1. cv, 1. tjedan

20.9
2018

(1.)

$$5x - 2y + 2z = 10$$
$$x + 4y - 2z = 2$$
$$2x + y + 7z = 1$$

$$x^{(k+1)} = \frac{1}{5} \cdot (2y^{(k)} - 2z^{(k)} + 10)$$
$$y^{(k+1)} = \frac{1}{4} \cdot (-x^{(k)} + 2z^{(k)} + 2)$$
$$z^{(k+1)} = \frac{1}{7} \cdot (-2x^{(k)} - y^{(k)} + 1)$$

$x^{(1)} = \frac{1}{5}($

Jacobi
$x_0 = y_0 = 0$
$\varepsilon = 0{,}01$

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	0	0	0
1	2	0,5	0,14
2	~~2,1428~~ 2,1428	~~0,25~~ 0,0715	~~-0,5~~ -0,5
3	2,2286	-0,2857	-0,4796

Gauss-Seidelova
$x_0 = y_0 = z_0 = 0$
$\varepsilon = 0{,}01$

k	x^k	y^k	z^k
0	0	0	0
1	2 √	0 √	-0,4
2	2,1714	-0,2572	-0,4408
3			
4			

(2.)

$$3x + 4y = 1$$
$$2x - y = 2 \quad *$$

$$-y = 2 - 2x$$
$$y = 2x - 2$$

$$3x + 4 \cdot (2x - 2) = 1$$
$$3x + 8x - 8 = 1$$
$$11x = 9$$
$$x = \frac{9}{11}$$

$$13x + 10y = 7$$
$$10x + 17y = 2$$

$*$ $Ax = b$

$$A^T \cdot A x = A^T \cdot b$$

$$A = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$$

$$A^T \cdot A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ 10 & 17 \end{pmatrix}$$

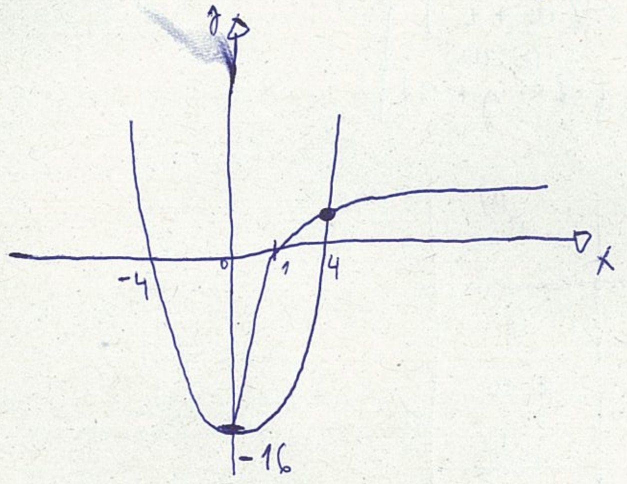
$$A^T \cdot b = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

27.9.2018

2. cv., 2 týždeň

① // metóda pólení

$x^2 - 16 = 3\ln x$, $\varepsilon = 0{,}01$, → kde je to najväčšie riešenie

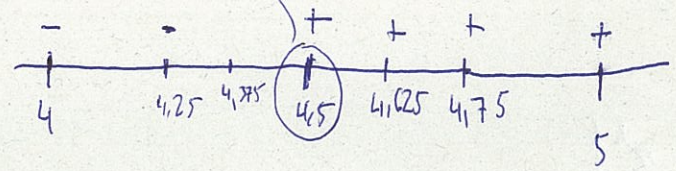


dosadíme a riešime znamienko (najprv 4 a 5)

$f(x) = x^2 - 16 - 3\ln x$

$\langle 4, 5 \rangle$ $\frac{4+5}{2} = 4{,}5$

- (+)
+ (+)



② // newtonova metóda

$x^2 - 16 - 3\ln x$

$\frac{3}{x} = 3 \cdot x^{-1} \to 3x^{-2}$

1) zderivujeme: $f'(x) = 2x - \frac{3}{x} > 0$

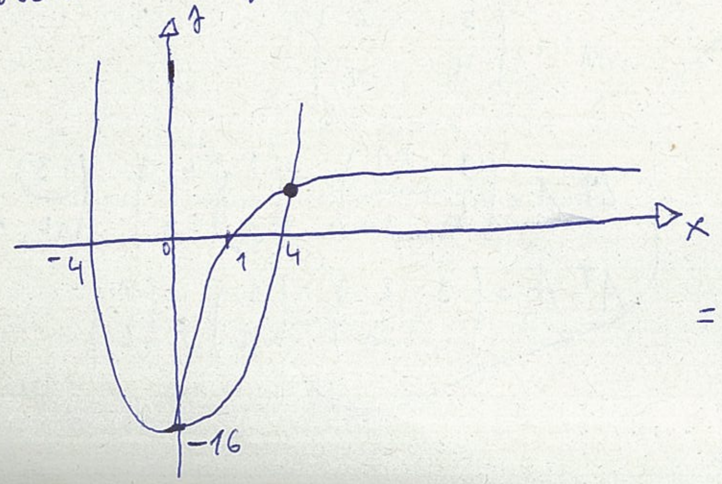
2) znova: $f''(x) = 2 + 3x^{-2} = 2 + \frac{3}{x^2} > 0$

z prvej a druhej derivácie riešime tvar funkcie

$2 \cdot 4 - \frac{3}{4} = 7{,}25$ + → kladné znam.

$2 \cdot 5 - \frac{3}{5} = 10 - \frac{3}{5} =$ + kladné znam.

→ funkcia konvexne rastie lebo obe derivácie majú kladné znamienko



$$x_{k+1} = x_k - \frac{x_k^2 - 16 - 3\ln x_k}{x_k - \frac{3}{x_k}} =$$

? neviem

$$= 5 - \frac{5^2 - 16 - 3\ln 5}{2 \cdot 5 - \frac{3}{5}} =$$

$x_1 = 4{,}5562$

$x_2 = 4{,}5314$

$x_3 = 4{,}5313$

③ $x_{k+1} = e^{\frac{x^2 - 15}{3}}$ $\langle 4, 5 \rangle$ – metóda prostej iterácie

i	x
0	0
1	4, ~~8279 · 10^-3~~
2	4, ~~827987 · 10^-3~~

zlá metóda

$X_k = \sqrt{3\ln X_k + 16}$

i	x
0	5
1	4,5638
2	4,5337

treba prevrátiť

$x = g(x)$

$x_{k+1} = g(x_k)$

$|g'(x)| < 1$ musí ge

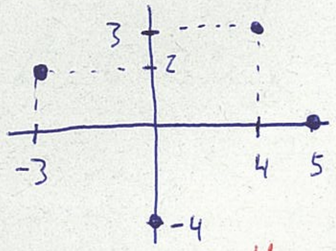
$x_{k+1} = e^{\frac{x^2 - 15}{3}}$ $\langle 4, 5 \rangle$

i	x
0	
1	
2	

4.10.2018

3. cv, 3 týždeň

nájdite interpolačný polynóm



x_i	-3	0	4	5
y_i	2	-4	3	0

Lagrangeova metóda →

$$P(x) = 2\frac{(x-0)\cdot(x-4)\cdot(x-5)}{(-3-0)\cdot(-3-4)(-3-5)} - 4\cdot\frac{(x+3)(x-4)(x-5)}{(0+3)(0-4)(0-5)} + 3\,\frac{(x+3)(x-0)(x-5)}{(4+3)(4-0)(4-5)} + 0$$

