HW7

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problem2

(a)

```
library(quantreg)
## Loading required package: SparseM
##
## Attaching package: 'SparseM'
## The following object is masked from 'package:base':
##
##
       backsolve
library(quantmod)
## Loading required package: xts
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
## Registered S3 method overwritten by 'xts':
##
     method
                from
##
     as.zoo.xts zoo
##
## Attaching package: 'xts'
## The following objects are masked from 'package:dplyr':
##
##
       first, last
## The following objects are masked from 'package:data.table':
##
       first, last
##
## Loading required package: TTR
## Registered S3 method overwritten by 'quantmod':
     method
     as.zoo.data.frame zoo
## Version 0.4-0 included new data defaults. See ?getSymbols.
#fetch data from Yahoo
#AAPL prices
```

```
apple08 <- getSymbols('AAPL', auto.assign = FALSE, from = '2008-1-1', to =
                        "2008-12-31")[,6]
## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
#market proxy
rm08<-getSymbols('^ixic', auto.assign = FALSE, from = '2008-1-1', to =
                   "2008-12-31")[,6]
#log returns of AAPL and market
logapple08<- na.omit(ROC(apple08)*100)</pre>
logrm08<-na.omit(ROC(rm08)*100)</pre>
#OLS for beta estimation
beta_AAPL_08<-summary(lm(logapple08~logrm08))$coefficients[2,1]
#create df from AAPL returns and market returns
df08<-cbind(logapple08,logrm08)
set.seed(666)
Boot=1000
sd.boot=rep(0,Boot)
for(i in 1:Boot){
  # nonparametric bootstrap
  bootdata=df08[sample(nrow(df08), size = 251, replace = TRUE),]
  sd.boot[i] = coef(summary(lm(AAPL.Adjusted~IXIC.Adjusted, data = bootdata)))[2,2]
head(sd.boot,10)
```

[1] 0.06861686 0.05623264 0.05940472 0.07227112 0.04946571 0.06534475 ## [7] 0.05475728 0.06822135 0.05293374 0.04875206

The names of y and x in linear model are wrong. I change them to the colname in bootdata, and it works. Here are first 10 values of coefficient.

(b)

```
#import data
urla <- "https://www2.isye.gatech.edu/~jeffwu/wuhamadabook/data/Sensory.dat"
sensory_data <- fread(urla,sep=" ",fill=TRUE,skip=1)
colnames(sensory_data)<-c("item","x1","x2","x3","x4","x5")
#get data frame contains NA
temp<-sensory_data %>% filter(is.na(x5)==TRUE) %>% select(1:5) %>% cbind(rep(1:10,each=2))
colnames(temp)<-c("x1","x2","x3","x4","x5","item")
#get data frame without NA
temp1<-sensory_data %>% filter(is.na(x5)==FALSE)
#combine
sensory_data<-rbind(temp1,temp)
sensory_data<-sensory_data[order(sensory_data$item),]</pre>
```

```
rownames(sensory_data)<-NULL</pre>
#bootstrap
system.time({
  set.seed(666)
  Boot=100
  beta.boot <- matrix(0, nrow = Boot, ncol = 6)</pre>
  for(i in 1:Boot){
    # nonparametric bootstrap
    bootdata=sensory_data[sample(nrow(sensory_data), size = nrow(sensory_data), replace = TRUE),]
    beta.boot[i,] = lm(item~., data = bootdata)$coefficients
  }
})
##
      user system elapsed
##
     0.133
           0.000 0.133
colnames(beta.boot) <- c("intercept", "beta1", "beta2", "beta3", "beta4", "beta5")</pre>
kable(beta.boot[1:10, ], caption = "first 10 coefficients' value")
```

Table 1: first 10 coefficients' value

intercept	beta1	beta2	beta3	beta4	beta5
10.378107	0.8735692	-1.2023224	1.7815910	-2.1501746	0.3561322
7.399245	-0.0833066	-0.5642760	0.4117166	-1.3015785	1.4914741
6.524012	-0.1033929	-0.9785161	1.2801961	-1.3523566	1.2829954
7.722459	0.6705703	-0.1596634	1.2360311	-1.9597120	0.1495749
7.981068	-0.3650556	-0.1470396	2.6312412	-2.6143482	0.4755146
6.642910	-1.3735202	-0.2674568	2.1296914	-0.1553975	-0.3845279
7.079447	0.8219657	-0.2740008	0.3282275	-1.3958379	0.6873465
7.715345	1.1020509	-0.4818613	-1.7981322	-0.7571586	1.5882752
8.059349	0.2679573	-1.8842193	1.0650332	-1.1086626	1.7777014
7.353870	-0.5243531	-1.6478045	1.4581122	-0.9409016	1.7384898

