HW2 Report

Part 1: Infrastructure Setup

Code Implementation

- **train.py**: Implements training loop with:
    - Character-level tokenization
    - Loss masking (only compute loss after '=' token)
    - Support for masking first N tokens (`-mask_first_n` flag)
    - Automatic saving of final model and training curves
- **inference.py**: Implements text generation with:
    - Model loading from checkpoints
    - Temperature and top-k sampling
    - Interactive mode for testing

Modifications from Original Codebase

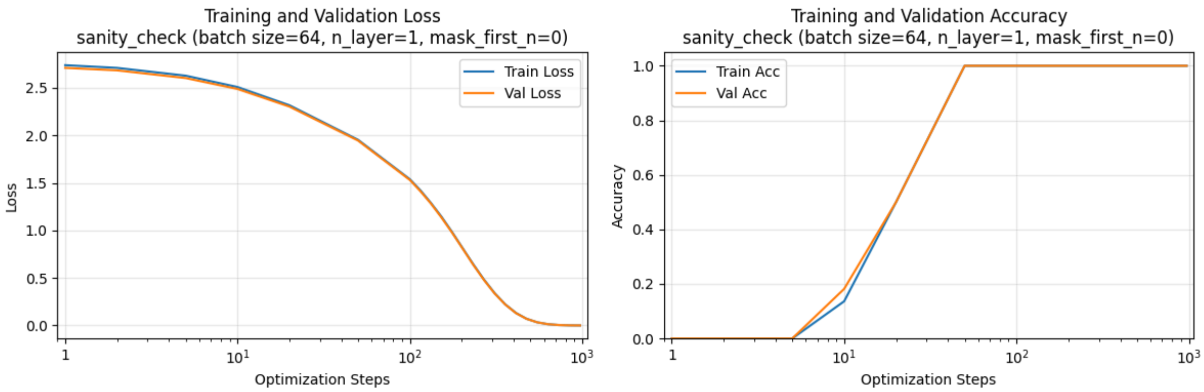
1. **Custom Dataset Class**: `AlgorithmicDataset` with loss masking for equation format
2. **Character Tokenizer**: Simple tokenizer for mathematical expressions
3. **Accuracy Metrics**: Added accuracy computation alongside loss
4. **Visualization**: Automatic generation of training curves

Sanity Check Results

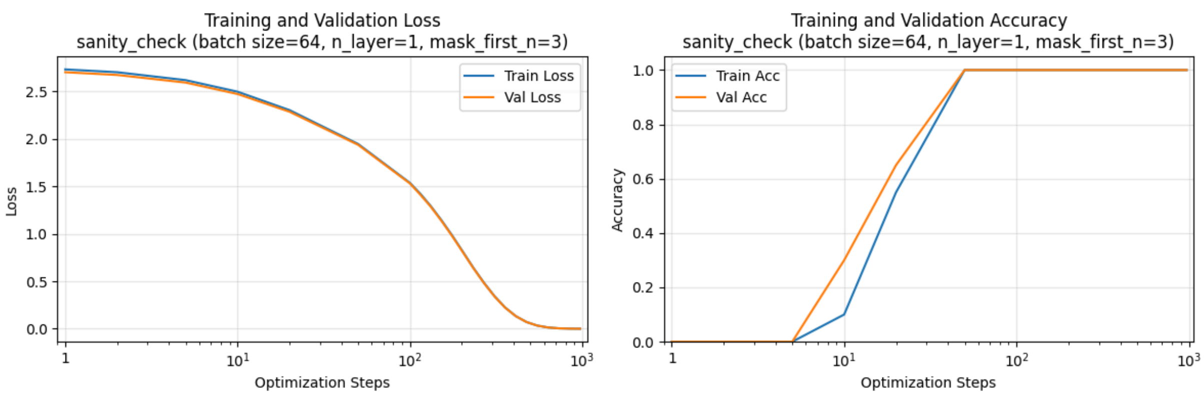
```
# Test 1: Basic memorization
python train.py --data_dir data/sanity_check --out_dir out/sanity_check --n_layer 1 --n_embd 32 --n_head 4 --max_steps 1000 --log_interval 100 --batch_size 64 --learning_rate 3e-4 --seed 42 --eval_points_per_decade 16

# Test 2: Masked first 3 tokens
python train.py --data_dir data/sanity_check --out_dir out/sanity_check_masked --n_layer 1 --n_embd 32 --n_head 4 --max_steps 1000 --log_interval 100 --batch_size 64 --learning_rate 3e-4 --seed 42 --eval_points_per_decade 16 --mask_first_n 3
```

