

# 筆記

Lien Chiang

## 1 離散分佈

### 1.1 Geometric Distribution 幾何分佈

#### 1.1.1 密度函數

$$f(x) = q^{x-1}p$$

#### 1.1.2 累積函數

$$F(x) = p \frac{1 - q^x}{1 - q} = p \frac{1 - q^x}{p} = 1 - q^x$$

#### 1.1.3 期望值

$$\begin{aligned} E[X] &= \sum_{x=1}^{\infty} xq^{x-1}p = p \sum_{x=1}^{\infty} xq^{x-1} = p \sum_{x=1}^{\infty} \frac{dq^x}{dq} \\ &= p \frac{d}{dq} \left( q \frac{1}{1-q} \right) = p \frac{q'(1-q) - q(1-q)'}{(1-q)^2} \\ &= p \frac{(1-q) + q}{p^2} = p \frac{1}{p^2} = \frac{1}{p} \end{aligned}$$

### 1.1.4 變異數

先算  $Var(X)$

$$\begin{aligned} Var(X) &= E[(X - \mu)^2] = E[(X^2 - 2\mu X + \mu^2)] = E[X^2] - 2\mu E[X] + \mu^2 \\ &= E[X^2] - 2E[X]^2 + E[X]^2 = E[X^2] - E[X]^2 \end{aligned}$$

再算其中的  $E[X^2]$

$$E[X^2] = \sum_{x=1}^{\infty} x^2 q^{x-1} p = p \sum_{x=1}^{\infty} x^2 q^{x-1}$$

不過由於我們無法計算出  $x^2 q^{x-1}$  的積分，式子的  $x^2 q^{x-1}$  無法代換成  $\frac{df(q)}{dq}$  的形式。相反的  $x(x-1)q^{x-2} = \frac{dq^x}{dq}$  卻成立，我們可以利用該式對  $E[X]$  做些手腳：算  $E[X(X-1)]$  作為替代。

$$\begin{aligned} E[X(X-1)] &= \sum_{x=1}^{\infty} x(x-1)q^{x-1}p = pq \sum_{x=1}^{\infty} x(x-1)q^{x-2} = pq \sum_{x=1}^{\infty} \frac{dq^x}{dq^2} = pq \frac{d}{dq^2} \left( \frac{1}{1-q} \right) \\ &= pq \left( \frac{0(1-q) - 1(-1)}{(1-q)^2} \right)' = pq \left( \frac{1}{(1-q)^2} \right)' = pq \left( \frac{0(1-q)^2 - 1 \cdot 2(1-q)(-1)}{(1-q)^4} \right) \\ &= pq \frac{2p}{p^4} = \frac{2q}{p^2} \end{aligned}$$

$$\begin{aligned} E[X^2] &= E[X(X-1)] + E[X] \\ &= \frac{2q}{p^2} + \frac{1}{p} = \frac{2q+p}{p^2} = \frac{2(1-p)+p}{p^2} = \frac{2-p}{p^2} \end{aligned}$$

$$\begin{aligned} Var(X) &= E[X^2] - E[X]^2 \\ &= \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2} = \frac{q}{p^2} \end{aligned}$$

### 1.1.5 動差生成函數

技巧：把  $x-1$  提出來

$$\begin{aligned} m_x(t) &= E[e^{tX}] = \sum_{x=1}^{\infty} e^{tx} q^{x-1} p = p \sum_{x=1}^{\infty} e^{tx} q^{x-1} = p \sum_{x=1}^{\infty} e^{t(x-1)} e^t q^{x-1} = pe^t \sum_{x=1}^{\infty} (qe^t)^{x-1} \\ &= pe^t \frac{1}{1-qe^t} = \frac{pe^t}{1-qe^t} \end{aligned}$$