Newtonova metóda ↓

1. diferencie — diferencie 2. rádu ↓ — 3 diferencie

-3	2
0	-4
4	3
5	0

$$\frac{-4-2}{0+3} = \frac{-6}{+3} = -2$$

$$\frac{3+4}{4+0} = \frac{7}{4}$$

$$\frac{0-3}{5-4} = -3$$

$$\frac{\frac{7}{4}+2}{4-(-3)} = \frac{\frac{15}{4}}{7} = \frac{15}{28}$$

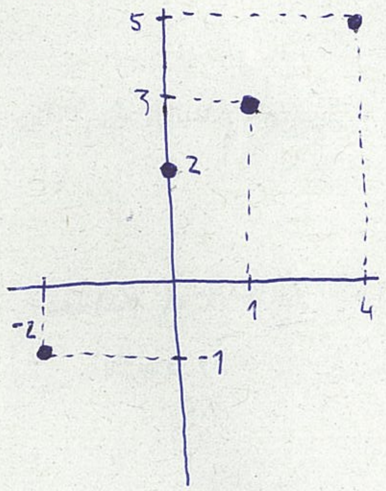
$$\frac{-3-\frac{7}{4}}{5-0} = \frac{-\frac{19}{4}}{5} = -\frac{19}{20}$$

$$\frac{-\frac{19}{20}-\frac{15}{28}}{5-(-3)} = \frac{-\frac{52}{35}}{8} = -\frac{52}{280} = -\frac{13}{70}$$