Both tests successfully converged with train loss → 0. You can run the `inference.py` on the model checkpoint for two sanity checks.

Sanity Curves



Masked Sanity Curves



Challenges Faced

- Debugging loss masking

Part 2.1: Data Generation

Process Description

We generate complete datasets for modular arithmetic:

```
# For each operation and modulus p:
- Addition: a + b = c where c = (a + b) % p
- Subtraction: a - b = c where c = (a - b) % p
- Division: a / b = c where a = (b * c) % p
```

Division uses the reformulation where `a/b ≡ c (mod p)` means `a ≡ b×c (mod p)`.

Dataset Statistics

Task	Modulus	Total Examples	Train	Val	Test
Addition	97	9,409	6,586	1,411	1,412
Subtraction	97	9,409	6,586	1,411	1,412
Division	97	9,312	6,518	1,396	1,398
Addition	113	12,769	8,938	1,915	1,916
Subtraction	113	12,769	8,938	1,915	1,916

Part 2.2: Addition and Subtraction Experiments ✓

Configuration

```
{
    'max_steps': 100000,
    'learning_rate': 1e-3,
    'log_interval': 1000,
    'n_embd': 128,
    'n_head': 4,
    'eval_points_per_decade': 16,
    'batch_size': 64
}
```

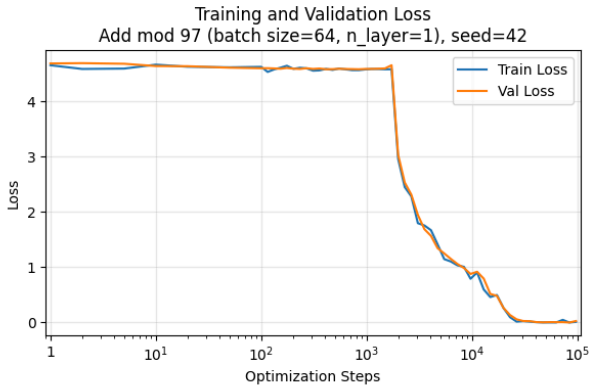
Results (3 Random Seeds: 42, 123, 456)

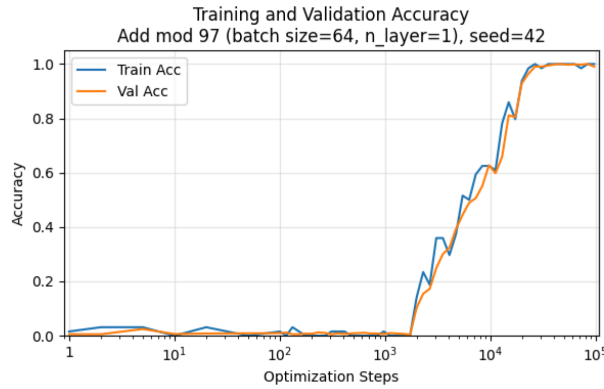
1-Layer Model Results

Task	Metric	Seed 42	Seed 123	Seed 456
Add p=97	Final Train Loss	0%	0%	0%
Add p=97	Final Train Acc	100%	100%	100%
Add p=97	Final Test Acc	99.04%	99.36%	99.07%
Add p=113	Final Train Loss	0%	0%	0%
Add p=113	Final Train Acc	100%	100%	100%
Add p=113	Final Test Acc	97.27%	97.26%	97.78%
Sub p=97	Final Train Loss	0%	0%	0%
Sub p=97	Final Train Acc	100%	100%	100%
Sub p=97	Final Test Acc	97.08%	91.92%	97.15%
Sub p=113	Final Train Loss	2%	0%	0%
Sub p=113	Final Train Acc	100%	100%	100%

