## **Chapter 1**

## A real fluid-structure interaction problem: Finite Reynolds number flow in an oscillating elastic ring.

Our first "real" fluid-structure interaction problem: We study the finite-Reynolds number internal flow generated by the motion of an oscillating elastic ring and compare the results against asymptotic predictions.

```
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
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//LIC//
// Driver for 2D Navier Stokes flow interacting with an elastic ring
// Oomph-lib include files
#include "generic.h"
#include "navier_stokes.h"
#include "beam.h"
//Need to include templated meshes, so that all functions
//are instantiated for our particular element types.
#include "meshes/quarter_circle_sector_mesh.h"
#include "meshes/one_d_lagrangian_mesh.h"
using namespace std;
using namespace oomph;
/// Namespace for physical parameters
namespace Global_Physical_Variables
 // Independent parameters:
 /// Square of Womersly number (a frequency parameter)
```

```
double Alpha_sq=50.0;
/// Density ratio of the solid and the fluid
double Density_ratio=1.0;
/// External Pressure
double Pext=0.0;
/// Poisson ratio
double Nu=0.49;
/// Nondimensional thickness of the beam
double H=0.05;
/// Perturbation pressure
double Pcos=0.0;
// Dependent parameters:
/// Reynolds number
double Re;
/// Reynolds x Strouhal number
double ReSt;
/// Timescale ratio (non-dimensation density)
double Lambda_sq;
/// Stress ratio
double O;
/// Set the parameters that are used in the code as a function
/// of the Womersley number, the density ratio and \ensuremath{\mathrm{H}}
void set_params()
 cout « "\n\n======= " «std::endl;
 cout « "\nSetting parameters. \n\n";
cout « "Prescribed: Square of Womersley number: Alpha_sq = "
     « Alpha_sq « std::endl;
 cout « "
 cout « "
                  Poisson ratio:
                                                Nu = "
 cout « "
     « Nu « std::endl;
 cout « " Pressure perturbation: Pcos = "
      « Pcos « std::endl;
 Q = "
 Lambda_sq=1.0/12.0*pow(H,3)*Density_ratio;
Timescale ratio:

Lambda_sq = "
     « Lambda_sq « std::endl;
 Re=Alpha_sq;
cout « "
             Reynolds number:
                                                Re = "
     « Re « std::endl;
 ReSt=Re;
cout « "
             Womersley number: ReSt = "
 «std::endl;
}
/// Non-FSI load function, a constant external pressure plus
for(unsigned i=0;i<2;i++)</pre>
  {load[i] = (Pext - Pcos*cos(2.0*xi[0]))*N[i];}
}
/// FSI Ring problem: a fluid-structure interaction problem in which
/// a viscous fluid bounded by an initially circular beam is set into motion /// by a small sinusoidal perturbation of the beam (the domain boundary).
```

```
class FSIRingProblem : public Problem
  /// There are very few element types that will work for this problem.
  /// Rather than passing the element type as a template parameter to the
 /// problem, we choose instead to use a typedef to specify the /// particular element fluid used.
 typedef AlgebraicElement<RefineableQCrouzeixRaviartElement<2> > FLUID_ELEMENT;
 /// Typedef to specify the solid element used
 typedef FSIHermiteBeamElement SOLID_ELEMENT;
public:
  /// Constructor: Number of elements in wall mesh, amplitude of the
  /// initial wall deformation, amplitude of pcos perturbation and its duration.
 FSIRingProblem(const unsigned &nelement_wall,
                            const double& eps_ampl, const double& pcos_initial,
                            const double& pcos_duration);
  /// Update after solve (empty)
 void actions_after_newton_solve() {}
 /// Update before solve (empty)
 void actions_before_newton_solve() {}
  /// Update the problem specs before checking Newton
  /// convergence
 void actions_before_newton_convergence_check()
   // Update the fluid mesh -- auxiliary update function for algebraic
    // nodes automatically updates no slip condition.
   Fluid_mesh_pt->node_update();
  /// Update the problem specs after adaptation:
  void actions_after_adapt()
     // The functions used to update the no slip boundary conditions
     // must be set on any new nodes that have been created during the
     // mesh adaptation process.
