PROPERTY OF THE PHILIPPINE SCIENCE HIGH SCHOOL SYSTEM

Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	4.0	Graphs of Polynomial and Rational Functions
Lesson Code	4.3	End Behavior of the Graph of Polynomial Function
Time Frame		30 minutes

After completing this learning guide, you should be able to

1. describe the end behavior of the graph of a polynomial function; and
2. identify the leading coefficient and degree of a polynomial function given its graph

Behavior of function is not far from how we deal with our actions in life. We have ups and downs. The journey either goes to the right way (good outcome) or left way (bad outcome). Whichever decisions we make, we always conclude that if we choose to think positive, we hope for the best result and our life goes up and turns right. Otherwise, negative actions lead to destruction and end up falling down.

In this section we will discuss the end behavior of the graph of a polynomial function.

10 MINUTES

The following graphs are parent functions of polynomial functions from degree 1 to 4.

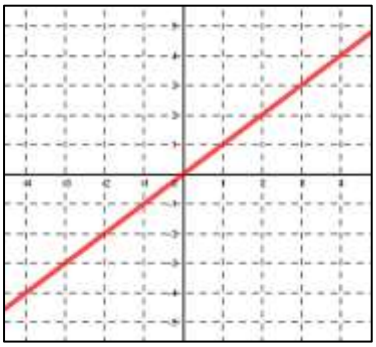


Figure 1. Graph of $f(x) = x$

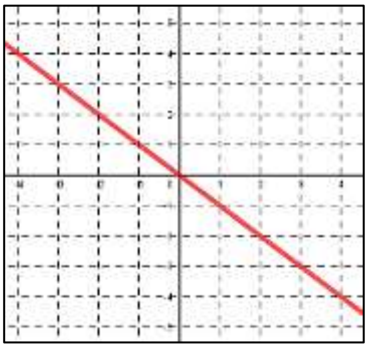


Figure 2. Graph of $f(x) = -x$

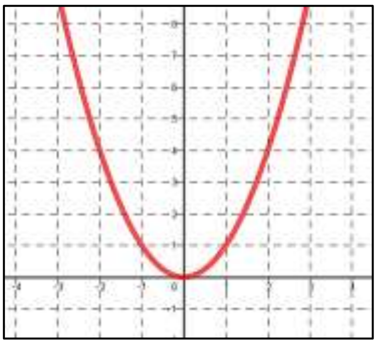


Figure 3. Graph of $f(x) = x^2$

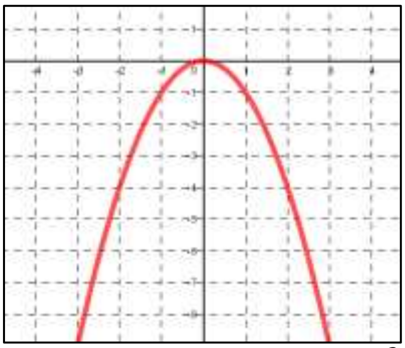


Figure 4. Graph of $f(x) = -x^2$

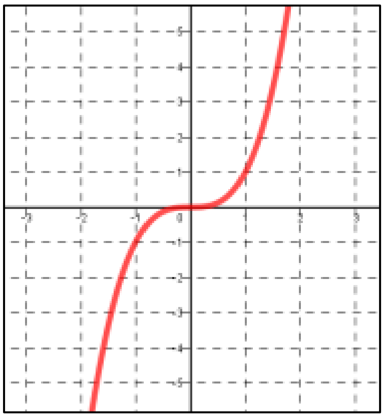


Figure 5. Graph of $f(x) = x^3$

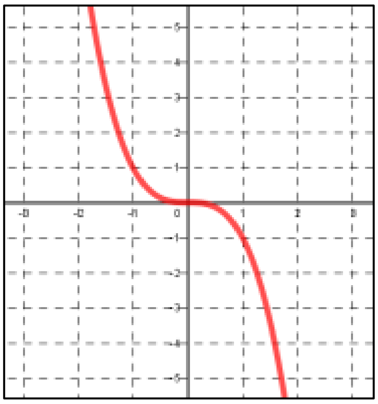


Figure 6. Graph of $f(x) = -x^3$

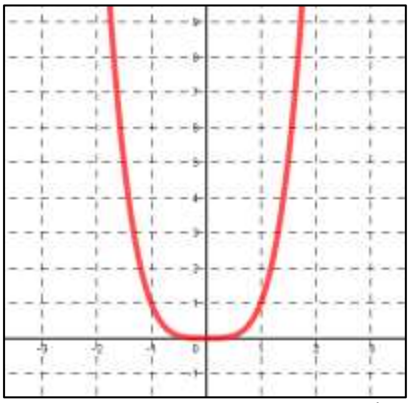


Figure 7. Graph of $f(x) = x^4$

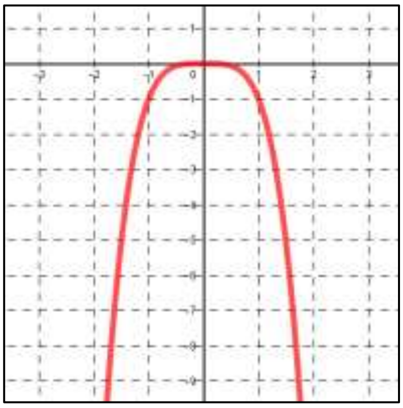


Figure 8. Graph of $f(x) = -x^4$

The graphs shown above follow similar trend as to the end behavior of the graphs of the polynomial functions.

The following graphs have the same trend in the end behavior:

GRAPH	END BEHAVIOR	SIMILARITY
