Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	5.0	*Other Types of Functions*
Lesson Code	5.1.1	*Square Root Functions 1*
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

By the end of this learning guide, the student will have been able to
1. define square root function (half parabola and semicircle); and
2. determine its properties (domain and range; increasing and decreasing on intervals).

HOOK

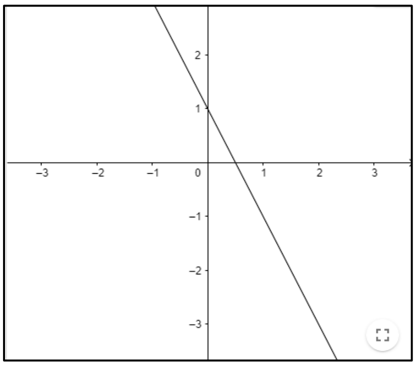
Time Allocation: 4 minutes
Actual Time Allocation: ______ minutes

In a previous learning guide, we have discussed how to identify whether a given relation is a function or not. We were also able to name the domain and range of some functions. To recall these topics, let us answer the exercises below. (This activity is not graded.)

A. Determine if the following given represents a FUNCTION or NOT. If it is a function, find the domain and range.

1. $y = 2x + 3$
2. $x^2 + y^2 = 4$
3. $y = x^2 - 1$
4. $y = x^3 - 3x^2 + 3x + 1$
5. $x = y^4$

B. Which of the following graphs describe/s a function?

1.


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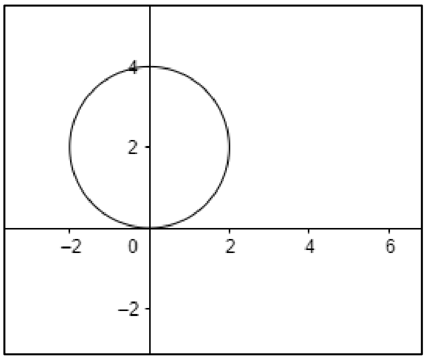

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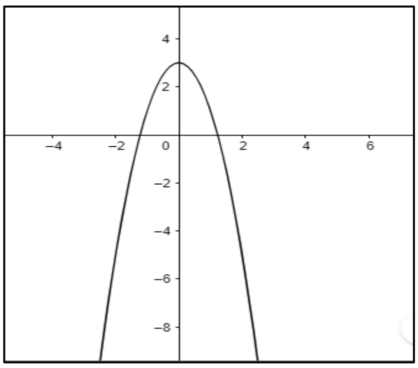

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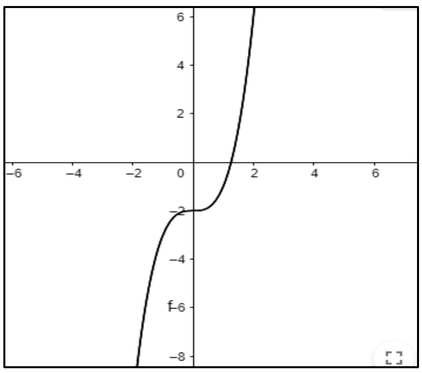


To check your answers, refer to the answer key provided on the last page of this learning guide.

Now, we are ready to study other types of functions. In this lesson, we will start exploring square root functions by describing them and determining their properties.

Are you ready?

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

THE SQUARE ROOT FUNCTION

Let us consider the square root of a nonnegative real number *x* which is a number *y* such that $y^2 = x$. Having this example, 4 and -4 are the square roots of 16 since $4^2 = 16$ and $(-4)^2 = 16$.

Given the two roots, we will use the nonnegative root of a nonnegative number *x* for the function to be real and it is written as a function defined by

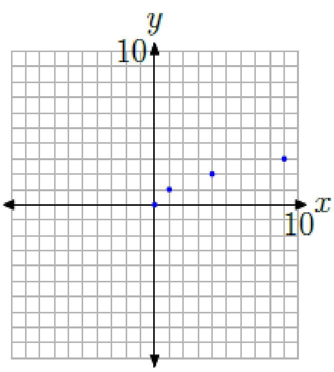
$$f(x) = \sqrt{x}$$

Let us start this lesson by presenting the graph of the function. We will also study and explore the properties of this function.

Illustrating the Graph of the Square Root Function

Let us first create a table of values that will satisfy the equation of the function. We are to plot the points on a rectangular coordinate system.

