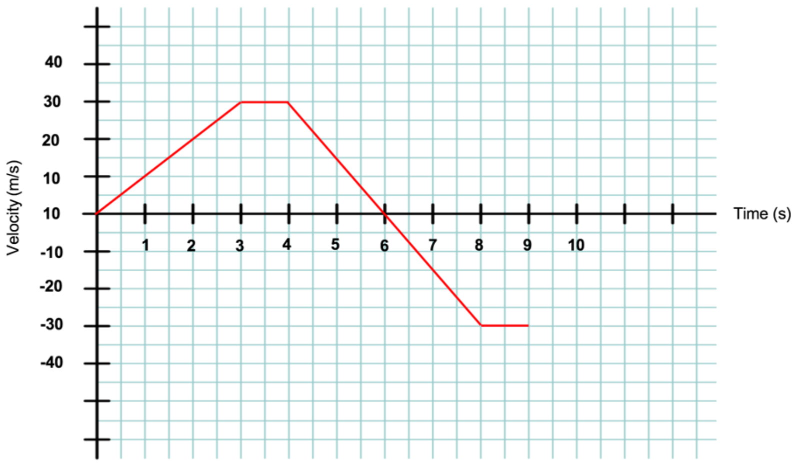
Subject Code PHY 1 **Physics 1**
Module Code 2.0 **Motion Graphs**
Lesson Code 2.4 **Area Under Motion Graphs**
Time Frame 30 minutes

<table>
<tr><th>Components</th><th>Tasks</th><th>TA[1] (min)</th><th>ATA[2] (min)</th></tr>
<tr><td>Target</td><td>By the end of this learning guide, the student should be able to:

identify the significance of the area under v-t and a-t graphs
determine the displacement and change in velocity from v-t and a-t graphs, respectively
</td><td>1</td><td></td></tr>
<tr><td>Hook</td><td>In the previous lessons, you learned how to analyze motion graphs by examining their slope. The slope of an x-t graph tells you about the velocity of an object at a specified period. In contrast, the slope of a v-t graph indicates the acceleration of a moving body at a specific time interval. Aside from slopes, did you know that the area under a motion graph can also tell us essential information about the motion of an object? Before we go into the details, take a look at the following velocity-time graph.



The graph and the x-axis of this v-t graph bound a particular area and can be represented by various geometric figures. This bound area is merely a combination of triangles and rectangles. The following figure shows you the different triangles and quadrilaterals (either rectangle or trapezoid) formed by the v-t curve and the x-axis.</td><td>5</td><td></td></tr>
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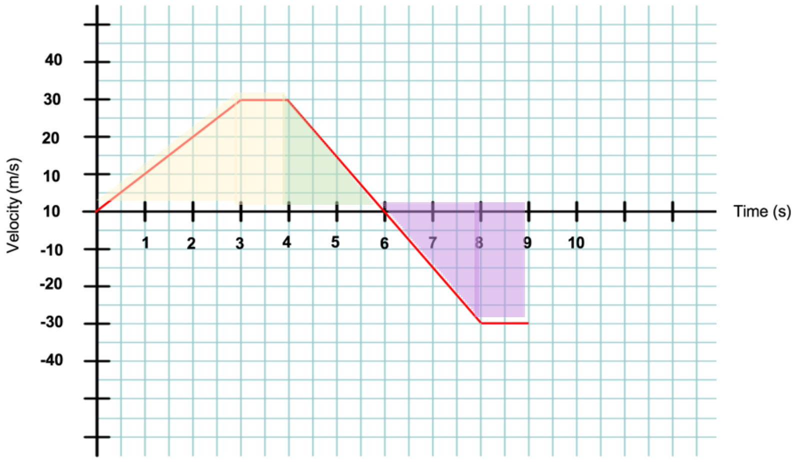
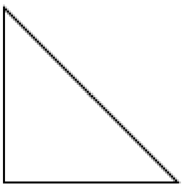
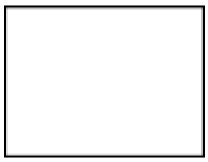
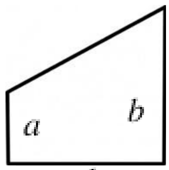
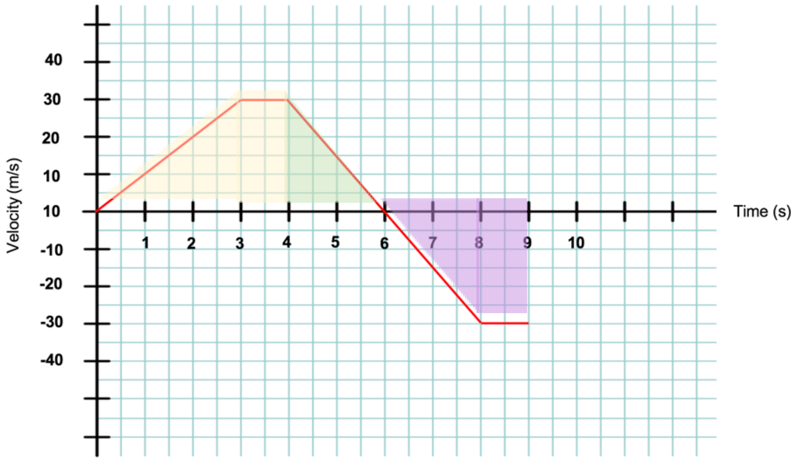
[1] Time allocation suggested by the teacher.
[2] Actual time allocation spent by the student (for information purposes only).