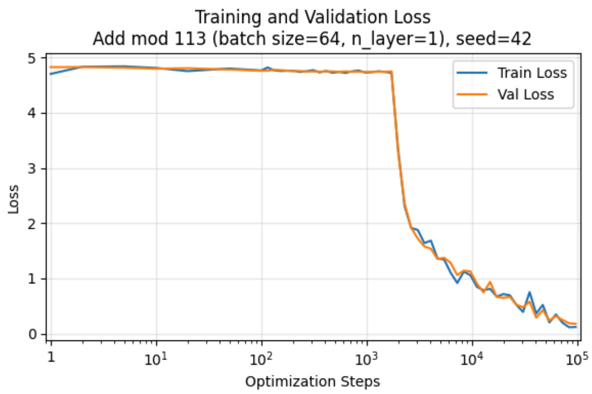
Sub p=113	Final Test Acc	91.49%	94.06%	93.54%

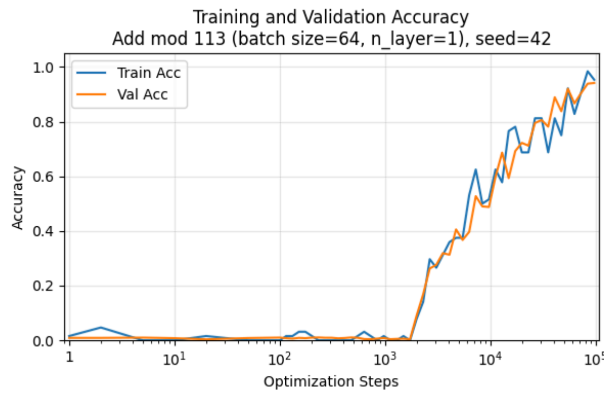
Add mod 97, 1 layer seed 42



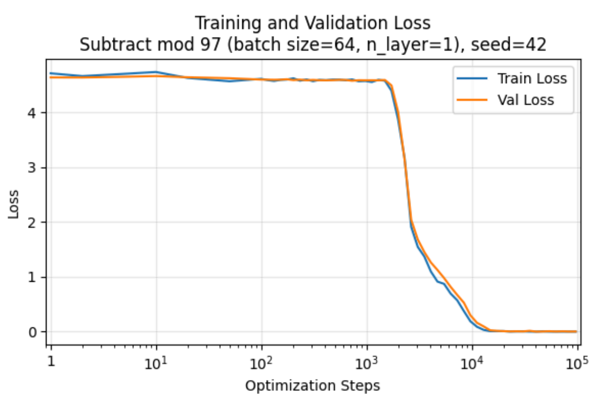


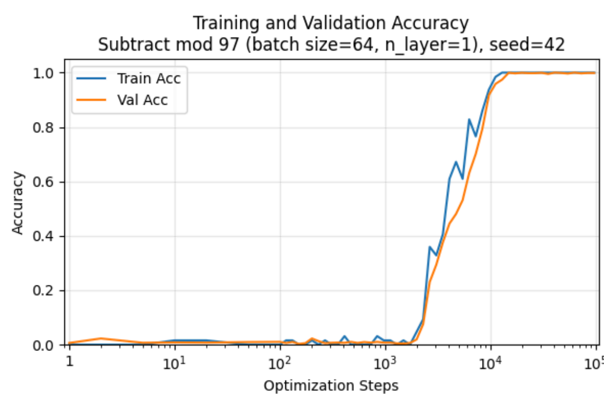
Add mod 113, 1 layer seed 42



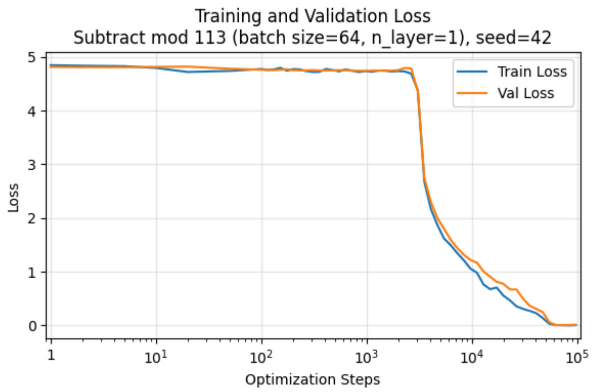


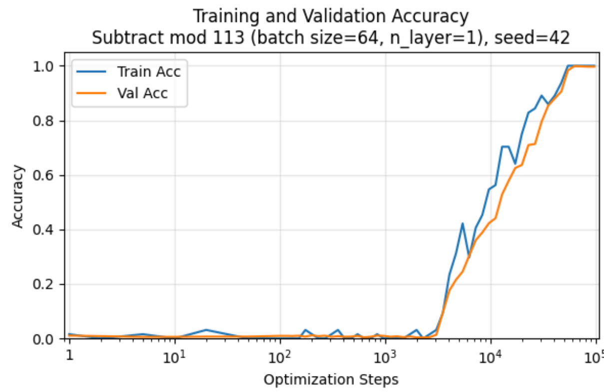
Sub mod 97, 1 layer seed 42





Sub mod 113, 1 layer seed 42

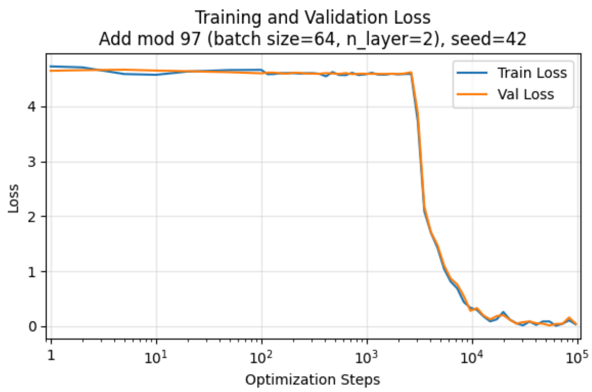


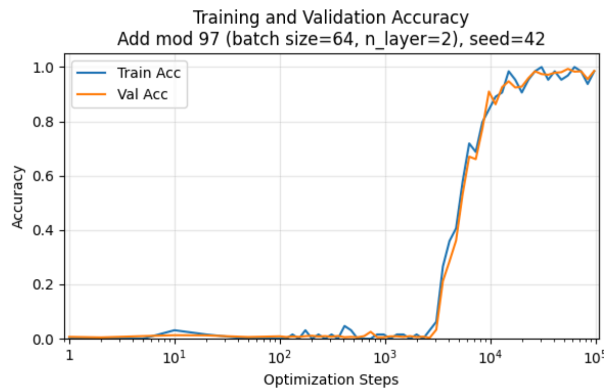


2-Layer Model Results