Figures 1 & 5	Rises to the right Falls to the left	Both have positive leading coefficients Both have odd degree
Figures 2& 6	Rises to the left Falls to the right	Both have negative leading coefficients Both have odd degree
Figures 3 & 7	Rises at both ends	Both have positive leading coefficients Both have even degree
Figures 4 & 8	Falls at both ends	Both have negative leading coefficients Both have even degree

TIP (The Important Point)

End Behavior of the graph of polynomial function

A polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + ax + a_0$ has the following end behaviors:

- If a_n is positive and n is odd
  - The graph rises to the right and falls to the left
- If a_n is negative and n is odd
  - The graph rises to the left and falls to the right
- If a_n is positive and n is even
  - The graph rises at both ends
- If a_n is negative and n is even
  - The graph falls at both ends

Describe the end behavior of the polynomial functions in examples 1 and 2.

Example 1. $g(x) = 2x^3 + 4x + 1$

Solution.
$a_n = 2$ (positive)
$n = 3$ (odd)

The graph has a similar end behavior with the parent function $f(x) = x^3$
Thus, the end behavior of $g(x)$ rises to the right and it falls to the left.

Example 2. $g(x) = -5x^3 + x^4 + x$

Solution.
$a_n = 1$ (positive)
$n = 4$ (even)

The graph has a similar end behavior with the parent function $f(x) = x^2$
Thus, the end behavior of $g(x)$ rises at both ends.

Example 3.
The graph of $g(x)$ is shown below. Describe the leading coefficient and degree of the polynomial function.

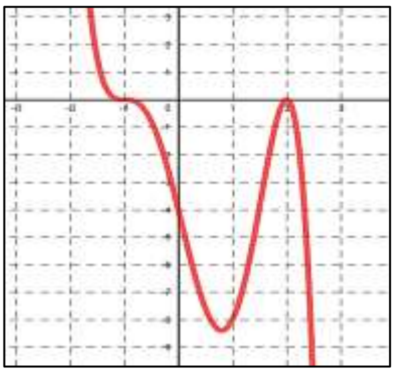


Figure 6

Solution.
Rises to the left
Falls to the right
The graph has a similar end behavior with the parent function $f(x) = -x^3$
Thus, a_n is negative and n is odd.

Example 4.
The graph of $g(x)$ is shown below. Describe the leading coefficient and degree of the polynomial function.

