

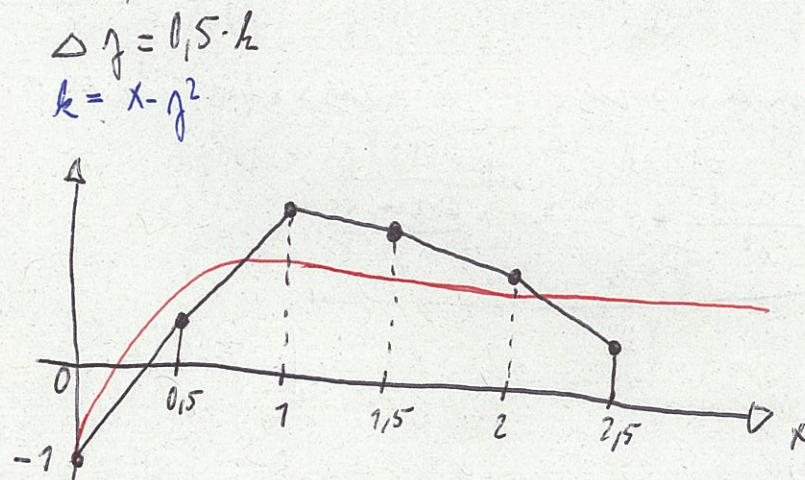
18.10.2018

Zakladni Eulerova metoda

5. květen

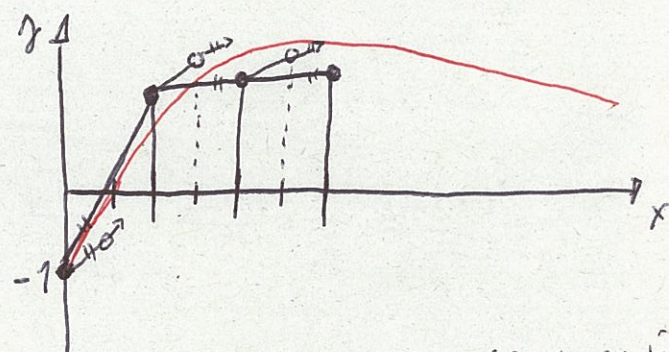
$$y' = \frac{5}{x^2+1} - x, \quad y(0) = -1, \quad h = 0,5 \leftarrow \text{skok}$$

X	y	k
0	-1	2,5
0,5	0,25	4,27
1	2,385	-0,24
1,5	2,235	-0,67
2	1,9	-0,92
2,5	1,44	



Eulerova metoda 1. modifik.

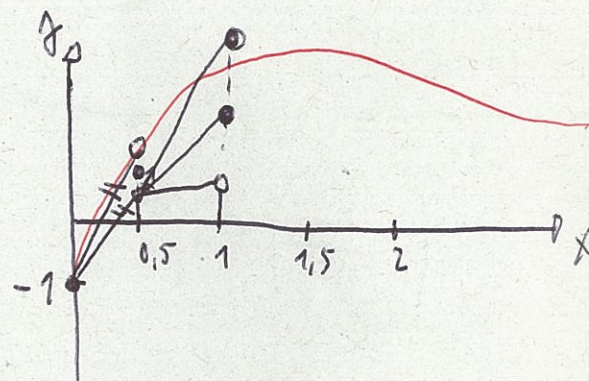
X	y	k ₁	x ^P	y ^P	k ₂
0	-1	2,5	0,25	-0,375	5,57
0,5	1,07	1,83	0,75	1,53	0,75
1	1,75	0,67	1,25	1,6	0,15
1,5	1,53



* = P (pomocný)

Eulerova metoda 2. modifikace

X	y	k ₁	x*	y*	k ₂	$\frac{k_1+k_2}{2}$
0	-1	2,5	0,5	0,25	4,205	3,35
0,5	0,68	2,92	1	2,14	-0,10	1,4
1	1,38	0,72	1,5	1,74	-0,26	0,23
1,5	1,5



6. květen

PA

- máme 3 body

→ jednoduší (klasická) pravděpodobnost

a) podrobně 12

b) podrobně 3 různé čísla, mezi nimi je 6

c) 2 rovnaké čísla + 1 jiné číslo je menší

$$|\Omega| = 6^3 = 216$$

$$\Omega = \{111, 112, \dots, 666\}$$

a) 165, 156, 675, 657, 576, 567

336, 363, 633

444

453, ...

246, ...

