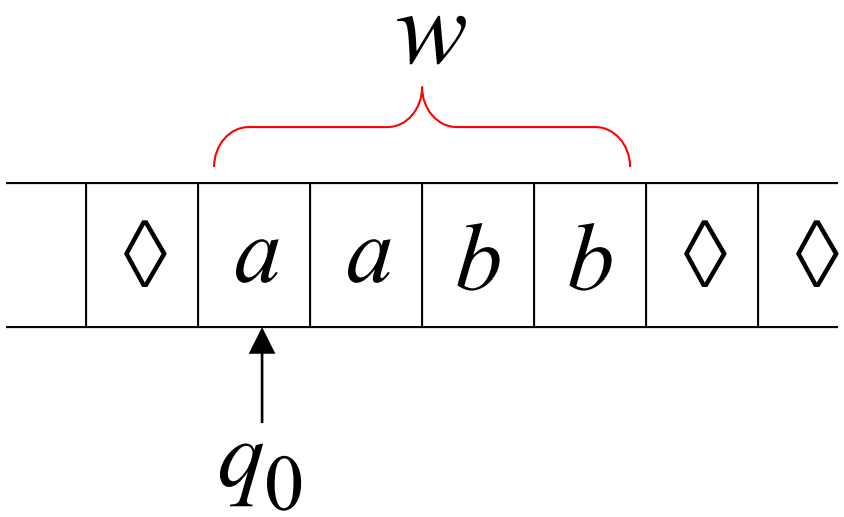
Equivalent notation:

$$q_2\ xayb \overset{*}{\succ} xxy\ q_1\ b$$

Initial configuration: $q_0\ w$

เริ่มต้นที่หัวอ่านที่ตัวแรกของ string เสมอ

Input string



The Accepted Language

For any Turing Machine M

ผลลัพธ์ language

$$L(M) = \{w : \quad q_0\, w \overset{*}{\succ} x_1\, q_f\, x_2\}$$

Initial state

Final state

ตอนแรก → what on tape

↓

just keep going to final state

Standard Turing Machine

The machine we described is the standard:

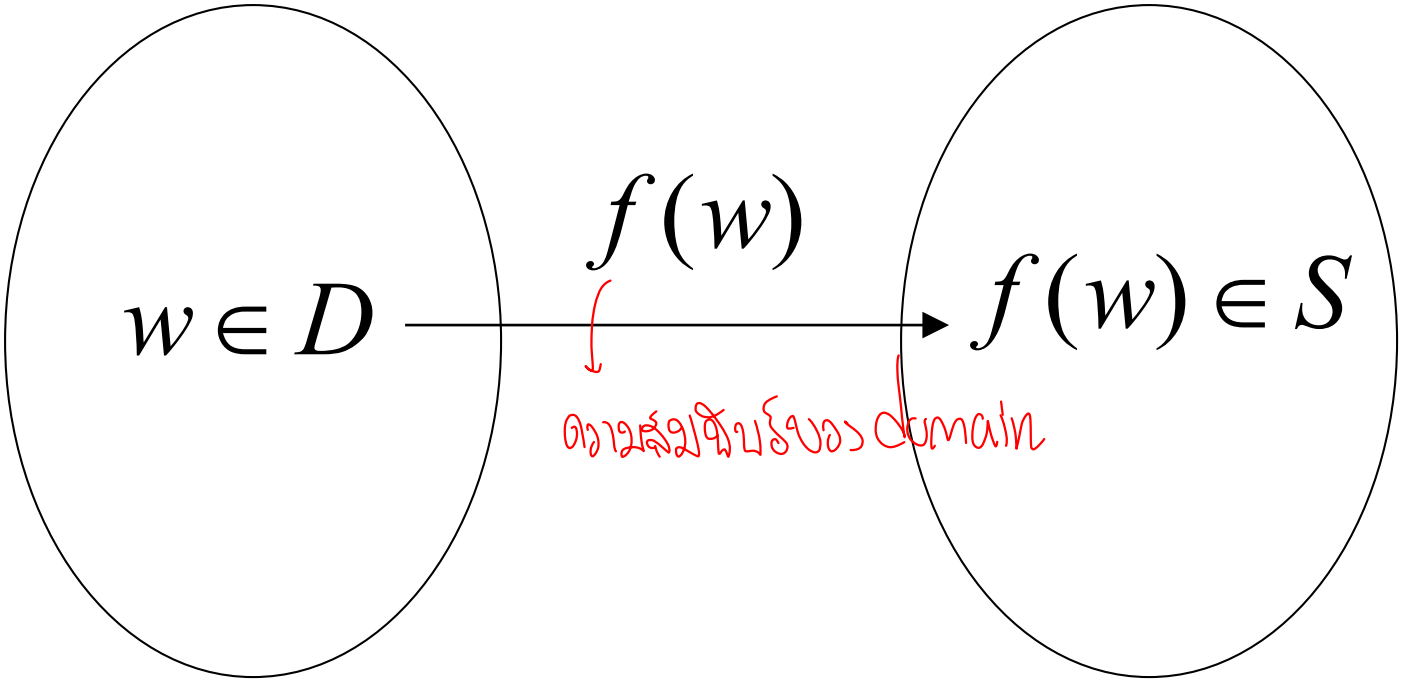
- Deterministic
- Infinite tape in both directions
- Tape is the input/output file

Computing Functions with Turing Machines

A function $f(w)$ has:

Domain: D

Result Region: S



A function may have many parameters:

Example: Addition function

$$f(x, y) = x + y$$

Integer Domain

Decimal: 5

Binary: 101

Unary: 11111

We prefer **unary** representation:

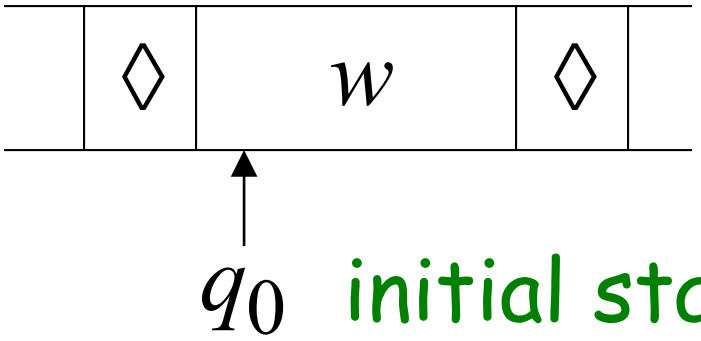
easier to manipulate with Turing machines

Definition:

A function f is computable if there is a Turing Machine M such that:

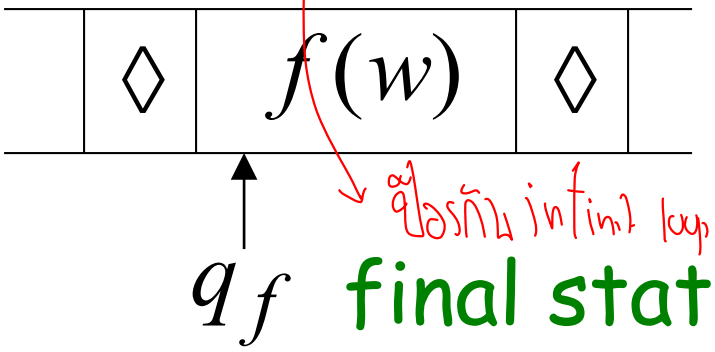
function ที่จะสามารถคำนวณได้

Initial configuration



ยอมรับ function อยู่บนเทป & ตัวอักษรอยู่ด้านซ้ายสุดเสมอ

Final configuration



For all $w \in D$ Domain

In other words:

A function f is computable if there is a Turing Machine M such that:

$$q_0\, w \overset{*}{\succ} q_f\, f(w)$$

ข้อความอยู่ที่ไหน?

Initial Configuration

Final Configuration

For all $w \in D$ Domain

Example

The function $f(x, y) = x + y$ is computable

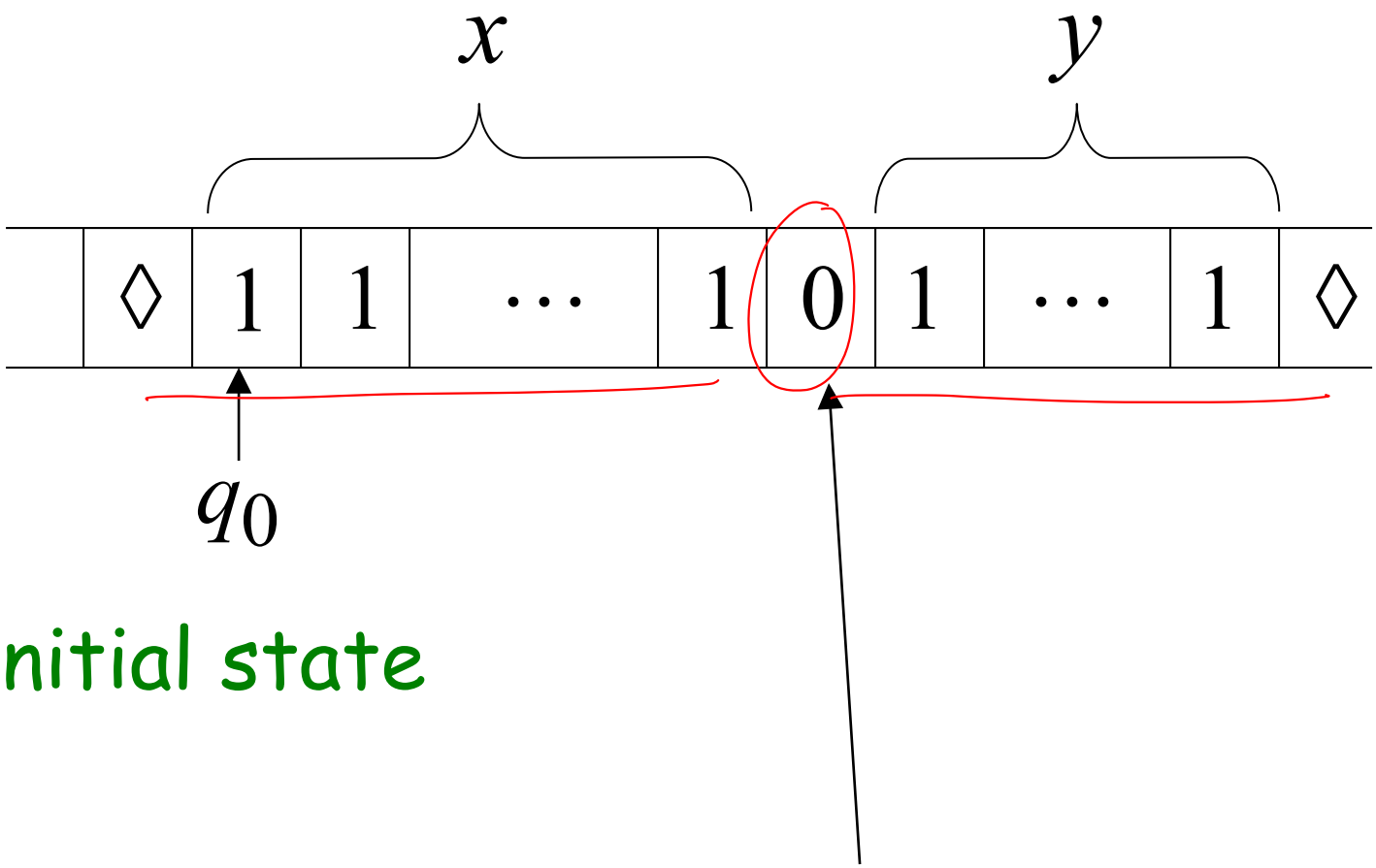
x, y are integers

Turing Machine:

Input string: $x0y$ unary

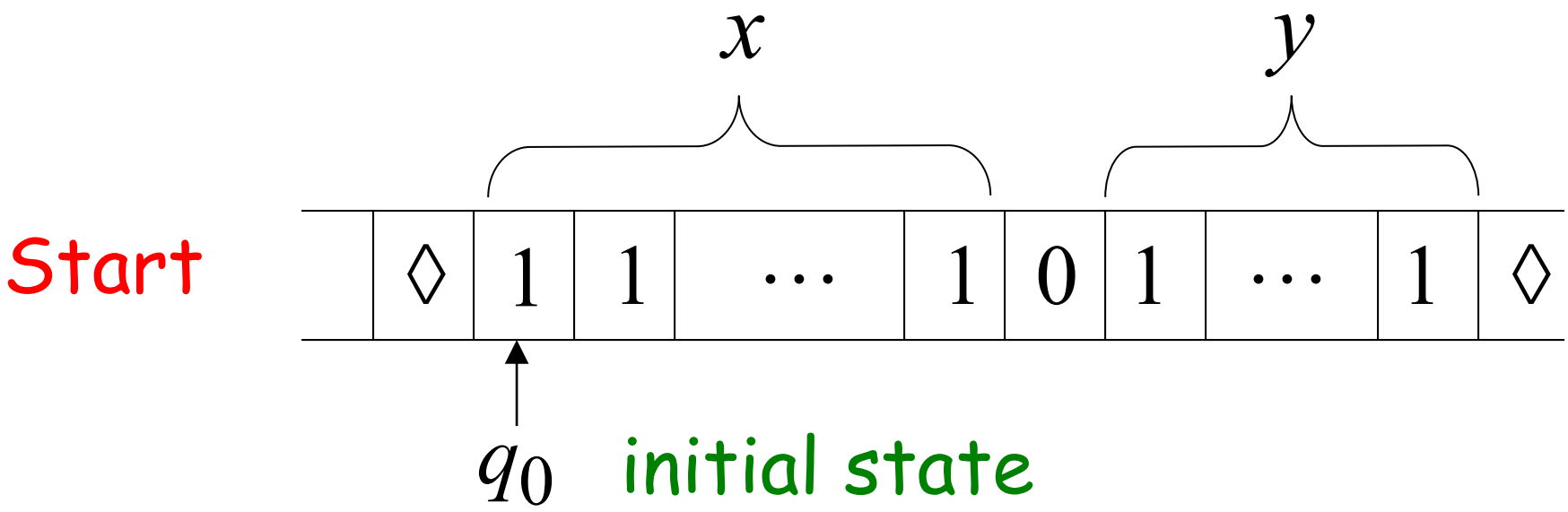
Output string: $xy0$ unary

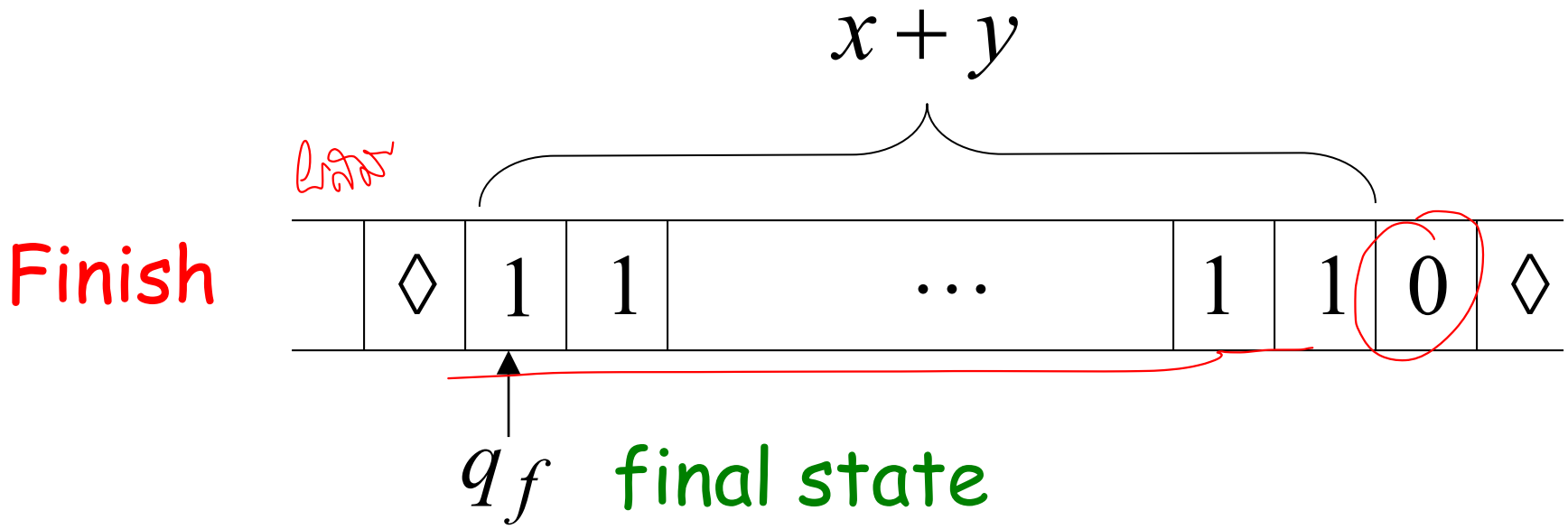
Start



initial state

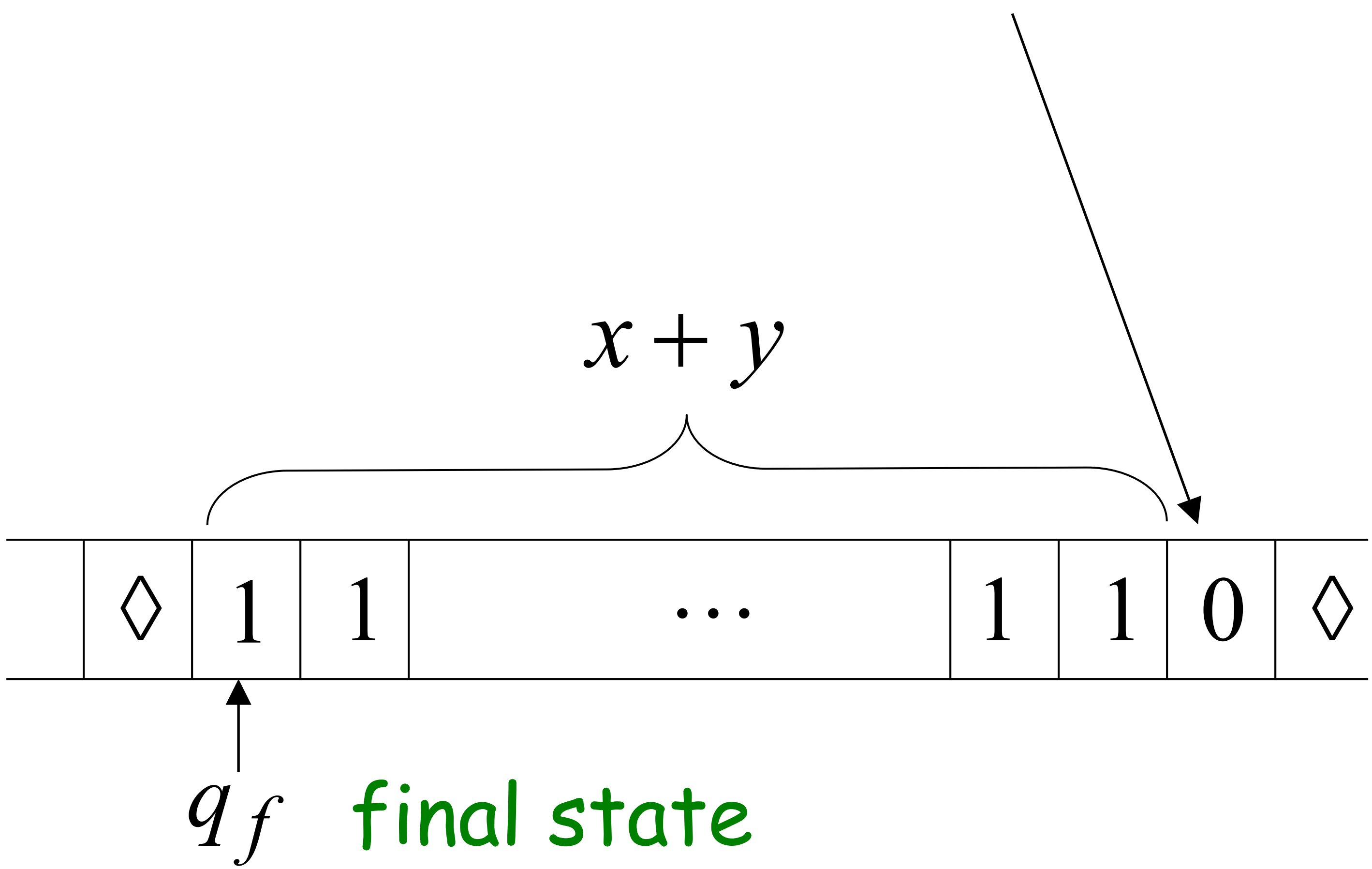
The 0 is the delimiter that separates the two numbers


x
y
Start
◊ 1 1 ⋯ 1 0 1 ⋯ 1 ◊
q_0 initial state


$x + y$
Finish
◊ 1 1 ⋯ 1 1 0 ◊
q_f final state

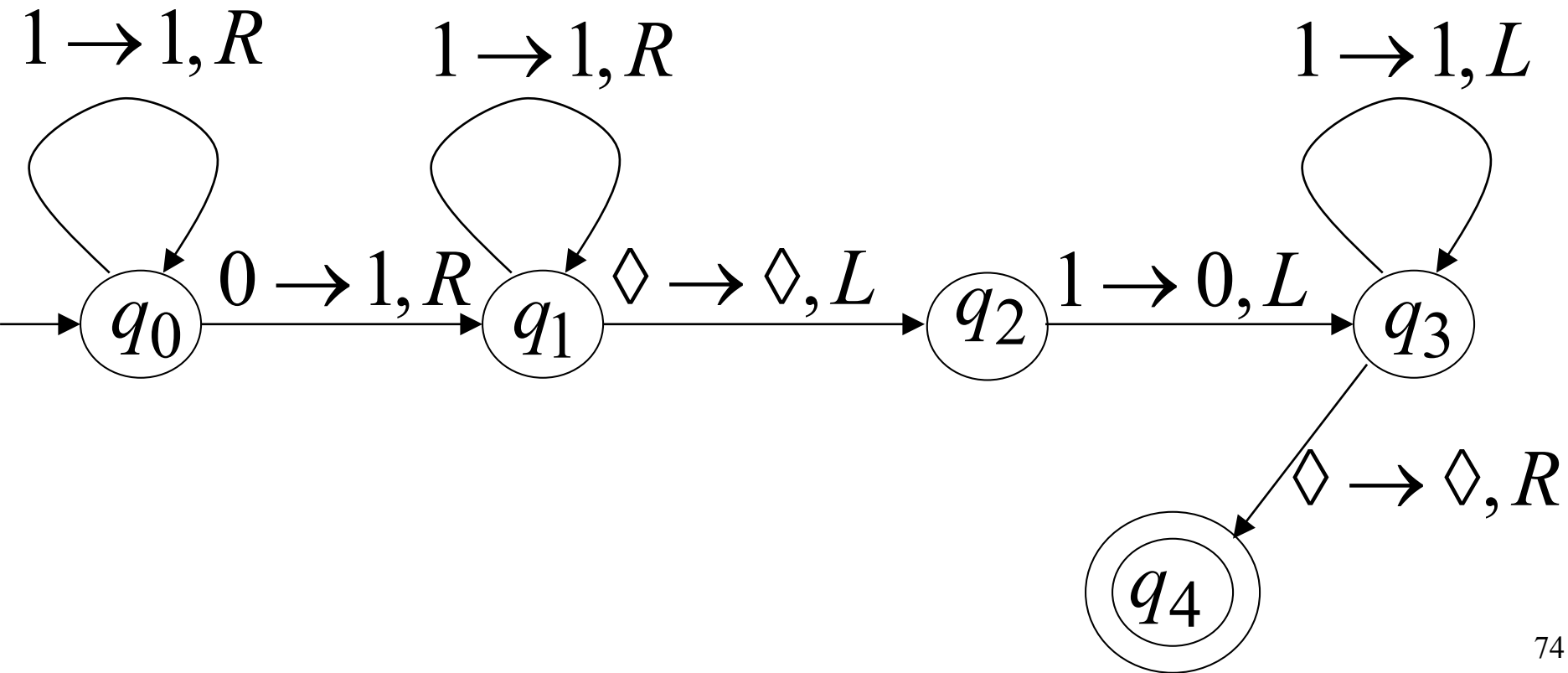
The 0 helps when we use the result for other operations

Finish



final state

Turing machine for function $f(x, y) = x + y$

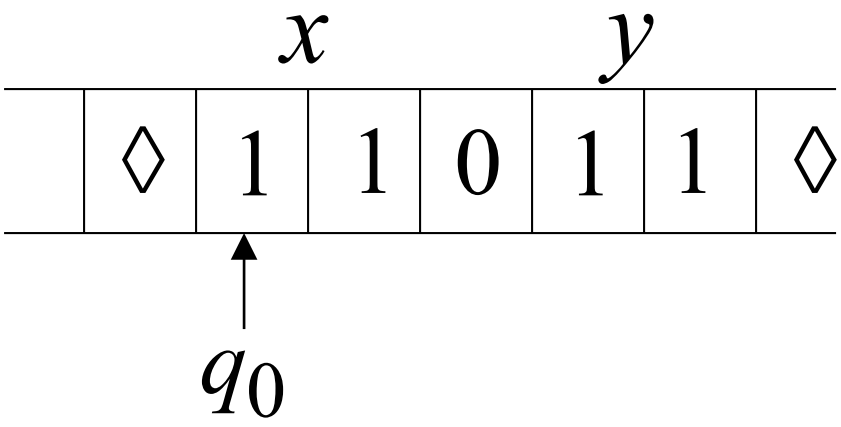


Execution Example:

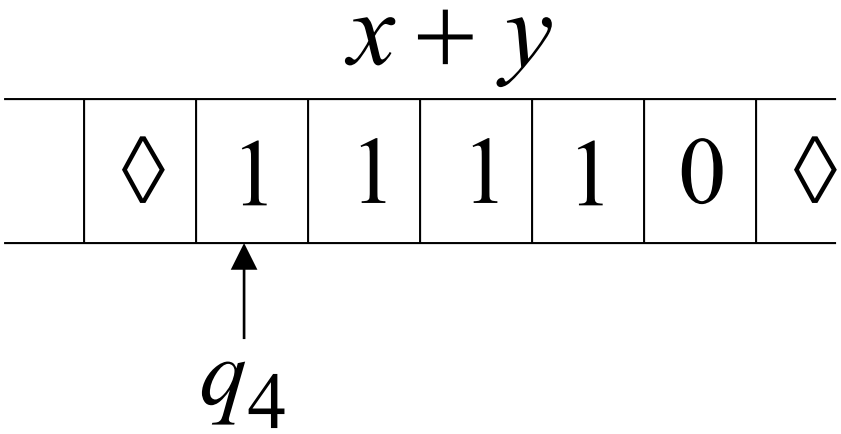
$x = 11 \quad (2)$

$y = 11 \quad (2)$

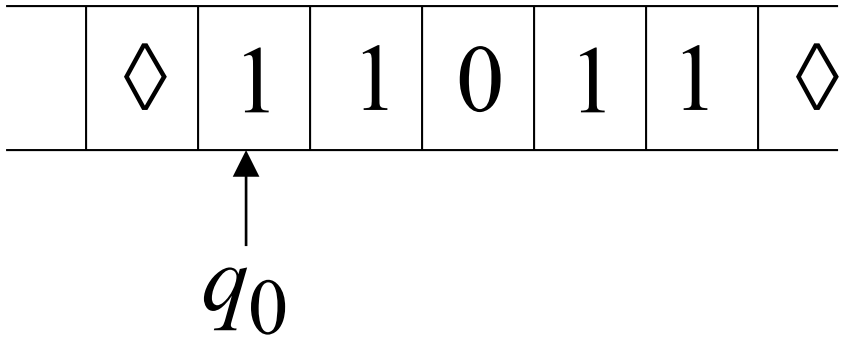
Time 0

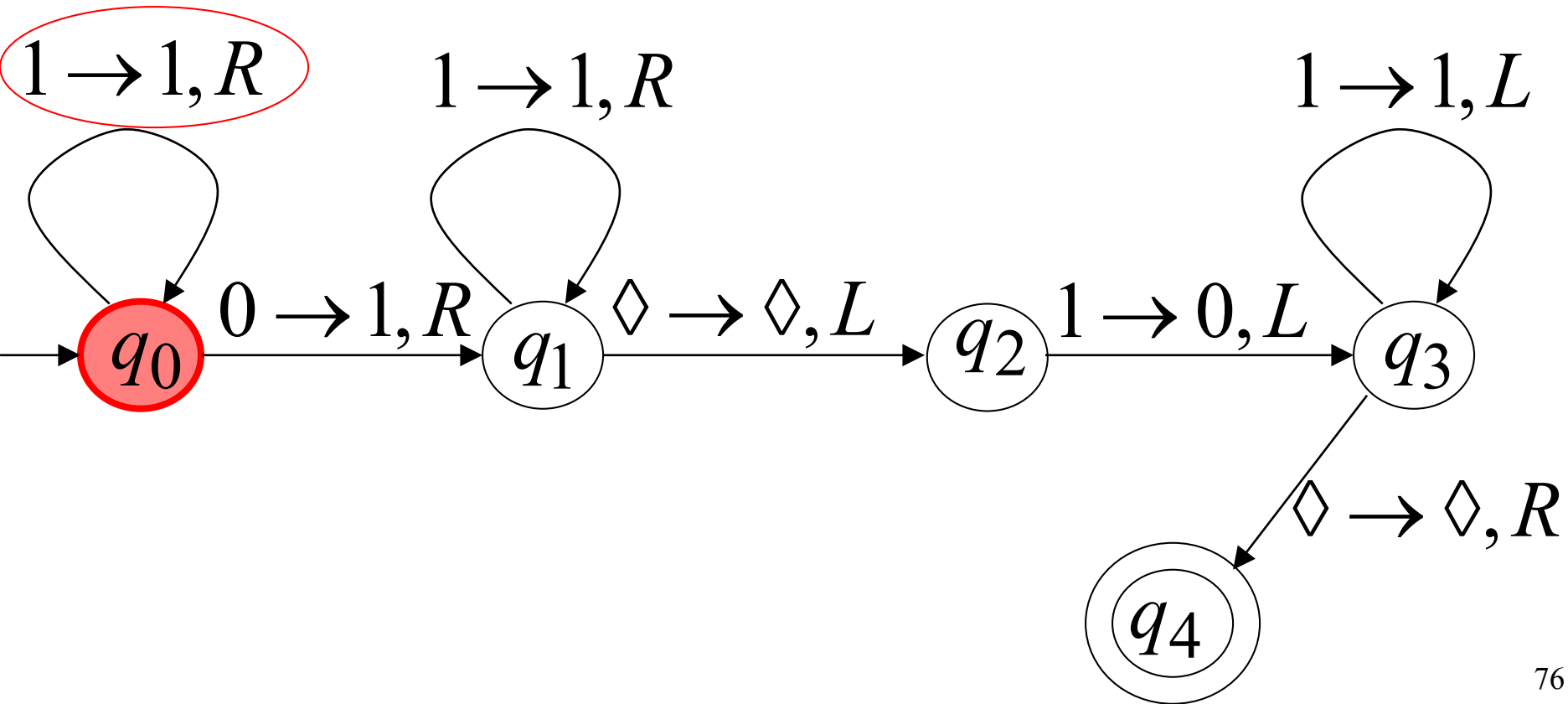


Final Result

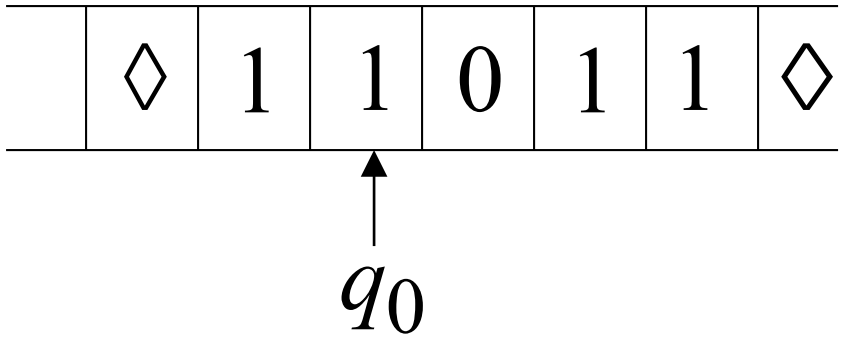


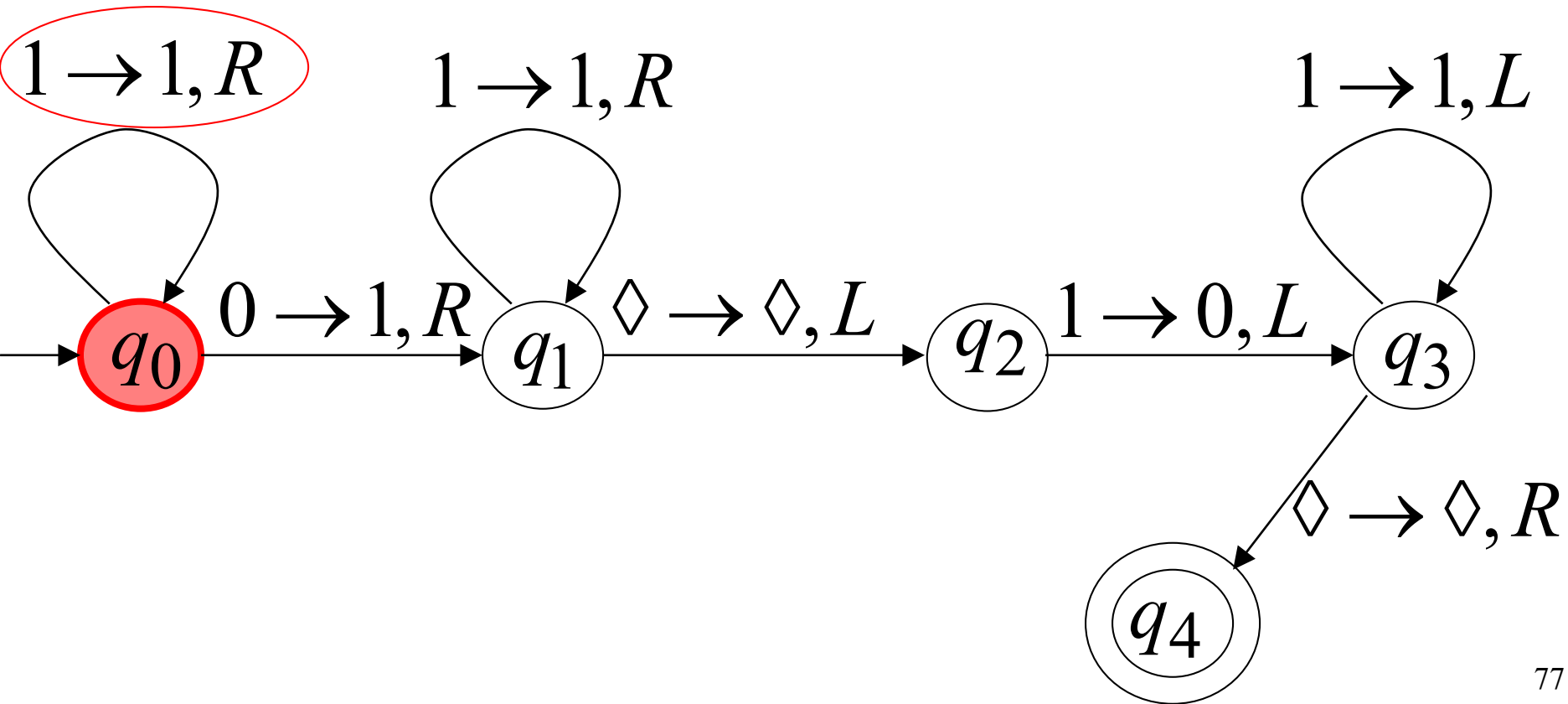
Time 0


◊ 1 1 0 1 1 ◊
q_0

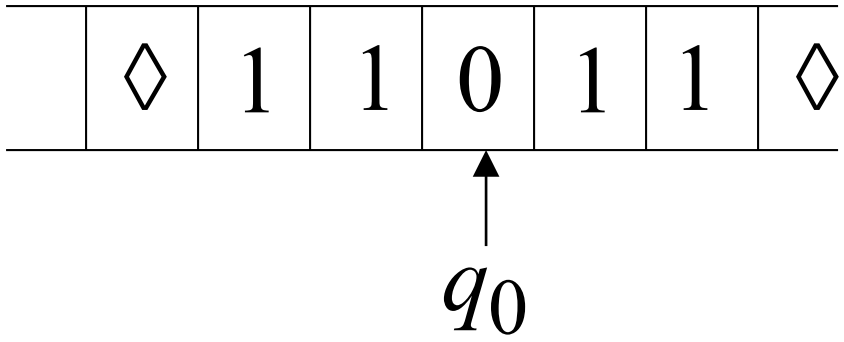

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

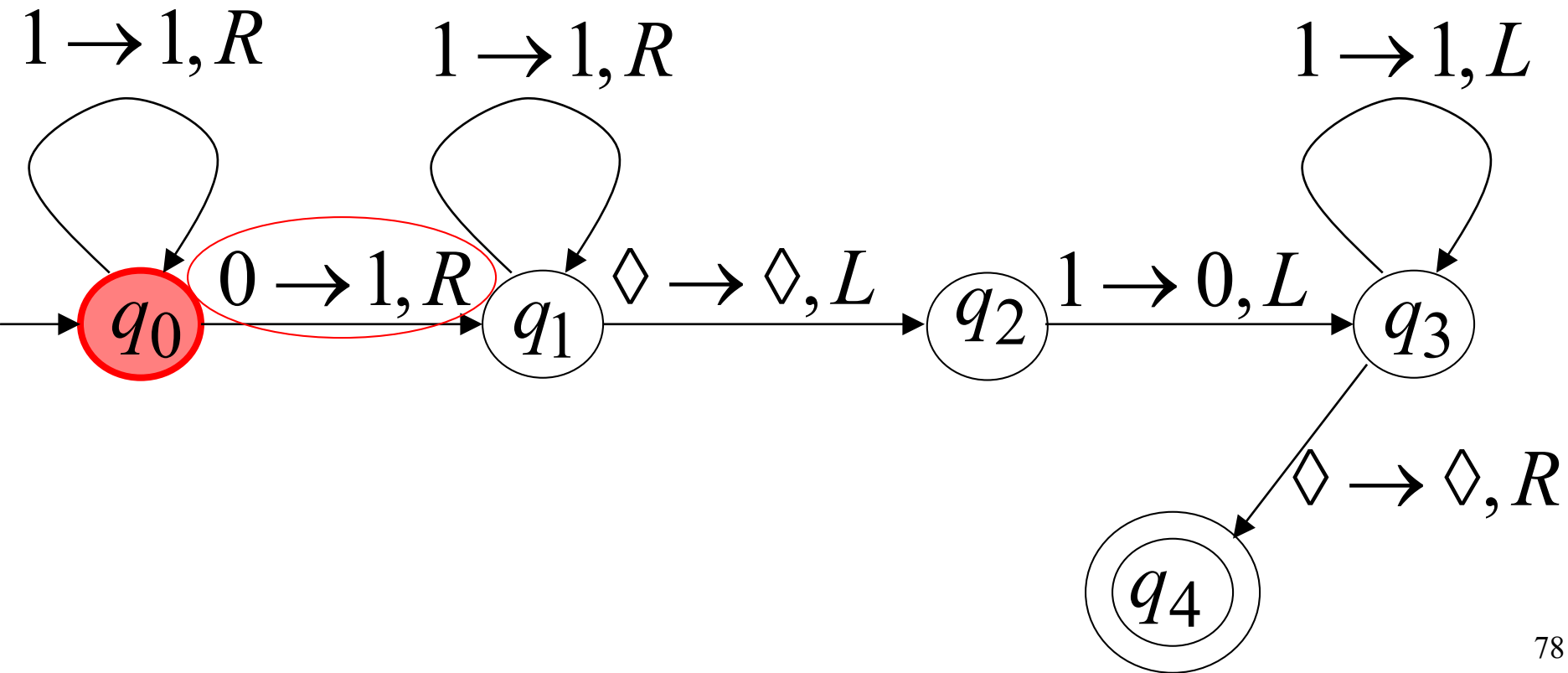
Time 1


◊ 1 1 0 1 1 ◊
q_0

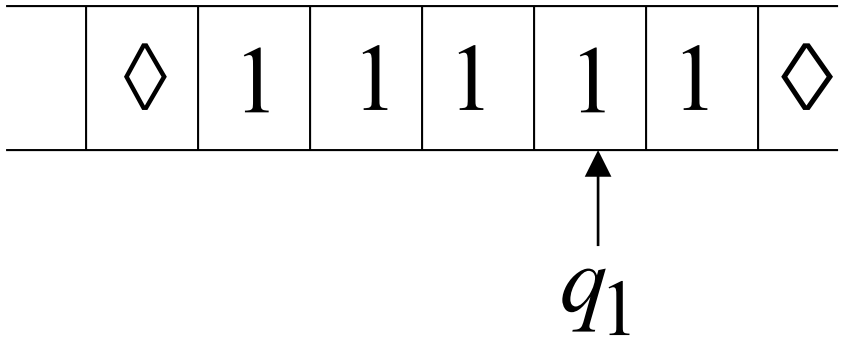