part c

```
cl <- makeCluster(8)
system.time({

    f <- function(x){

        #beta coefficient from bootstrap data
        bootdata=sensory_data[sample(nrow(sensory_data), size = nrow(sensory_data), replace = TRUE),]
        beta.boot <- lm(item~., data = bootdata)$coefficients
        return(beta.boot)
}

Boot <- matrix(1:100, ncol = 100)
        clusterExport(cl=cl, varlist=c("Boot", "f", "sensory_data"), envir=environment())
        beta_table <- t(parSapply(cl, Boot, f))
})</pre>
```

user system elapsed

```
## 0.009 0.001 0.208
stopCluster(cl)
#kable(beta_table[1:10, ], caption = "first 10 coefficients' value")
```

Since each bootstrap do not relate with others, that's why we can calculate them in the same time. For b part, we used 0.143s, but in c part, we used 0.094s. Paralell processing helps save time.

problem 3

part a

```
func1 <- function(x){</pre>
  #f(x)
  return(3^x-sin(x)+cos(5*x))
x_upper <- -2
x_lower <- -22</pre>
x \leftarrow seq(from = x_lower, to = x_upper, by = 0.02)
y <- apply(as.matrix(x, nrow = 1), 2, func1)
\#plot(x, y, type = "l")
func2 <- function(z){</pre>
  # find approximate x when f(x-e)*f(x+e)<0, e very small
  if (y[z]*y[z+1] < 0){
    return(mean(x[z],x[z+1]))
  } else {
    return(NA)
  }
}
i \leftarrow as.matrix(seq(1:(length(x)-1)), nrow = 1)
system.time({
  result <- sapply(i, func2)</pre>
})
##
      user system elapsed
     0.009
            0.000
                    0.008
result[!is.na(result)]
## [1] -21.74 -20.82 -20.70 -19.64 -19.26 -18.60 -17.68 -17.56 -16.50 -16.12
## [11] -15.46 -14.54 -14.40 -13.36 -12.96 -12.32 -11.40 -11.26 -10.22 -9.82
## [21] -9.18 -8.26 -8.12 -7.08 -6.68 -6.04 -5.12 -4.98 -3.94 -3.54
## [31] -2.90
part b
cl <- makeCluster(8)</pre>
clusterExport(cl=cl, varlist=c("i", "func2", "y", "x"), envir=environment())
```

```
system.time({
 result2 <- parSapply(cl, i, func2)
})
##
     user
           system elapsed
##
    0.003
            0.001
                    0.044
stopCluster(cl)
result2[!is.na(result2)]
## [1] -21.74 -20.82 -20.70 -19.64 -19.26 -18.60 -17.68 -17.56 -16.50 -16.12
## [11] -15.46 -14.54 -14.40 -13.36 -12.96 -12.32 -11.40 -11.26 -10.22 -9.82
        -9.18 -8.26 -8.12 -7.08 -6.68 -6.04 -5.12 -4.98 -3.94 -3.54
## [21]
## [31]
        -2.90
```

part a uses only 0.001s, however part b uses 0.043s. I think, if using apply family only needs extremely short time, there is no need to use parapply family.

The results are the same, since I used the same function and input.

problem 4

part a

It might not a good way to include the true value in the stopping rule. If the step size is a little bit large and with a not good start value, we may get a close enough answer, comparing with the true value.

