Chapter 1

A simple fluid-structure interaction problem: Finite Reynolds number flow, driven by an oscillating ring.

This is a very simple fluid-structure interaction problem: We study the finite-Reynolds number internal flow generated by an oscillating ring. The wall motion only has a single degree of freedom: The ring's average radius, which needs to be adjusted to conserve mass. [This is a warm-up problem for the full fluid structure interaction problem discussed in the next example]. We compare the predictions for the flow field against asymptotic results.

The nodal positions in the fluid domain is updated by MacroElements.

```
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
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//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC//
//LIC//======
// Driver for 2D Navier Stokes flow, driven by oscillating ring
// with pseudo-elasticity: The mean radius of ring is adjusted to
// allow conservation of volume (area).
// Oomph-lib includes
#include "generic.h"
#include "navier_stokes.h"
//Need to instantiate templated mesh
#include "meshes/quarter_circle_sector_mesh.h"
//Include namespace containing Sarah's asymptotics
#include "osc_ring_sarah_asymptotics.h"
using namespace std;
using namespace oomph;
using namespace MathematicalConstants;
```

```
/// Namespace for physical parameters
namespace Global_Physical_Variables
 /// Reynolds number
double Re=100.0; // ADJUST_PRIORITY
 /// Reynolds \mathbf{x} Strouhal number
double ReSt=100.0; // ADJUST_PRIORITY
/// Driver for oscillating ring problem: Wall performs oscillations /// that resemble eigenmodes of freely oscillating ring and drives /// viscous fluid flow. Mean radius of wall is adjustable and
/// responds to a pressure value in the fluid to allow for
/// mass conservation.
//=====
template<class ELEMENT>
class OscRingNStProblem : public Problem
public:
 /// Constructor: Pass timestep and function pointer to the
 /// solution that provides the initial conditions for the fluid
OscRingNStProblem(const double& dt,
                   FiniteElement::UnsteadyExactSolutionFctPt IC_fct_pt);
 /// Destructor (empty)
 ~OscRingNStProblem(){}
 /// Get pointer to wall as geometric object
GeomObject* wall_pt()
   return Wall_pt;
 /// Update after solve (empty)
void actions_after_newton_solve(){}
 /// Update the problem specs before solve (empty)
void actions_before_newton_solve(){}
 /// Update the problem specs before checking Newton
 /// convergence: Update the fluid mesh and re-set velocity
 /// boundary conditions -- no slip velocity on the wall means
 /// that the velocity on the wall is dependent.
 void actions_before_newton_convergence_check()
  // Update the fluid mesh -- auxiliary update function for algebraic
  // nodes automatically updates no slip condition.
  fluid_mesh_pt()->node_update();
 /// Update the problem specs after adaptation:
 /// Set auxiliary update function that applies no slip on all /// boundary nodes and choose fluid pressure dof that drives
 /// the wall deformation
 void actions_after_adapt()
   // Ring boundary: No slip; this also implies that the velocity needs
  // to be updated in response to wall motion. This needs to be reset
// every time the mesh is changed -- there's no mechanism by which
   // auxiliary update functions are copied to newly created nodes.
   // (that's because unlike boundary conditions, they don't
   // occur exclusively at boundaries)
   unsigned ibound=1;
```

```
unsigned num_nod= fluid_mesh_pt()->nboundary_node(ibound);
    for (unsigned inod=0;inod<num_nod;inod++)</pre>
      fluid_mesh_pt()->boundary_node_pt(ibound,inod)->
       set_auxiliary_node_update_fct_pt(
FSI_functions::apply_no_slip_on_moving_wall);
   // Set the reference pressure as the constant pressure in element \boldsymbol{0}
  dynamic_cast<PseudoBucklingRingElement*>(Wall_pt)
   ->set_reference_pressure_pt(fluid_mesh_pt()->element_pt(0)
                                 ->internal_data_pt(0));
 /// Run the time integration for ntsteps steps
void unsteady_run(const unsigned &ntsteps, const bool& restarted,
                   DocInfo& doc info);
 /// Set initial condition (incl previous timesteps) according
 /// to specified function.