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

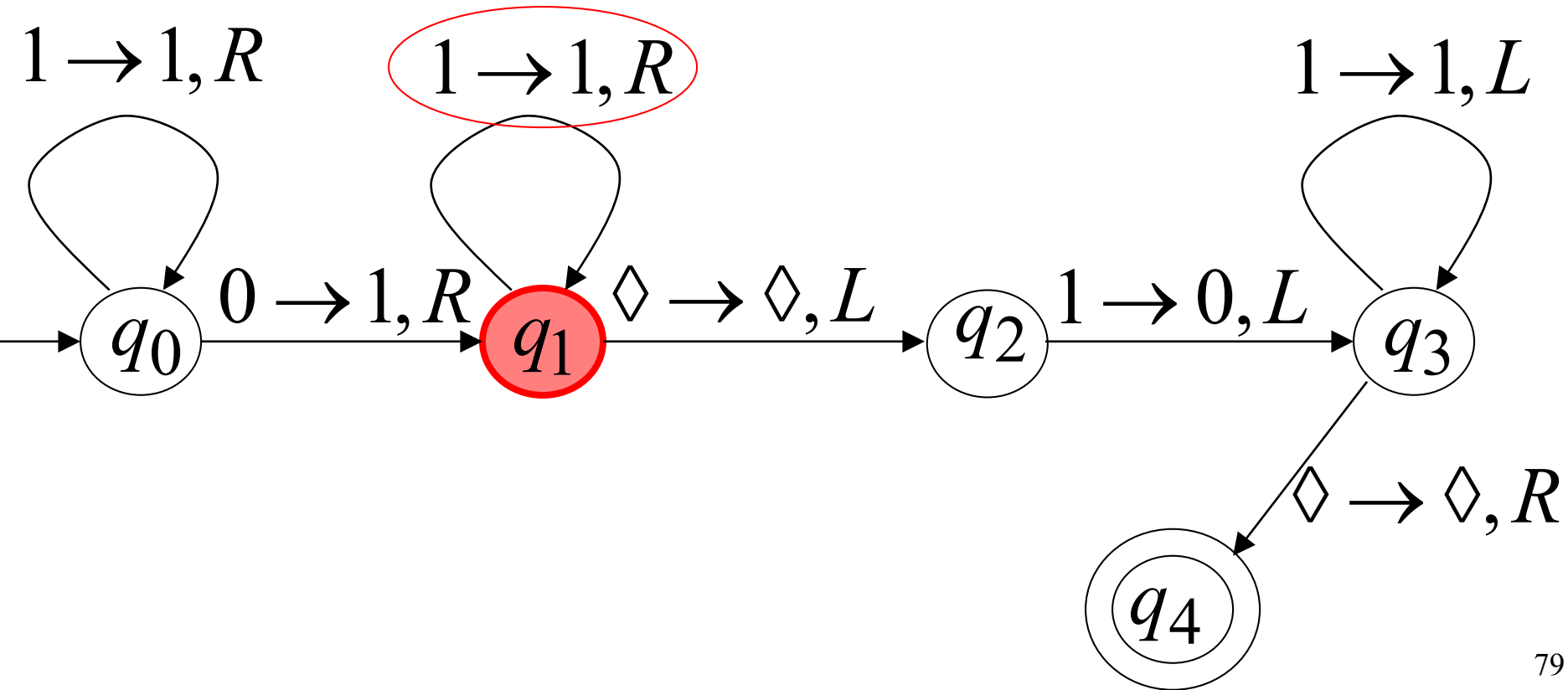
Time 2


◊ 1 1 0 1 1 ◊
q_0

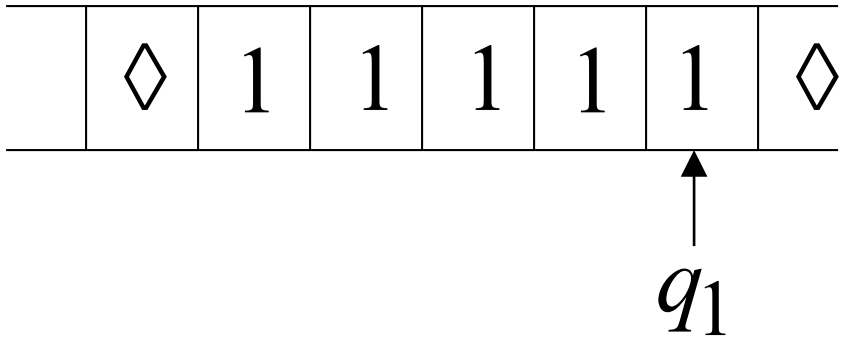

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

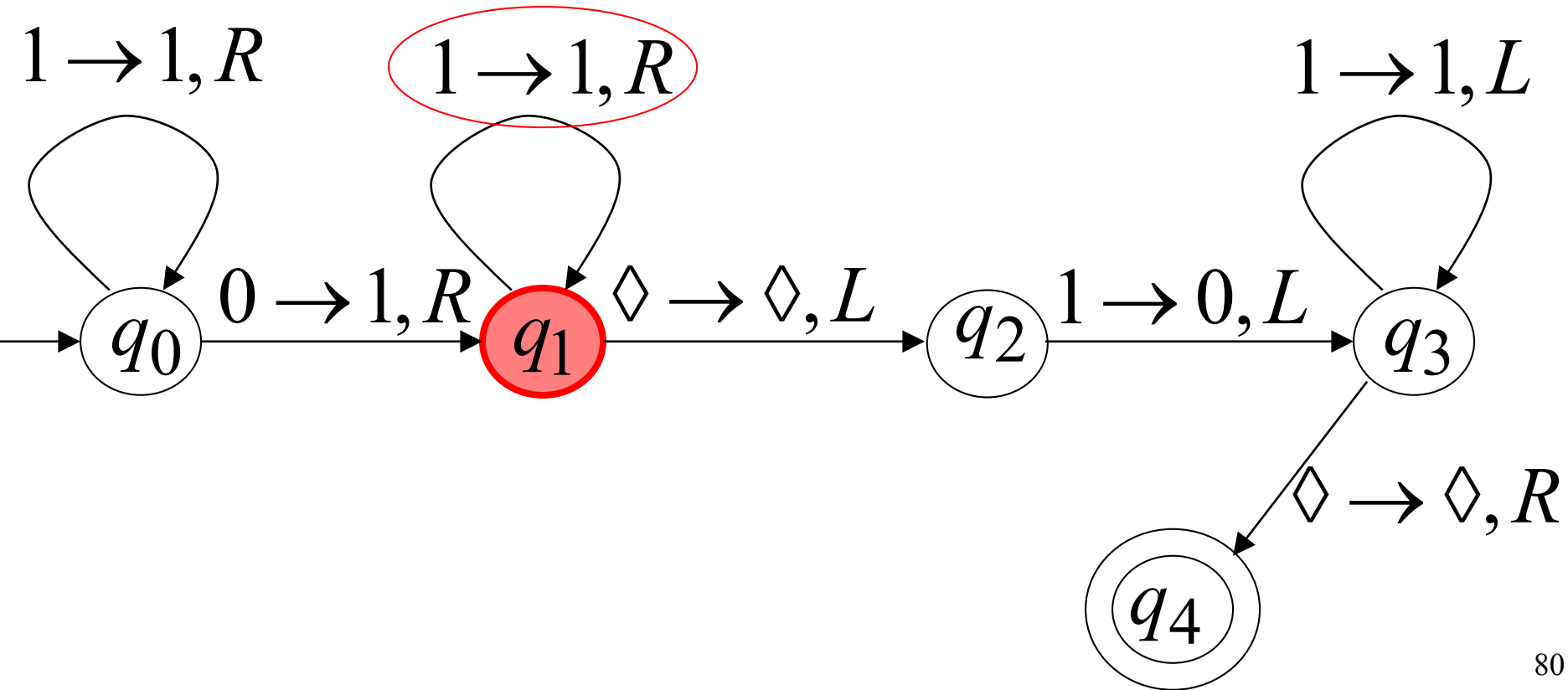
Time 3


◊ 1 1 1 1 1 ◊
q_1

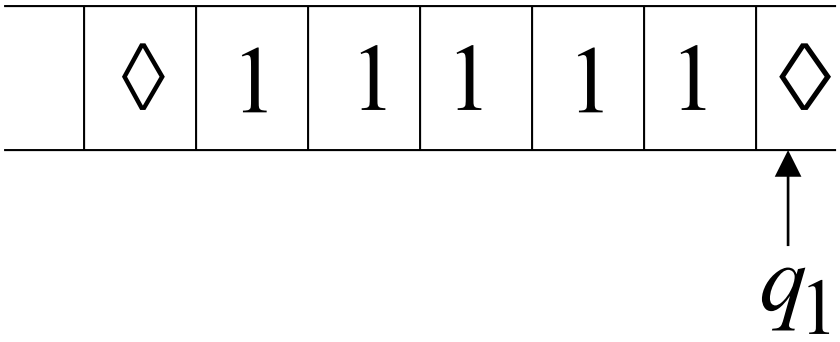

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

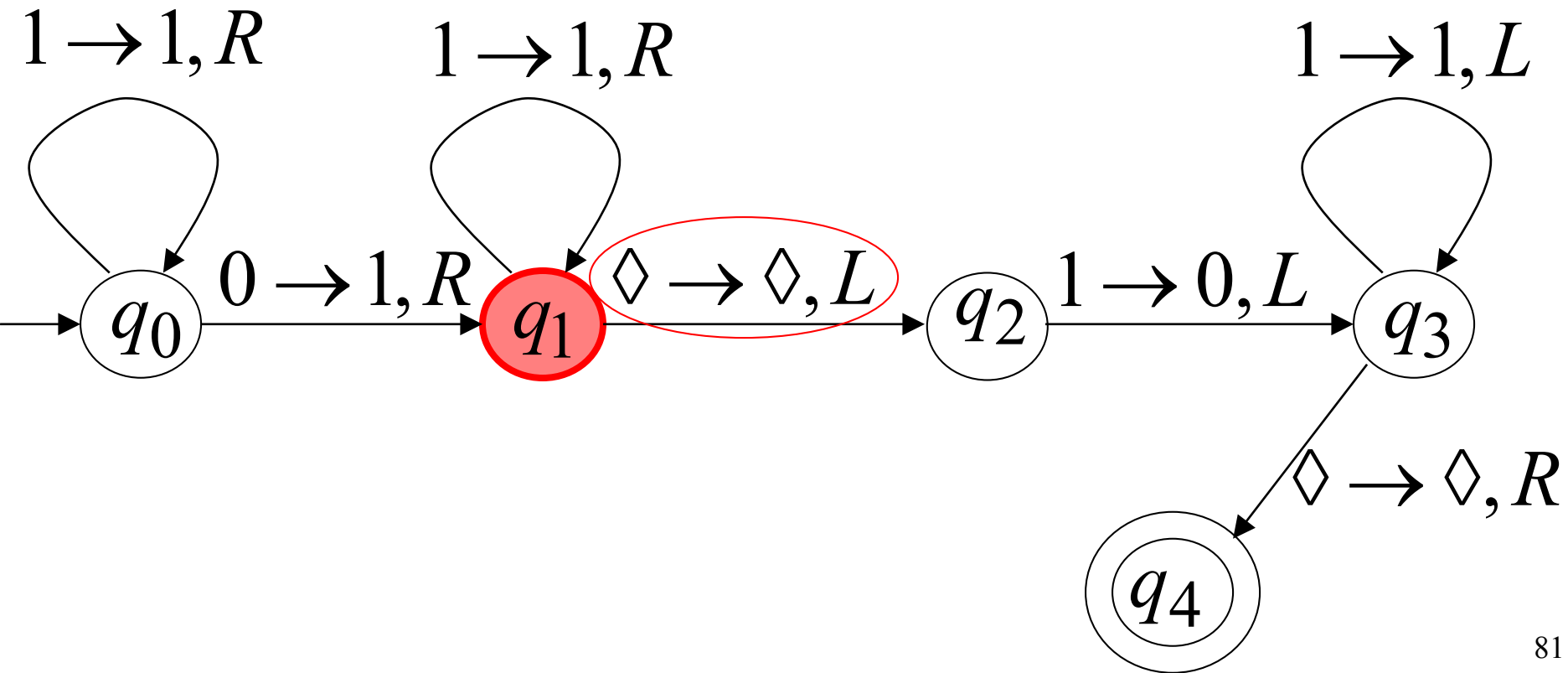
Time 4


◊ 1 1 1 1 1 ◊
q_1

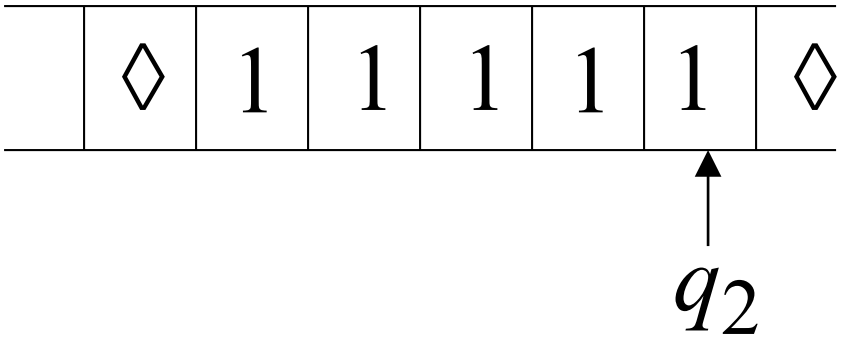

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