     // There is no mechanism by which auxiliary update functions
     // are copied to newly created nodes.
     // (because, unlike boundary conditions, they don't occur exclusively
     // at boundaries)
     // The no-slip boundary is boundary 1 of the mesh
     // Loop over the nodes on this boundary and reset the auxilliary
     // node update function
     unsigned n_node = Fluid_mesh_pt->nboundary_node(1);
     for (unsigned n=0;n<n_node;n++)</pre>
        {\tt Fluid\_mesh\_pt->} boundary\_node\_pt (1,n) -> {\tt set\_auxiliary\_node\_update\_fct\_pt} \ (1,n) -> {\tt set\_auxiliary\_node\_update\_f
          FSI_functions::apply_no_slip_on_moving_wall);
     // (Re-)setup fsi: Work out which fluid dofs affect wall elements
     // the correspondance between wall dofs and fluid elements is handled
      // during the remeshing, but the "reverse" association must be done
      // separately.
     // We need to set up the interaction every time because the fluid element
     // adjacent to a given solid element's integration point may have changed
     // We pass the boundary between the fluid and solid meshes and pointers
      // to the meshes. The interaction boundary is boundary 1 of the 2D
      // fluid mesh.
     FSI_functions::setup_fluid_load_info_for_solid_elements<FLUID_ELEMENT,2>
       (this,1,Fluid_mesh_pt,Wall_mesh_pt);
  /// Doc solution: Pass number of timestep, i (we append to tracefile
  /// after every timestep but do a full doc only at certain intervals),
  /// DocInfo object and tracefile
 void doc_solution(const unsigned& i, DocInfo& doc_info, ofstream& trace_file);
  /// Do dynamic run
 void dynamic_run();
  /// Setup initial condition for both domains
 void set initial condition();
  /// Setup initial condition for wall
 void set_wall_initial_condition();
 /// Setup initial condition for fluid
 void set fluid initial condition();
```

```
/// Element used for documenting displacement
SOLID_ELEMENT* Doc_displacement_elem_pt;
 /// Pointer to wall mesh
OneDLagrangianMesh<SOLID_ELEMENT> *Wall_mesh_pt;
 /// Pointer to fluid mesh
AlgebraicRefineableQuarterCircleSectorMesh<FLUID_ELEMENT> *Fluid_mesh_pt;
 /// Pointer to geometric object that represents the undeformed wall shape
GeomObject* Undef_geom_pt;
 /// Pointer to wall timestepper
Newmark<2>* Wall_time_stepper_pt;
 /// Pointer to fluid timestepper
BDF<2>* Fluid_time_stepper_pt;
 /// Pointer to node on coarsest mesh on which velocity is traced
Node* Veloc_trace_node_pt;
 /// Amplitude of initial deformation
double Eps_ampl;
 /// Initial pcos
double Pcos_initial;
/// Duration of initial pcos
double Pcos_duration;
};
/// Setup initial condition: When we're done here, all variables
/// represent the state at the initial time.
void FSIRingProblem::set_initial_condition()
cout « "Setting wall ic" « std::endl;
set_wall_initial_condition();
cout « "Setting fluid ic" « std::endl;
set_fluid_initial_condition();
}
/// Setup initial condition for fluid: Impulsive start
void FSIRingProblem::set_fluid_initial_condition()
 // Update fluid domain: Careful!!! This also applies the no slip conditions
 // on all nodes on the wall! Since the wall might have moved since
 // we created the mesh; we're therefore imposing a nonzero
 // velocity on these nodes. Must wipe this afterwards (done
 // by setting *all* velocities to zero) otherwise we get
// an impulsive start from a very bizarre initial velocity // field! [Yes, it took me a while to figure this out...]
Fluid_mesh_pt->node_update();
// Assign initial values for the velocities; // pressures don't carry a time history and can be left alone.