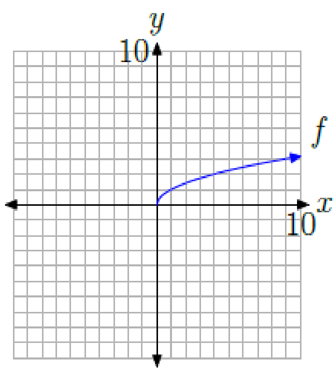
We have mentioned earlier that for us to have a function that is real, we take only the square roots of nonnegative number so we will not use any negative *x*-values in our table. That is $x \geq 0$. Also, for us to have integral *y* – values, we will use perfect square numbers for our *x*-values such as 0, 1, 4, 9, and so on. We have three figures shown below. ***Figure 1***(b) shows the table of values for $f(x) = \sqrt{x}$. ***Figure 1***(a) presents each of the points in ***Figure 1***(b) plotted on the coordinate plane. ***Figure 1***(c) shows the function's graph.



(a)

x	$f(x) = \sqrt{x}$
0	0
1	1
4	2
9	3

(b)



(c)

Image Source: http://msenux2.redwoods.edu/IntAlgText/chapter9/section1.pdf

To present the graph of $f(x) = \sqrt{x}$ shown in ***Figure 1***, we have utilized the point plotting approach**.** We can conclude that when we take the square root, we do the inverse of squaring.

Half – Parabola Function

Let us explore the relation $\boldsymbol{y^2 = x}$ to discover the ***half parabola function***.

Study **Figure 2**. The table shows some values of x and y given the equation $y^2 = x$. To help us locate points easily in the coordinate plane, we have used perfect square numbers for our x – *values* to generate *y-values* that are integers.

We can observe from the table that we have two y – values for each x – value. This means that $y^2 = x$ follows a one-to-many correspondence. ***Figure 3*** shows the graph of $y^2 = x$ and it is a parabola that opens to the right. We can see that the graph fails the vertical line test. Thus, the relation $y^2 = x$ is not a function.

x	*y*
0	*0*
1	*-1*
	1
4	*-2*
	2
9	*-3*
	3

Figure 2. x and y values for $y^2 = x$

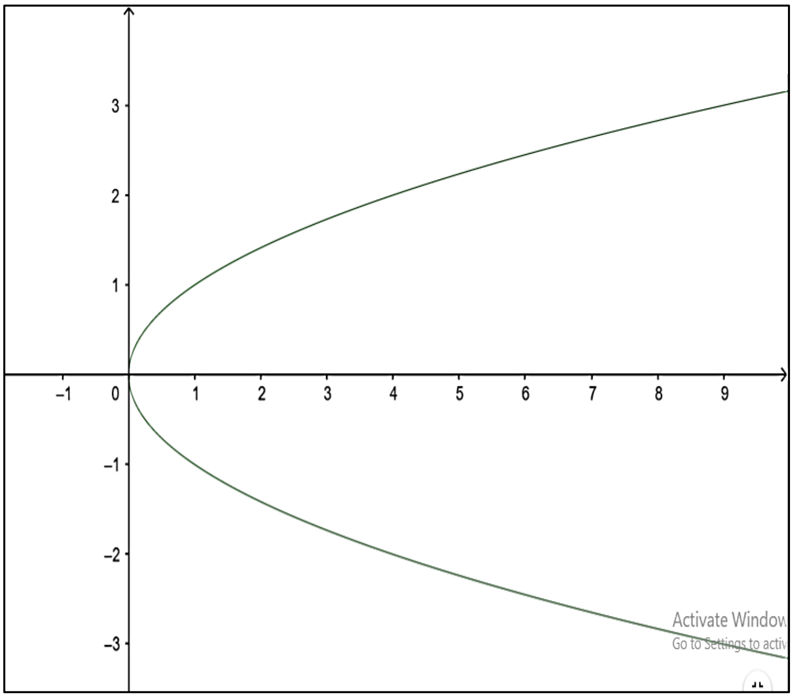


Figure 3. Graph of $y^2 = x$

The relation $y^2 = x$ is equivalent to $y = \pm\sqrt{x}$. We can have two relations which are $y = \sqrt{x}$ and $y = -\sqrt{x}$. If we are to show the graphs of the two relations, we'll have