$$P(x) = 2 - 2(x+3) + \frac{15}{28}(x+3)(x-0) - \frac{13}{70}(x+3)(x-0)(x-4)$$

SPLAJ nr 34 MAT3

metóda najmenších štvorcov



počet uzlov, ostatné sú sumy

priamka

x	y	x^2	$x\cdot y$
-2	-1	4	2
0	2	0	0
1	3	1	3
4	5	16	20
Σ 3	9	21	25

$$4\cdot c_0 + 3\cdot c_1 = 9$$

$$3c_0 + 21c_1 = 25$$

$$c_0 = \frac{38}{25}$$

$$c_1 = \frac{73}{75}$$

$$y = \frac{38}{25} + \frac{73}{75}x$$

parabola

x^3	x^4	$x^2\cdot y$
-8	16	-4
0	0	0
1	1	3
64	256	80
Σ 57	273	79

počet uzlov

$$4c_0 + 3c_1 + 21c_2 = 9$$

$$3c_0 + 21c_1 + 57c_2 = 25$$

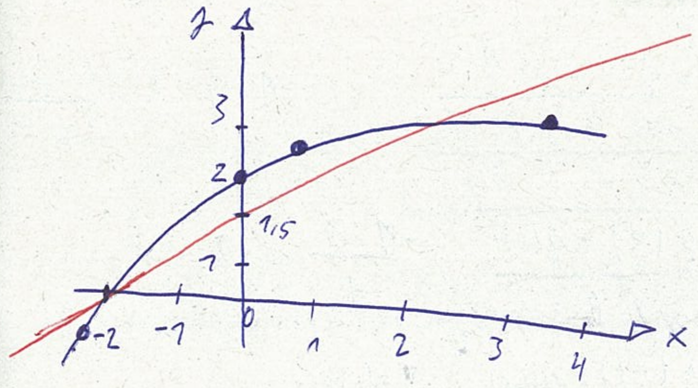
$$21c_0 + 57c_1 + 273c_2 = 79$$

$$c_0 = \frac{97}{50}$$

$$c_1 = \frac{123}{100}$$

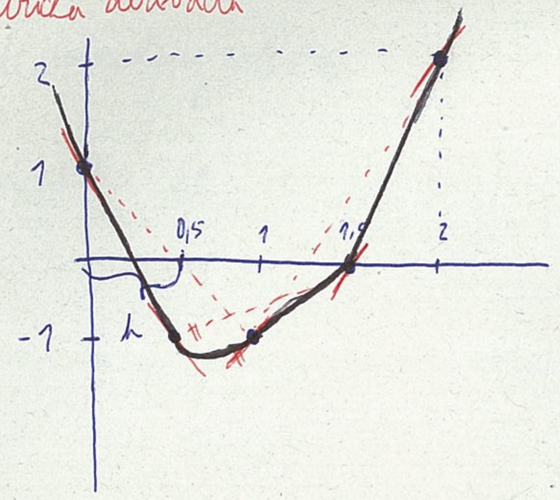
$$c_2 = \frac{7}{60}$$

$$y = \frac{97}{50} + \frac{123}{100}x - \frac{7}{60}x^2$$



Numerická derivácia

4cv, 4 týždeň



pre krajné

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

pre stredné

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

stredné body

$$f'(0,5) = \frac{f(1) - f(0)}{2 \cdot 0,5} = \frac{-\cancel{0} - 1}{1} = -2$$

$$f'(1) = \frac{f(1,5) - f(0,5)}{1} = \frac{0-(-1)}{1} = 1$$

$$f'(1,5) = \frac{f(2) - f(1)}{1} = \frac{2-(-1)}{1} = 3$$

krajné body

$$f'(0) = \frac{f(0,5) - f(0)}{0,5} = \frac{-1-1}{0,5} = -4$$

$$f'(2) = \frac{f(2) - f(1,5)}{0,5} = \quad = 4$$

Numerická integrácia

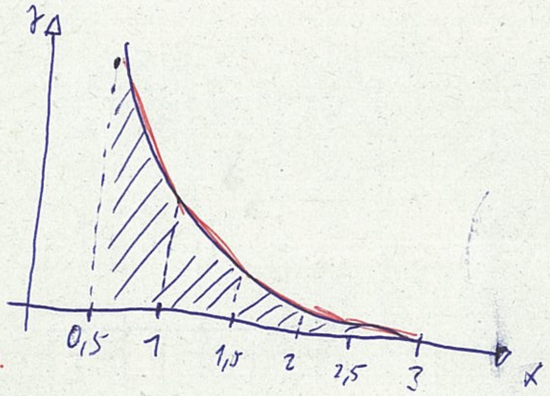
Pr

$$\int_{0,5}^{2,5} \frac{\sin x}{x^2}\,dx$$

$h = 0,5$

$$L_4 = 0,5 \cdot \left(\frac{1}{2} \frac{\sin 0,5}{0,5^2} + \frac{\sin 1}{1^2} + \frac{\sin 1,5}{1,5^2} + \frac{\sin 2}{2^2} + \frac{1}{2} \cdot \frac{\sin 2,5}{2,5^2} \right) =$$

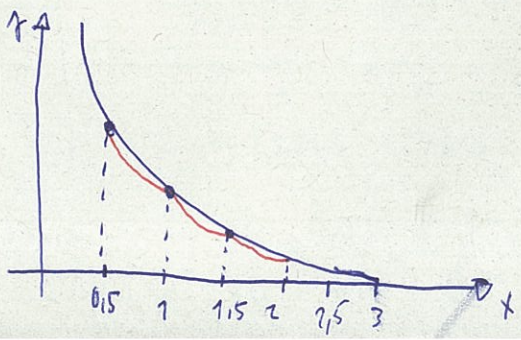
$$= \underline{\underline{1,25943}}$$



SIMPSONOVA M.

$$S_4 = \frac{0,5}{3} \cdot \left(\frac{\sin(0,5)}{0,5^2} + 4 \cdot \left(\frac{\sin(1)}{1^2} \right) + 2 \cdot \left(\frac{\sin(1,5)}{1,5^2} \right) + 4 \cdot \left(\frac{\sin(2)}{2,5^2} \right) \right) =$$

$$= 1,195883$$



$$\left(\sin x \cdot \frac{1}{x^2} \right)' = \sin x \cdot x^{-2} = \cos x \cdot x^{-2} + \sin x \cdot (-2) \cdot x^{-3} =$$

$$= \underbrace{\frac{\cos x}{x^2} - \frac{2 \cdot \sin x}{x^3}} \qquad 0,5 \leq x \leq 2,5$$

$$f''(x) = -\sin x \cdot x^{-2} + \cos x \cdot (-2) x^{-3} + \cos x \cdot (-2) x^{-3} + \sin x \cdot 6 x^{4}$$

$$= \underbrace{-\frac{\sin x}{x^2} - 4 \frac{\cos x}{x^3} + \frac{6 \sin x}{x^4}}$$

$$| \qquad | \leq 132$$

4 32 96

18.10.2018

5 cv. 5. týždeň

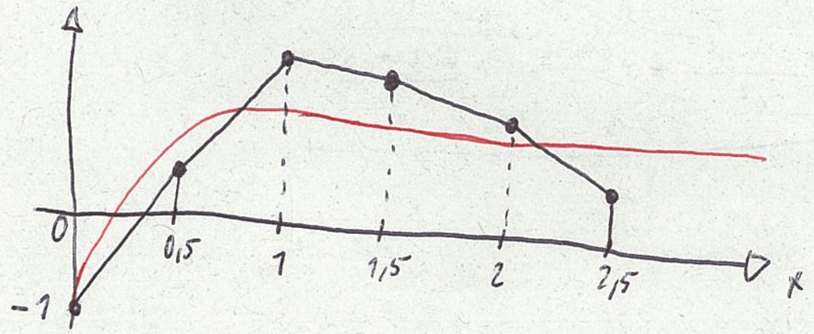
Základná Eulerova metóda

$$y' = \frac{5}{y^2+1} - x \;;\; y(0) = -1 \;;\; h = 0{,}5 \leftarrow \text{krok}$$

$\Delta y = 0{,}5 \cdot k$

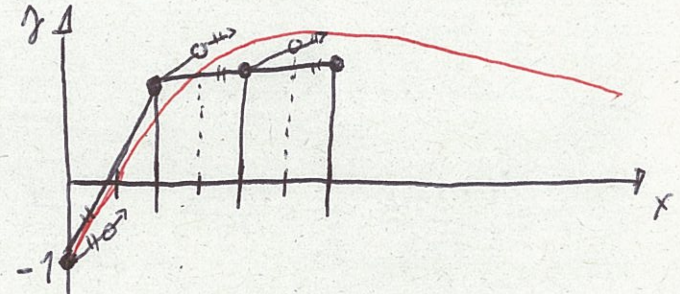
$k = x - y^2$

X	y	k
0	-1	2,5
0,5	0,25	4,21
1	2,355	-0,24
1,5	2,235	-0,67
2	1,9	-0,92
2,5	1,44	



Eulerova metóda 1. modif.

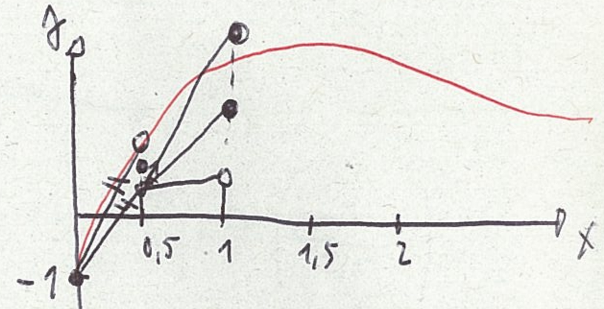
X	y	k_1	x^P	y^P	k_2
0	-1	2,5	0,25	-0,375	5,57
0,5	1,07	1,83	0,75	1,53	0,75
1	1,75	0,61	1,25	1,6	0,15
1,5	1,53	...	...	...	



$* = P$ (POMOCNÝ)

Eulerova metóda 2. modifikácia

X	y	k_1	x^*	y^*	k_2	$\frac{k_1+k_2}{2}$
0	-1	2,5	0,5	0,25	4,205	3,35
0,5	0,68	2,92	1	2,14	-0,10	1,4
1	1,38	0,72	1,5	1,74	-0,26	0,23
1,5	1,5	...	...			