void set_initial_condition();
/// Doc the solution
void doc_solution(DocInfo& doc_info);
 /// Access function for the fluid mesh
MacroElementNodeUpdateRefineableQuarterCircleSectorMesh<ELEMENT>* fluid_mesh_pt()
   return Fluid_mesh_pt;
 /// Dump problem data.
void dump_it(ofstream& dump_file, DocInfo doc_info);
 /// Read problem data.
void restart(ifstream& restart_file);
/// Write header for trace file
void write_trace_file_header();
 /// Function pointer to set the intial condition
FiniteElement::UnsteadyExactSolutionFctPt IC_Fct_pt;
 /// Pointer to wall
GeomObject* Wall_pt;
 /// Pointer to fluid mesh
MacroElementNodeUpdateRefineableQuarterCircleSectorMesh<ELEMENT>* Fluid_mesh_pt;
 /// Pointer to wall mesh (contains only a single GeneralisedElement)
Mesh* Wall_mesh_pt;
 /// Trace file
ofstream Trace_file;
 /// Pointer to node on coarsest mesh on which velocity is traced
Node* Veloc_trace_node_pt;
/// Pointer to node in symmetry plane on coarsest mesh at /// which velocity is traced \,
Node* Sarah_veloc_trace_node_pt;
/// Constructor: Pass (constant) timestep and function pointer to the solution
/// that provides the initial conditions for the fluid.
template<class ELEMENT>
OscRingNStProblem<ELEMENT>::OscRingNStProblem(const double& dt,
FiniteElement::UnsteadyExactSolutionFctPt IC_fct_pt) : IC_Fct_pt(IC_fct_pt)
\ensuremath{//} Period of oscillation
double T=1.0;
//Allocate the timestepper
add_time_stepper_pt(new BDF<4>);
 // Initialise timestep -- also sets the weights for all timesteppers
 // in the problem.
```

```
initialise_dt(dt);
// Parameters for pseudo-buckling ring
double eps_buckl=0.1;  // ADJUST_PRIORITY
double ampl_ratio=-0.5;  // ADJUST_PRIORITY
double ampl_ratio=-0.5; // ADJUST_PRI
unsigned n_buckl=2; // ADJUST_PRIORITY
double r_0=1.0;
 // Build wall geometric element
Wall_pt=new PseudoBucklingRingElement(eps_buckl,ampl_ratio,n_buckl,r_0,T,
                                                                        time_stepper_pt());
// Fluid mesh is suspended from wall between these two Lagrangian
// coordinates:
double xi_lo=0.0;
double xi_hi=2.0*atan(1.0);
// Fractional position of dividing line for two outer blocks in fluid mesh
double fract mid=0.5;
 // Build fluid mesh
\verb|Fluid_mesh_pt=new| MacroElementNodeUpdateRefineableQuarterCircleSectorMesh<| ELEMENT>( | Continue of the c
  Wall_pt, xi_lo, fract_mid, xi_hi, time_stepper_pt());
// Set error estimator
Z2ErrorEstimator* error_estimator_pt=new Z2ErrorEstimator;
Fluid_mesh_pt->spatial_error_estimator_pt()=error_estimator_pt;
 // Fluid mesh is first sub-mesh
add_sub_mesh(Fluid_mesh_pt);
// Build wall mesh
Wall mesh pt=new Mesh;
// Wall mesh is completely empty: Add Wall element in its GeneralisedElement
 // incarnation
Wall_mesh_pt->add_element_pt(dynamic_cast<GeneralisedElement*>(Wall_pt));
// Wall mesh is second sub-mesh
add_sub_mesh(Wall_mesh_pt);
// Combine all submeshes into a single Mesh
build_global_mesh();
// Extract pointer to node at center of mesh (this node exists
// at all refinement levels and can be used to doc continuous timetrace
// of velocities)
unsigned nnode=fluid_mesh_pt()->finite_element_pt(0)->nnode();
\label{lem:veloc_trace_node_pt} Veloc\_trace\_node\_pt=fluid\_mesh\_pt() -> finite\_element\_pt(0) -> node\_pt(nnode-1) \end{substitute} \footnote{\columnwidth}{.}
// Extract pointer to node in symmetry plane (this node exists // at all refinement levels and can be used to doc continuous timetrace \,
// of velocities)
unsigned nnode_1d=dynamic_cast<ELEMENT *> (
  fluid_mesh_pt()->finite_element_pt(0))->nnode_1d();
Sarah_veloc_trace_node_pt=fluid_mesh_pt()->
  finite_element_pt(0)->node_pt(nnode_1d-1);
// The "pseudo-elastic" wall element is "loaded" by a pressure.