255, 525, 552

$$P(A) = \frac{25}{216} = \underline{\underline{0,1157}}$$

$$b) \left(\frac{1}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \right) + \left(\frac{5}{6} \cdot \frac{1}{6} \cdot \frac{4}{6} \right) + \left(\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{1}{6} \right) =$$

$$= \frac{20}{216} + \frac{20}{216} + \frac{20}{216} = \frac{60}{216} = \underline{\underline{0,277}}$$

c)

$$C = \{221, 332, 331, 443, 442, 441, 554, 553, 552, 551, \dots\}$$

$$15 \cdot 3 \Rightarrow |C| = 45$$

$$P(C) = \frac{45}{216} = \underline{\underline{0,2083}}$$

P1) 1 koda, ako je pravdepodobnosť, že súčet náhodných čísel bude 11 → Distribúcia prav.

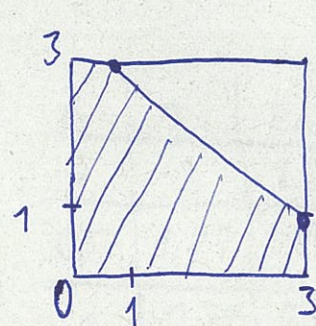
q.v. 6

$$\Omega = \{6, 16, 26, 36, \dots, 126, \dots, 111114, \dots\}$$

$$|A| = 70$$

$$P(A) = \frac{1}{36} + 4 \cdot \frac{1}{216} + 6 \cdot \frac{1}{1296} + 4 \cdot \frac{1}{7776} + 1 \cdot \frac{1}{46656} = 0,05746$$

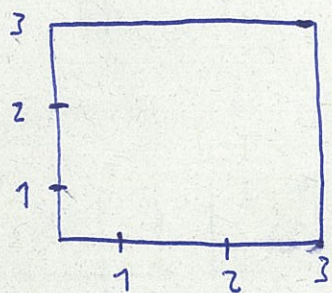
P2) bodu náhodne vybranú z čísla v intervale $<0,3>$ ako je pravdepodobnosť, že ich súčet bude menší alebo rovný 4



a) $P(A) = \frac{7}{9}$

b) $P(B)$

↓
súčet bude menší alebo rovný 4



1.

Risť: $H_1 A: \dots 50\%$ a toho sú 2% červení
 $H_2 B: \dots 40\%$ 4% červení
 $H_3 C: \dots 10\%$ 7% červení

$P(A) = 0,5$
 $P(B) = 0,4$
 $P(C) = 0,1$

1. Ako je prav. že 1 svetlo bude červené?
2. Náhodne vybrané svetlo je červené, P, že je od C

$P(H_1) = 0,5$
 $P(H_2) = 0,4$
 $P(H_3) = 0,1$

$P(\bar{c} | H_1) = 0,02$
 $P(\bar{c} | H_2) = 0,04$
 $P(\bar{c} | H_3) = 0,07$

$P(\bar{c}) = P(H_1) \cdot P(\bar{c} | H_1) + P(H_2) \cdot P(\bar{c} | H_2) + P(H_3) \cdot P(\bar{c} | H_3) =$

$$= 0,5 \cdot 0,02 + 0,4 \cdot 0,04 + 0,1 \cdot 0,07 = 0,033$$

$$P(H_3 | \bar{c}) = \frac{P(H_3) \cdot P(\bar{c} | H_3)}{P(\bar{c})} = \frac{0,1 \cdot 0,07}{0,033} = \frac{7}{33} = \underline{\underline{0,2121}}$$

2.

32 hod. 4 dni

36 hod. 8 dni

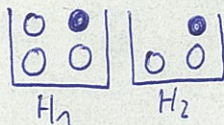
1) ako je prav. že vybrané karta bude eso
 2) ako je prav. že podoben a pritom bolíden

$P(E) = P(H_1) \cdot P(E | H_1) + P(H_2) \cdot P(E | H_2) = \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{2}{9} = \frac{1}{16} + \frac{2}{18} = 0,17361$
 $P(H_1) = \frac{1}{2}$ $P(E | H_1) = \frac{4}{32} = \frac{1}{8}$
 $P(H_2) = \frac{1}{2}$ $P(E | H_2) = \frac{8}{36} = \frac{2}{9}$

PRÁVDA

$$P(H_2 | E) = \frac{P(H_2) \cdot P(E | H_2)}{P(E)} = \frac{\frac{1}{2} \cdot \frac{2}{9}}{0,17361} = \underline{\underline{0,64}}$$

