b) $X \sim Po\left(\frac{1}{5}\right)$

$P(0) = \frac{\left(\frac{1}{5}\right)^0}{0!} \cdot e^{-\frac{1}{5}} = 0{,}8187$

$P(1) = \frac{\left(\frac{1}{5}\right)^1}{1!} \cdot e^{-\frac{1}{5}} = 0{,}1637$

$P(2) = \frac{\left(\frac{1}{5}\right)^2}{2!} \cdot e^{-\frac{1}{5}} = 0{,}01637$

$P(X>2) = 1-(p(0)+p(1)+p(2)) = 1-(0{,}8187+0{,}1637+0{,}01637) = 0{,}0011$

c) $X \sim Po\left(\frac{1}{15}\right)$

$P(0) = \frac{\left(\frac{1}{15}\right)^0}{0!} \cdot e^{-\frac{1}{15}} = 0{,}9355$

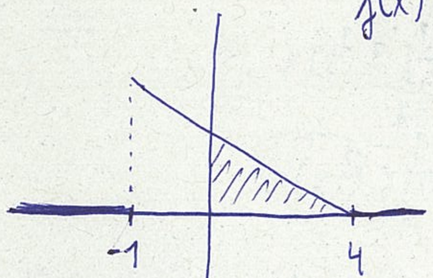
$P(1) = \frac{\left(\frac{1}{15}\right)^1}{1!} \cdot e^{-\frac{1}{15}} = 0{,}0623$

$P(2) = \left(\frac{\frac{1}{15}}{2!}\right)^2 \cdot e^{-\frac{1}{15}} = 0{,}00207$

$P(X>2) = 1-[P(0)+P(1)+P(2)] = 0{,}000046979$

Pr3

$f(x)$ $P(X \geq 0)$



$\frac{5 \cdot f(-1)}{2} = 1$

$f(-1) = \frac{2}{5}$

$f(0) = f(-1) \cdot \frac{4}{5} = \frac{8}{25}$

$P(X \geq 0) = \frac{8}{25} \cdot \frac{4}{2} = \frac{16}{25}$

$P(X \geq 0)$

$P(X \geq 4) = 0$

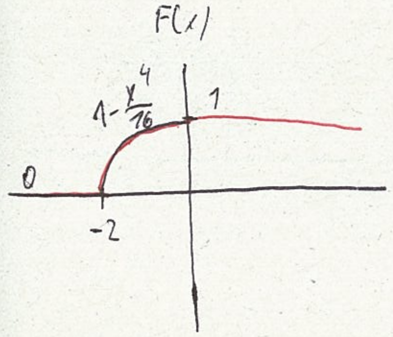
$P(X \leq -2) = 0$

$P(X \leq 10) = 1$

$P(X \leq 0) = 1 - \frac{16}{25} = \frac{9}{25}$

22.11.2018

10 cv. 10 bodov

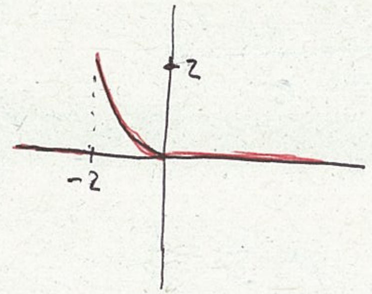


1) $P(X \leq -1)$; $P(-0{,}2 \leq X \leq -0{,}1)$
2) $f(x)$
3) EX
4) DX, σ

1.) $P(X \leq -1) = \frac{15}{16}$, $P(-0{,}2 \leq X \leq -0{,}1) = F(-0{,}1) - F(0{,}2) = 0{,}99999375 - 0{,}9999 = 0{,}00009375$

2.) $f(x) = F'(x)$

$f(x) = \begin{cases} 0 & \text{pre } x \leq -2 \\ -\frac{x^3}{4} & \text{pre } x \in \langle -2, 0) \\ 0 & \text{pre } x \geq 0 \end{cases}$



$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g'} = \frac{4x^3 \cdot 16 - x^4 \cdot 0}{256} = -\frac{x^3}{4}$

3.) $EX = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-2}^{0} x \cdot \left(-\frac{x^3}{4}\right) dx = \int_{-2}^{0} -\frac{x^4}{4} = -\frac{1}{4} \int_{-2}^{0} x^4 = -\frac{1}{4}\left[\frac{x^5}{5}\right]_{-2}^{0} = -\frac{1}{4}\left(0 + \frac{32}{5}\right) = \frac{8}{5}$

