1. cv, 1. tjedan

20.9
2018

1.

$$5x - 2y + 2z = 10$$
$$x + 4y - 2z = 2$$
$$2x + y + 7z = 1$$

$$x^{(k+1)} = \frac{1}{5}\cdot\left(2y^{(k)} - 2z^{(k)} + 10\right)$$
$$y^{(k+1)} = \frac{1}{4}\cdot\left(-x^{(k)} + 2z^{(k)} + 2\right)$$
$$z^{(k+1)} = \frac{1}{7}\cdot\left(-2x^{(k)} - y^{(k)} + 1\right)$$

$x^{(1)} = \frac{1}{5}\,($

Jacobi

$x_0 = y_0 = 0$

$\varepsilon = 0{,}01$

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	0	0	0
1	2	0,5	0,14
2	~~2,1428~~ 2,1428	~~0,25~~ 0,0715	~~-0,5~~ -0,5
3	2,2286	-0,2857	-0,4796

Gauss-Seidelova

$x_0 = y_0 = z_0 = 0$

$\varepsilon = 0{,}01$

k	x^k	y^k	z^k
0	0	0	0
1	2 ✓	0 ✓	-0,4
2	2,1714	-0,2572	-0,4408
3			
4			

2.

$$3x + 4y = 1$$
$$2x - y = 2 \quad *$$

$$-y = 2 - 2x$$
$$y = 2x - 2$$

$$3x + 4\cdot(2x - 2) = 1$$
$$3x + 8x - 8 = 1$$
$$11x = 9$$
$$x = \frac{9}{11}$$

$*$ $Ax = b$

$$A^T \cdot A x = A^T \cdot b$$

$$A = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$$

$$A^T \cdot A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ 10 & 17 \end{pmatrix}$$

$$A^T \cdot b = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

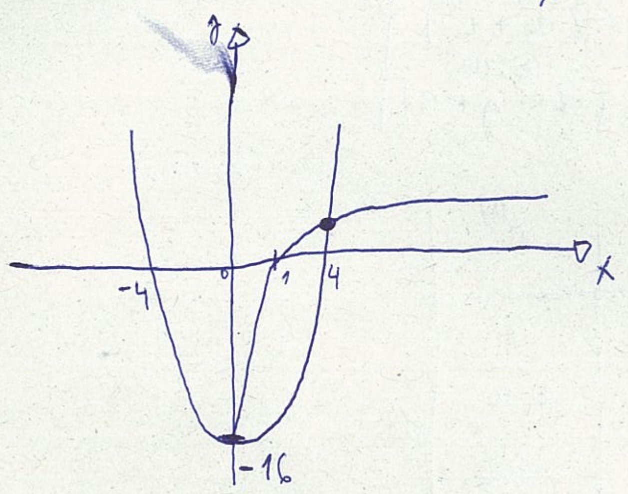
$$13x + 10y = 7$$
$$10x + 17y = 2$$

27.9.2018

2. cv., 2. týždeň

① // metóda pólení

$x^2 - 16 = 3\ln x$, $\varepsilon = 0{,}01$, → kde je to najväčšie riešenie

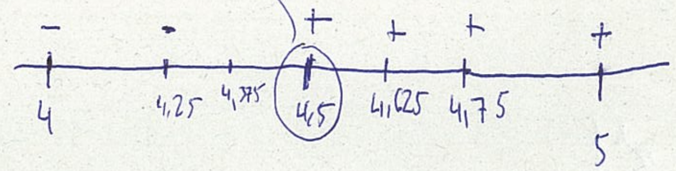


dosadíme a riešime znamienka (najprv 4 a 5)

$f(x) = x^2 - 16 - 3\ln x$

$\langle 4,5 \rangle$ $\frac{4+5}{2} = 4{,}5$

− (+)
+ (+)