6 cv. 6. týždeň

Př

→ jednoduchá (klasická) pravdepodobnosť

- máme 3 kocky

a) padne súčet 12

b) padnú 3 rôzne čísla, medzi nimi je 6

c) 2 rovnaké čísla + 1 iné ktoré je menšie

$|\Omega| = 6^3 = 216$

$\Omega = \{111, 112, \dots, 666\}$

a) 165, 156, 615, 651, 516, 561

336, 363, 633

444

453...

246...

255, 525, 552

$P(A) = \frac{25}{216} = \underline{\underline{0{,}1157}}$

b) $\left(\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{4}{6}\right) + \left(\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{4}{6}\right) + \left(\frac{5}{6}\cdot\frac{4}{6}\cdot\frac{1}{6}\right) =$

$= \frac{20}{216} + \frac{20}{216} + \frac{20}{216} = \frac{60}{216} \xrightarrow{181} = \underline{\underline{0{,}27\bar{7}}}$

c)

$C = \{221, 332, 331, 443, 442, 441, 554, 553, 552, 551 \dots\}$

$15 \cdot 3 \Rightarrow |C| = 45$

$P(C) = \frac{45}{216} = \underline{\underline{0{,}208\bar{3}}}$

Pr 1 kocka, aká je pravdepodobnosť že súčet nahádzaných čísel bude 11 →Distribučná prav.

opm $\boxed{6}$

$\Omega = \{6, 16, 26, 36, \dots, 126, \dots, 11111 6 \dots\}$

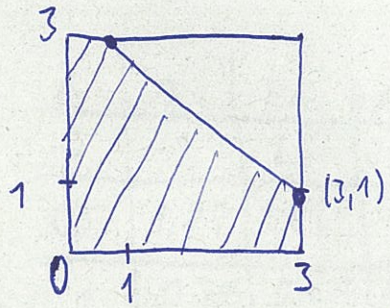
⋮

$|A| = 16$

$$P(A) = \frac{1}{36} + 4\cdot\frac{1}{216} + 6\cdot\frac{1}{1296} + 4\cdot\frac{1}{7776} + 1\cdot\frac{1}{46656} = 0{,}05146$$

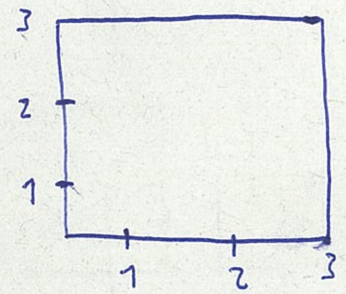
Pr2

budú náhodne vybrané 2 čísla z intervalu $\langle 0;3 \rangle$ aká je pravdepodobnosť že ich súčet bude menší alebo rovný 4



a) $P(A) = \frac{7}{9}$

b) $P(B)$
↓
súčin bude menší alebo rovný ako 4



7cv. 7 týždeň

1.

Pištoľ H_1 A: … 50% z toho sú 2% červené
H_2 B: … 40% 4% červených
H_3 C: … 10% 7% červených

$P(A) = 0{,}5$
$P(B) = 0{,}4$
$P(C) = 0{,}1$

1. Aká je prav. že 1 sviečka bude červená?
2. náhodne vybraná sviečka je červená, Pst že je od C

$P(H_1) = 0{,}5$
$P(H_2) = 0{,}4$
$P(H_3) = 0{,}1$

$P(č|H_1) = 0{,}02$
$P(č|H_2) = 0{,}04$
$P(č|H_3) = 0{,}07$

$P(č) = P(H_1)\,P(č|H_1) + P(H_2)\cdot P(č|H_2) + P(H_3)\cdot P(č|H_3) =$

$= 0{,}5\cdot 0{,}02 + 0{,}4\cdot 0{,}04 + 0{,}1\cdot 0{,}07 = 0{,}033$

$$P(H_3|č) = \frac{P(H_3)\cdot P(č|H_3)}{P(č)} = \frac{0{,}1\cdot 0{,}07}{0{,}033} = \frac{7}{33} = \underline{\underline{0{,}2\overline{1}}}$$

2.

32 kariet 4 esá

36 kariet 8 es

1) Aká je prav. že vybraná karta bude eso
2) aká je prav. že pochádza z prvého balíčka

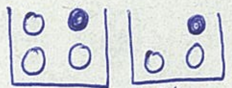
$P(E) = P(H_1)\cdot P(E|H_1) + P(H_2)\cdot P(E|H_2) = \frac{1}{2}\cdot\frac{1}{8} + \frac{1}{2}\cdot\frac{2}{9} = \frac{1}{16} + \frac{2}{18} = \underline{\underline{0{,}1736\overline{1}}}$

$P(H_1) = \frac{1}{2}$ $\quad P(E|H_1) = \frac{4}{32} = \frac{1}{8}$

$P(H_2) = \frac{1}{2}$ $\quad P(E|H_2) = \frac{8}{36} = \frac{2}{9}$

$$P(H_2|E) = \frac{P(H_2)\cdot P(E|H_2)}{P(E)} = \frac{\frac{1}{2}\cdot\frac{2}{9}}{0{,}1736\overline{1}} = \underline{\underline{0{,}64}}$$

Pr



H_1 H_2

Biela gulička -> som si vytiahol

P = ? že v tej istej krabici bude náhodne vytiahnutá biela

$P(H_1) = \frac{1}{2}$ $\quad P(B \mid H_1) = \frac{3}{4}$

$P(H_2) = \frac{1}{2}$ $\quad P(B \mid H_2) = \frac{2}{3}$

$$\overbrace{P(H_1) \cdot P(B \mid H_1) + P(H_2) \cdot P(B \mid H_2)}^{P(B)} = \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} = \frac{3}{8} + \frac{2}{6} = \frac{17}{24}$$

$$P(H_1 \mid B) = \frac{P(H_1) \cdot P(B \mid H_1)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{17}{24}} = \frac{9}{17}$$

$$P(H_2 \mid B) = \frac{P(H_2) \cdot P(B \mid H_2)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{17}{24}} = \frac{8}{17}$$

8cv. 8 týždeň

písomka vo 7 týždni: 5,6 -> koniec, max 3x

1. $\Omega = \{5, 6, 15, 25, 35, 45, 16, 26, 36, 46, 111, \dots, \} = \{5,6\} \cup \{(a,b) : a \in \{1,2,3,4\}, b \in \{5,6\}\} \cup \{(a,b,c) : a,b \in \{1,2,3,4\}, c \in \{1,2,3,4,5,6\}\}$

$|\Omega| = 2 + 8 + 4 \cdot 4 \cdot 6 = \underline{\underline{106}}$

2. n1: $\underset{\{5,6\}}{\frac{2}{6}} + \frac{3 \cdot 2}{6^2} + \frac{3 \cdot 3 \cdot 5}{6^3} = \frac{17}{24} = 0{,}708\overline{3}$