4.) $DX = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - (EX)^2 = \int_{-2}^{0} x^2 \cdot \left(-\frac{x^3}{4}\right) dx - \left(-\frac{8}{5}\right)^2 = \int_{-2}^{0} -\frac{x^5}{4}\,dx - \frac{64}{25} =$

$= -\frac{1}{4} \cdot \int_{-2}^{0} x^5\,dx - \frac{64}{25} = -\frac{1}{4} \cdot \left[\frac{x^6}{6}\right]_{-2}^{0} - \frac{64}{25} = -\frac{1}{4} \cdot \left(-\frac{64}{6}\right) - \frac{64}{25} = \frac{64}{24} - \frac{64}{25} = \frac{8}{75} = 0{,}1066$

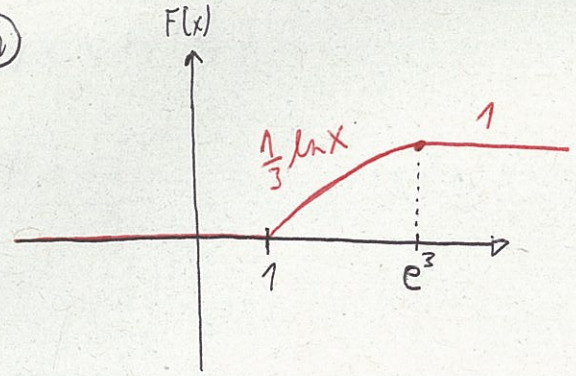
$\sigma = \sqrt{DX} = \sqrt{\frac{8}{75}} = 0{,}3265$

$1{,}5 - 0 = 1{,}5$

$Dx = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - (EX)^2 = \int_{1}^{e^3} x^2 \cdot \frac{1}{3x}$

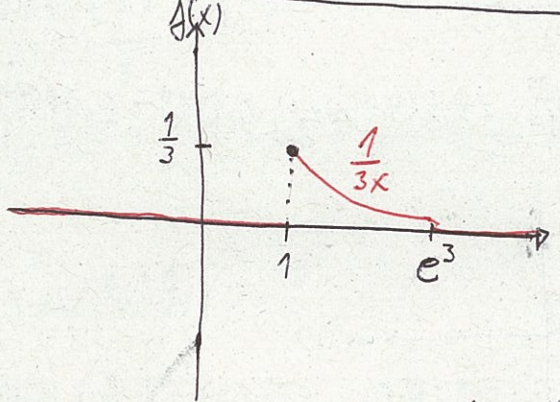
$\frac{x^{\cancel{x}}}{\cancel{3}x}$ $\frac{x}{2x} = \frac{1}{2}$

Pr



1. $P(X \leq 2)$, $P(10 \leq X \leq 12)$
2. $f(x)$
3. EX
4. DX, σ

2)



1) $\frac{1}{3} \ln(2) = 0{,}231049$

$\frac{1}{3} \cdot \ln(10) = 0{,}76752$

$\frac{1}{3} \cdot \ln(12) = 0{,}8283$

$\rightarrow P(10 \leq X \leq 12) = \underline{\underline{0{,}06078}}$

3)

$x \cdot \frac{1}{3x} = \frac{x}{3x}$

$$EX = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_1^{e^3} \frac{x}{3x} = \frac{1}{3} \int_1^{e^3} 1\, dx = \left[\frac{x}{3}\right]_1^{e^3} = \left[\frac{e^3}{3} - \frac{1}{3}\right] = \underline{\underline{6{,}36185}}$$

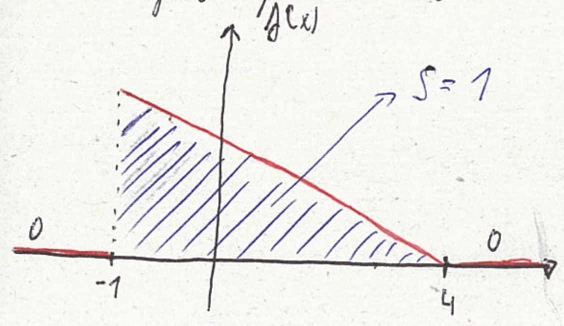
4.)

$$DX = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx - (EX)^2 = \int_1^{e^3} x^2 \cdot \frac{1}{3x} dx - 6{,}36185^2 = \int \frac{x}{3} dx - 6{,}36185^2 = \left[\frac{x^2}{6}\right]_1^{e^3} - 6{,}36185^2 =$$

$$= \frac{e^6}{6} - \frac{1}{6} - 6{,}36185^2 = \underline{\underline{26{,}59838562}}$$

$$\sigma = \sqrt{DX} = \sqrt{26{,}59838562} = \underline{\underline{5{,}157362273}}$$