// Use the "constant" pressure component in Crouzeix Raviart
 // fluid element as that pressure.
dynamic cast<PseudoBucklingRingElement*>(Wall pt)
  ->set_reference_pressure_pt(fluid_mesh_pt()->element_pt(0)
                                                       ->internal_data_pt(0));
// Set the boundary conditions for this problem:
// All nodes are free by default -- just pin the ones that have
// Dirichlet conditions here.
// Bottom boundary:
unsigned ibound=0;
    unsigned num_nod= fluid_mesh_pt()->nboundary_node(ibound);
    for (unsigned inod=0;inod<num nod;inod++)</pre>
        // Pin vertical velocity
           fluid_mesh_pt()->boundary_node_pt(ibound,inod)->pin(1);
       }
// Ring boundary: No slip; this also implies that the velocity needs
 // to be updated in response to wall motion
ibound=1;
  {
```

```
unsigned num_nod=fluid_mesh_pt()->nboundary_node(ibound);
   for (unsigned inod=0;inod<num_nod;inod++)</pre>
     // Which node are we dealing with?
    Node* node_pt=fluid_mesh_pt()->boundary_node_pt(ibound,inod);
     // Set auxiliary update function pointer to apply no-slip condition
     // to velocities whenever nodal position is updated
     node_pt->set_auxiliary_node_update_fct_pt(
      FSI_functions::apply_no_slip_on_moving_wall);
     // Pin both velocities
     for (unsigned i=0; i<2; i++)</pre>
       node_pt->pin(i);
      }
  }
 // Left boundary:
   {\tt unsigned num\_nod=fluid\_mesh\_pt()->nboundary\_node(ibound);}
   for (unsigned inod=0;inod<num_nod;inod++)</pre>
    // Pin horizontal velocity
       fluid_mesh_pt()->boundary_node_pt(ibound,inod)->pin(0);
   }
 // Complete the build of all elements so they are fully functional
 //Find number of elements in mesh
unsigned n_element = fluid_mesh_pt()->nelement();
 // Loop over the elements to set up element-specific
 // things that cannot be handled by constructor
 for (unsigned i=0;i<n_element;i++)</pre>
   // Upcast from FiniteElement to the present element
  ELEMENT *el_pt = dynamic_cast<ELEMENT*>(fluid_mesh_pt()->element_pt(i));
   //{
m Set} the Reynolds number, etc
   el_pt->re_pt() = &Global_Physical_Variables::Re;
  el_pt->re_st_pt() = &Global_Physical_Variables::ReSt;
 //Attach the boundary conditions to the mesh
cout «"Number of equations: " « assign_eqn_numbers() « std::endl;
 // Set parameters for Sarah's asymptotic solution
 // Amplitude of the oscillation
 SarahBL::epsilon=static_cast<PseudoBucklingRingElement *>(Wall_pt)->
 eps_buckl();
 // Womersley number is the same as square root of Reynolds number
 SarahBL::alpha=sqrt(Global_Physical_Variables::Re);
 // Amplitude ratio
SarahBL::A=static_cast<PseudoBucklingRingElement*>(Wall_pt)->ampl_ratio();
 // Buckling wavenumber
SarahBL::N=static_cast<PseudoBucklingRingElement*>(Wall_pt)->n_buckl_float();
 // Frequency of oscillation (period is one)
SarahBL::Omega=2.0*MathematicalConstants::Pi;
}
/// Set initial condition: Assign previous and current values
/// of the velocity from the velocity field specified via
/// the function pointer.