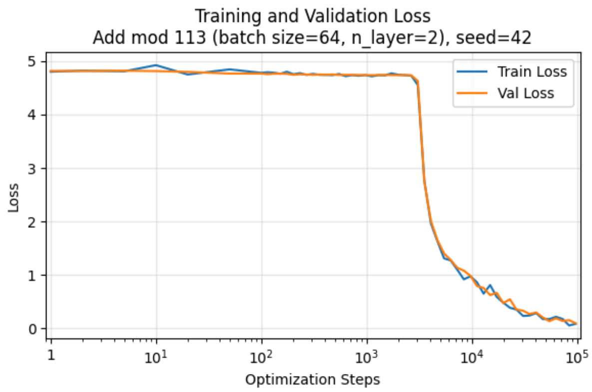
Task	Metric	Seed 42	Seed 123	Seed 456
Add p=97	Final Train Loss	0%	0%	0%
Add p=97	Final Train Acc	100.0%	100.0%	100.0%
Add p=97	Final Test Acc	99.78%	99.96%	99.78%
Add p=113	Final Train Loss	0%	0%	0%
Add p=113	Final Train Acc	100%	100%	100%
Add p=113	Final Test Acc	98.49%	98.04%	98.34%
Sub p=97	Final Train Loss	0%	0%	0%
Sub p=97	Final Train Acc	100%	100%	100%
Sub p=97	Final Test Acc	99.2%	99.31%	99.64%
Sub p=113	Final Train Loss	0%	0%	1.8%
Sub p=113	Final Train Acc	100%	100%	100%
Sub p=113	Final Test Acc	98.46%	98.44%	97.87%