② // newtonova metóda

$x^2 - 16 - 3\ln x$

$\frac{3}{x} = 3 \cdot x^{-1} \to 3x^{-2}$

1) zderivujeme: $f'(x) = 2x - \frac{3}{x} > 0$

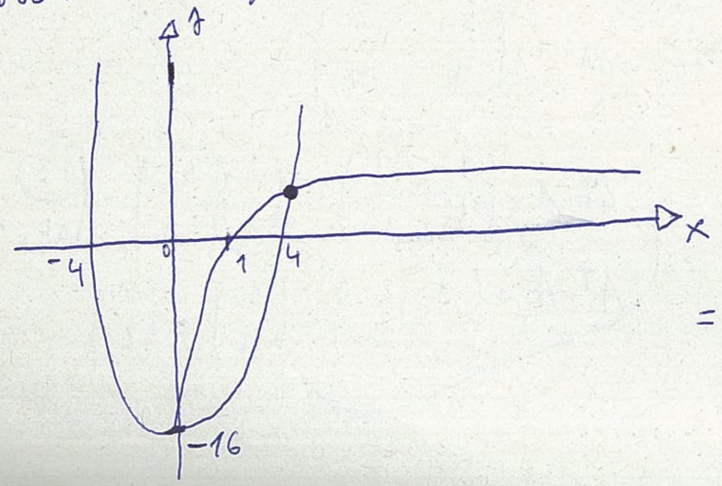
2) znova: $f''(x) = 2 + 3x^{-2} = 2 + \frac{3}{x^2} > 0$

z prvej a druhej derivácie riešime tvar funkcie

$2 \cdot 4 - \frac{3}{4} = 7{,}25 +$ → kladné znamienko

$2 \cdot 5 - \frac{3}{5} = 10 - \frac{3}{5} = +$ kladné znamienko

→ funkcia konvexne rastie lebo obe derivácie majú kladné znamienka



$$x_{k+1} = x_k - \frac{x_k^2 - 16 - 3\ln x_k}{x_k - \frac{3}{x_k}} =$$

?. neviem

$$= 5 - \frac{5^2 - 16 - 3\ln 5}{2 \cdot 5 - \frac{3}{5}} =$$

$x_1 = 4{,}5562$
$x_2 = 4{,}5314$
$x_3 = 4{,}5313$

③ $x_{k+1} = e^{\frac{x^2-15}{3}}$ $\langle 4,5 \rangle$ – metóda prostej iterácie

i	x
0	0
1	~~4,8279·10^-3~~
2	~~4,827987·10^-3~~

zlá metóda

$$x_k = \sqrt{3\ln x_k + 16}$$

i	x
0	5
1	4,5638
2	4,5337

treba previesť

$x = g(x)$

$x_{k+1} = g(x_k)$

$|g'(x)| < 1$ musí byť

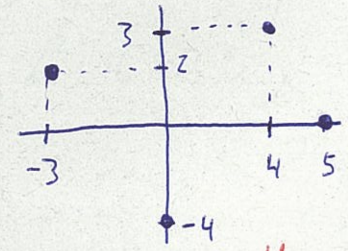
$x_{k+1} = e^{\frac{x^2-15}{3}}$ $\langle 4,5 \rangle$

i	x
0	
1	
2	

4.10.2018

3. cv, 3 týždeň

nájdite interpolačný polynóm



x_i	-3	0	4	5
y_i	2	-4	3	0

Lagrangeova metóda →

$$P(x) = 2\frac{(x-0)\cdot(x-4)\cdot(x-5)}{(-3-0)\cdot(-3-4)(-3-5)} - 4\cdot\frac{(x+3)(x-4)(x-5)}{(0+3)(0-4)(0-5)} + 3\,\frac{(x+3)(x-0)(x-5)}{(4+3)(4-0)(4-5)} + 0$$