/// Values are assigned so that the velocities and accelerations % \frac{1}{2}\left( \frac{1}{2}\right) =\frac{1}{2}\left( \frac{1}{2}\right) 
/// are correct for current time.
```

```
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::set_initial_condition()
// Elastic wall: We're starting from a given initial state in which
// the wall is undeformed. If set_initial_condition() is called again
 // after mesh refinement for first timestep, this needs to be reset.
dynamic_cast<PseudoBucklingRingElement*>(Wall_pt)->set_R_0(1.0);
// Backup time in global timestepper
double backed_up_time=time_pt()->time();
// Past history for velocities needs to be established for t=time0-deltat, \dots
// Then provide current values (at t=time0) which will also form
// the initial guess for first solve at t=time0+deltat
// Vector of exact solution values (includes pressure)
Vector<double> soln(3);
Vector<double> x(2);
//Find number of nodes in mesh
unsigned num_nod = fluid_mesh_pt()->nnode();
// Get continuous times at previous timesteps
int nprev_steps=time_stepper_pt()->nprev_values();
Vector<double> prev_time(nprev_steps+1);
for (int itime=nprev_steps;itime>=0;itime--)
  prev_time[itime] = time_pt() -> time(unsigned(itime));
// Loop over current & previous timesteps (in outer loop because
 // the mesh might also move)
for (int itime=nprev_steps;itime>=0;itime--)
  double time=prev_time[itime];
  // Set global time (because this is how the geometric object refers
   // to continous time
  time_pt()->time()=time;
  cout « "setting IC at time =" « time « std::endl;
  // Update the fluid mesh for this value of the continuous time
  // (The wall object reads the continous time from the same
   // global time object)
  fluid_mesh_pt()->node_update();
  // Loop over the nodes to set initial guess everywhere
  for (unsigned jnod=0; jnod<num_nod; jnod++)</pre>
    // Get nodal coordinates
    x[0]=fluid_mesh_pt()->node_pt(jnod)->x(0);
    x[1]=fluid_mesh_pt()->node_pt(jnod)->x(1);
    // Get intial solution
    (*IC\_Fct\_pt) (time, x, soln);
    // Loop over velocity directions (velocities are in soln[0] and soln[1]).
    for (unsigned i=0;i<2;i++)</pre>
      fluid_mesh_pt()->node_pt(jnod)->set_value(itime,i,soln[i]);
    // Loop over coordinate directions
    for (unsigned i=0;i<2;i++)</pre>
      fluid_mesh_pt()->node_pt(jnod)->x(itime,i)=x[i];
// Reset backed up time for global timestepper
time_pt()->time()=backed_up_time;
/// Doc the solution
```

```
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::doc_solution(DocInfo& doc_info)
ofstream some_file;
char filename[100];
// Number of plot points
unsigned npts;
npts=5;
// Output solution on fluid mesh
sprintf(filename, "%s/soln%i.dat", doc_info.directory().c_str(),
        doc_info.number());
 //some_file.precision(20);
 some_file.open(filename);
 unsigned nelem=fluid_mesh_pt()->nelement();
 for (unsigned ielem=0;ielem<nelem;ielem++)</pre>
  dynamic_cast<ELEMENT*>(fluid_mesh_pt()->element_pt(ielem))->
   full_output(some_file,npts);
 some_file.close();
 // Plot wall posn
 sprintf(filename, "%s/Wall%i.dat", doc_info.directory().c_str(),
        doc_info.number());
 some_file.open(filename);
 unsigned nplot=100;
 Vector<double > xi_wall(1);
 Vector<double > r_wall(2);
 for (unsigned iplot=0;iplot<nplot;iplot++)</pre>
   xi_wall[0]=0.5*Pi*double(iplot)/double(nplot-1);
   Wall_pt->position(xi_wall,r_wall);