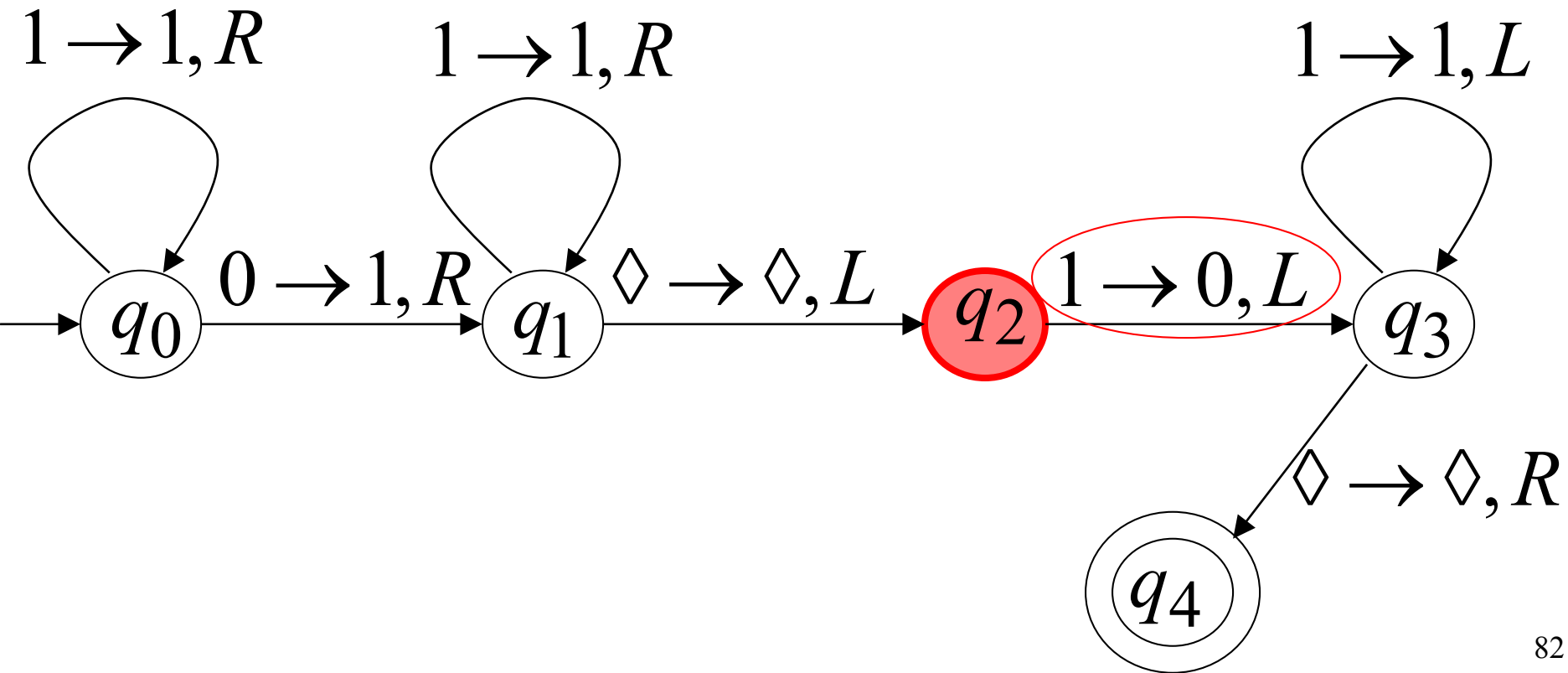
Time 5


◊ 1 1 1 1 1 ◊
q_1

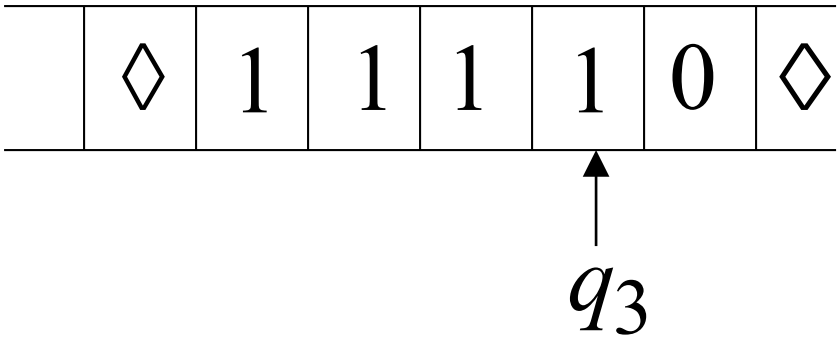

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

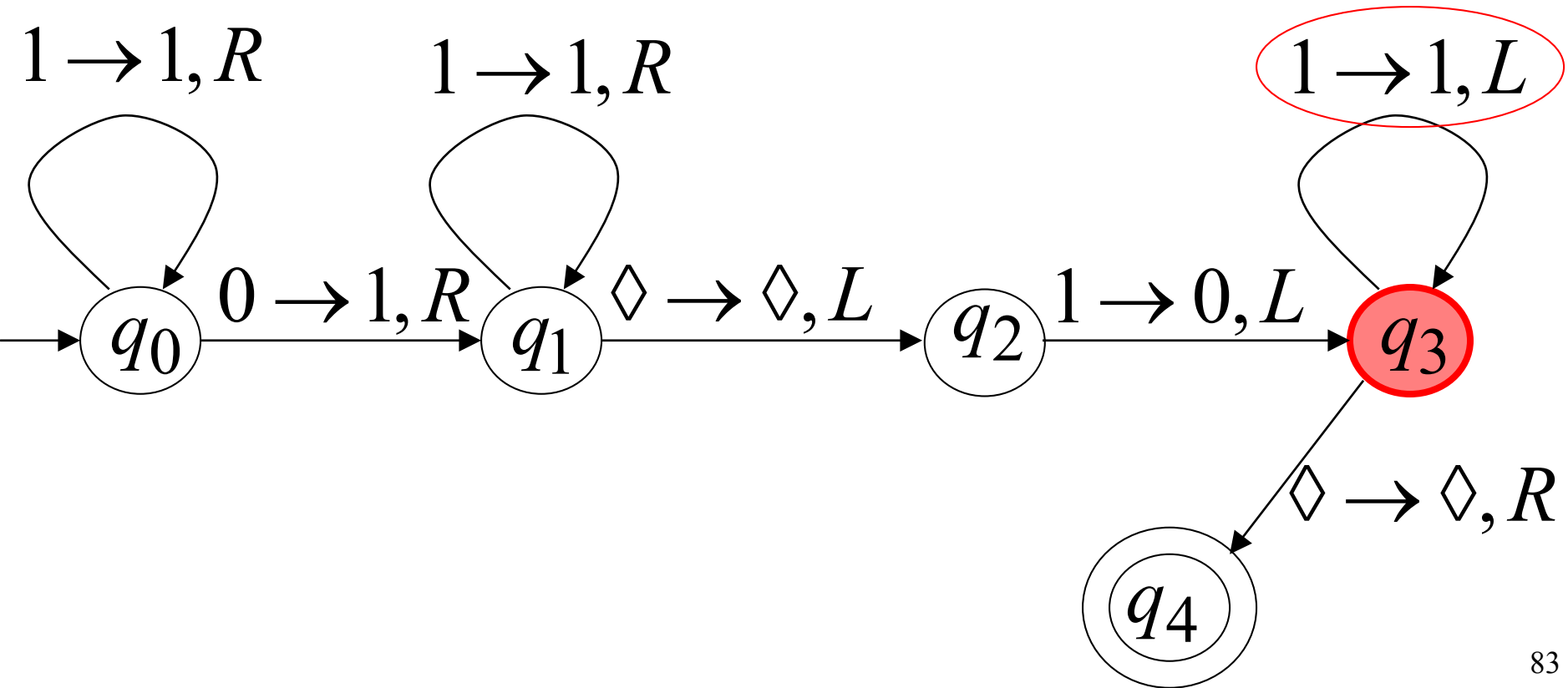
Time 6


◊ 1 1 1 1 1 ◊
q_2

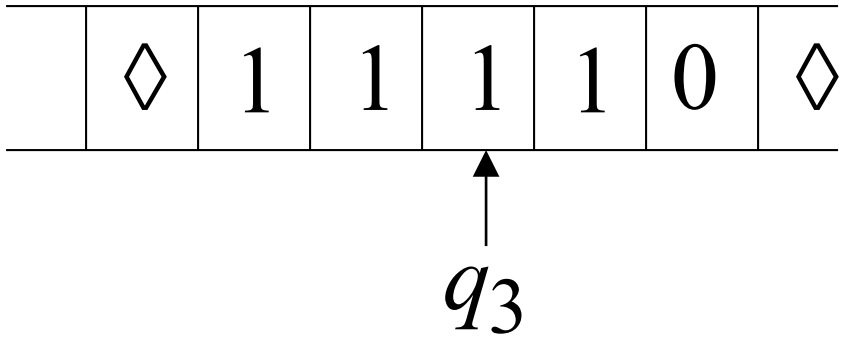

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

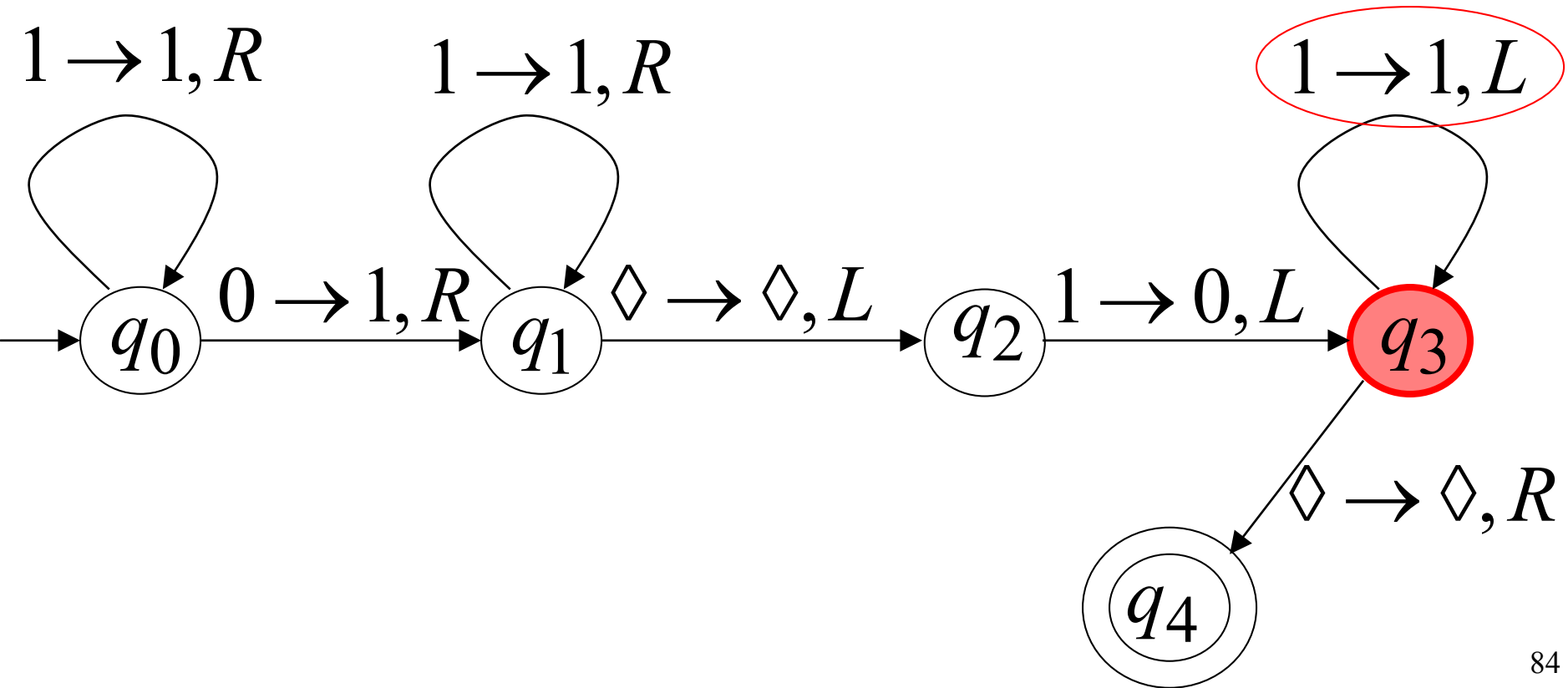
Time 7


◊ 1 1 1 1 0 ◊
q_3

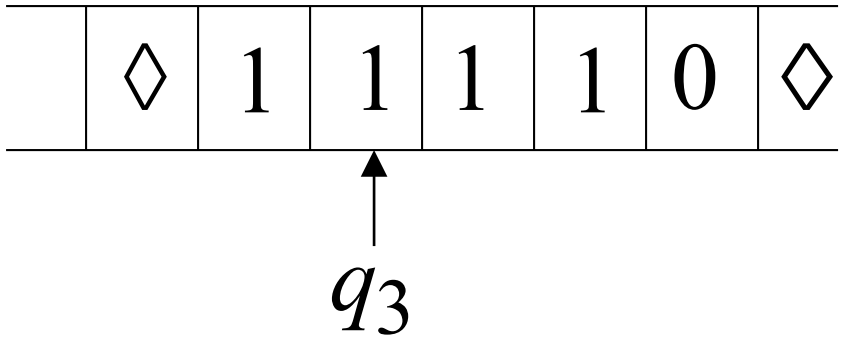

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

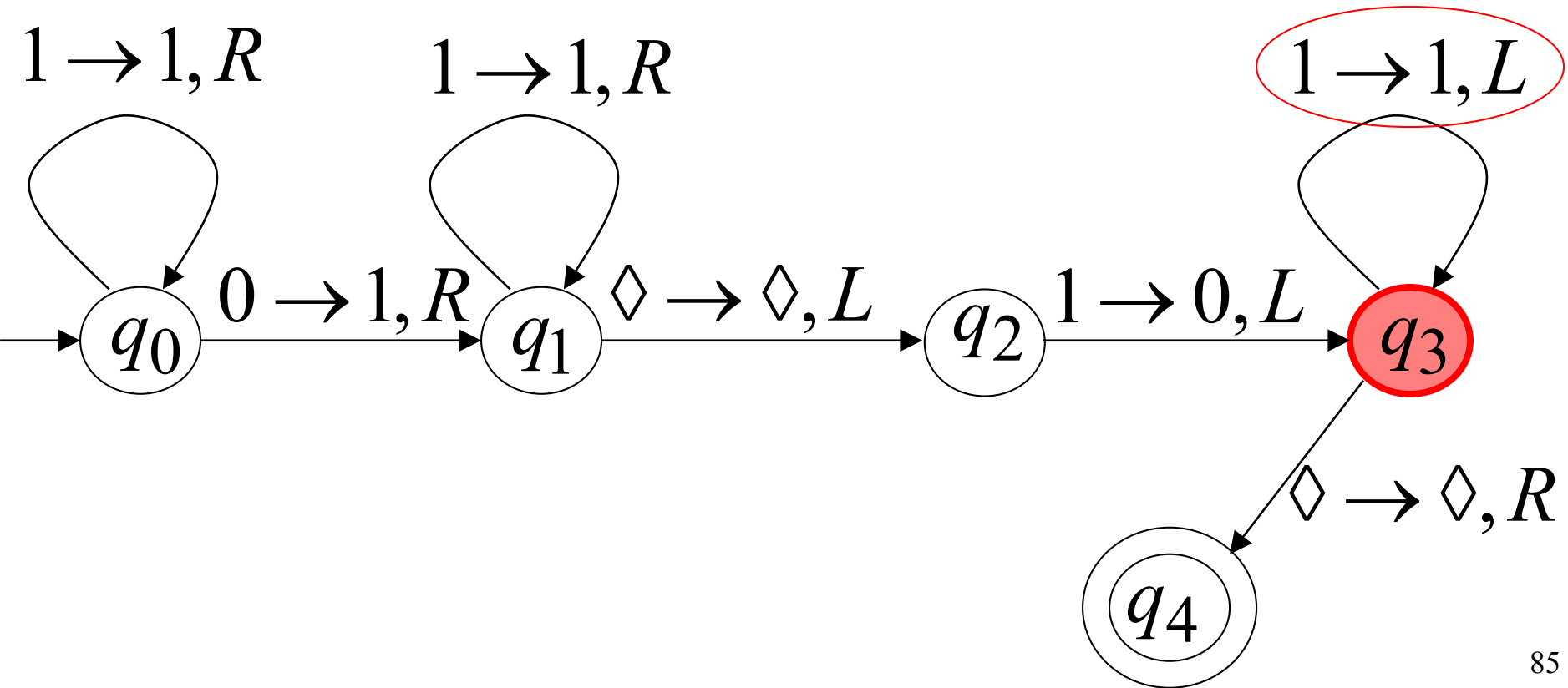