Pr nadväzujúci z predošlého cvika



$\frac{5 \cdot f(-1)}{2} = 1 \rightarrow f(-1) = \frac{2}{5}$

$y = k \cdot x + q$

$A[-1, \frac{2}{5}]$ $\quad \frac{2}{5} = -k + q$

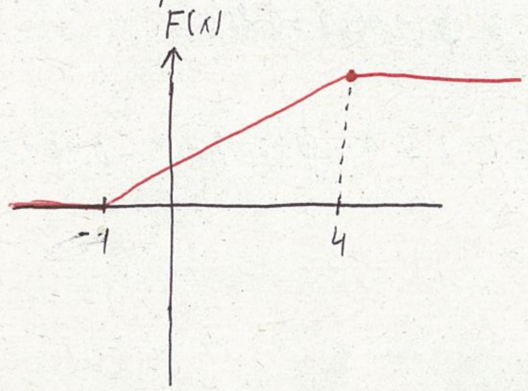
$B[4, 0]$ $\quad 0 = 4k + q$

$\underline{\underline{y = -\frac{2}{25} x + \frac{8}{25}}}$

$$\int \left(-\frac{2}{25} x + \frac{8}{25}\right) dx = \int \left(-\frac{2x}{25}\right) dx + \int \frac{8}{25} dx = -\frac{2}{25} \cdot \frac{x^2}{2} + \frac{8}{25} x + C = \underline{\underline{-\frac{x^2}{25} + \frac{8x}{25} + C}}$$

$$-\frac{(-1)^2}{25} + \frac{8 \cdot (-1)}{25} = -C \rightarrow \underline{\underline{C = \frac{9}{25} = 0{,}36}}$$

$$F(x) = \begin{cases} 0 & \text{pre } (-\infty, -1) \\ -\frac{x^2}{25} + \frac{8x}{25} + \frac{9}{25} & \text{pre } \langle -1, 4) \\ 1 & \text{pre } \langle 4, \infty) \end{cases}$$



29.11.2018

11cv. 11. týždeň

normální rozdelení

Pr

Lúlev ... 500[ml]

$X \sim N(\mu, \sigma^2)$

$\mu = 492\,[ml]$, $\sigma = 7{,}5\,[ml]$

norma: 485 - 515[ml]

a) aká je prav. že náh. nakúpená lúlev bude v norme ?

b) objem je menší ako 500 ml

c) objem je väčší ako 510 ml

d) nad akú hodnotu sa objem dostane s PST α -0,01

---

a) $P(485 \leq X \leq 515) = P\left(\frac{485-492}{7{,}5} \leq U \leq \frac{515-492}{7{,}5}\right) = P(-0{,}9\overline{3} \leq U \leq 3{,}0\overline{6}) =$

$U = \frac{X-\mu}{\sigma}$

$= \Phi(3{,}06) - \Phi(-0{,}9\overline{3}) = \Phi(3{,}06) - 1 + \Phi(0{,}9\overline{3}) \doteq$

$\doteq 0{,}9987 - 1 + 0{,}8239 = \underline{\underline{0{,}8226}} \quad (\doteq 0{,}82)$

b) $P(X \leq 500) = P\left(U \leq \frac{500-492}{7{,}5}\right) = P\left(U \leq \frac{16}{15}\right) \doteq P(U \leq 1{,}066) \doteq \Phi(1{,}066) = 0{,}856$

c) $1 - P(X \leq 510) = 1 - P\left(U \leq \frac{510-492}{7{,}5}\right) = 1 - P(U \leq 2{,}4) = 1 - \Phi(2{,}4) = 1 - 0{,}9918 = 0{,}0082$

d)

$P(X \geq d) = 0{,}01$

$P\left(\frac{X-492}{7{,}5} \geq \frac{d-492}{7{,}5}\right) = 0{,}01$

$1 - \Phi\left(\frac{d-492}{7{,}5}\right) = 0{,}01$

$\frac{d-492}{7{,}5} = 2{,}33$

$d - 492 = 2{,}337$

$d = 2{,}33 \cdot 7{,}5 + 492$

$d = \underline{\underline{509{,}475}}$

Pr →

Bi + NORM !