//Find number of nodes in fluid mesh
unsigned n_node = Fluid_mesh_pt->nnode();
 // Loop over the nodes to set initial guess everywhere
 for(unsigned n=0;n<n_node;n++)</pre>
   // Loop over velocity directions: Impulsive initial start from
   // zero velocity!
   for (unsigned i=0;i<2;i++)</pre>
     Fluid_mesh_pt->node_pt(n)->set_value(i,0.0);
 // Do an impulsive start with the assigned velocity field
```

```
Fluid_mesh_pt->assign_initial_values_impulsive();
}
/// Setup initial condition: Impulsive start either from
/// deformed or undeformed wall shape.
void FSIRingProblem::set_wall_initial_condition()
 // Geometric object that specifies the initial conditions:
 // A ring that is bucked in a 2-lobed mode
GeomObject* ic_geom_object_pt=
 new PseudoBucklingRing(Eps_ampl,Global_Physical_Variables::H,2,2,
Wall_time_stepper_pt);
// Assign period of oscillation of the geometric object
static_cast<PseudoBucklingRing*>(ic_geom_object_pt)->set_T(1.0);
 //Set initial time (to deform wall into max. amplitude)
 double time=0.25;
 // Assign initial radius of the object
 \verb|static_cast| < \verb|PseudoBucklingRing*| > (ic_geom_object_pt) -> set_R_0 (1.00); \\
 // Setup object that specifies the initial conditions:
SolidInitialCondition* IC_pt = new SolidInitialCondition(ic_geom_object_pt);
 // Assign values of positional data of all elements on wall mesh
 // so that the wall deforms into the shape specified by IC object.
 SolidMesh::Solid_IC_problem.set_static_initial_condition(
 this, Wall_mesh_pt, IC_pt, time);
/// Document solution: Pass number of timestep, i; we append to trace file
/// at every timestep and do a full doc only after a certain number
/// of steps.
void FSIRingProblem::doc_solution(const unsigned& i,
 DocInfo& doc_info, ofstream& trace_file)
 // Full doc every nskip steps
 unsigned nskip=1; // ADJUST
  // If we at an integer multiple of nskip, full documentation.
  if (i%nskip==0)
   //Otherwise, just output the trace file
  else
   {
   doc info.disable_doc();
   cout « "Only trace for time "
        « time_stepper_pt()->time_pt()->time() « std::endl;
  // If we are at a full documentation step, output the fluid solution
 if (doc info.is doc enabled())
   //Variables used in the output file
   ofstream some_file; char filename[100];
    //Construct the output filename from the doc_info number and the
   //output directory
sprintf(filename, "%s/soln%i.dat", doc_info.directory().c_str(),
           doc info.number());
    //Open the output file
   some_file.open(filename);
    /// Output the solution using 5x5 plot points
   Fluid_mesh_pt->output(some_file,5);
   //Close the output file
   some file.close();
  //Temporary vector to give the local coordinate at which to document
  //the wall displacment
 Vector<double> s(1,1.0);
  // Write to the trace file:
 trace_file « time_pt()->time()
   //Document the displacement at the end of the the chosen element
             « " " « Doc_displacement_elem_pt->interpolated_x(s,1)
             « " " « Veloc_trace_node_pt->x(0)
             " " « Veloc_trace_node_pt->x(1)
             « " " « Veloc_trace_node_pt->value(0)
             « " " « Veloc_trace_node_pt->value(1)
```

```
« " " « Fluid_mesh_pt->nelement()
             « " " « ndof()
             « " " « Fluid_mesh_pt->nrefinement_overruled()
             " " « Fluid_mesh_pt->max_error()
             « " " « Fluid_mesh_pt->min_error()
             « " " « Fluid_mesh_pt->max_permitted_error()
             « " " « Fluid_mesh_pt->min_permitted_error()
             « " " « Fluid_mesh_pt->max_keep_unrefined();
  // Output the number of the corresponding full documentation
  // file number (or -1 if no full doc was made)
 //{\it End} the trace file
 trace_file « std::endl;
 // Increment counter for full doc
 if (doc_info.is_doc_enabled()) {doc_info.number()++;}
/// Constructor for FSI ring problem. Pass number of wall elements /// and length of wall (in Lagrangian coordinates) amplitude of
/// initial deformation, pcos perturbation and duration.