Interestingly, the shaded region of this *v-t* graph tells an essential information about the motion of the object. To do this, we need to recall some basic formulas in geometry, such as the area of triangles, rectangles, and trapezoids. We will be using these formulas to find out the area bounded by a *v-t* or *a-t* graph to give us further details about the motion of an object.

		
Area of a Triangle	Area of a Rectangle	Area of a Trapezoid
$A = \frac{1}{2}bh$	$A = lw$	$A = \frac{1}{2}(a+b)\,h$

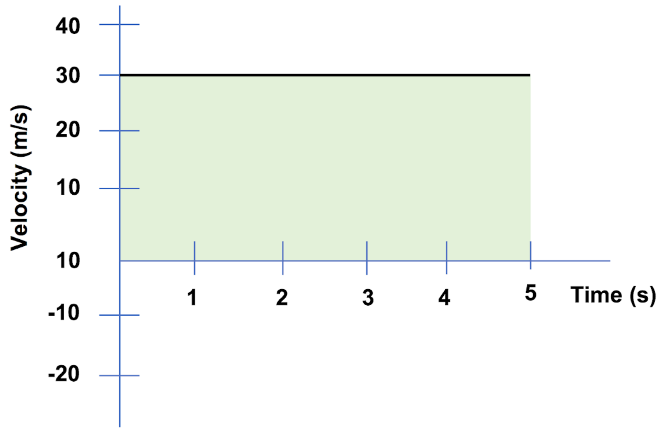
Ignite	Let us find the area bounded by the *v-t* graph in our previous example. To find the total area, we need to decompose the area bounded by the *v-t* graph into constituent triangles, rectangles, and trapeziums. As shown, the restricted area is composed of two trapezoids and one triangle. To find the total area bounded by the *v-t* graph, we need to calculate the individual areas of these figures.	12	



The dimensions of these geometric figures correspond to the distance bounded along the x- and y-axes, respectively (together with the units). The following table shows the step-by-step procedure for calculating the area bounded by the v-t graph.

Region	**Solution**
First trapezoid	$A = \frac{1}{2}(a+b)h$ $A = \frac{1}{2}(4\ s + 1\ s)\left(30\frac{m}{s}\right)$ $A = \frac{1}{2}(5\ s)\left(30\frac{m}{s}\right)$ $A = \frac{1}{2}(150\ m)$ $A = 75\ m$
Right Triangle	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(2\ s)\left(30\frac{m}{s}\right)$ $A = \frac{1}{2}(60\ m)$ $A = 30\ m$
Second Trapezoid	$A = \frac{1}{2}(a+b)h$ $A = \frac{1}{2}(1\ s + 3\ s)\left(-30\frac{m}{s}\right)$ $A = \frac{1}{2}(4\ s)\left(-30\frac{m}{s}\right)$ $A = -60\ m$

Adding the areas of the figures in the bounded area, we have $A = 75\ m + 30\ m - 60\ m = 45\ m$. Notice that the unit is in meters and is a unit of displacement. The area bounded by a *v-t* graph is the object's displacement. This can be further explained by using the following *v-t* graph.



