# Counting

# Topics

- Rule of sum
- Rule of product
- Permutation
- Combination
- Pigeonhole Principle
- Inclusion-Exclusion Principle

# Rule of Sum

กฎการบวก งานที่เสร็จแล้วแต่ละท่อนๆ มารวมกัน

- Sequence of task

$$T_1, T_2, T_3, \ldots, T_n$$

- Can be done in $w_1, w_2, w_3, \ldots, w_n$ ways respectively,
- No tasks can be perform simultaneously

- Then , the number of ways to do one of these tasks is

$$w_1 + w_2 + w_3 + \ldots + w_n$$

# Rule of Product

กฎการคูณ

งานยังไม่เสร็จจนกว่าจะทำทุกอัน

- Sequence of tasks ,

$$T_1, T_2, T_3, \ldots, T_n$$

- Can be done in $w_1, w_2, w_3, \ldots, w_n$ ways respectively and every task arrives after the occurrence of the previous task

- Then , the number of ways to perform tasks is

$$w_1 \times w_2 \times w_3 \times \ . \ . \ . \times w_n$$

# Questions

$$\underline{10^4} + \underline{10^5} + 10^6$$

- How many 4 to 6 digit pin codes are there?

- The minimum password length is 6 and the maximum is 8. The password can consist of either an uppercase letter or a digit. There must be at least one digit in the password. How many different passwords are there?

$$P_n = P_{11} - P_6$$

$$36^6 - 26^6$$
$$36^7 - 26^7$$
$$36^8 - 26^8$$

# Permutation

การเรียง

- A permutation is an arrangement of some elements in which order matters. In other words a Permutation is an ordered Combination of elements.

# Examples

- From a set S ={x, y, z} by taking two at a time, all permutations are – xy,yx,xz,zx,yz,zy

ลองหยิบมาเรียง

- We have to form a permutation of three digit numbers from a set of numbers S={1,2,3}. Different three digit numbers will be formed when we arrange the digits. The permutation will be = 123, 132, 213, 231, 312, 321

# Number of Permutation

- Number of permutation of n different things taken ‘r’ at a time is denote by $n_p$

$$n_{\mathrm{p}} = \frac{n!}{(n-r)!}$$

$P^{n}_{r} = C^{n}_{r} \, r!$

- Where n! = 1 x 2 x 3 x 4 x … x (n-1) x n

# Examples 1

6 5 4 3 2 1

- From a bunch of 6 different cards, how many ways we can permute it?

- Solution – As we are taking 6 cards at a time from a deck of 6 cards, the permutation will be

ได้เลขซ้อนกัน

$$^{6}P_{6} = 6! / (6-6)! = 720$$

# Examples 2

ການລຽງ
ຊ້ຳອອກ!

READ
RE

$$\frac{6!}{2!2!}$$

- In how many ways can the letters of the word 'READER' be arranged?

- Solution – There are 6 letters word (2 E, 1 A, 1D and 2R.) in the word 'READER’. The permutation will be

$$6!/[(2!)(1!)(1!)(2!)]=180.$$

# Examples 3

$\underline{3}\ \ \underset{1}{\underline{3}}\ \ \underline{2}\ \ \underset{1}{\underline{2}}\ \ \underline{1}\ \ \underset{1}{\underline{1}} = 3!3!$

- In how ways can the letters of the word 'ORANGE' be arranged so that the consonants occupy only the even positions?

พยัญชนะ =

- Solution – There are 3 vowels and 3 consonants in the word 'ORANGE'. Number of ways of arranging the consonants among themselves = ${}^{3}P_{3}=3!=6$ . The remaining 3 vacant places will be filled up by 3 vowels in ${}^{3}P_{3}=3!=6$ ways. Hence, the total number of permutation is $6×6=36$

# Combination

การหยิบมาเฉยๆ

- A combination is selection of some given elements in which order does not matter. The number of all combinations of n things, taken r at a time is –

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$\binom{n}{r} \qquad \binom{4}{2} = \frac{4\times3}{2\times1}$$

ขอค่าดอบมากี่ตัว

$$\frac{4!}{2!\,(2!)} = \frac{\cancel{4}\times3\times2}{\cancel{2\times2}} = 6$$

ค่าเท่ากัน

# Examples 1

- Find the number of subsets of the set {1,2,3,4,5,6} having 3 elements.
- Solution - The cardinality of the set is 6 and we have to choose 3 elements from the set. Here, the ordering does not matter. Hence, the number of subsets will be

$$\frac{\cancel{6} \times 5 \times 4}{\cancel{3 \times 2} \times 1} = 20$$

$$^6C_3 = 6!/3!(6-3)! = 20$$

# Questions

$C_{ช3ญ2}$ $C_{ช4ญ1}$ $C_{ช5ญ0}$

- From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

- Out of 7 consonants (พยัญชนะ) and 4 vowels (ตัวอักษร), how many words of 3 consonants and 2 vowels can be formed?

