Algorithm Design and Analysis
Assignment 6
Wu Jiabao, 523030910241

3. Given a *dominating set problem* instance $(G = (V, E), D \subset V)$, we iterate each vertex in V to check whether it is incident with a vertex in D. It takes $O(|V|(|V| + |E|))$ time, so the *dominating set problem* is in NP.

   Considering a VertexCover instance $(G = (V, E), k)$. For each edge $e = (u, v)$ in V, we construct a vertex x_e and link it to u and v, resulting a new graph $G' = (V', E')$. Let $k = k$ and $(G' = (V', E'), k)$ is a *dominating set problem* instance.

   If VertexCover has a yes instance, for each edge $e \in E$, it has at least one endpoint in the cover set C. Thus for the corresponding $u, v, x_e \in V'$, we have $u \in C \vee v \in C$. Thus u, v, x_e must be dominated. Therefore, the *dominating set problem* has a yes instance.

   If the *dominating set problem* has a yes instance, for any vertex x in the dominating set D, if $x \notin V$, remove x and add its adjacent vertex into D. Since x is only incident to its endpoints in G in this case, the operation before will keep D still a dominating set. After the operation D only contains vertices in V. Since D is dominating set and all vertices in $V \setminus D$ are dominated, we have all edges are covered. Thus the VertexCover problem is a yes instance.

   Therefore, we have VertexCover $\leq_k$ *dominating set problem*, *dominating set problem* is NP-complete.

4. Firstly we prove the *set cover problem* is NP-hard. Consider a VertexCover instance $(G = (V, E), k)$. We will construct a SetCover instance $(U, \mathcal{T}, k)$. Label the edges in E with $1, 2, \ldots, |E|$ in arbitraty order. Construct a ground set $U = E$. For each $v_i \in V$, construct a subset $S_i = \{e \in E \mid v_i \text{ is an endpoint of } e\}$. Let $k = k$.

   Thus we get a VertexCover instance $(U, \mathcal{T}, k)$. Obviously the conversion is under polynomial time. Denote the set of chosen subsets as A.

   Picking $S_i \in A$ represents picking v_i in the vertex cover.

   If the VertexCover has a yes instance, for any edge e in E, it has at least one endpoint chosen. Thus $e \in \bigcup_{i=1}^{k} A_i$ for all $e \in E$, which gives the *set cover problem* a yes instance.

   If the *set cover problem* has a yes instance, we can select k subsets from $\mathcal{T}$ to cover E. Thus we can pick k vertices to make all the edges have at least one endpoint selected.

   Thus we have VertexCover $\leq_k$ SetCover, *set cover problem* is NP-hard.

   Then we prove the *max-k-coverage problem* is also NP-hard. The construction is the same as above.

   If the VertexCover has a yes instance, we can select k subsets to make $\bigcup_{i=1}^{k} A_i = E$. Since $|E|$ is the maximum size of coverage, the *max-k-coverage problem* also gives a yes instance.

   If the *max-k-coverage problem* has a yes instance, we can find k vertices to cover the maximum amount of edges, which is $|E|$. Thus the VertexCover has a yes instance.

   Thus we have VertexCover $\leq_k$ *max-k-coverage problem*, *max-k-coverage problem* is NP-hard.

5. The homework takes me 3 hours. Collaborators: Wang Kun, Li Haochen.