some_file « r_wall[0] « " " « r_wall[1] « std::endl;
 some_file.close();
 // Doc Sarah's asymptotic solution
 sprintf(filename, "%s/exact_soln%i.dat", doc_info.directory().c_str(),
        doc_info.number());
 some_file.open(filename);
 fluid_mesh_pt()->output_fct(some_file,npts,
                      time_stepper_pt()->time_pt()->time(),
                      SarahBL::full_exact_soln);
some file.close();
 // Get position of control point
 Vector<double> r(2);
Vector<double> xi(1);
 xi[0]=MathematicalConstants::Pi/2.0;
wall_pt()->position(xi,r);
 // Get total volume (area) of computational domain, energies and average
 // pressure
 //----
double area=0.0;
double press_int=0.0;
double diss=0.0;
 double kin_en=0.0;
nelem=fluid_mesh_pt()->nelement();
 for (unsigned ielem=0;ielem<nelem;ielem++)</pre>
  area+=fluid_mesh_pt()->finite_element_pt(ielem)->size();
  press_int+=dynamic_cast<ELEMENT*>(fluid_mesh_pt()->element_pt(ielem))
    ->pressure_integral();
  diss+=dynamic_cast<ELEMENT*>(fluid_mesh_pt()->element_pt(ielem))->
   dissipation();
  kin_en+=dynamic_cast<ELEMENT*>(fluid_mesh_pt()->element_pt(ielem))->
```

```
kin_energy();
// Total kinetic energy in entire domain
double global_kin=4.0*kin_en;
// Max/min refinement level
unsigned min_level;
unsigned max_level;
fluid_mesh_pt()->get_refinement_levels(min_level, max_level);
// Total dissipation for quarter domain
double time=time_pt()->time();
double diss_asympt=SarahBL::Total_Diss_sarah(time)/4.0;
// Asymptotic predictions for velocities at control point
Vector<double> x_sarah(2);
Vector<double> soln_sarah(3);
x_sarah[0]=Sarah_veloc_trace_node_pt->x(0);
x_sarah[1]=Sarah_veloc_trace_node_pt->x(1);
SarahBL::exact_soln(time,x_sarah,soln_sarah);
// Doc
Trace_file « time_pt()->time()
          « " " « SarahBL::Kin_energy_sarah(time_pt()->time())
           « " " « static_cast<PseudoBucklingRingElement*>(Wall_pt)->r_0()
           « " " « area
           « " " « press_int/area
           « " " « diss
           « " " « diss_asympt
           « " " « Veloc_trace_node_pt->x(0)
           « " " « Veloc_trace_node_pt->x(1)
           « " " « Veloc_trace_node_pt->value(0)
           « " " « Veloc_trace_node_pt->value(1)
             " " « fluid_mesh_pt()->nelement()
           " " " « ndof()
           « " " « min_level
           « " " « max_level
           « " " « fluid_mesh_pt()->nrefinement_overruled()
           « " " « fluid_mesh_pt()->max_error()
           « " " « fluid_mesh_pt() ->min_error()
             " " « fluid_mesh_pt()->max_permitted_error()
           « " " « fluid_mesh_pt()->min_permitted_error()
           « " " « fluid_mesh_pt()->max_keep_unrefined()
           « " " « doc_info.number()
           « " " « Sarah_veloc_trace_node_pt->x(0)
             " " « Sarah_veloc_trace_node_pt->x(1)
           « " " « Sarah_veloc_trace_node_pt->value(0)
           « " " « Sarah_veloc_trace_node_pt->value(1)
           « " " « x_sarah[0]
           " " « x_sarah[1]
           « " " « soln_sarah[0]
           « " " « soln_sarah[1]
           « static_cast<PseudoBucklingRingElement*>(Wall_pt)->r_0()-1.0
           « std::endl;
// Output fluid solution on coarse-ish mesh
// Extract all elements from quadtree representation
Vector<Tree*> all_element_pt;
fluid_mesh_pt()->forest_pt()->
 stick_all_tree_nodes_into_vector(all_element pt);
// Build a coarse mesh
Mesh* coarse_mesh_pt = new Mesh();