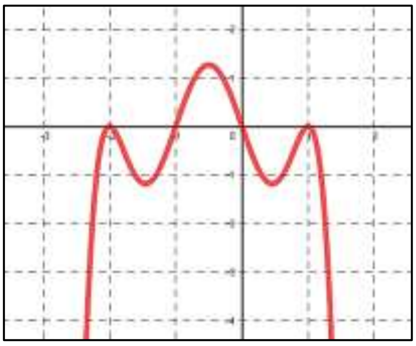


Figure 7

Solution.
Falls at both ends
The graph has a similar end behavior with the parent function $f(x) = -x^2$
Thus, a_n is negative and n is even.

Example 5. *Finding a polynomial with specific graph*

Which among the four functions in the choices has its graph shown in Figure 8?

a) $f(x) = -\frac{1}{2}x^2(x+1)(x-2)^2$
b) $f(x) = \frac{1}{2}x^2(x+1)^2(x-2)$
c) $f(x) = -x^2(x+1)(x-2)^2$
d) $f(x) = x(x+1)^2(x-2)^2$

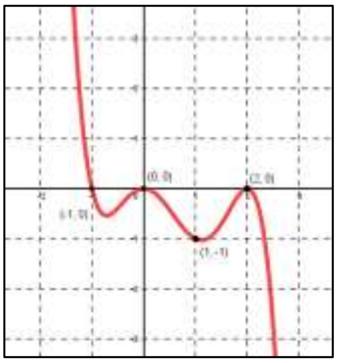


Figure 8: [x:y=1:1]

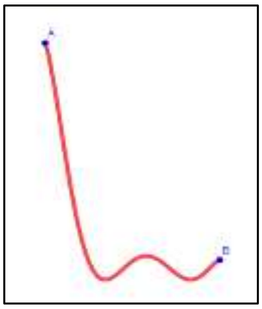
Solution.
The graph has the same end behaviour with a parent graph $f(x) = -x^3$.
The leading coefficient a_n is negative and the degree is odd. In this case, options (b) and (d) are eliminated from the choices.

Applying the theorem on multiplicity, -1 has odd multiplicity while 0 and 2 have even multiplicities. This theorem was met by options (a) and (c).
The graph passes through $(1, -1)$. If we substitute $x = 1$, we obtain:
(a) $f(1) = -\frac{1}{2}(1)^2(1+1)(1-2)^2 = -\frac{1}{2}(1)(2)(1) = -1$
(b) $f(1) = -x^2(x+1)(x-2)^2 == -(1)(2)(1) = -2$

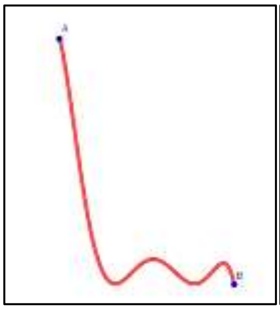
Thus, $f(x) = -\frac{1}{2}x^2(x+1)(x-2)^2$ has the graph in Figure 8.

Real Life Application.

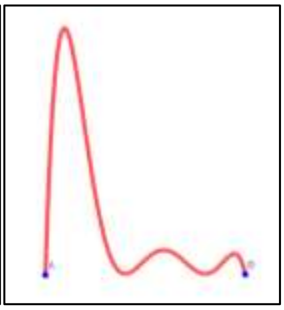
The roller coaster ride has two options for entrance (A) and exit (B) as shown below. Which among these ride adventures is similar to the path function $f(x) = -x^4 - x^3 + 7x^2 + x - 6$?



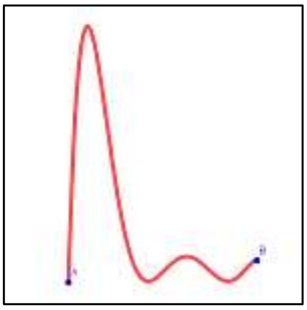
1



2



3



4

NAVIGATE

Answer the following problems below.
Believe in yourself that you can do it.