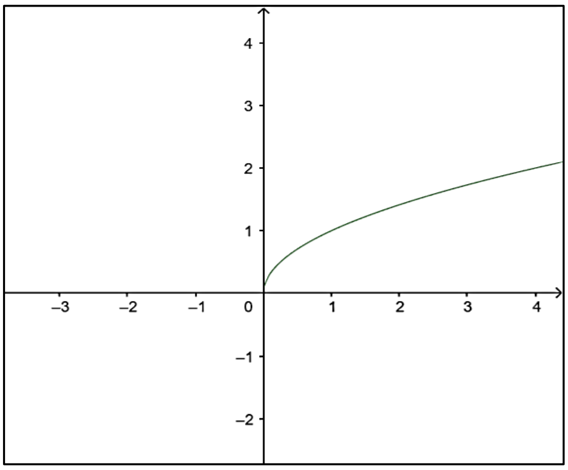


Figure 4. Graph of $f(x) = \sqrt{x}$

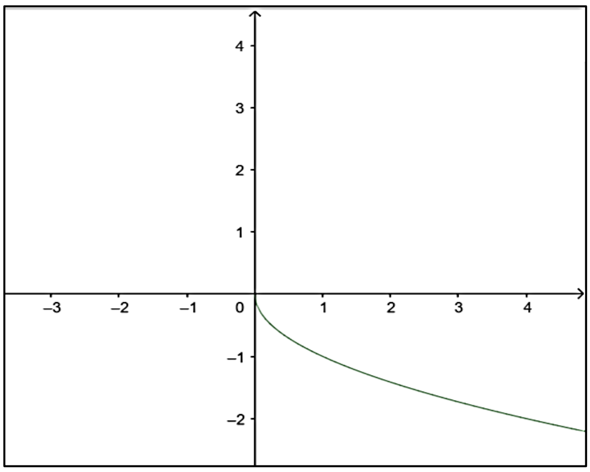


Figure 5. Graph of $f(x) = -\sqrt{x}$

By observing the graphs shown above, we can say that these relations can now be considered functions since they are one – to – one correspondences and pass the vertical line tests.

Think about this!
Considering the two functions below, what will be their domain and range?.

$$\boldsymbol{y} = \sqrt{\boldsymbol{x}} \text{ and } \boldsymbol{y} = -\sqrt{\boldsymbol{x}}$$

Semi-circular Function

Consider the standard form of equation of the circle centered at the origin below.

$$x^2 + y^2 = r^2, \text{ where } r \text{ is the radius.}$$

We have discussed circles in the previous quarter and we know that its graph is a relation since it is a one – to – many correspondence. In addition, the graph of a circle fails the vertical line test.

Rewriting the equation in terms of y, we have

$$y^2 = r^2 - x^2 \Longrightarrow y = \pm\sqrt{r^2 - x^2}$$

We now have, two relations which are $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$.

Let $r = 1$. Observe the graphs of $f(x) = \sqrt{1 - x^2}$ and $f(x) = -\sqrt{1 - x^2}$ as shown below.

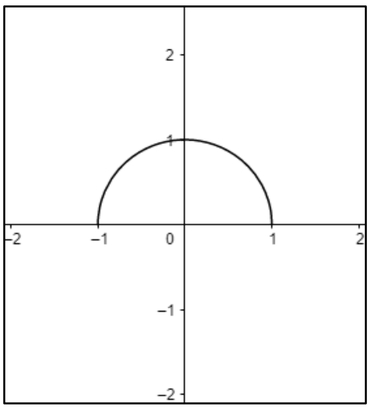


Figure 6. Graph of $f(x) = \sqrt{1 - x^2}$

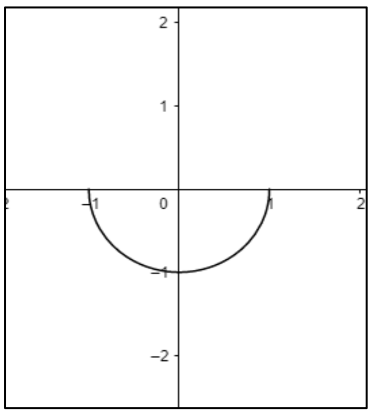


Figure 7. Graph of $f(x) = -\sqrt{1 - x^2}$

Notice that these graphs can now be considered as functions.

Think about this!
Determine the intervals on which $f(x) = \sqrt{1 - x^2}$ and $f(x) = -\sqrt{1 - x^2}$ are increasing or decreasing.

Properties of Square Root Function

Let $f(x) = \sqrt{x}$, where $x \geq 0$, then it has the following properties:

1. The domain of $f(x)$ is $[0, +\infty)$.
2. The range of $f(x)$ is $[0, +\infty)$.
3. The x - and y – intercepts are both $(0,0)$.
4. It is an increasing function.
5. It is a one – to – one function and has an inverse.

Let us study the following examples.

Example 1 Let $\boldsymbol{f(x) = \sqrt{x+2}}$**,** determine the domain and range; and the interval which the function *f* is increasing or decreasing.

Solution

To determine the domain of *f*, we need to set the radicand to be ≥ 0. Then, we solve for x.

$$x + 2 \geq 0$$
$$\boldsymbol{x \geq -2}$$

Thus, the *domain* of *f* is $[\boldsymbol{-2}, +\infty)$**.**

For us to determine the range of $\boldsymbol{f}$, we must take the *x*- values within the domain of the function $\boldsymbol{f}$. As x takes values from -2 to $+\infty$, $\sqrt{x+2}$ takes values from 0 to $+\infty$. Therefore, the *range* of $\boldsymbol{f}$ is $[0, +\infty)$.