$(\neq 1, 5/6)$
$\neq 5$
$\neq 6$
$(2/3/4/5/6)$

$(\neq 1, \neq 1, \neq 1$
$2 \quad 2 \quad 2$
$3 \quad 3 \quad 1$
$4 \quad 4 \quad 6$

súčet = 6 $\{6, 15, 114, 141, 411, \underbrace{123, \dots,}_{6} 222$

$$\frac{1}{6} + \frac{1}{6^2} + \frac{10}{6^3} = \frac{13}{54} \doteq 0{,}24\overline{07}$$

Pr hádzanie 2 špeciálnymi kockami $\quad p(x), F(x), EX, DX, \sigma = ?$

A: 1, 1, 1, 1, 2, 3
B: 1, 3, 4, 4, 4, 4

"X = A - B"

$X = 0, -2, -3, 1, -1, 2$

$p(-3) = \frac{4 \cdot 4}{6 \cdot 6} = \frac{16}{36}$ $\quad p(0) = \frac{4+1}{36} = \frac{5}{36}$

$p(-2) = \frac{4+4}{36} = \frac{8}{36}$ $\quad p(1) = \frac{1}{36}$

$p(-1) = \frac{1+4}{36} = \frac{5}{36}$ $\quad p(2) = \frac{1}{36}$

$EX = \sum_{x_i} x_i \, p(x_i)$

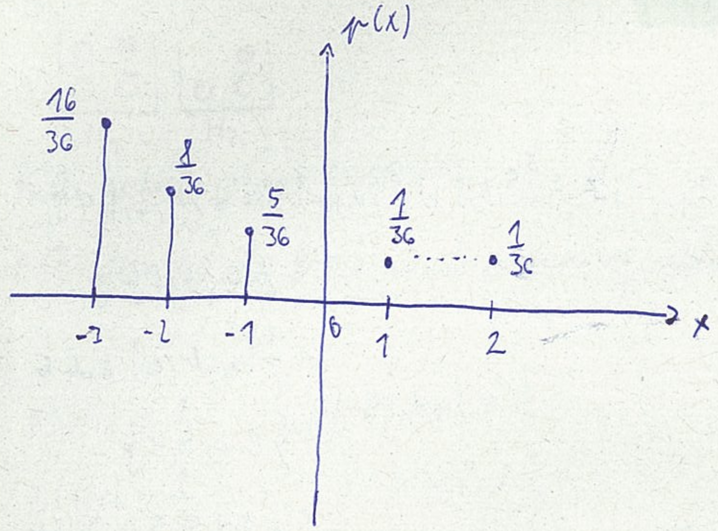
$EX = \frac{16}{36} \cdot (-3) + \frac{8}{36} \cdot (-2) + \frac{5}{36} \cdot (-1) + \frac{1}{36} + \frac{2}{36}$

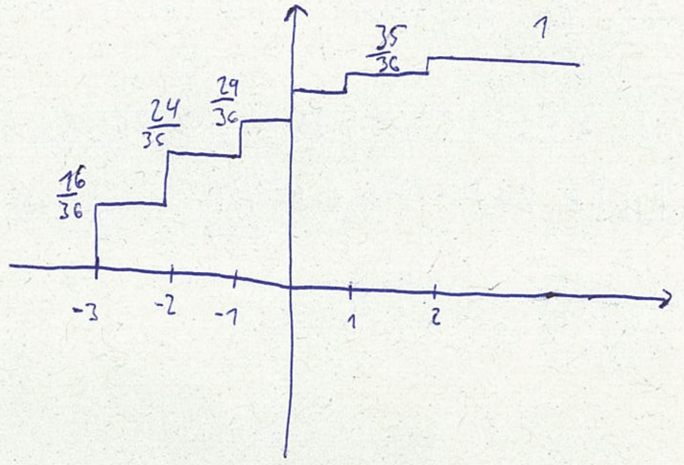
$= -\frac{11}{6} = \underline{\underline{-1{,}8\overline{3}}}$

$DX = \sum_{x_i} x_i^2 \, p(x_i) - (EX)^2 = *$

$* = 9 \cdot \frac{16}{36} + 4 \cdot \frac{8}{36} + 1 \cdot \frac{5}{36} + 1 \cdot \frac{1}{36} + 4 \cdot \frac{1}{36} - \left(-\frac{11}{6}\right)^2 = \frac{31}{6} - \left(-\frac{11}{6}\right)^2 = \underline{\underline{\frac{65}{36}}}$

$\sigma = \sqrt{DX} = \sqrt{\frac{65}{36}} = 1{,}3437$





$F(-\pi) = 0$

$F(-2) = \frac{24}{36}$

$F(0{,}1) = \frac{34}{36}$

$F(\sqrt{2}) = \frac{35}{36}$

---

1 h ... 2 autá

$P(k) = P(X=k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$

a) za 1 hodinu žiadne
b) za 10 minút viac než 2
c) za 8 hodín 16 áut

---

a)

$X \sim Po(2)$

$P(k) = \frac{2^k}{k!} \cdot e^{-2}$

$P(0) = \underline{\underline{e^{-2}}}$

b)

$X \sim Po\left(\frac{1}{3}\right)$

$P(0) = e^{-\frac{1}{3}} = 0{,}7165$

$P(1) = \frac{\left(\frac{1}{3}\right)^1}{1!} \cdot e^{-\frac{1}{3}} = 0{,}2388$

$P(2) = \frac{\left(\frac{1}{3}\right)^2}{2!} \cdot e^{-\frac{1}{3}} = 0{,}039$

$P(X > 2) = 1 - (P(0) + P(1) + P(2)) = 1 - 0{,}9943 = 0{,}0049$

c) $X \sim Po(16)$

$P(16) = \frac{16^{16}}{16!} \cdot e^{-16} = 0{,}09921$

15. 11. 2018

9cv. 9. týždeň

Pr 1

10 otázok a, b, c

úspech: aspoň 7 správne

prav. úspechu pri náhodnej odpovedi

$X \sim Bi\left(10; \frac{1}{3}\right)$

$p(k) = \binom{n}{k} \cdot p^k (1-p)^{n-k}$

$p(7) = \binom{10}{7} \cdot \left(\frac{1}{3}\right)^7 \cdot \left(1 - \frac{1}{3}\right)^3 = \frac{320}{19683} = 0{,}01626$