Do It Yourself

15 MINUTES

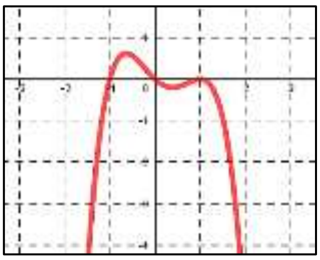
Note: Items marked with an asterisk (*) are graded

Describe the end behavior of the following polynomial functions:

1. $f(x) = -3x^3 - 4x^2 + x - 1$	*2. $f(x) = 5x^4 + x^2 - 6$
3. $f(x) = -x^3 + 4x^5 + 3x - 4$	*4. $f(x) = 5 - 2x^4 - 2x$

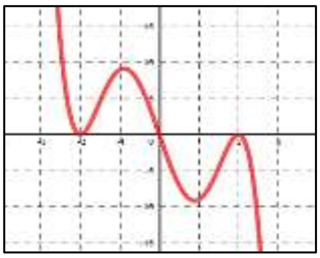
Describe the leading coefficient and degree of the polynomial functions:

5.



a_n: ___________
n: ___________

*6.



a_n: ___________
n: ___________

CRITICAL THINKING

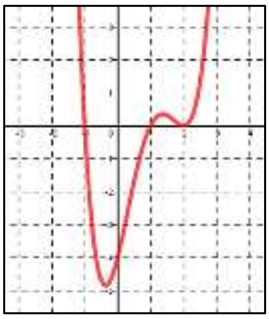
Match each graph to the following functions:

7. $f(x) = (x-2)^2(x-1)(x+1)$ __________

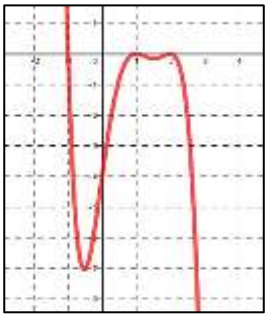
*8. $f(x) = -(x-2)^2(x+1)^2(x-1)$ __________

9. $f(x) = -(x-2)^2(x-1)^2(x+1)^2$ __________

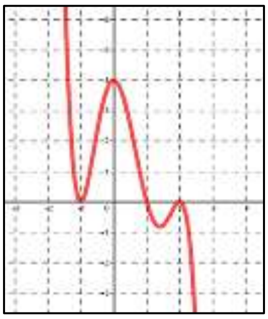
*10. $f(x) = -(x-2)^2(x-1)(x+1)^2$ __________



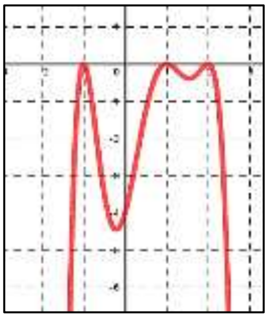
A



B



C



D

Answer to Real Life Application: 3

KNOT

Before this lesson ends, *Keep* *Note* of these *Outstanding* *Thoughts*:

A polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + ax + a_0$ has the following end behaviors:

- If a_n is positive and n is odd
  - The graph rises to the right and falls to the left
- If a_n is negative and n is odd
  - The graph rises to the left and falls to the right
- If a_n is positive and n is even
  - The graph rises at both ends
- If a_n is negative and n is even
  - The graph falls at both ends

ℳΔTΠi∫ƒμN ℳΔTΠi∫ƒμN ℳΔTΠI ∫ƒμN ℳΔTΠi∫ƒμN ℳΔTΠi∫ƒμN ℳΔTΠi∫ƒμN

References:

[1] Albarico, J.M. (2013). THINK Framework. (Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Rex Bookstore, Inc.)

[2] International Geogebra Institute. (2020). *GeoGebra*. https://www.geogebra.org/

[3] Swokowski, E., & Cole, J. (2008). *Algebra and Trigonometry with Analytic Geometry, 12th Edition*. Thomson Learning, Inc.

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ANSWER KEY:

1. Rises to the left and falls to the right
3. Rises to the right and falls to the left
5. a_n is negative, n is even
7. A
9. D