Newtonova metóda ↓

1. diferencie — diferencie 2. rádu — 3 diferencie

x_i	y_i
-3	2
0	-4
4	3
5	0

$$\frac{-4-2}{0+3} = \frac{-6}{+3} = -2 \qquad \frac{3+4}{4+0} = \frac{7}{4} \qquad \frac{0-3}{5-4} = -3$$

$$\frac{\frac{7}{4}+2}{4-(-3)} = \frac{\frac{15}{4}}{7} = \frac{15}{28} \qquad \frac{-3-\frac{7}{4}}{5-0} = \frac{-\frac{19}{4}}{5} = -\frac{19}{20}$$

$$\frac{-\frac{19}{20}-\frac{15}{28}}{5-(-3)} = \frac{-\frac{52}{35}}{8} = -\frac{52}{280} = -\frac{13}{70}$$

$$P(x) = 2 - 2(x+3) + \frac{15}{28}(x+3)(x-0) - \frac{13}{70}(x+3)(x-0)(x-4)$$

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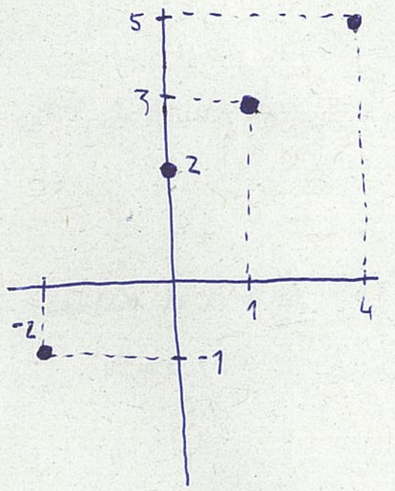
metóda najmenších štvorcov



x	y	x^2	$x\cdot y$
-2	-1	4	2
0	2	0	0
1	3	1	3
4	5	16	20
Σ 3	9	21	25

*

počet riadkov, ostatné sú sumy

priamka

$$4\cdot c_0 + 3\cdot c_1 = 9$$
$$3c_0 + 21c_1 = 25$$
$$c_0 = \frac{38}{25}$$
$$c_1 = \frac{73}{75}$$

$$y = \frac{38}{25} + \frac{73}{75}x$$

parabola

*

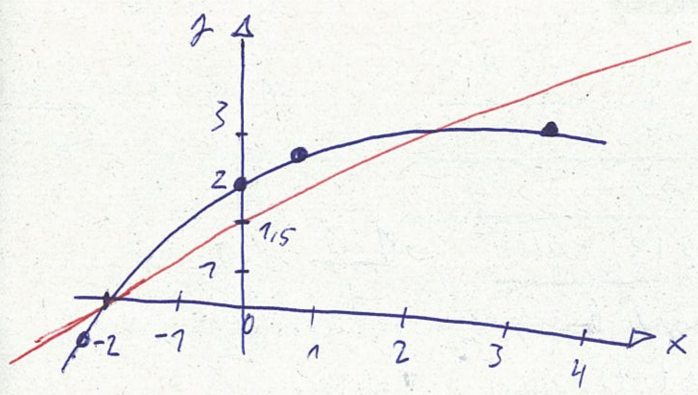
x^3	x^4	$x^2\cdot y$
-8	16	-4
0	0	0
1	1	3
64	256	80
Σ 57	273	79

počet riadkov

$$4c_0 + 3c_1 + 21c_2 = 9$$
$$3c_0 + 21c_1 + 57c_2 = 25$$
$$21c_0 + 57c_1 + 273c_2 = 79$$

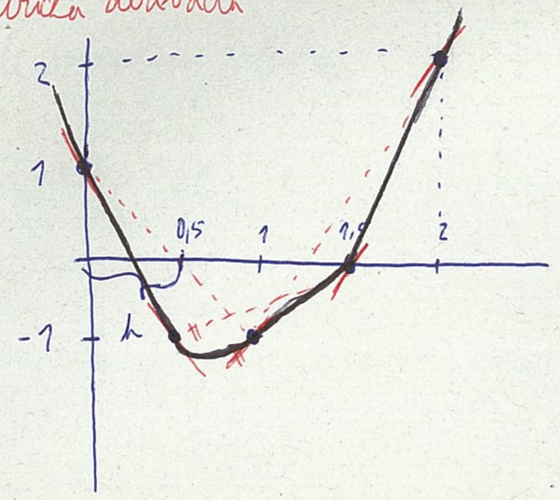
$$c_0 = \frac{97}{50} \qquad c_1 = \frac{123}{100} \qquad c_2 = \frac{7}{60}$$