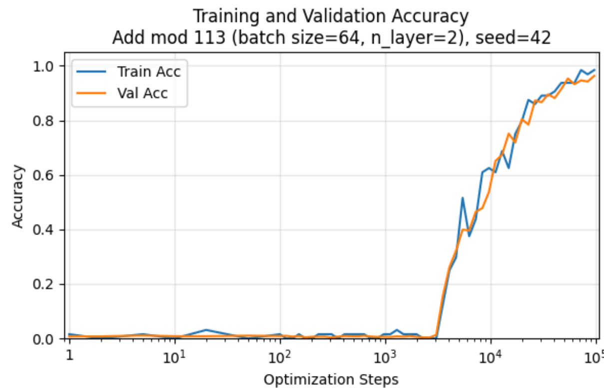
Add mod 97, 2 layer seed 42



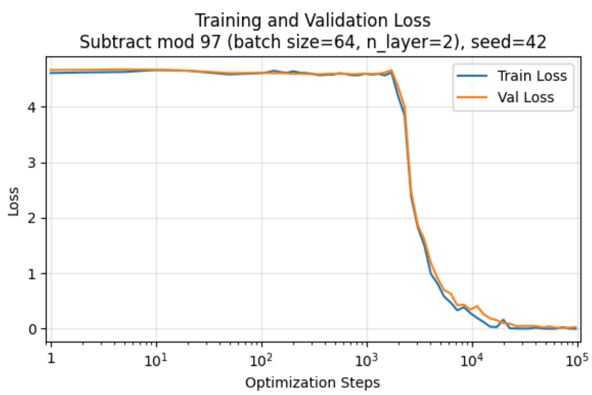


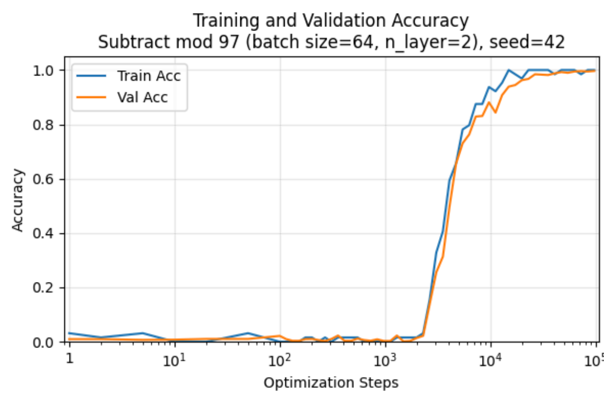
Add mod 113, 2 layer seed 42



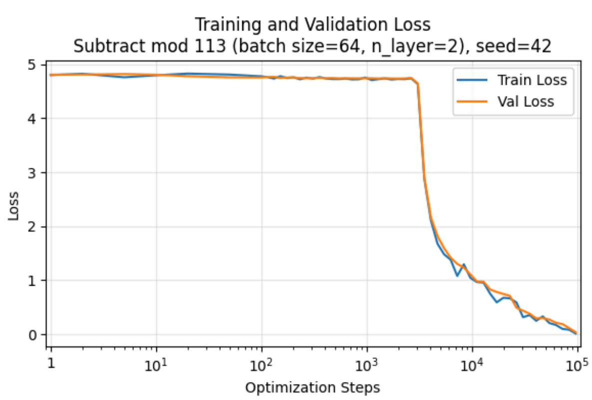


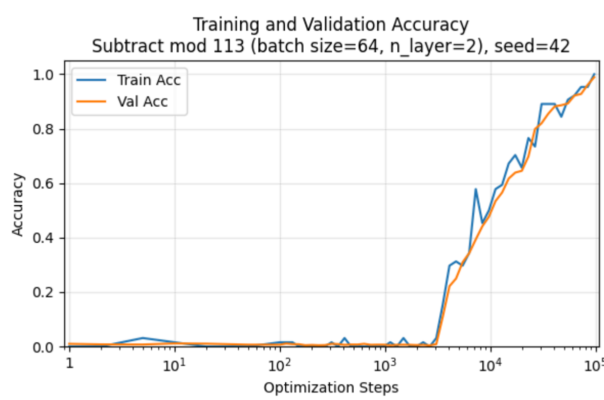
Sub mod 97, 2 layer seed 42





Sub mod 113, 2 layer seed 42



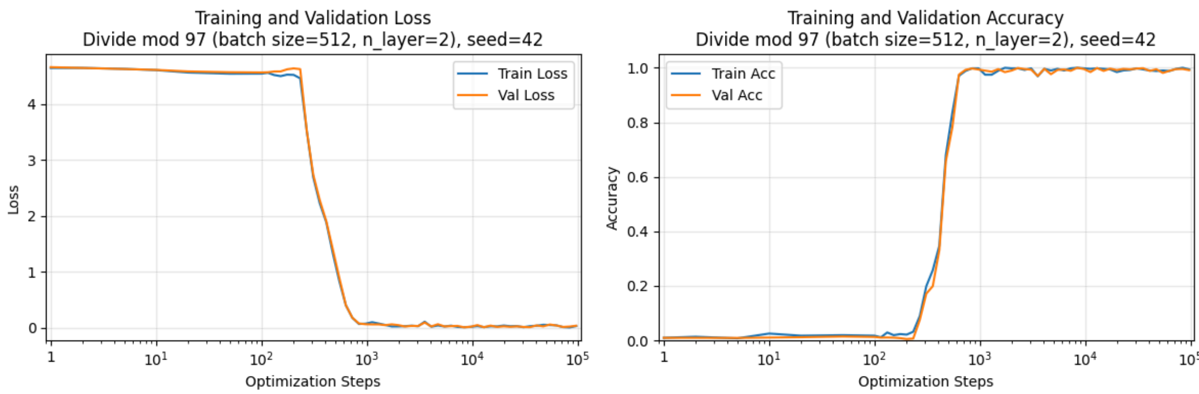


Model Checkpoints

Final models saved at: `out/[task]_mod[p]_layer[n]_seed[n]/final_model.pt`

Part 2.3: Grokking

We trained on the modulo division task for p = 97, and the training curves are as follows:



Detailed training configurations:

```
{
    'n_layer': 2,
    'n_embd': 128,
    'n_head': 4,
    'batch_size': 512,
    'learning_rate': 1e-3,
    'weight_decay': 1.0,
    'beta_1': 0.9,
    'beta_2': 0.98
    'max_steps': 100000
}
```

Grokking Results

- Final **validation loss:** 0.0972, **validation accuracy:** 0.9913.
- Final **training loss:** 0.0160, **training accuracy:** 1.0000

Model Checkpoint

`out/divide_mod97_layer2_batch512/final_model.pt`

Inference Instructions

```
python inference.py --checkpoint out/divide_mod97_layer2_batch512/final_model.pt --pro
mpts "30/74=" --max_new_tokens 2
```

Part 2.4: Ablation Study

Motivation

We investigated how batch size affects grokking speed and reliability, as batch size influences gradient noise and learning dynamics.