$p(8) = \binom{10}{8} \cdot \left(\frac{1}{3}\right)^8 \cdot \left(1 - \frac{1}{3}\right)^2 = 0{,}0003048 = \frac{20}{6561}$

$p(9) = \binom{10}{9} \cdot \left(\frac{1}{3}\right)^9 \cdot \left(\frac{2}{3}\right) = \frac{20}{59049} = 0{,}0003387$

$p(10) = \binom{10}{10} \cdot \left(\frac{1}{3}\right)^{10} = \frac{1}{59049} = 0{,}000016935$

$p(7) + p(8) + p(9) + p(10) = \underline{\underline{0{,}01966}} = p(X \geq 7)$

$X \sim Bi(30; 0{,}1966)$

$1 - p(0)$

$P(0) = \binom{30}{0} \cdot (0{,}01966)^0 - (1 - 0{,}01966)^{30}$

$P(0) = 0{,}55119$

$P(X \geq 1) = \underline{\underline{0{,}44881}}$

Pr 2

1 dieťa ... 2,5h

a) za 8h ... nič
b) za 1/2 h ... viac ako 2 pôrody
c) za 10 min ... viac ako 2 defekty

---

a) $X \sim Po(3{,}2)$

$X \sim Po(\lambda)$

2,5 ... 1
8 ... x

$x = \frac{8}{2{,}5} = 3{,}2$

a) $p(k) = \frac{3{,}2^k}{k!} \cdot e^{-3{,}2}$

$p(0) = \frac{3{,}2^0}{0!} \cdot e^{-3{,}2}$

$p(0) = e^{-3{,}2} = \underline{\underline{0{,}0407}}$

b) $X \sim Po\left(\frac{1}{5}\right)$

$P(0) = \frac{\left(\frac{1}{5}\right)^0}{0!} \cdot e^{-\frac{1}{5}} = 0{,}8187$

$P(1) = \frac{\left(\frac{1}{5}\right)^1}{1!} \cdot e^{-\frac{1}{5}} = 0{,}1637$

$P(2) = \frac{\left(\frac{1}{5}\right)^2}{2!} \cdot e^{-\frac{1}{5}} = 0{,}01637$

$P(X>2) = 1-(p(0)+p(1)+p(2)) = 1-(0{,}8187+0{,}1637+0{,}01637) = 0{,}0011$

c) $X \sim Po\left(\frac{1}{15}\right)$

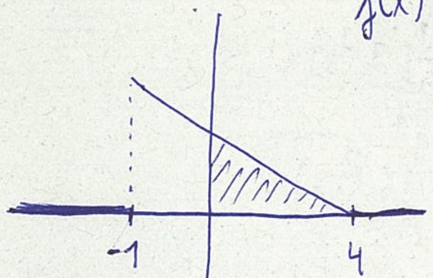
$P(0) = \frac{\left(\frac{1}{15}\right)^0}{0!} \cdot e^{-\frac{1}{15}} = 0{,}9355$

$P(1) = \frac{\left(\frac{1}{15}\right)^1}{1!} \cdot e^{-\frac{1}{15}} = 0{,}0623$

$P(2) = \left(\frac{\frac{1}{15}}{2!}\right)^2 \cdot e^{-\frac{1}{15}} = 0{,}00207$

$P(X>2) = 1-[P(0)+P(1)+P(2)] = 0{,}000046979$

Pr3



$f(x)$ $\quad P(X\geq 0)$

$\frac{5 \cdot f(-1)}{2} = 1$

$f(-1) = \frac{2}{5}$

$f(0) = f(-1) \cdot \frac{4}{5} = \frac{8}{25}$

$P(X\geq 0) = \frac{8}{25} \cdot \frac{4}{2} = \frac{16}{25}$

$P(X\geq 0)$

$P(X\geq 4) = 0$

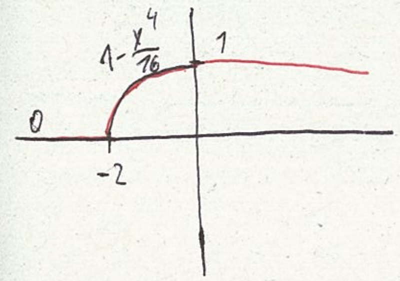
$P(X\leq -2) = 0$

$P(X\leq 10) = 1$

$P(X\leq 0) = 1-\frac{16}{25} = \frac{9}{25}$

22.11.2018

10 cv. 10 bodov

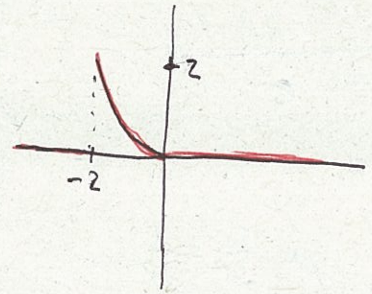


1) $P(X\leq -1)$; $P(-0{,}2 \leq X \leq -0{,}1)$
2) $f(x)$
3) EX
4) DX, σ

1.) $P(X\leq -1) = \frac{15}{16}$, $P(-0{,}2 \leq X \leq -0{,}1) = F(-0{,}1) - F(0{,}2) = 0{,}99999375 - 0{,}9999 = 0{,}00009375$

2.) $f(x) = F'(x)$

$$f(x) = \begin{cases} 0 & \text{pre } x \leq -2 \\ -\frac{x^3}{4} & \text{pre } x \in \langle -2, 0) \\ 0 & \text{pre } x \geq 0 \end{cases}$$



$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2} = \frac{4x^3 \cdot 16 - x^4 \cdot 0}{256} = -\frac{x^3}{4}$

3.) $EX = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{-2}^{0} x \cdot \left(-\frac{x^3}{4}\right)dx = \int_{-2}^{0} -\frac{x^4}{4} = -\frac{1}{4}\int_{-2}^{0} x^4 = -\frac{1}{4}\left[\frac{x^5}{5}\right]_{-2}^{0} = -\frac{1}{4}\left(0+\frac{32}{5}\right) = -\frac{8}{5}$

4.) $DX = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - (EX)^2 = \int_{-2}^{0} x^2 \left(-\frac{x^3}{4}\right)dx - \left(-\frac{8}{5}\right)^2 = \int_{-2}^{0} -\frac{x^5}{4}\,dx - \frac{64}{25} =$

$= -\frac{1}{4} \int_{-2}^{0} x^5\,dx - \frac{64}{25} = -\frac{1}{4} \left[\frac{x^6}{6}\right]_{-2}^{0} - \frac{64}{25} = -\frac{1}{4}\left(-\frac{64}{6}\right) - \frac{64}{25} = \frac{64}{24} - \frac{64}{25} = \frac{8}{75} = 0{,}1066$