Time 8


◊ 1 1 1 1 0 ◊
q_3

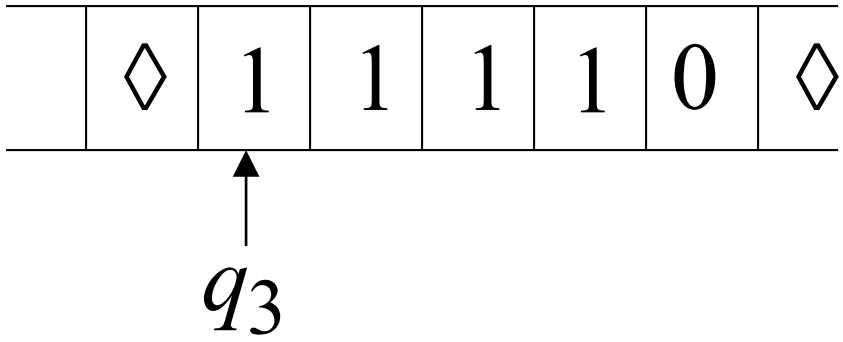

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

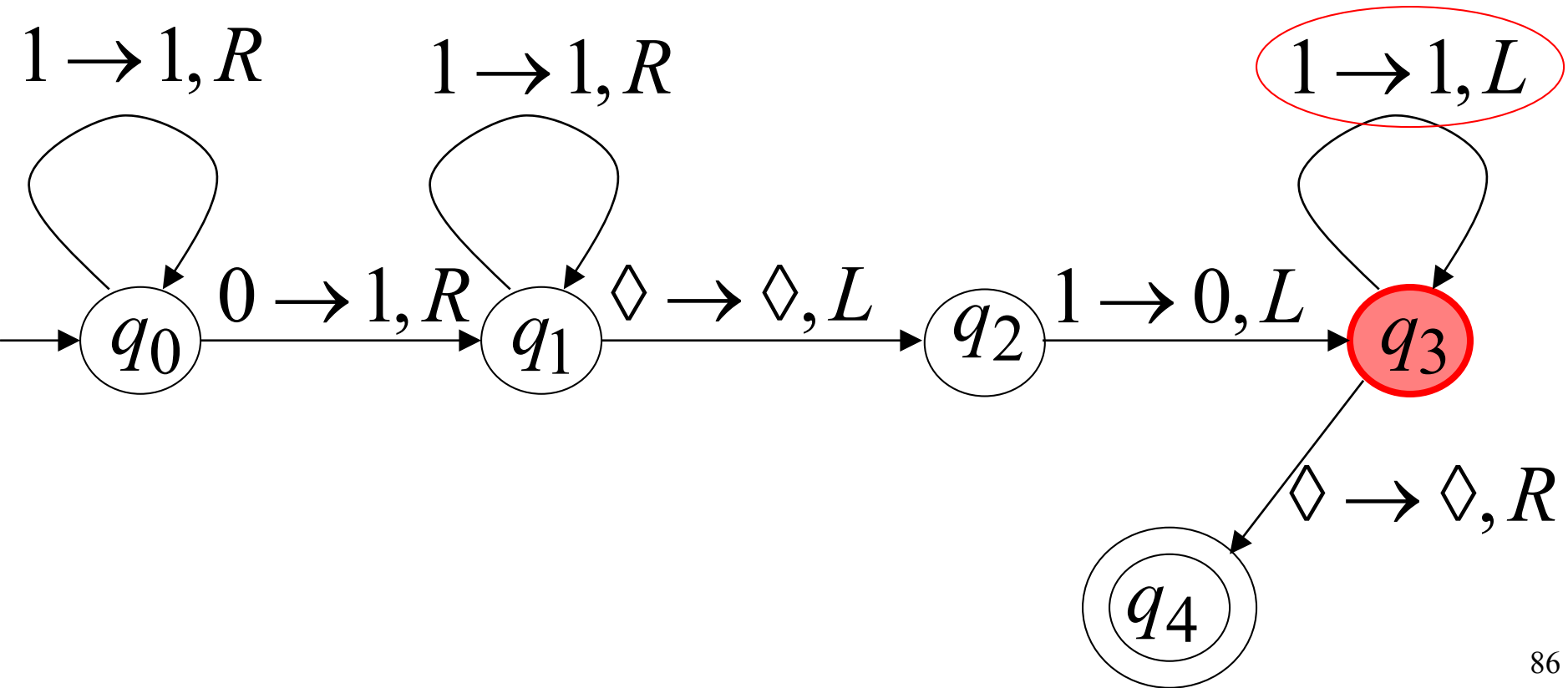
Time 9


◊ 1 1 1 1 0 ◊
q_3

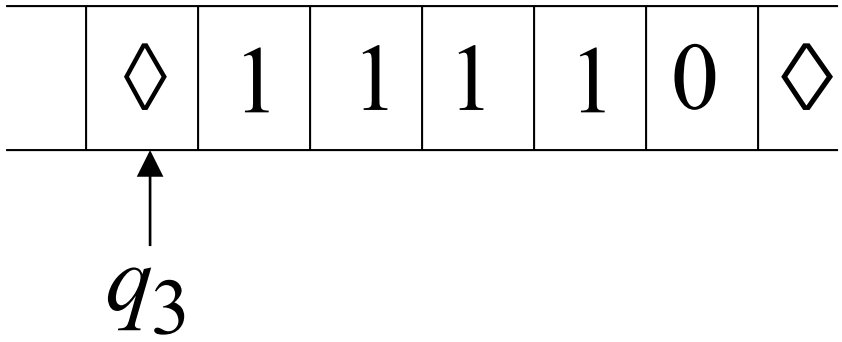

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

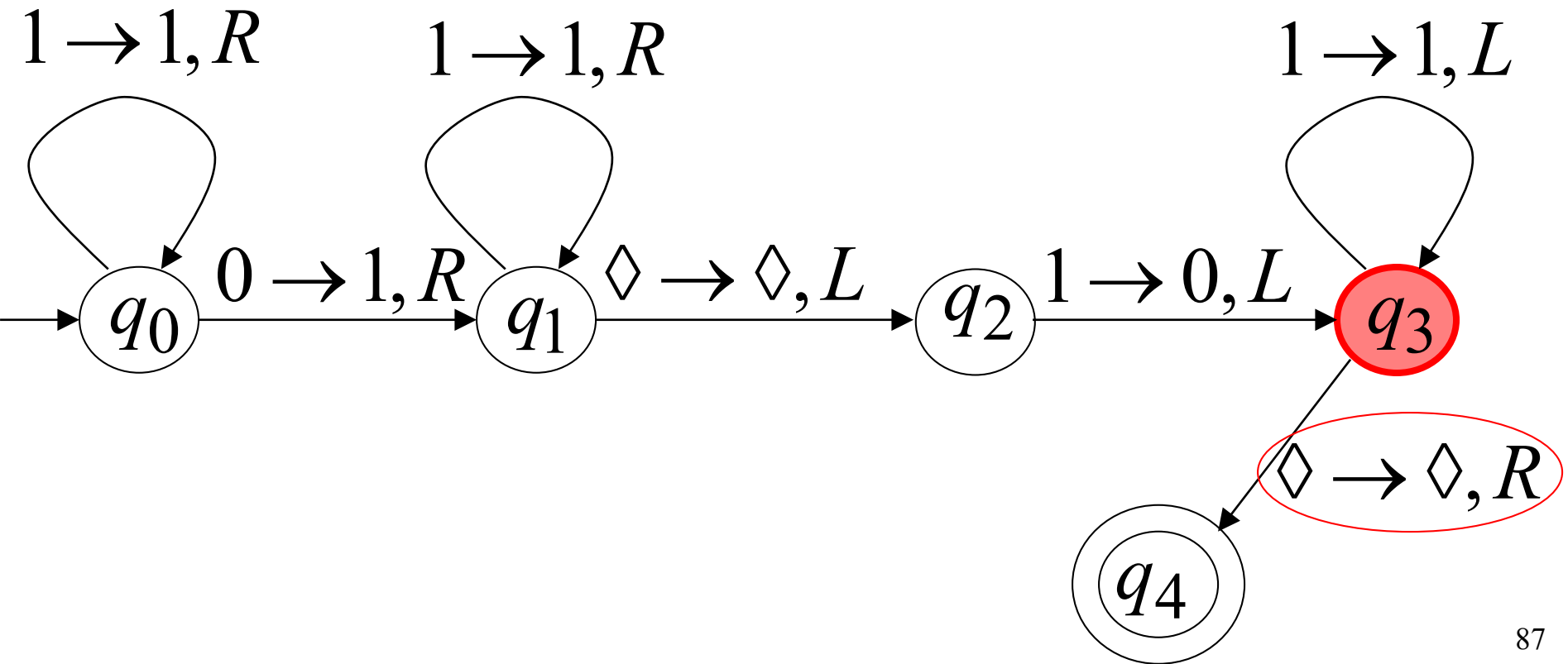
Time 10


◊ 1 1 1 1 0 ◊
q_3

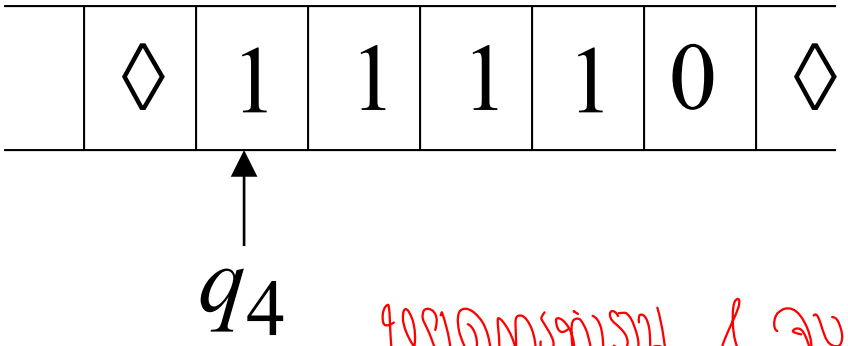

$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

Time 11

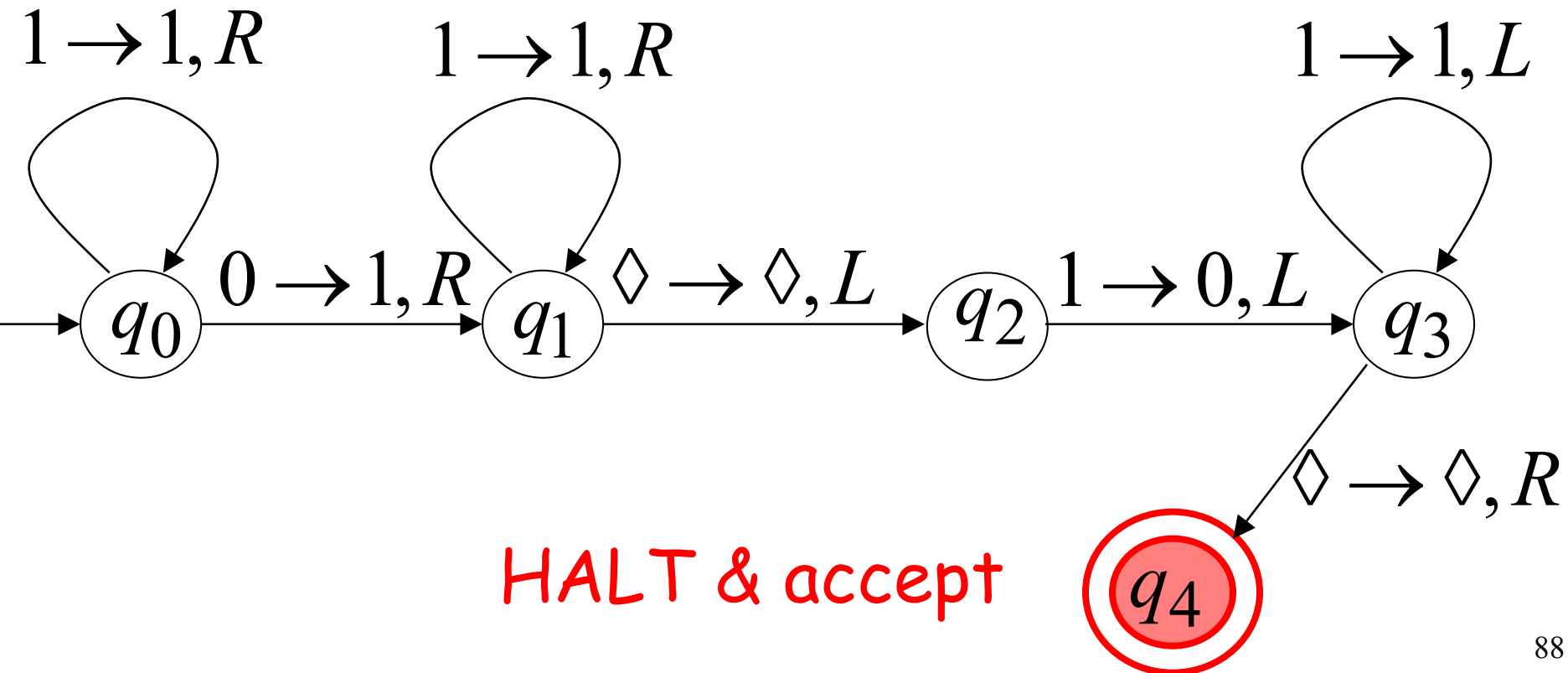

◊ 1 1 1 1 0 ◊
q_3