Experimental Setup

- **Fixed**:

```
{
    'n_layer': 2,
    'n_embd': 128,
    'n_head': 4,
    'learning_rate': 1e-3,
    'weight_decay': 1.0,
    'beta_1': 0.9,
    'beta_2': 0.98
    'max_steps': 100000
    'p': 97
}
```

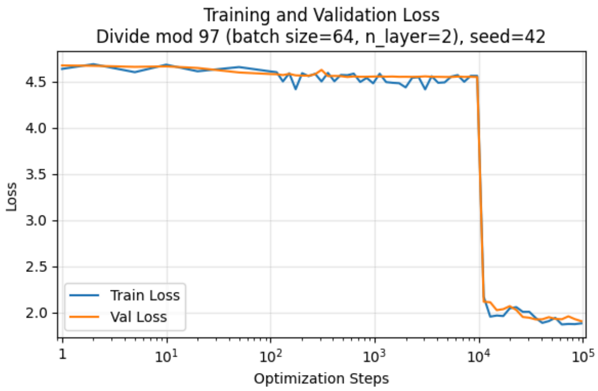
- **Varied**: batch_size ∈ {64, 128, 256, 512}
- We tested across three different seeds to ensure reliability. For simplicity we report only the seed=42 plots here, but plots for other seeds are available at `out/divide_mod97_layer2_<suffix>` . Result path to batch size mapping is as follows:

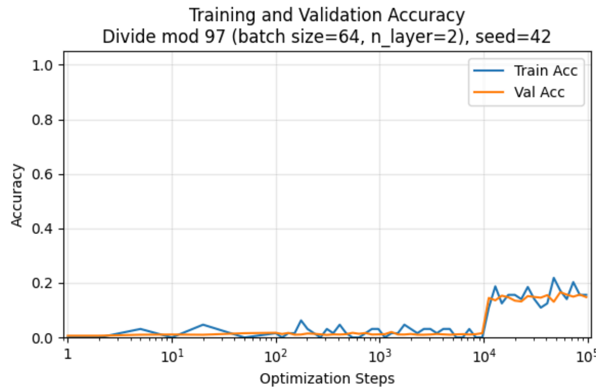
Batch Size	Path
64	`out/divide_mod97_layer2_seed{seed}_batch{batch}`
128	`out/divide_mod97_layer2_seed{seed}_batch{batch}`
256	`out/divide_mod97_layer2_seed{seed}_batch{batch}`
512	`out/divide_mod97_layer2_seed{seed}_batch{batch}`

Results

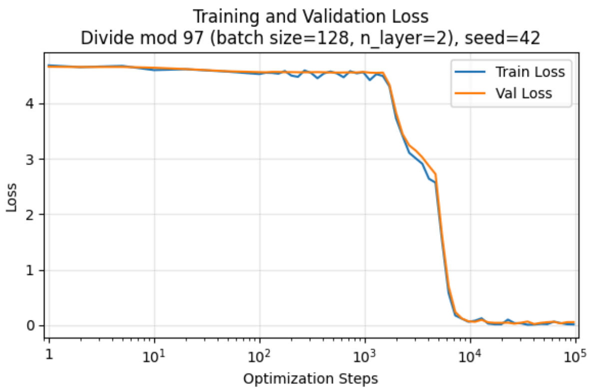
Batch Size	Steps to Grok*	Final Training Loss	Final Training Accuracy	Final Test Accuracy
64	Not Grokking**	1.7755	18.25%	17.06%
128	~9000	0.0191	100%	97.85%
256	~10000	0.0032	100%	99.24%
512	~800	0.0007	100%	99.57%