$$\boxed{y = \frac{97}{50} + \frac{123}{100}x - \frac{7}{60}x^2}$$



Numerická derivácia

4cv, 4 týždeň



pre krajné

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

pre stredné

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

stredné body

$$f'(0,5) = \frac{f(1) - f(0)}{2\cdot 0,5} = \frac{-1-1}{1} = -2$$

$$f'(1) = \frac{f(1,5) - f(0,5)}{1} = \frac{0-(-1)}{1} = 1$$

$$f'(1,5) = \frac{f(2) - f(1)}{1} = \frac{2-(-1)}{1} = 3$$

krajné body

$$f'(0) = \frac{f(0,5) - f(0)}{0,5} = \frac{-1-1}{0,5} = -4$$

$$f'(2) = \frac{f(2) - f(1,5)}{0,5} = \quad = 4$$

Numerická integrácia

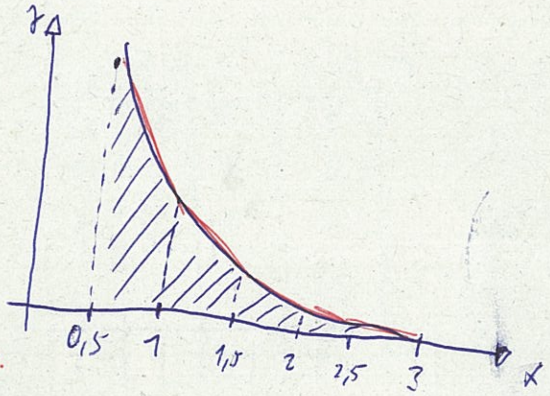
Pr.

$$\int_{0,5}^{2,5} \frac{\sin x}{x^2}\,dx$$

$h = 0,5$

$$L_4 = 0,5\cdot\left(\frac{1}{2}\frac{\sin 0,5}{0,5^2} + \frac{\sin 1}{1^2} + \frac{\sin 1,5}{1,5^2} + \frac{\sin 2}{2^2} + \frac{1}{2}\cdot\frac{\sin 2,5}{2,5^2}\right) =$$

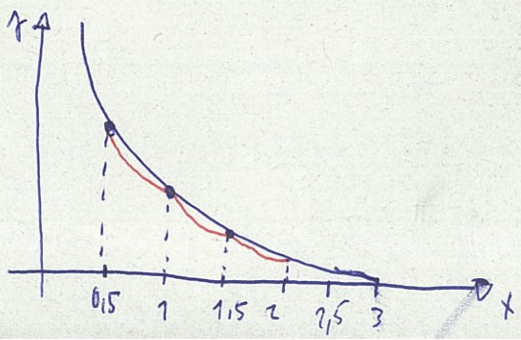
$$= \underline{\underline{1,25943}}$$



SIMPSONOVA M.

$$S_4 = \frac{0,5}{3}\cdot\left(\frac{\sin(0,5)}{0,5^2} + 4\cdot\left(\frac{\sin(1)}{1^2}\right) + 2\cdot\left(\frac{\sin(1,5)}{1,5^2}\right) + 4\cdot\left(\frac{\sin(2)}{2,5^2}\right)\right) =$$

$$= 1,195883$$



$$\left(\sin x\cdot\frac{1}{x^2}\right)' = \sin x\cdot x^{-2} = \cos x\cdot x^{2} + \sin x\cdot(-2)\cdot x^{-3} =$$

$$= \underbrace{\frac{\cos x}{x^2} - \frac{2\cdot\sin x}{x^3}}$$

$0,5 \le x \le 2,5$

$$f''(x) = -\sin x\cdot x^{-2} + \cos x\cdot(-2)x^{-3} + \cos x\cdot(-2)x^{-3} + \sin x\cdot 6x^{4}$$

$$= \underbrace{-\frac{\sin x}{x^2} - 4\frac{\cos x}{x^3} + \frac{6\sin x}{x^4}}$$

$$|\qquad| \le 132$$

4 32 96