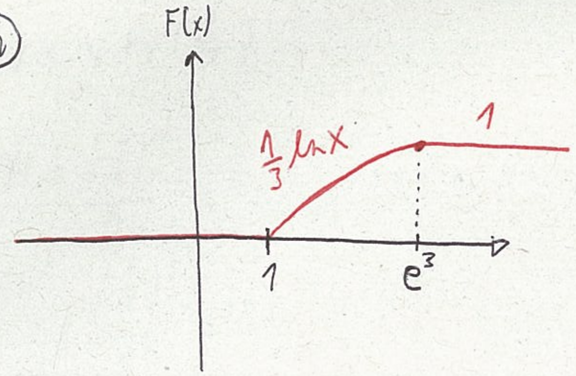
$\sigma = \sqrt{DX} = \sqrt{\frac{8}{75}} = 0{,}3265$

$1{,}5 - 0 = 1{,}5$

$Dx = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - (EX)^2 = \int_{1}^{e^3} x^2 \cdot \frac{1}{3x}$

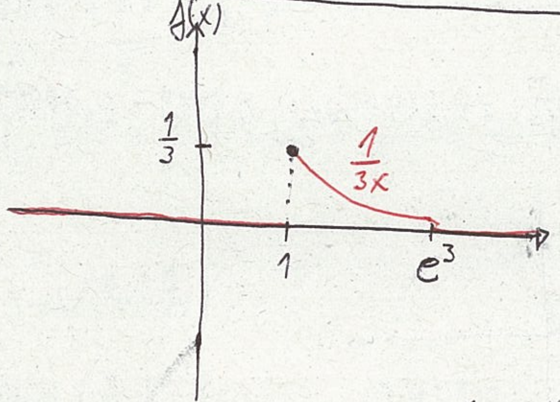
$\frac{x^{\cancel{x}}}{\cancel{3}x} \quad \frac{x}{2x} = \frac{1}{2}$

Pr



① $P(X \leq 2)$, $P(10 \leq X \leq 12)$
② $f(x)$
③ EX
④ DX, σ

2)



1) $\frac{1}{3}\ln(2) = 0{,}231049$

$\left.\begin{array}{l} \frac{1}{3}\cdot\ln(10) = 0{,}76752 \\ \frac{1}{3}\cdot\ln(12) = 0{,}8283 \end{array}\right\} \longrightarrow P(10 \leq X \leq 12) = \underline{\underline{0{,}06078}}$

3)

$x \cdot \frac{1}{3x} = \frac{x}{3x}$

$$EX = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_1^{e^3} \frac{x}{3x} = \frac{1}{3}\int_1^{e^3} 1\,dx = \left[\frac{x}{3}\right]_1^{e^3} = \left[\frac{e^3}{3} - \frac{1}{3}\right] = \underline{\underline{6{,}36185}}$$

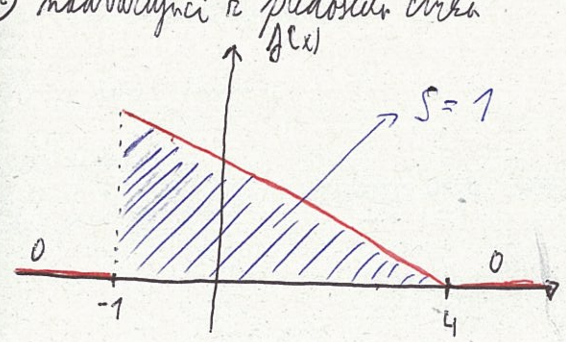
4.)

$$DX = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - (EX)^2 = \int_1^{e^3} x^2 \cdot \frac{1}{3x}\,dx - 6{,}36185^2 = \int \frac{x}{3}\,dx - 6{,}36185^2 = \left[\frac{x^2}{6}\right]_1^{e^3} - 6{,}36185^2 =$$

$$= \frac{e^6}{6} - \frac{1}{6} - 6{,}36185^2 = \underline{\underline{26{,}59838562}}$$

$$\sigma = \sqrt{DX} = \sqrt{26{,}59838562} = \underline{\underline{5{,}157362273}}$$

Pr nadväzujúci z predošlého cvika



$$\frac{5 \cdot f(-1)}{2} = 1 \rightarrow f(-1) = \frac{2}{5}$$

$y = k \cdot x + q$

$A\left[-1, \frac{2}{5}\right]$ $\quad \frac{2}{5} = -k + q$

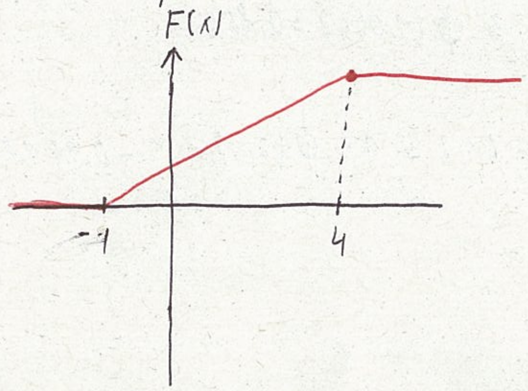
$B[4, 0]$ $\quad 0 = 4k + q$

$$\underline{\underline{y = -\frac{2}{25}x + \frac{8}{25}}}$$

$$\int\left(-\frac{2}{25}x + \frac{8}{25}\right)dx = \int\left(-\frac{2x}{25}\right)dx + \int\frac{8}{25}\,dx = -\frac{2}{25}\cdot\frac{x^2}{2} + \frac{8}{25}x + C = \underline{\underline{-\frac{x^2}{25} + \frac{8x}{25} + C}}$$

$$-\frac{(-1)^2}{25} + \frac{8\cdot(-1)}{25} = -C \longrightarrow \underline{\underline{C = \frac{9}{25} = 0{,}36}}$$

$$F(x) = \begin{cases} 0 & \text{pre } (-\infty, -1) \\ -\frac{x^2}{25} + \frac{8x}{25} + \frac{9}{25} & \text{pre } \langle -1, 4) \\ 1 & \text{pre } \langle 4, \infty) \end{cases}$$



29.11.2018

11cv. 11. týždeň

normální rozdelení

(Pr)

Lúšev ... 500[ml]

$X \sim N(\mu, \sigma^2)$

$\mu = 492\,[ml]$, $\sigma = 7{,}5\,[ml]$

norma: 485 - 515[ml]

a) aká je prav. že náhodne nakúpená fľaša bude v norme?