This *v-t* graph shows an object moving at constant velocity for five seconds. Since the area bounded by the v-t graph is a rectangle, we can just simply multiply velocity equal to 30 m/s (v) and time interval of 5 s (Δt) to determine its displacement (Δx).

$$\Delta x = v\Delta t$$

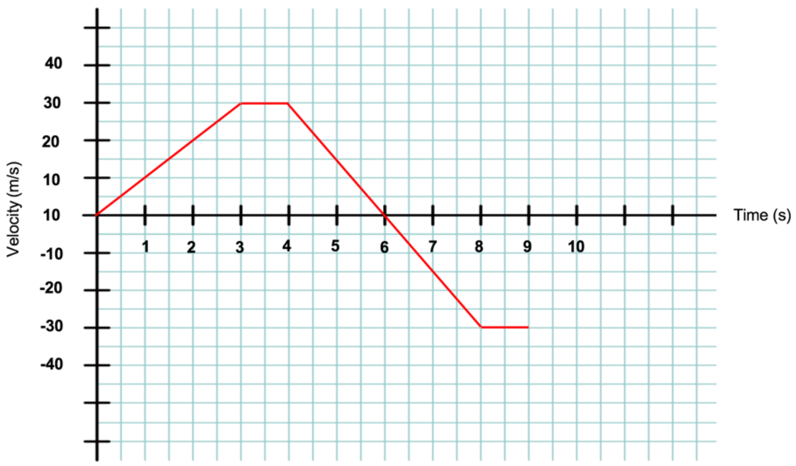
The above equation can also be derived mathematically from our definition of velocity that is

$$v = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0}$$

This calculation means that the area bounded by a v-t graph tells us the ***displacement*** *of an object.*

Therefore, the *v-t* graph shows us that the total displacement of the object is equal to 45 m.

In addition, you can also solve for the displacement traveled by the object described in the abovementioned *v-t* graph by decomposing the figure into triangles and rectangles.

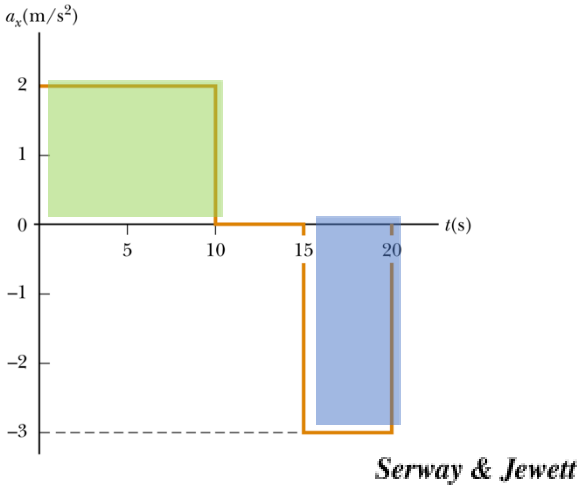


Region	Solution
First triangle (between 0 to 3 seconds)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(3\ s)\left(30\frac{m}{s}\right)$ $A = \frac{1}{2}(90\ m)$ $A = 45\ m$
First rectangle (between 3 to 4 seconds)	$A = lw$ $A = (1\ s)(30\ m/s)$ $A = 30\ m$
Second triangle (between 4 to 6 seconds)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(2\ s)\left(30\frac{m}{s}\right)$ $A = \frac{1}{2}(60\ m)$ $A = 30\ m$
Third triangle (between 6 to 8 seconds)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(2\ s)\left(-30\frac{m}{s}\right)$ $A = \frac{1}{2}(-60\ m)$ $A = -30\ m$
Second rectangle (between 8 to 9 seconds)	$A = lw$ $A = (1\ s)(-30\ m/s)$ $A = -30\ m$

Notice that even if we decompose them into triangles and rectangles alone, the calculated displacement remains 45 m.

Should we decide to ignore the negative sign in the second trapezium, it still provides us with vital information about the *v-t* graph. This time, it tells us the ***total distance*** traveled by the object. In this case, the total distance traveled is equal to 165 m. That is from the equation $A = 75\ m + 30\ m + 65\ m$. Note that distance and displacement are different terms we discussed in a previous lesson.

This principle can also be extended to *a-t* graphs. Let's take the following *a-t* graph as our example.