We can see that $f(x)$ increases as the value of x increases. Hence, the function is *increasing* on the interval $[\boldsymbol{-2}, +\infty)$.

Example 2 Let $\boldsymbol{f(x) = -\sqrt{x-5} - 6}$**,** determine the domain and range; and the interval which the function *f* is increasing or decreasing.

Solution

To determine the domain of *f*, we set $x - 5 \geq 0$, then solve for *x*.

$$x - 5 \geq 0$$
$$\boldsymbol{x \geq 5}$$

Thus, the *domain* of *f* is $[\boldsymbol{5}, +\infty)$**.**

For us to determine the range of $\boldsymbol{f}$, we must take the *x*- values within the domain of the function $\boldsymbol{f}$. As x takes values from 5 to $+\infty$, $-\sqrt{x-5} - 6$ takes values from -6 to $-\infty$. Therefore, the *range* of $\boldsymbol{f}$ is $(-\infty, -6]$.

We can see that $f(x)$ decreases as the value of x increases. Hence, the function is *decreasing* on the interval $[\boldsymbol{5}, +\infty)$.

Example 3 Let $\boldsymbol{f(x) = \sqrt{4 - x^2}}$**,** determine the domain and range; and the interval which the function *f* is increasing or decreasing.

Solution

To determine the domain of *f*, we set the radicand to be ≥ 0. We now then solve the resulting inequality.

$$4 - x^2 \geq 0$$
$$\boldsymbol{-x^2 \geq -4}$$
$$\boldsymbol{x^2 \leq 4}$$

Solving for *x*,

$$-2 \leq \boldsymbol{x \leq 2}$$

Therefore, the *domain* of *f* is $[\boldsymbol{-2}, \boldsymbol{2}]$**.** In determining the range of f, we'll see that as x takes the value of -2 to 2, $\sqrt{4 - x^2}$ takes values from 0 to 2. Thus, the *range* of the function is $[0, 2]$.

This function is increasing on the interval $[-2,0]$ and decreasing on the interval $[0,2]$.

NAVIGATE

Time Allocation: 8 minutes
Actual Time Allocation: ______ minutes

Note: Items marked with an asterisk (*) will be graded.

Directions: Determine the a) domain, b) range, and c) the interval where *f(x)* is increasing or decreasing for each given square root function.

1. $f(x) = \sqrt{x} + c$

* 2. $f(x) = -2\sqrt{x - 7}$

3. $f(x) = \sqrt{x - 4} - 2$

* 4. $f(x) = -\sqrt{3 - x} + 1$

5. $f(x) = -\sqrt{5 - x^2}$

* 6. $f(x) = \sqrt{\frac{1}{9} - x^2}$

7. $f(x) = -\sqrt{x + a} + b$

* 8. $f(x) = \sqrt{2x + 3} - 2$

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: ______ minutes

To summarize this lesson,

- The square root function is a function denoted by $\boldsymbol{f(x) = \sqrt{x}}$.
- In this lesson, we have identified half – parabolas and semi-circular functions to be square root functions.
- For half-parabolas, we have $f(x) = \sqrt{x}$ and $f(x) = -\sqrt{x}$.
- For semi-circular functions, we have $f(x) = \sqrt{r^2 - x^2}$ and $f(x) = -\sqrt{r^2 - x^2}$ where *r* is the radius.
- Let $f(x) = \sqrt{x}$, then it has the following properties:
  1. The domain of $f(x)$ is $[0, +\infty)$.
  2. The range of $f(x)$ is $[0, +\infty)$.
  3. The x - and y – intercepts are both (0,0).
  4. It is an increasing function.
  5. It is a one – to – one function.

References:

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ANSWER KEY

Hook

A. 1. Function, Domain: $x \epsilon R$ and Range: $y \epsilon R$
   2. Not function
   3. Function, Domain: $x \epsilon R$ and Range: $y \geq -1$
   4. Function, Domain: $x \epsilon R$ and Range: $y \epsilon R$
   5. Not Function

B. 1. Function
   2. Function
   3. Not Function
   4. Function

Check Your Understanding!

1. a. $x \geq 0$	b. $y \geq c$	c. increasing, $x \geq 0$
3. a. $x \geq 4$	b. $y \geq -2$	c. increasing, $x \leq 4$
5. a. $-\sqrt{5} \leq x \leq \sqrt{5}$	b. $-\sqrt{5} \leq y \leq 0$	c. decreasing, $-\sqrt{5} \leq x \leq 0$; increasing, $0 \leq x \leq \sqrt{5}$
7. a. $x \geq -a$	b. $y \leq b$	c. decreasing, $x \geq -a$