10% losov vyhráva

500 losov

Aká je prav. že z 500 losov bude 50-80 vyhrávajúcich

$X \sim Bi(500; 0{,}1)$

$\mu = n \cdot p = 500 \cdot 0{,}1 = 50$

$\sigma = \sqrt{n \cdot p(1-p)} = \sqrt{500 \cdot 0{,}1 \cdot 0{,}9} =$

$\doteq \underline{\underline{6{,}7082}}$

korekcia pri nahradzovaní

$P(50 \leq X \leq 80) = P(50-0{,}5 < X < 80+0{,}5) =$

$= P(49{,}5 < X < 80{,}5) = P\left(\frac{49{,}5-50}{6{,}7082} < U < \frac{80{,}5-\cancel{80}\,50}{6{,}7082}\right)$

$\doteq P(-0{,}074 < U < 4{,}546) = \Phi(4{,}546) - 1 + \Phi(0{,}074) \doteq$

$= 1 - 1 + 0{,}528 \doteq \underline{\underline{0{,}528}}$

12. cviko 12 týždeň

Štatistické testy

(Pr) životnosť súčiastky je 500 hodín

$X \sim N(500, \sigma^2)$

$\mu = 500[h]$, $\sigma = 25[h]$

novou technológiou sa otestovalo 50 súčiastok

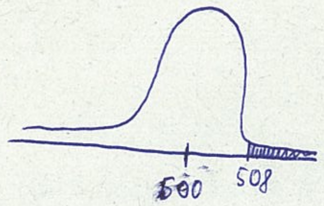
50 súčiastok ... priemerná životnosť $508[h]$ → je to štatisticky významné?

H_0 ... nie je štat. význam $\qquad \alpha = 0{,}05$ ← hraničná pravdepodob.

H_1 ... je to štat. významná zmena

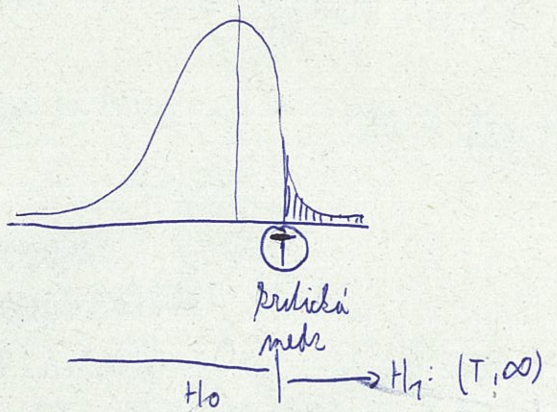
$\bar{X} \sim N(\bar{\mu}, \bar{\sigma}^2) \qquad \bar{\mu} = \mu = 500$

$\bar{\sigma} = \frac{\sigma}{\sqrt{50}} = 3{,}5355$



riešenie cez P hodnotu

(1) $P(\bar{X} \geq 508) = P\left(U > \frac{508-500}{3{,}5355}\right) = 1 - \Phi(2{,}263) = 1 - 0{,}987 = 0{,}012 < \alpha \Rightarrow H_1$ prijímame



(2) aká je to hranica čo odrezáva 0,05 ← pravo stranný test

$P(\bar{X} > T) = 0{,}05$

$P(\bar{X} < T) = 0{,}95$

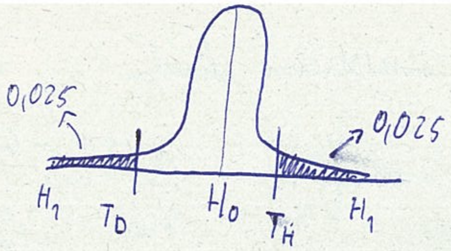
pozrieme sa do tabuľky → $\Phi(1{,}65) = 0{,}95$

$P\left(U < \frac{T-500}{3{,}5355}\right) = 0{,}95$

$\frac{T-500}{3{,}5355} = 1{,}65 \quad / \cdot 3{,}5355$

$T - 500 = 5{,}8336 \rightarrow T = 505{,}8336$

(Pr) obojstranný test



súčiastka: $\qquad \alpha = 0{,}05$

A: 492 H_1 $\qquad$ B: 496 H_0 $\qquad$ C: 505 H_0

$P(\bar{X} > T_H) = 0{,}025$

$P(\bar{X} < T_D) = 0{,}975$

$P\left(U < \frac{T_H - 500}{3{,}5355}\right) = 0{,}975 \qquad T_H = 506{,}93$

$T_D = 493{,}07$

$\Phi\left(\frac{T_H - 500}{3{,}5355}\right) = 0{,}975$

$\frac{T_H - 500}{3{,}5355} = 1{,}96$ $\qquad$ kritický obor $(-\infty; 493{,}07) \cup (506{,}93; \infty)$