//Loop over all elements and check if their refinement level is
//equal to the mesh's minimum refinement level
nelem=all_element_pt.size();
for (unsigned ielem=0;ielem<nelem;ielem++)</pre>
  Tree* el_pt=all_element_pt[ielem];
  if (el_pt->level() ==min_level)
  {
    coarse_mesh_pt->add_element_pt(el_pt->object_pt());
// Output fluid solution on coarse mesh
sprintf(filename,"%s/coarse_soln%i.dat",doc_info.directory().c_str(),
        doc info.number());
```

```
some_file.open(filename);
nelem=coarse_mesh_pt->nelement();
 for (unsigned ielem=0;ielem<nelem;ielem++)</pre>
  dynamic_cast<ELEMENT*>(coarse_mesh_pt->element_pt(ielem))->
   full_output(some_file,npts);
 some_file.close();
// Write restart file
sprintf(filename,"%s/restart%i.dat",doc_info.directory().c_str(),
       doc_info.number());
some_file.open(filename);
dump_it(some_file,doc_info);
some_file.close();
}
/// Dump the solution
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::dump_it(ofstream& dump_file,DocInfo doc_info)
// Dump refinement status of mesh
//fluid_mesh_pt()->dump_refinement(dump_file);
// Call generic dump()
Problem::dump(dump_file);
}
/// Read solution from disk
//============
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::restart(ifstream& restart_file)
 // Refine fluid mesh according to the instructions in restart file
//fluid_mesh_pt()->refine(restart_file);
// Rebuild the global mesh
//rebuild_global_mesh();
// Read generic problem data
Problem::read(restart_file);
    // Complete build of all elements so they are fully functional
// finish_problem_setup();
//Assign equation numbers
 « std::endl« std::endl;
/// Driver for timestepping the problem: Fixed timestep but
/// guaranteed spatial accuracy. Beautiful, innit?
///
//=======
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::unsteady_run(const unsigned& ntsteps,
                                            const bool& restarted,
                                           DocInfo& doc_info)
{
 // Open trace file
 char filename[100];
 sprintf(filename, "%s/trace.dat", doc_info.directory().c_str());
Trace_file.open(filename);
// Max. number of adaptations per solve
unsigned max_adapt;
 // Max. number of adaptations per solve
 if (restarted)
  max adapt=0;
```

```
else
  max_adapt=1;
// Max. and min. error for adaptive refinement/unrefinement
fluid_mesh_pt()->max_permitted_error()= 0.5e-2;
fluid_mesh_pt()->min_permitted_error()= 0.5e-3;
// Don't allow refinement to drop under given level
fluid_mesh_pt()->min_refinement_level()=2;
// Don't allow refinement beyond given level
fluid_mesh_pt()->max_refinement_level()=6;
// Don't bother adapting the mesh if no refinement is required
// and if less than ... elements are to be merged.
fluid_mesh_pt()->max_keep_unrefined()=20;
// Get max/min refinement levels in mesh
unsigned min_refinement_level;
unsigned max_refinement_level;
\verb|fluid_mesh_pt()->| get_refinement_levels(min_refinement_level, |
                                        max_refinement_level);
// Doc refinement targets
fluid_mesh_pt()->doc_adaptivity_targets(cout);
// Write header for trace file
write_trace_file_header();
// Doc initial condition
doc solution (doc info);
doc_info.number()++;
// Switch off doc during solve
doc_info.disable_doc();
// If we set first to true, then initial guess will be re-assigned
// after mesh adaptation. Don't want this if we've done a restart.