Just like what we did in the previous example (*v-t* graph), we need to decompose the total area bounded by the given *a-t* graph into smaller and simpler geometric figures. It can be gleaned that the given graph bounds two rectangles – one rectangle above the *x*-axis (shaded green) and another rectangle below the *x*-axis (shaded blue).

Now, we need to calculate the areas bounded by these rectangles. Applying the same procedure employed to *v-t* graphs, we will come up with the following solution.

Region	**Solution**
First rectangle	$A = lw$ $A = \left(2\frac{m}{s^2}\right)(10\ s)$ $A = 20\ m/s$
Second Rectangle	$A = lw$ $A = \left(-3\frac{m}{s^2}\right)(5\ s)$ $A =- 15\ m/s$

If we add this together, we get 5 m/s. Notice that it has a unit of velocity. Recalling the definition of acceleration, when you multiply acceleration by time interval you get the change in velocity. Thus, the area bounded by an a-t graph is equal to the change in velocity of a moving body. In the above example the change in velocity during the 20-s travel is 5 m/s.

Refer again to our sample *a-t* graph and focus on the motion of the object between 10 to 15 seconds, you notice that the acceleration-time graph coincides with the *x*-axis, which implies zero acceleration. It already indicates that there will be no observable change in velocity during this time period.

In some cases, the *v-t* or *a-t* graphs may include curved sections like parabolas. In this case, our previous methods for finding the area bounded by the graph may not be a convenient procedure. Instead, we will use a more sophisticated mathematical technique in calculus known as **integration.** You will learn more about this as you progress to higher physics courses.

Navigate

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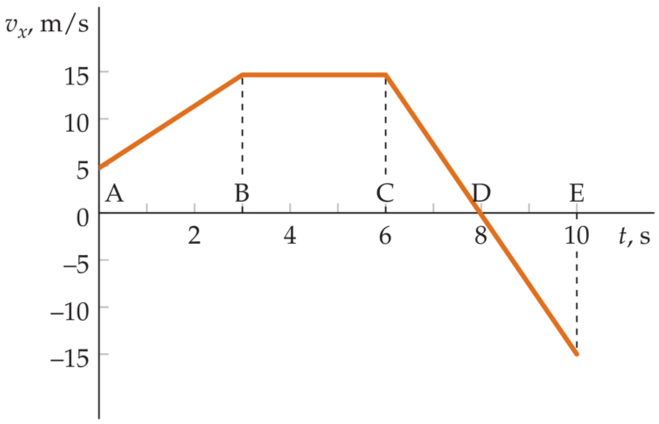
Let's see if you understood the lesson on areas under a motion graph by working on the following exercises. Write your answers on a clean sheet of paper. Follow your teacher's instructions regarding submission.

1. A v-t graph of a moving object is shown below. Based on this v-t graph, determine the displacement and the total distance traveled by the object.



Tipler & Mosca

2. A rigid ball traveling in a straight line (the x-axis) hits a solid wall and suddenly rebounds during a brief instant. The v-t graph in the following figure shows this ball's velocity as a function of time. During the first 20.0 s of its motion, find (a) the total distance the ball moves and (b) its displacement (Young & Freedman, 2012).

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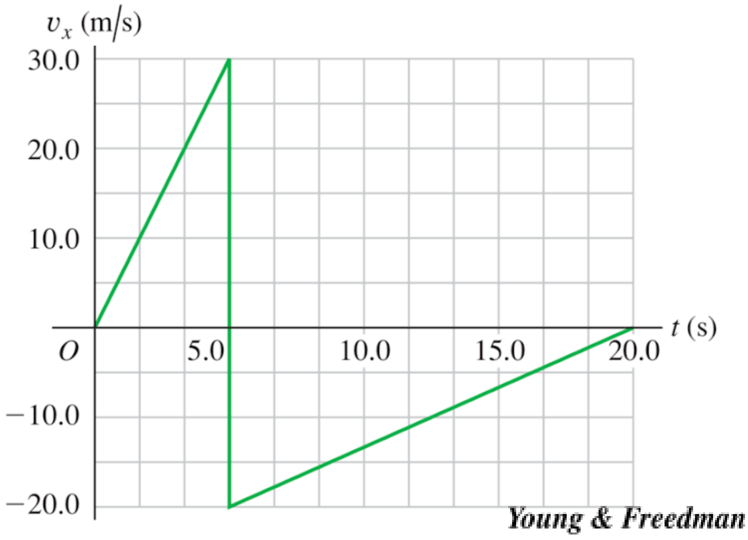
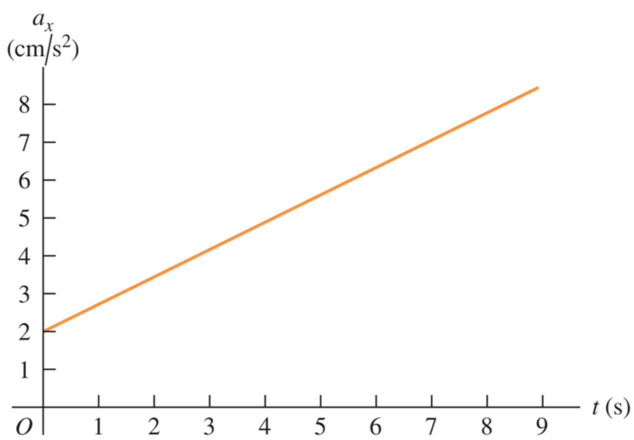