FSIRingProblem::FSIRingProblem(const unsigned& N,
               const double& eps_ampl, const double& pcos_initial,
               const double& pcos_duration) :
Eps_ampl(eps_ampl), Pcos_initial(pcos_initial),
Pcos_duration(pcos_duration)
 // Create timesteppers
 // Allocate the wall timestepper and add it to the problem's vector
 // of timesteppers
 Wall_time_stepper_pt = new Newmark<2>;
add_time_stepper_pt(Wall_time_stepper_pt);
 // Allocate the fluid timestepper and add it to the problem's Vector
 // of timesteppers
 Fluid_time_stepper_pt = new BDF<2>;
add_time_stepper_pt(Fluid_time_stepper_pt);
 // Create the wall mesh
 // Undeformed wall is an elliptical ring
Undef_geom_pt = new Ellipse(1.0,1.0);
//Length of wall in Lagrangian coordinates double L = 2.0*atan(1.0);
 //Now create the (Lagrangian!) mesh
 Wall mesh pt = new
 OneDLagrangianMesh<SOLID_ELEMENT>(N,L,Undef_geom_pt,Wall_time_stepper_pt);
 // Set the boundary conditions for wall mesh (problem)
 // Bottom boundary: (Boundary 0)
 // No vertical displacement
 Wall_mesh_pt->boundary_node_pt(0,0)->pin_position(1);
 // Zero slope: Pin type 1 dof for displacement direction 0 \,
Wall_mesh_pt->boundary_node_pt(0,0)->pin_position(1,0);
 // Top boundary: (Boundary 1)
 // No horizontal displacement
Wall_mesh_pt->boundary_node_pt(1,0)->pin_position(0);
 // Zero slope: Pin type 1 dof for displacement direction 1 \,
Wall_mesh_pt->boundary_node_pt(1,0)->pin_position(1,1);
 // Create the fluid mesh:
 // Fluid mesh is suspended from wall between the following Lagrangian
 // coordinates:
double xi_lo=0.0;
double xi hi=L;
 // Fractional position of dividing line for two outer blocks in mesh
double fract mid=0.5;
 //Create a geometric object that represents the wall geometry from the
```

```
//wall mesh (one Lagrangian, two Eulerian coordinates).
MeshAsGeomObject *wall_mesh_as_geometric_object_pt
 = new MeshAsGeomObject(Wall_mesh_pt);
// Build fluid mesh using the wall mesh as a geometric object
Fluid_mesh_pt = new AlgebraicRefineableQuarterCircleSectorMesh<FLUID_ELEMENT >
 (wall_mesh_as_geometric_object_pt,
  xi_lo,fract_mid,xi_hi,Fluid_time_stepper_pt);
// Set the error estimator
Z2ErrorEstimator* error_estimator_pt=new Z2ErrorEstimator;
Fluid_mesh_pt->spatial_error_estimator_pt() = error_estimator_pt;
// Extract pointer to node at center of mesh
unsigned nnode=Fluid_mesh_pt->finite_element_pt(0)->nnode();
\label{lem:lement_pt} \mbox{Veloc\_trace\_node\_pt=Fluid\_mesh\_pt->finite\_element\_pt(0)->node\_pt(nnode-1);} \\
// Set the fluid boundary conditions
// Bottom boundary (boundary 0):
 unsigned n_node = Fluid_mesh_pt->nboundary_node(0);
 for (unsigned n=0;n<n_node;n++)</pre>
   // Pin vertical velocity
   Fluid_mesh_pt->boundary_node_pt(0,n)->pin(1);
// Ring boundary (boundary 1):
// No slip; this also implies that the velocity needs
// to be updated in response to wall motion
 unsigned n_node = Fluid_mesh_pt->nboundary_node(1);
for (unsigned n=0;n<n_node;n++)</pre>
   // Which node are we dealing with?