$1 \to 1, R$
$1 \to 1, R$
$1 \to 1, L$
$0 \to 1, R$
$\Diamond \to \Diamond, L$
$1 \to 0, L$
$\Diamond \to \Diamond, R$
q_0
q_1
q_2
q_3
q_4

Time 12



ผลลัพธ์ 4 จบ



Another Example

The function $f(x) = 2x$ is computable

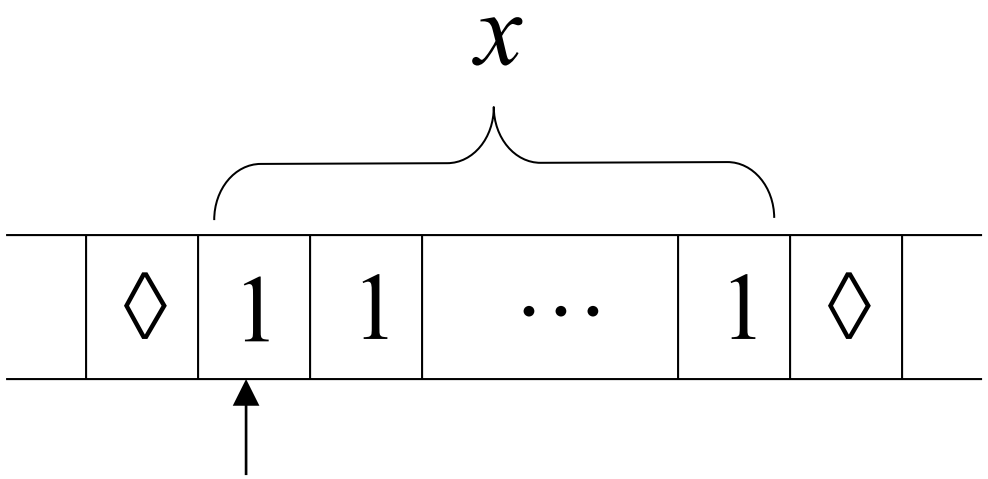
x is integer

Turing Machine:

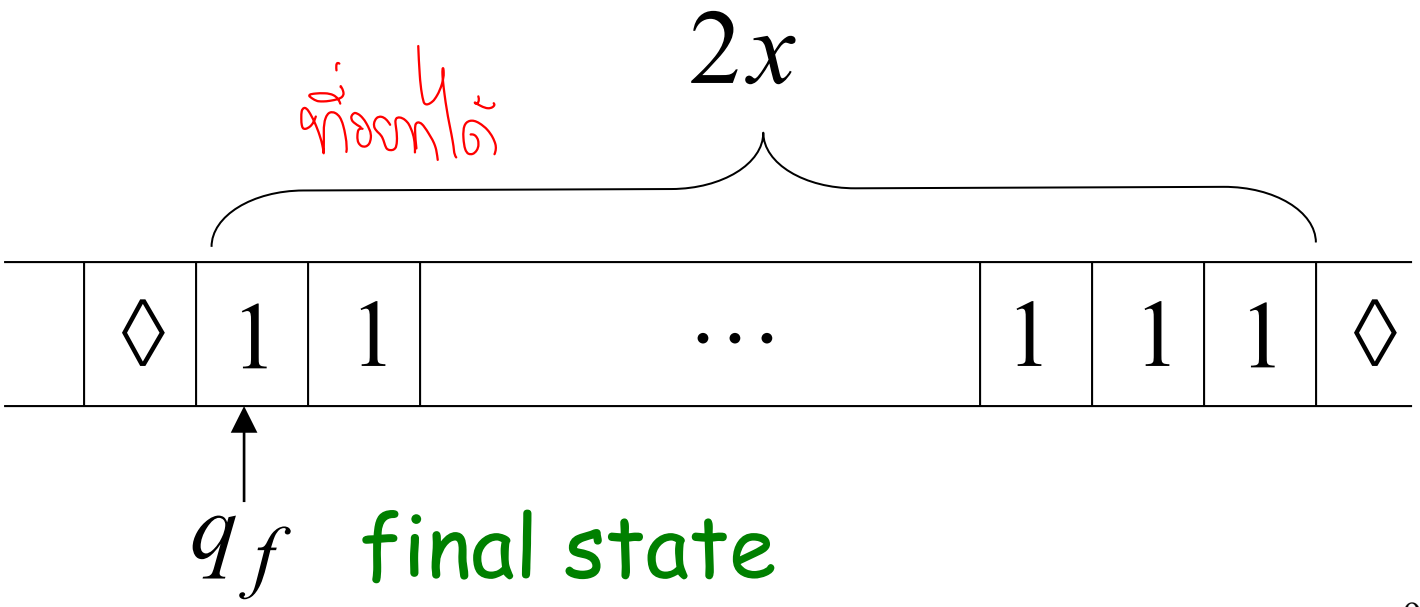
Input string: x unary

Output string: xx unary

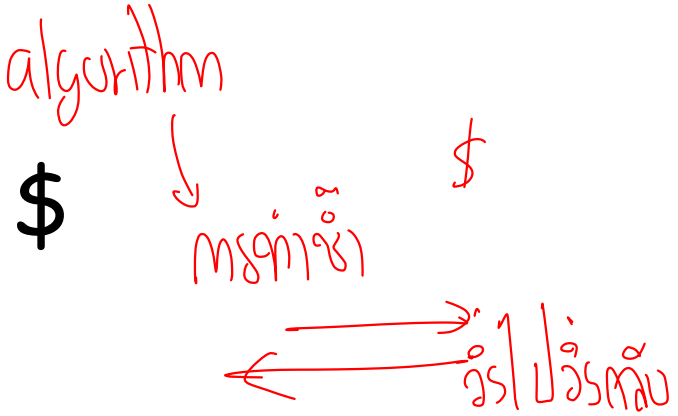
Start



Finish



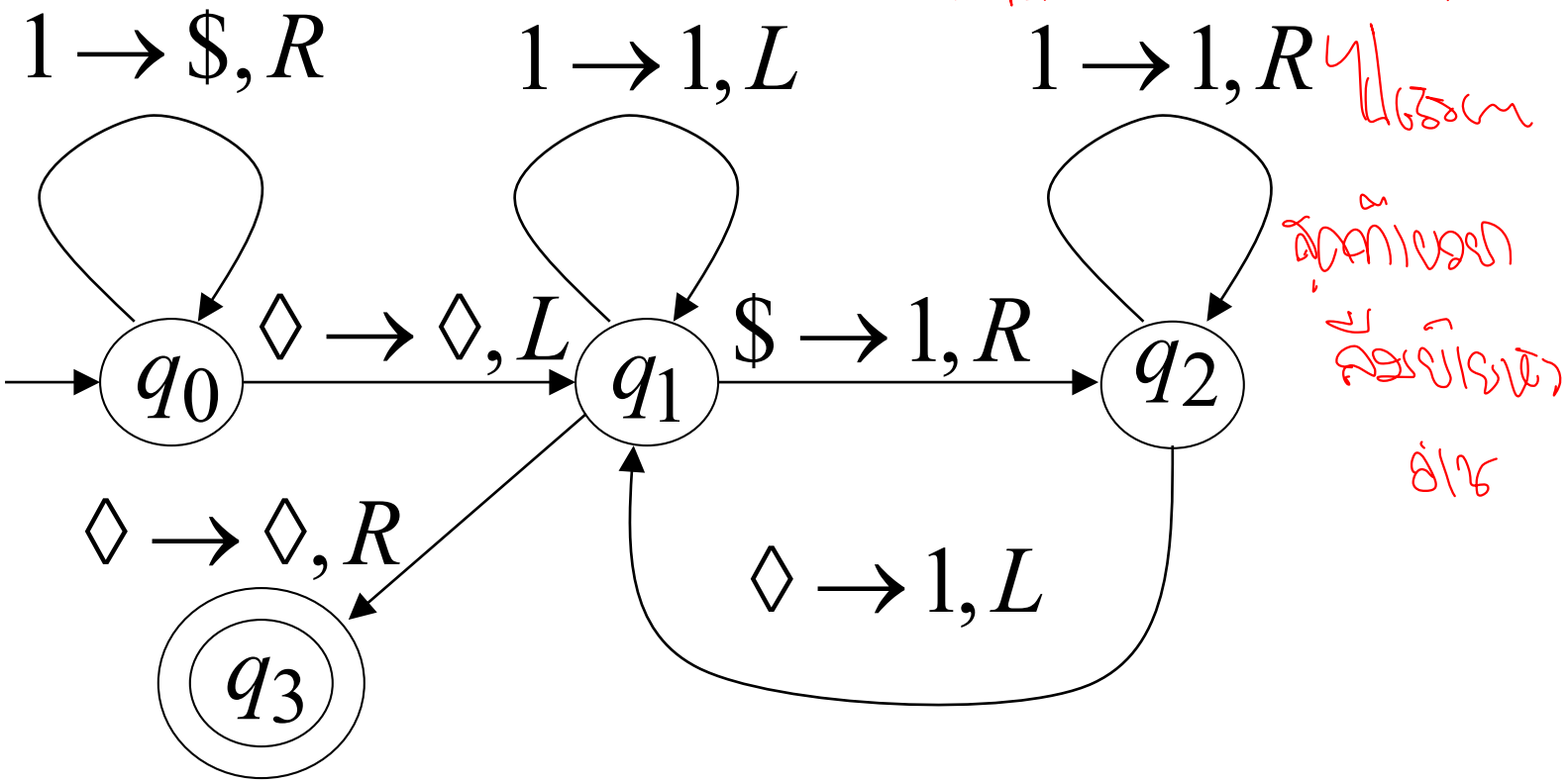
Turing Machine Pseudocode for $f(x) = 2x$