Pr



Bele guliča → som ni opihali

$P = ?$ če n. ky nity krotitj kude n. krotitj opihaličel bele

$$P(H_1) = \frac{1}{2} \quad P(B|H_1) = \frac{3}{4}$$

$$P(H_2) = \frac{1}{2} \quad P(B|H_2) = \frac{2}{3}$$

$$P(H_1) \cdot P(B|H_1) + P(H_2) \cdot P(B|H_2) = \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} = \frac{3}{8} + \frac{2}{6} = \frac{17}{24}$$

$$P(H_1|B) = \frac{P(H_1) \cdot P(B|H_1)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{17}{24}} = \frac{9}{17}$$

$$P(H_2|B) = \frac{P(H_2) \cdot P(B|H_2)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{17}{24}} = \frac{8}{17}$$

Šco: 8 kječa

pisanka ro 7 kječa: 5, 6 → konec, max 3x

$$\Omega = \{5, 6, 15, 25, 35, 45, 16, 26, 36, 46, 111, \dots\} = \{5, 6\} \cup \{(a, b) : a \in \{1, 2, 3, 4\},$$

$$|\Omega| = 2 + 8 + 4 \cdot 4 \cdot 6 = 106$$

$$b \in \{5, 6\} \cup$$

$$\{(a, b, c) : a, b \in$$

$$\{1, 2, 3, 4\}$$

$$c \in \{1, 2, 3, 4, 5, 6\}$$

$$n1 : \frac{2}{6} + \frac{32}{6^2} + \frac{3 \cdot 3 \cdot 5}{6^3} = \frac{17}{24} = 0,708\bar{3}$$

$\{5, 6\}$ $(\neq 1, 5/6)$ $(\neq 1, \neq 1, \neq 1)$
 $\neq 5$ $\neq 6$ $\frac{2}{3} \quad \frac{2}{3} \quad \frac{2}{3}$
 $(2/3/4/5/6)$ $\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}$
 $\frac{4}{4} \quad \frac{4}{4} \quad \frac{4}{4}$

$$\text{mud} = 6 \quad \{6, 15, 114, 141, 411, 123, \dots\}^{222}$$

$$\frac{1}{6} + \frac{1}{6^2} + \frac{10}{6^3} = \frac{13}{54} = 0,2407$$

Pr: kječa 2 specialni kodami $p(x), F(x), EX, DX, \sigma = ?$

A: 1, 1, 1, 1, 2, 3

B: 1, 3, 4, 4, 4, 4

"X = A - B"

X = 0, -2, -3, 1, -1, 2

$$p(-3) = \frac{4 \cdot 4}{6 \cdot 6} = \frac{16}{36}$$

$$p(-2) = \frac{4+4}{36} = \frac{8}{36}$$

$$p(-1) = \frac{1+4}{36} = \frac{5}{36}$$

$$p(0) = \frac{4+1}{36} = \frac{5}{36}$$

$$p(1) = \frac{1}{36}$$

$$p(2) = \frac{1}{36}$$

$$EX = \sum x_i \cdot p(x_i)$$

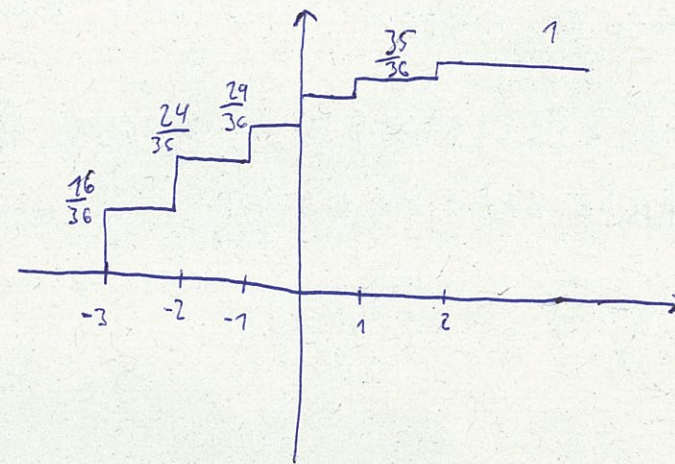
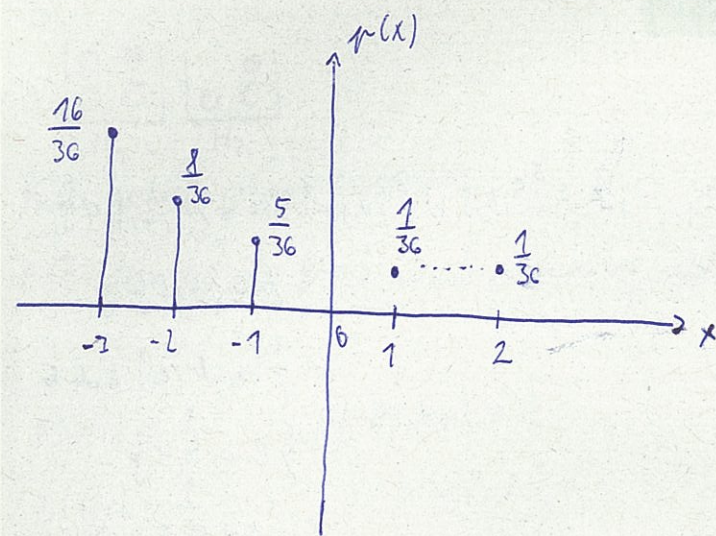
$$EX = \frac{16}{36} \cdot (-3) + \frac{8}{36} \cdot (-2) + \frac{5}{36} \cdot (-1) + \frac{1}{36} + \frac{2}{36}$$