  $C^7_3 \, C^4_2$

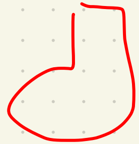
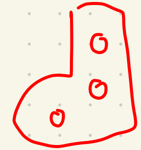
- How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?

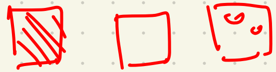
  $\underline{2 \times C^5_2}\ \ \frac{1}{5} = 40$

# Pigeonhole Principle

มาจากกรณีนกที่โชคร้ายที่สุด

- In 1834, German mathematician, Peter Gustav Lejeune Dirichlet, stated a principle which he called the drawer principle. Now, it is known as the pigeonhole principle.
- Pigeonhole Principle states that if there are fewer pigeon holes than total number of pigeons and each pigeon is put in a pigeon hole, then there must be at least one pigeon hole with more than one pigeon. If n pigeons are put into m pigeonholes where n > m, there's a hole with more than one pigeon.

 x5  x7  x 2 หยิบอย่างน้อยกี่ข้างให้เป็นคู่

มองกรณีที่โชคร้ายที่สุดคือ ได้สีละข้าง  x1

ตรงนี้ไม่ว่าหยิบ $\boxed{?}$ ก็ได้ 1 คู่

∴ ต้องหยิบทั้งหมด 4 ข้าง

# Examples

โชคร้ายสุดคือคนละตัวอักษรเลย 26 คน ∴ 27 คนจะมีตัวอักษรซ้ำกัน

- There must be at least two people in a class of 30 whose names start with the same alphabet.

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■ × 5

□ × 12

▣ × 7

อยากได้ครบ 3 สี

โชคร้าย ■ 5 □ 12

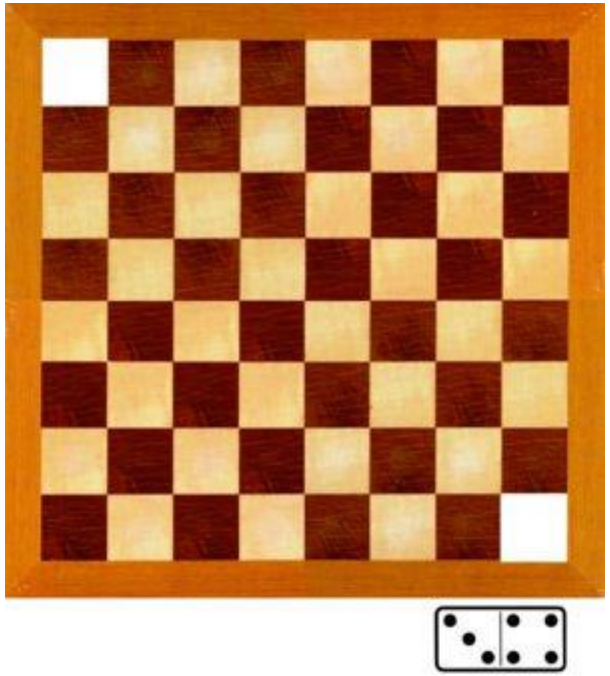
โชคร้าย □ 12 ▣ 7

หยิบเพิ่มอีก 1 ก็ได้ครบแล้ว

∴ หยิบทั้งหมด 12+7+1 = 20 อัน

# Questions: Dominoes on a Chess Board

- Assume we remove two of the diagonally opposite corners of a chess board. Then it is impossible to cover the chess board with dominoes whose size is two board squares each.



# Inclusion – Exclusion Principle

- The Inclusion-exclusion principle computes the cardinal number of the union of multiple non-disjoint sets. For two sets A and B, the principle states −

$$|A \cup B| = |A| + |B| - |A \cap B|$$

- For three sets A, B and C, the principle states −

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

# Problem 1 : How many integers from 1 to 50 are multiples of 2 or 3 but not both?

Solution

- From 1 to 100, there are 50/2=25
- numbers which are multiples of 2. There are 50/3=16
- numbers which are multiples of 3. There are 50/6=8
- numbers which are multiples of both 2 and 3.So,
- $|A|=25$ , $|B|=16$ and $|A \cap B|=8$
- $|A \cup B|=|A|+|B|-|A \cap B|=25+16-8=33$

# Problem 2

- In a group of 50 students 24 like cold drinks and 36 like hot drinks and each student likes at least one of the two drinks. How many like both coffee and tea?

Solution

- Let X be the set of students who like cold drinks and Y be the set of people who like hot drinks.
- So,
- $|X \cup Y|=50$, $|X|=24, |Y|=36$
- $|X \cap Y|=|X|+|Y|-|X \cup Y|=24+36-50=60-50=10$
- Hence, there are 10 students who like both tea and coffee.

# Questions

ขาเองเถอะ

- How many of the integers n with 1 <= n <= 150 are relatively prime to 70?