bool first;
bool shift;
if (restarted)
  first=false;
  shift=false;
  // Move time back by dt to make sure we're re-solving the read-in solution
  time_pt()->time()-=time_pt()->dt();
else
  first=true;
  shift=true;
//Take the first fixed timestep with specified tolerance for Newton solver
double dt=time_pt()->dt();
unsteady_newton_solve(dt,max_adapt,first,shift);
// Switch doc back on
doc_info.enable_doc();
// Doc initial solution
doc_solution(doc_info);
doc info.number()++;
// Now do normal run; allow for one mesh adaptation per timestep
//Loop over the remaining timesteps
for(unsigned t=2;t<=ntsteps;t++)</pre>
  // Switch off doc during solve
  doc_info.disable_doc();
  //Take fixed timestep
  first=false;
  unsteady_newton_solve(dt,max_adapt,first);
  // Switch doc back on
  doc_info.enable_doc();
  // Doc solution
```

```
//if (icount%10==0)
          doc_solution(doc_info);
          doc_info.number()++;
  // Close trace file
 Trace_file.close();
}
/// Write trace file header
//=========
template<class ELEMENT>
void OscRingNStProblem<ELEMENT>::write_trace_file_header()
  // Doc parameters in trace file
 Trace_file « "# err_max " « fluid_mesh_pt()->max_permitted_error() « std::endl;
Trace_file « "# err_min " « fluid_mesh_pt()->min_permitted_error() « std::endl;
Trace_file « "# Re " « Global_Physical_Variables::Re « std::endl;
  Trace_file « "# St " « Global_Physical_Variables::ReSt/
                                                        Global_Physical_Variables::Re « std::endl;
 Trace_file « "# dt " « time_stepper_pt()->time_pt()->dt() « std::endl;
Trace_file « "# initial # elements " « mesh_pt()->nelement() « std::endl;
Trace_file « "# min_refinement_level "
                         « fluid_mesh_pt()->min_refinement_level() « std::endl;
  Trace_file « "# max_refinement_level "
                          « fluid_mesh_pt()->max_refinement_level() « std::endl;
Trace_file « "VARIABLES=\"time\",\"V_c_t_r_l\",\"e_k_i_n\"";
Trace_file « ",\"e_k_i_n_(_a_s_y_m_p_t_)\",\"R_0\",\"Area\"";
Trace_file « ",\"Average pressure\",\"Total dissipation (quarter domain)\"";
Trace_file « ",\"axymptotic dissipation (quarter domain)\"";
Trace_file « ",\"x<sub>!</sub><tup>\track)</sup>\"";
Trace_file « ",\"x<sub>2</sub><sup>(track)</sup>\"";
Trace_file « ",\"u<sub>!</sub><tup>\track)</sup>\"";
Trace_file « ",\"u<sub>2</sub><sup>(track)</sup>\"";
Trace_file « ",\"u\sub>2</sub><sup>(track)</sup>\"";
Trace_file « ",\"N\sub>element</sub>\"";
Trace_file « ",\"N\sub>dof</sub>\"";
Trace_file « ",\"max. refinement level\"";
 Trace_file « ",\"max. refinement level\"";
Trace_file « ",\"min. refinement level\"";
Trace_file « ",\"# of elements whose refinement was over-ruled\"";
 Trace_file « ",\"# of elements whose refinement was over-ru
Trace_file « ",\"max. error\"";
Trace_file « ",\"min. error\"";
Trace_file « ",\"min. permitted error\"";
Trace_file « ",\"min. permitted error\"";
Trace_file « ",\"max. permitted # of unrefined elements\"";
Trace_file « ",\"doc number\"";
Trace_file « ",\"x<sub>1</sub><sup>(track2 FE)</sup>\"";
 Trace_file « ",\"x<sub>1</sub><sup>(track2 FE)</sup>\"";
Trace_file « ",\"x<sub>2</sub><sup>(track2 FE)</sup>\"";
Trace_file « ",\"u<sub>1</sub><sup>(track2 FE)</sup>\"";
Trace_file « ",\"u<sub>2</sub><sup>(track2 FE)</sup>\"";
Trace_file « ",\"x<sub>2</sub><sup>(track2 FE)</sup>\"";
Trace_file « ",\"x<sub>2</sub><sup>(track2 Sarah)</sup>\"";
Trace_file « ",\"x<sub>2</sub><sup>(track2 Sarah)</sup>\"";
Trace_file « ",\"u<sub>1</sub><sup>(track2 Sarah)</sup>\"";
Trace_file « ",\"u<sub>2</sub><sup>(track2 Sarah)</sup>\"";
Trace_file « ",\"u<sub>2</sub><sup>(track2 Sarah)</sup>\"";
Trace_file « ",\"x<sub>0</sub>-1\"";
Trace_file « ",\"x<sub>0</sub>-1\"";
 Trace_file « std::endl;
.