$$= \frac{-11}{6} = -1,8\bar{3}$$

$$DX = \sum x_i^2 \cdot p(x_i) - (EX)^2 = *$$

$$\sigma = \sqrt{DX} = \sqrt{\frac{65}{36}} = 1,3437$$

$$* = 9 \cdot \frac{16}{36} + 4 \cdot \frac{8}{36} + 1 \cdot \frac{5}{36} + 1 \cdot \frac{1}{36} + 4 \cdot \frac{1}{36} - \left(-\frac{11}{6}\right)^2 = \frac{31}{6} - \left(-\frac{11}{6}\right)^2 = \frac{65}{36}$$



$$F(-1) = 0$$

$$F(-2) = \frac{24}{36}$$

$$F(0,1) = \frac{34}{36}$$

$$F(1/2) = \frac{35}{36}$$

1 h ... 2 autai

a) na 1 hodinu riadne

b) na 10 minút viac než 2

c) na 8 hodín 16 aut

$$P(k) = P(X=k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$$

a)

$$X \sim P_0(2)$$

$$P(k) = \frac{2^k}{k!} \cdot e^{-2}$$

$$P_0 = \underline{\underline{e^{-2}}}$$

b)

$$X \sim P_0\left(\frac{1}{3}\right)$$

$$P(0) = e^{-\frac{1}{3}} = 0,7765$$

$$P(1) = \frac{\left(\frac{1}{3}\right)^1}{1!} \cdot e^{-\frac{1}{3}} = 0,2388$$

$$P(2) = \frac{\left(\frac{1}{3}\right)^2}{2!} \cdot e^{-\frac{1}{3}} = 0,039$$

$$P(X > 2) = 1 - (P(0) + P(1) + P(2)) = 1 - 0,9943 = 0,0049$$

$$c) X \sim P_0(16)$$

$$P(16) = \frac{16^{16}}{16!} \cdot e^{-16} = 0,09461$$

Pr1

10 otázok a, b, c

úspech : aspoň 7 správne

prav. úspechu pri náhodných odpovediach

$$X \sim Bi(10; \frac{1}{3})$$

$$p(k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$$

$$p(7) = \binom{10}{7} \cdot \left(\frac{1}{3}\right)^7 \cdot \left(1 - \frac{1}{3}\right)^3 = \frac{320}{19683} = 0,01626$$

$$p(8) = \binom{10}{8} \cdot \left(\frac{1}{3}\right)^8 \cdot \left(1 - \frac{1}{3}\right)^2 = 0,0003048 = \frac{20}{6561}$$

$$p(9) = \binom{10}{9} \cdot \left(\frac{1}{3}\right)^9 \cdot \left(\frac{2}{3}\right) = \frac{20}{59049} = 0,0003387$$

$$p(10) = \binom{10}{10} \cdot \left(\frac{1}{3}\right)^{10} = \frac{1}{59049} = 0,000016935$$

$$p(7) + p(8) + p(9) + p(10) = \underline{\underline{0,01966}} = p(X \geq 7)$$

$$X \sim Bi(30; 0,1966)$$

$$1 - p(0)$$

$$P(0) = \binom{30}{0} \cdot (0,1966)^0 \cdot (1 - 0,1966)^{30}$$

$$P(0) = 0,55119$$

$$P(X \geq 1) = \underline{\underline{0,44881}}$$

Pr2

1 diela ... 2,5 h

a) na 8 h ... nič

b) na 1/2 h ... viac ako 2 diela

c) na 10 min ... viac ako 2 diela

$$X \sim P_0(\lambda)$$

$$\lambda = \frac{2,5}{8} = 0,3125$$

$$X = \frac{8}{2,5} = 3,2$$

$$p(k) = \frac{3,2^k}{k!} \cdot e^{-3,2}$$

$$p(0) = \frac{3,2^0}{0!} \cdot e^{-3,2}$$

$$p(0) = e^{-3,2} = \underline{\underline{0,0407}}$$