   Node* node_pt=Fluid_mesh_pt->boundary_node_pt(1,n);
   // Set auxiliary update function pointer
node_pt->set_auxiliary_node_update_fct_pt(
    FSI_functions::apply_no_slip_on_moving_wall);
   // Pin both velocities
   for(unsigned i=0;i<2;i++) {node_pt->pin(i);}
// Left boundary (boundary 2):
  unsigned n_node = Fluid_mesh_pt->nboundary_node(2);
  for (unsigned n=0;n<n_node;n++)</pre>
    // Pin horizontal velocity
      Fluid_mesh_pt->boundary_node_pt(2,n)->pin(0);
// Add the submeshes and build global mesh
// Wall mesh
add_sub_mesh(Wall_mesh_pt);
//Fluid mesh
add_sub_mesh(Fluid_mesh_pt);
// Combine all submeshes into a single Mesh
build_global_mesh();
// Finish problem setup
//Find number of elements in fluid mesh
unsigned n_element = Fluid_mesh_pt->nelement();
// Loop over the fluid elements to set up element-specific
// things that cannot be handled by constructor
for (unsigned e=0;e<n_element;e++)</pre>
  // Upcast from FiniteElement to the present element
  FLUID ELEMENT *el pt
```

```
= dynamic_cast<FLUID_ELEMENT*>(Fluid_mesh_pt->element_pt(e));
   //Set the Reynolds number, etc
   el_pt->re_pt() = &Global_Physical_Variables::Re;
  el_pt->re_st_pt() = &Global_Physical_Variables::ReSt;
  el_pt->evaluate_shape_derivs_by_direct_fd();
    el_pt->evaluate_shape_derivs_by_chain_rule();
   el_pt->enable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
        el_pt->disable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
     else
       el_pt->enable_always_evaluate_dresidual_dnodal_coordinates_by_fd();
  //el_pt->evaluate_shape_derivs_by_direct_fd();
//Loop over the solid elements to set physical parameters etc.
unsigned n_wall_element = Wall_mesh_pt->nelement();
for (unsigned e=0;e<n_wall_element;e++)</pre>
  ///Cast to proper element type
SOLID_ELEMENT *el_pt = dynamic_cast<SOLID_ELEMENT*>(
Wall_mesh_pt->element_pt(e));
   //Set physical parameters for each element:
   el_pt->h_pt() = &Global_Physical_Variables::H;
  el_pt->lambda_sq_pt() = &Global_Physical_Variables::Lambda_sq;
  //Function that specifies the external load Vector
el_pt->load_vector_fct_pt() = &Global_Physical_Variables::pcos_load;
   // Function that specifies the load ratios
  el_pt->q_pt() = &Global_Physical_Variables::Q;
   //Assign the undeformed beam shape
  el_pt->undeformed_beam_pt() = Undef_geom_pt;
// Establish control displacment: (even if no displacement control is applied
// we still want to doc the displacement at the same point)
// Choose element: (This is the last one)
Doc_displacement_elem_pt = dynamic_cast<SOLID_ELEMENT*>(
 Wall_mesh_pt->element_pt(n_wall_element-1));
// Setup fsi: Work out which fluid dofs affect the wall elements
// the correspondance between wall dofs and fluid elements is handled // during the remeshing, but the "reverse" association must be done
// separately.
// We pass the boundary between the fluid and solid meshes and pointers
// to the meshes. The interaction boundary is boundary 1 of the
 // 2D fluid mesh.