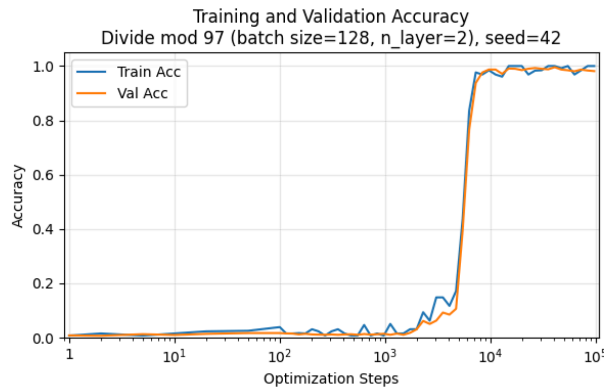
Batch size=64



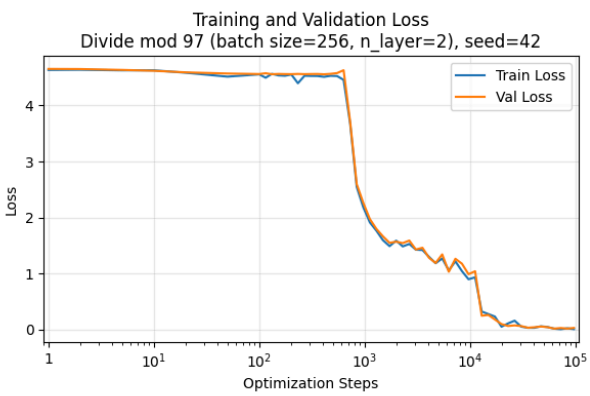


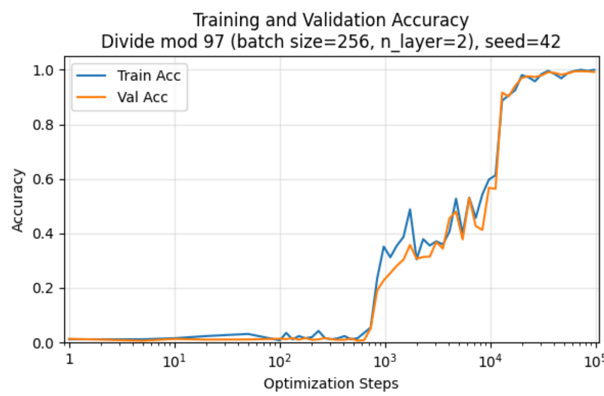
Batch size=128



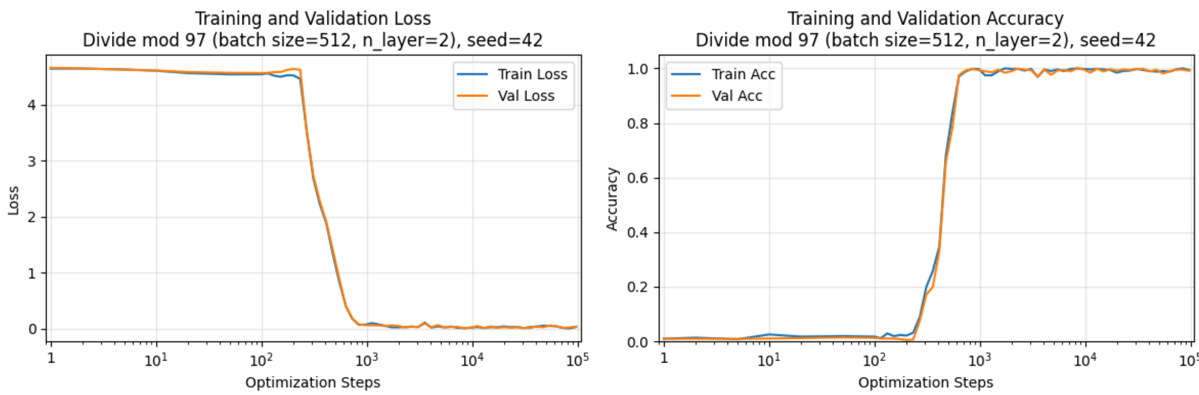


Batch size=256





Batch size=512



*We mark the steps-to-grok as the number of steps the model takes to reach a generally high and stable test accuracy.

**We find that on batch size 64 there's no sign of grokking at all, i.e. the test accuracy keeps bouncing in a very low range and do not present signs of increasing.