b) objem je menší ako 500 ml

c) objem je väčší ako 510 ml

d) nad akú hodnotu sa objem dostane s PST α - 0,01

---

a) $P(485 \leq X \leq 515) = P\left(\frac{485-492}{7{,}5} \leq U \leq \frac{515-492}{7{,}5}\right) = P(-0{,}9\overline{3} \leq U \leq 3{,}0\overline{6}) =$

$U = \frac{X-\mu}{\sigma}$

$= \Phi(3{,}06) - \Phi(-0{,}9\overline{3}) = \Phi(3{,}06) - 1 + \Phi(0{,}9\overline{3}) \doteq$

$\doteq 0{,}9987 - 1 + 0{,}8239 = \underline{\underline{0{,}8226}}$ $(\doteq 0{,}82)$

b) $P(X \leq 500) = P\left(U \leq \frac{500-492}{7{,}5}\right) = P\left(U \leq \frac{16}{15}\right) \doteq P(U \leq 1{,}066) \doteq \Phi(1{,}066) = 0{,}856$

c) ~~[illegible]~~ $1 - P(X \leq 510) = 1 - P\left(U \leq \frac{510-492}{7{,}5}\right) = 1 - P(U \leq 2{,}4) = 1 - \Phi(2{,}4) = 1 - 0{,}9918 = 0{,}0082$

d) ~~[illegible]~~

~~[illegible]~~

$P(X \geq d) = 0{,}01$

$P\left(\frac{X-492}{7{,}5} \geq \frac{d-492}{7{,}5}\right) = 0{,}01$

$1 - \Phi\left(\frac{d-492}{7{,}5}\right) = 0{,}01$

$\frac{d-492}{7{,}5} = 2{,}33$

$d - 492 = 2{,}337$

$d = 2{,}33 \cdot 7{,}5 + 492$

$d = \underline{\underline{509{,}475}}$

(Pr) → ~~[illegible]~~ Bi + NORM !

10% losov výherných

500 losov

Aká je prav. že z 500 losov bude 50-80 vyhrávajúcich

$X \sim Bi(500; 0{,}1)$

$\mu = n \cdot p = 500 \cdot 0{,}1 = 50$

$\sigma = \sqrt{n \cdot p(1-p)} = \sqrt{500 \cdot 0{,}1 \cdot 0{,}9} =$

$\doteq \underline{\underline{6{,}7082}}$

korekcia pri nahradzovaní

↓

$P(50 \leq X \leq 80) = P(50-0{,}5 < X < 80+0{,}5) =$

$= P(49{,}5 < X < 80{,}5) = P\left(\frac{49{,}5-50}{6{,}7082} < U < \frac{80{,}5-\cancel{80}\,50}{6{,}7082}\right)$

$\doteq P(-0{,}074 < U < 4{,}546) = \Phi(4{,}546) - 1 + \Phi(0{,}074) \doteq$

$= 1 - 1 + 0{,}528 \doteq \underline{\underline{0{,}528}}$

12. cvičko 12 týždeň

Štatistické testy

Pr. životnosť súčiastky je 500 hodín

$X \sim N(500, \sigma^2)$

$\mu = 500[h]$, $\sigma = 25[h]$

novou technológiou sa otestovalo 50 súčiastok

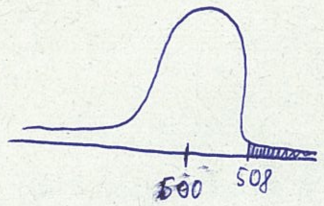
50 súčiastok ... priemerná životnosť 508[h] → je to štatisticky významné?

H_0... nie je štat. význam $\alpha = 0{,}05$ ← hraničná pravdepodob.

H_1... je to štat. význam. zmena

$\bar{X} \sim N(\bar{\mu}, \bar{\sigma}^2)$ $\bar{\mu} = \mu = 500$

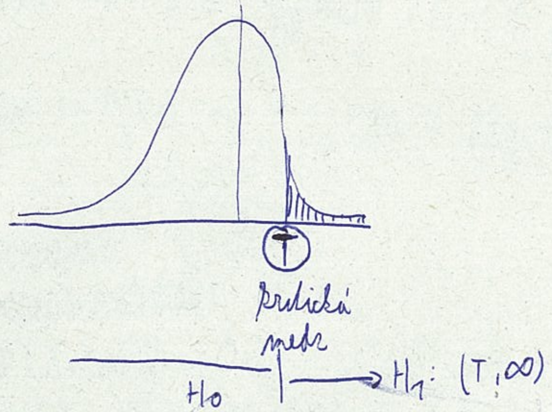
$\bar{\sigma} = \frac{\sigma}{\sqrt{50}} = 3{,}5355$



riešenie cez P hodnotu

① $P(\bar{X} \geq 508) = P\left(U > \frac{508-500}{3{,}5355}\right) = 1 - \Phi(2{,}263) = 1 - 0{,}987 = 0{,}012 < \alpha \Rightarrow H_1$

prijímame



② Aká je tá hranica čo odhryzne 0,05 ← pravo stranný test

$P(\bar{X} > T) = 0{,}05$

$P(\bar{X} < T) = 0{,}95$

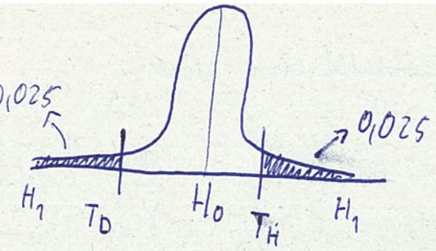
pozrieme sa do tabuľky → $\Phi(1{,}65) = 0{,}95$

$P\left(U < \frac{T-500}{3{,}5355}\right) = 0{,}95$

$\frac{T-500}{3{,}5355} = 1{,}65$ ← $/\cdot 3{,}5355$

$T - 500 = 5{,}8336 \rightarrow \underline{T = 505{,}8336}$

Pr. obojstranný test



súčiastky: $\alpha = 0{,}05$

A: 492 (H_1) B: 496 (H_0) C: 505 (H_0)

$P(\bar{X} > T_H) = 0{,}025$

$P(\bar{X} < T_D) = 0{,}975$

$P\left(U < \frac{T_H - 500}{3{,}5355}\right) = 0{,}975$ $T_H = 506{,}93$, $T_D = 493{,}07$

$\Phi\left(\frac{T_H - 500}{3{,}5355}\right) = 0{,}975$

$\frac{T_H - 500}{3{,}5355} = 1{,}96$ kritický obor $(-\infty; 493{,}07) \cup (506{,}93; \infty)$