- Replace every 1 with $
- Repeat:
  - Find rightmost $, replace it with 1
  - Go to right end, insert 1

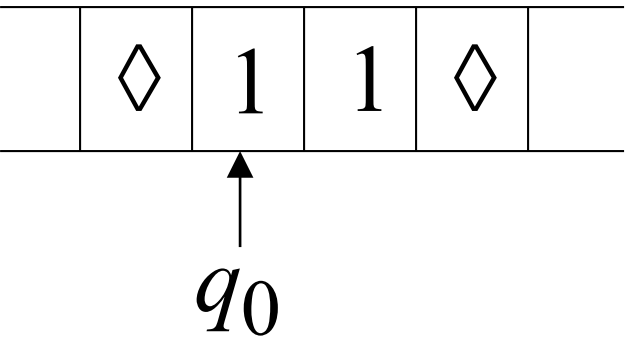
  Until no more $ remain

Turing Machine for $f(x) = 2x$

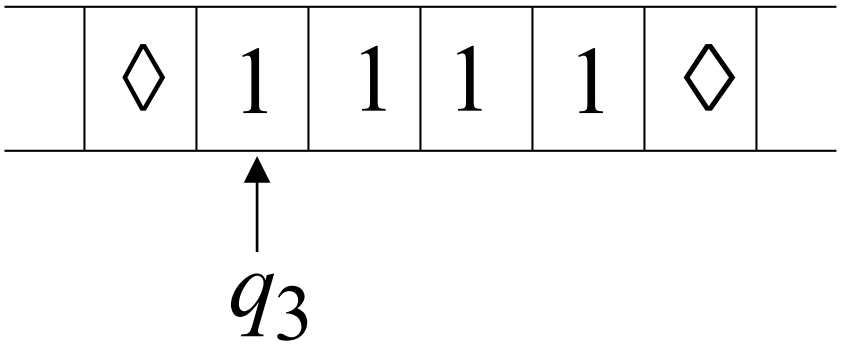


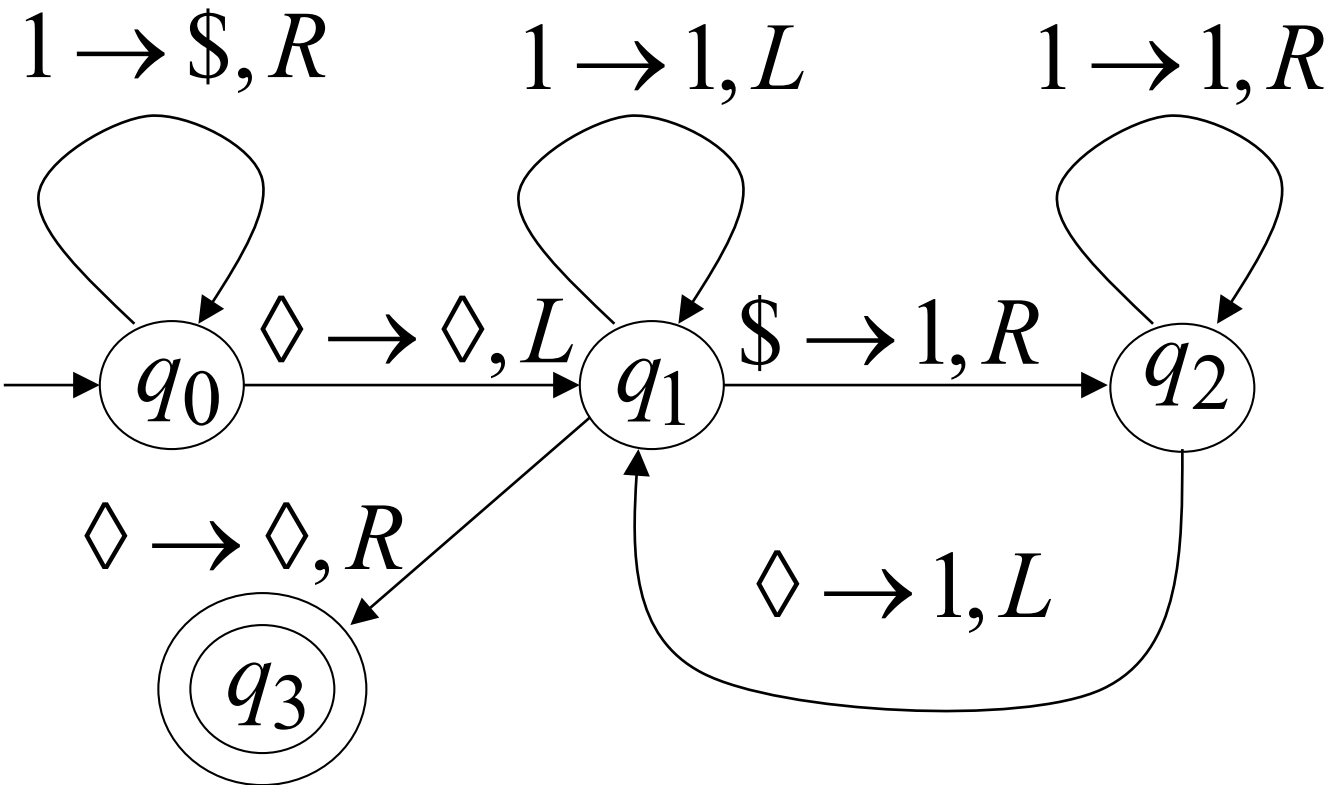
Example

Start



Finish





Another Example

function ccw discrete

The function

is computable

$$f(x,y)=\begin{cases}1 & \text{if } x>y\\ 0 & \text{if } x\le y\end{cases}$$

Turing Machine for

$$f(x,y)=\begin{cases}1 & \text{if } x>y\\ 0 & \text{if } x\le y\end{cases}$$

Input: $x0y$

Output: 1 or 0

Turing Machine Pseudocode:

- Repeat

  Match a 1 from x with a 1 from y

  Until all of x or y is matched

- If a 1 from x is not matched

  erase tape, write 1

  else

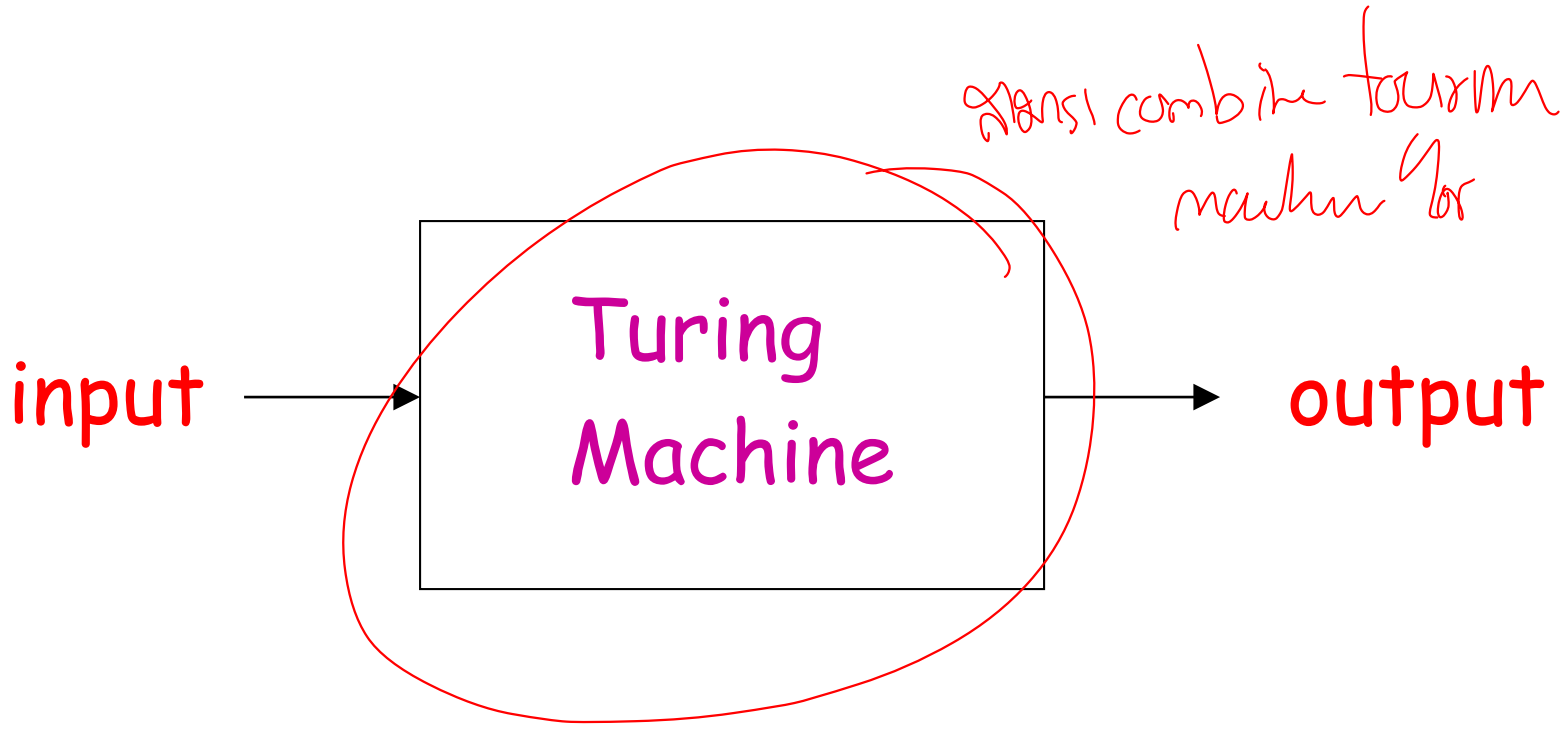
  erase tape, write 0

ถ้าของตัวบนยังเหลือ $(x > y)$

ถ้าของตัวล่างเหลือ $(x \leq y)$

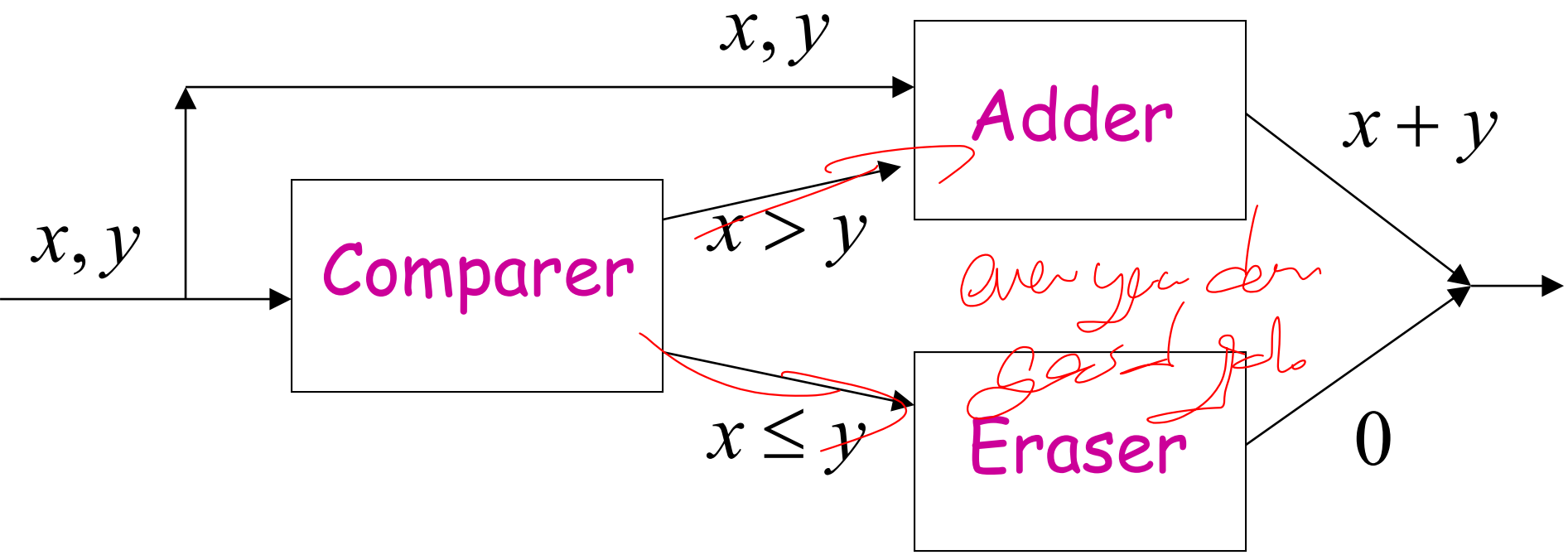
Combining Turing Machines

Block Diagram



Example:

$$f(x,y) = \begin{cases} x+y & \text{if } x > y \\ 0 & \text{if } x \le y \end{cases}$$



ans: why we have to learn