3. The following a-t graph describes the acceleration as a function of time for a stone rolling down a hill starting from rest. What is the stone’s change in velocity after (a) 2.5 s and (b) 7.5 s? (Young & Freedman, 2012)



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Here are some of the significant key ideas that you should remember about the area under motion graphs.

- The area bounded by a motion graph can provide us additional information about the motion of an object.
- The area bounded by a motion graph can be determined by decomposing the restricted area into simpler geometric figures like triangles, rectangles, and trapezoids.
- The area bounded by a velocity-time graph describes the object’s displacement. If signs of the component areas will be disregarded, the total area refers to the total distance traveled by the object.
- The area bounded by an acceleration-time graph a certain period describes the object’s change in velocity.
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References

1. Serway, R., & Jewett, J. (2004). *Physics for Scientists and Engineers.* Thomson Brooks/Cole.
2. Tipler, P., & Mosca, G. (2008). *Physics for Scientists and Engineers with Modern Physics.* W. H. Freeman and Company.
3. Young, H. & Freedman, R. (2012). *Sears and Zemansky's University Physics with Modern Physics (13th Edition).* Addison-Wesley.

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ANSWER KEY

NAVIGATE

1.

Sections	Solution
First trapezoid (between 0 to 3 s)	$A = \frac{1}{2}(a+b)\,h$ $A = \frac{1}{2}(5\ m/s\ +\ 15\ m/s)\,(3\ s)$ $A = \frac{1}{2}(20\ m/s)(3s)$ $A = \frac{1}{2}(60\ m)$ $A = 30\ m$
First rectangle (between 3 to 6 s)	$A = \ lw$ $A = \ (15\ m/s)(3s)$ $A = \ 45\ m$
First triangle (between 6 to 8 s)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(15\ m/s)\,(2\ s)$ $A = \frac{1}{2}(30\ m)$ $A = 15\ m$
Second triangle (between 8 to 10 s)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(-\ 15\ m/s)\,(2\ s)$ $A = \frac{1}{2}(-\ 30\ m)$ $A = \ -15\ m$

Displacement : 75 m

Distance : 105 m

2.

Sections	Solution
First triangle (between 0 to 5 s)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(5.0\ s)\,(30.0\ m/s)$ $A = \frac{1}{2}(150\ m)$ $A = 75\ m$
Second triangle (between 5 to 20 s)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}(15.0\ s)\,(-\ 20.0\ m/s)$ $A = \frac{1}{2}(-\ 300\ m)$ $A = \ -150\ m$

Displacement : -75 m

Distance : 225 m

3.

Sections	Solution
Change in velocity after 2.5 s	$A = \frac{1}{2}(a+b)h$ $A = \frac{1}{2}\left(2.0\ m/s^2 +\ 3.5\ m/s^2\right)(2.5\ s)$ $A = \frac{1}{2}\left(5.5\ m/s^2\right)(2.5\ s)$ $A = 6.9\ m/s$ (two SF)
Change in velocity after 7.5 s	$A = \frac{1}{2}(a+b)h$ $A = \frac{1}{2}\left(2.0\ m/s^2 +\ 7.0\ m/s^2\right)(7.5\ s)$ $A = \frac{1}{2}\left(9.0\ m/s^2\right)(7.5\ s)$ $A = 34\ m/s$ (two SF)