/// Demonstrate how to solve OscRingNSt problem in deformable domain
/// with mesh adaptation
int main(int argc, char* argv[])
```

```
{
  // Store command line arguments
 CommandLineArgs::setup(argc,argv);
  //Do a certain number of timesteps per period
 unsigned nstep_per_period=40; // 80; // ADJUST_PRIORITY
 unsigned nperiod=3;
  // Work out total number of steps and timestep
 unsigned nstep=nstep_per_period*nperiod;
 double dt=1.0/double(nstep_per_period);
  // Set up the problem: Pass timestep and Sarah's asymptotic solution for
  // generation of initial condition
  OscRingNStProblem<MacroElementNodeUpdateElement<RefineableQCrouzeixRaviartElement<2> > >
    problem(dt,&SarahBL::full_exact_soln);
  // Restart?
  bool restarted=false;
  // Pointer to restart file
 ifstream* restart_file_pt=0;
  // No restart
  if (CommandLineArgs::Argc!=2)
      cout « "No restart" « std::endl;
      restarted=false;
      // Refine uniformly
      problem.refine_uniformly();
      problem.refine_uniformly();
      // Set initial condition on uniformly refined domain (if this isn't done
      // then the mesh contains the interpolation of the initial guess
       // on the original coarse mesh -- if the nodal values were zero in
       // the interior and nonzero on the boundary, then the the waviness of
       \ensuremath{//} of the interpolated i.g. between the nodes on the coarse mesh
       \ensuremath{//} gets transferred onto the fine mesh where we can do better
      problem.set initial condition();
  }
// Restart
  else if (CommandLineArgs::Argc==2)
      restarted=true;
       // Open restart file
       restart_file_pt=new ifstream(CommandLineArgs::Argv[1],ios_base::in);
       if (restart_file_pt!=0)
           cout « "Have opened " « CommandLineArgs::Argv[1] «
                 for restart. " « std::endl;
      else
           std::ostringstream error_stream;
           error_stream « "ERROR while trying to open '
                                        « CommandLineArgs::Argv[2]
                                         « " for restart." « std::endl;
           throw OomphLibError(error_stream.str(),
                                                         OOMPH_CURRENT_FUNCTION.
                                                         OOMPH_EXCEPTION_LOCATION);
       // Do the actual restart
      problem.restart(*restart_file_pt);
  // Two command line arguments: do validation run
  if (CommandLineArgs::Argc==3)
        \label{eq:nstep=3} \mbox{nstep=3;} \\ \mbox{cout $\kappa$ "Only doing nstep steps for validation: " $\kappa$ nstep $\kappa$ std::endl;} \\ \mbox{total $\kappa$} \mbox{ and $\kappa$} \
  // Setup labels for output
 DocInfo doc info;
  // Output directory
```

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```
doc_info.set_directory("RESLT");
\ensuremath{//} Do unsteady run of the problem for nstep steps
problem.unsteady_run(nstep,restarted,doc_info);
// Validate the restart procedure
if (CommandLineArgs::Argc==3)
  // Build problem and restart
  // Set up the problem: Pass timestep and Sarah's asymptotic solution for
  // generation of initial condition
OscRingNStProblem<MacroElementNodeUpdateElement<
   RefineableQCrouzeixRaviartElement<2>>>
   restarted_problem(dt,&SarahBL::full_exact_soln);
  // Setup labels for output
  DocInfo restarted_doc_info;
  // Output directory
  restarted_doc_info.set_directory("RESLT_restarted");
  // Step number
  restarted_doc_info.number()=0;
  // Copy by performing a restart from old problem
  restart_file_pt=new ifstream("RESLT/restart2.dat");
  \ensuremath{//} Do the actual restart
  restarted_problem.restart(*restart_file_pt);
  // Do unsteady run of the problem for one step unsigned nstep=2;
  bool restarted=true;
  restarted_problem.unsteady_run(nstep,restarted,restarted_doc_info);
```

1.1 PDF file

A pdf version of this document is available.