FSI_functions::setup_fluid_load_info_for_solid_elements<FLUID_ELEMENT,2>
  (this,1,Fluid_mesh_pt,Wall_mesh_pt);
// Do equation numbering
cout « "# of dofs " « assign_eqn_numbers() « std::endl;
/// Solver loop to perform unsteady run
void FSIRingProblem::dynamic_run()
// Setup documentation
/// Label for output
DocInfo doc_info;
// Output directory
doc_info.set_directory("RESLT");
// Step number
doc_info.number()=0;
//Open a trace file
ofstream trace_file("RESLT/trace_ring.dat");
// Write header for trace file
```

```
trace_file « "VARIABLES=\"time\",\"V_c_t_r_l\"";
trace_file « ",\"x<sub>!c/sub><sup>(track)</sup>\"";
trace_file « ",\"x<sub>2c/sub><sup>(track)</sup>\"";
trace_file « ",\"u<sub>!c/sub><sup>(track)</sup>\"";
trace_file « ",\"u<sub>2c/sub><sup>(track)</sup>\"";
trace_file « ",\"u<sub>2c/sub><sup>(track)</sup>\"";
trace_file « ",\"N<sub>element</sub>\"";
trace_file « ",\"for under-refined elements\"";
trace_file « ",\"max. error\"";
trace_file « ",\"min. error\"";
trace_file « ",\"min. permitted error\"";
trace_file « ",\"min. permitted error\"";
trace_file « ",\"max. permitted # of unrefined elements\"";
trace_file « ",\"max. permitted # of unrefined elements\"";
trace_file « ",\"doc number\"";
trace_file « std::endl;
trace_file « "VARIABLES=\"time\",\"V_c_t_r_l\"";
trace_file « std::endl;
 // Initialise timestepping
 // Number of steps
unsigned nstep=300;
 // Nontrivial command line input: validation: only do three steps
if (CommandLineArgs::Argc>1)
   nstep=1;
   cout « "Only doing nstep steps for validation: " « nstep « std::endl;
 // Set initial timestep
double dt=0.004;
 // Set initial value for dt -- also assigns weights to the timesteppers
initialise_dt(dt);
 // Set physical parameters
using namespace Global_Physical_Variables;
 // Set Womersley number
Alpha_sq=100.0; // 50.0; // ADJUST
// Set density ratio
Density_ratio=10.0; // 0.0; ADJUST
 // Wall thickness
H=1.0/20.0;
 // Set external pressure
Pext=0.0;
 /// Perturbation pressure
Pcos=Pcos_initial;
 // Assign/doc corresponding computational parameters
set params();
 \ensuremath{//} Refine uniformly and assign initial conditions
 // Refine the problem uniformly
 refine_uniformly();
refine_uniformly();
 // This sets up the solution at the initial time
 set_initial_condition();
 // Set targets for spatial adptivity
 // Max. and min. error for adaptive refinement/unrefinement
Fluid_mesh_pt->max_permitted_error()=1.0e-2;
Fluid_mesh_pt->min_permitted_error()=1.0e-3;
 // Don't allow refinement to drop under given level
Fluid_mesh_pt->min_refinement_level()=2;
 // Don't allow refinement beyond given level
Fluid_mesh_pt->max_refinement_level()=6;
 // Don't bother adapting the mesh if no refinement is required
 ^{\prime}/ and if less than \dots elements are to be merged.
 Fluid_mesh_pt->max_keep_unrefined()=20;
 // Doc refinement targets
```

```
Fluid_mesh_pt->doc_adaptivity_targets(cout);
 // Do the timestepping
 // Reset initial conditions after refinement for first step only
//Output initial data
doc_solution(0,doc_info,trace_file);
     unsigned nel=Fluid_mesh_pt->nelement();
     for (unsigned e=0;e<nel;e++)</pre>
      std::cout « "\n\nEl: " « e « std::endl « std::endl;
FiniteElement* el_pt=Fluid_mesh_pt->finite_element_pt(e);
      unsigned n_dof=el_pt->ndof();
       Vector<double> residuals(n_dof);
      DenseDoubleMatrix jacobian(n_dof,n_dof);
       el_pt->get_jacobian(residuals, jacobian);
     exit(0);
 // Time integration loop
 for(unsigned i=1;i<=nstep;i++)</pre>
   // Switch doc off during solve
  doc_info.disable_doc();
   // Solve
   unsigned max_adapt=1;
  unsteady_newton_solve(dt,max_adapt,first);
   // Now we've done the first step
   first=false;
   // Doc solution
  doc_solution(i,doc_info,trace_file);
   /// Switch off perturbation pressure
   if (time_pt()->time()>Pcos_duration) {Pcos=0.0;}
}
/// Driver for fsi ring test problem
int main(int argc, char* argv[])
 // Store command line arguments
CommandLineArgs::setup(argc,argv);
// Number of elements
unsigned nelem = 13;
 /// Perturbation pressure
double pcos_initial=1.0e-6; // ADJUST
/// Duration of initial pcos perturbation double pcos_duration=0.3; // ADJUST
/// Amplitude of initial deformation
double eps_ampl=0.0; // ADJUST
//Set up the problem
FSIRingProblem problem (nelem, eps_ampl, pcos_initial, pcos_duration);
 // Do parameter study
problem.dynamic_run();
```

1.1 PDF file

## 1.1 PDF file

A pdf version of this document is available.