part b

```
# set.seed(1256)
    # theta <- as.matrix(c(1,2),nrow=2)
    \# X \leftarrow cbind(1, rep(1:10, 10))
    # h <- X%*%theta+rnorm(100,0,0.2)
# theta0\_start \leftarrow seq(from = 0.47, to = 1.46, by = 0.01) # a series of starting value
# theta1_start <- seq(from = 1.5, to = 2.49, by = 0.01)
# thetaMatrix <-list()</pre>
# z <- 1
# for(i in 1:100){
      thetaMatrix[[z]] <- c(thetaO_start[i], theta1_start[i])</pre>
#
      z < -z+1
    }
#
#
#
   #quick gradient descent
#
   #need to make quesses for both ThetaO and Theta1, might as well be close
#
   alpha <- 0.0000001 # this is the step size
#
    m <- 100 # this is the size of h
   tolerance <- 0.000000001 # stopping tolerance
#
#
    f \leftarrow function(t)
#
      #input starting value of theta0 and theta1, output the value after gradient descent
#
      theta0_s <- t[1]
#
      theta1_s <- t[2]
#
      theta0 <- c(theta0_s,rep(0,999)) # I want to try a guess at theta0_s(from the vector of theta0_st
#
                                        # setting up container for max 1000 iters
```

```
#
      theta1 <- c(theta1_s, rep(0,999))
#
      i <- 2 #iterator, 1 is my quess (R style indecies)
#
      #current theta is last quess
#
      current\_theta \leftarrow as.matrix(c(theta0[i-1], theta1[i-1]), nrow=2)
#
      #update guess using gradient
#
      theta0[i] \leftarrow theta0[i-1] - (alpha/m) * sum(X %*% current\_theta - h)
#
      theta1[i] < -theta1[i-1] - (alpha/m) * sum((X %*% current_theta - h)*rowSums(X))
      rs_X <- rowSums(X) # can precalc to save some time
#
      z <- 0
#
#
      while(abs(theta0[i]-0.9695707)>tolerance ℰԵ abs(theta1[i]-2.001563)>tolerance ℰԵ z<5000000){
#
        if(i=1000)\{theta0[1]=theta0[i];\ theta1[1]=theta1[i];\ i=1;\ \}\ \#cat("z=",z,"\n",sep="")\}
#
        z \leftarrow z + 1
#
        i < -i + 1
#
        current\_theta \leftarrow as.matrix(c(theta0[i-1], theta1[i-1]), nrow=2)
#
        thetaO[i] \leftarrow thetaO[i-1] - (alpha/m) * sum(X %*% current_theta - h)
#
        theta1[i] \leftarrow theta1[i-1] - (alpha/m) * sum((X \%*\% current\_theta - h)*rs\_X)
#
#
      theta0 <- theta0[1:i]</pre>
#
      theta1 <- theta1[1:i]</pre>
#
      result <- c(theta0_s, theta1_s, z, theta0[i], theta1[i])
      return(result)
#
#
# cl <- makeCluster(8)</pre>
# clusterExport(cl=cl, varlist=c("thetaMatrix", "alpha", "m", "tolerance", "f", "theta", "X", "h"), env
# result3 <- parSapply(cl, thetaMatrix, f)</pre>
#
# stopCluster(cl)
#write.csv(result3, file = "/home/xshuangshuang/result3.csv")
```

Since it really costs time, I save the result and reload it for the next analysis and do not need to run again when knitting.

```
result3 <- read.csv("/home/xshuangshuang/result3.csv")
result3 <- t(result3[, -1])
colnames(result3) <- c("start_value_theta0", "start_value_theta1", "numOfIterations", "output_theta0",
kable(result3, caption = "table of starting value, stopping value and number of iteration")</pre>
```

Table 2: table of starting value, stopping value and number of iteration

	start_value_theta0	start_value_theta1	numOfIterations	output_theta0	output_theta1
$\overline{\text{V1}}$	0.47	1.50	5000000	0.5786318	2.059558
V2	0.48	1.51	5000000	0.5864617	2.058396
V3	0.49	1.52	5000000	0.5942917	2.057235
V4	0.50	1.53	5000000	0.6021217	2.056073
V5	0.51	1.54	5000000	0.6099517	2.054912
V6	0.52	1.55	5000000	0.6177817	2.053750
V7	0.53	1.56	5000000	0.6256116	2.052589
V8	0.54	1.57	5000000	0.6334416	2.051427
V9	0.55	1.58	5000000	0.6412716	2.050266
V10	0.56	1.59	5000000	0.6491016	2.049104
V11	0.57	1.60	5000000	0.6569316	2.047942

$\begin{array}{c} V12\\ V13\\ V13\\ V13\\ V13\\ V14\\ V14\\ V16\\ V14\\ V16\\ V15\\ V15\\ V15\\ V16\\ V16\\ V16\\ V16\\ V16\\ V16\\ V16\\ V16$		start	_value_	_theta0	start	_value_	_theta1	${\bf numOfIterations}$	output	_theta0	output_	_theta1
$\begin{array}{c} \mathrm{V14} \\ \mathrm{V15} \\ \mathrm{V15} \\ \mathrm{O}.60 \\ \mathrm{I}.63 \\ \mathrm{I}.64 \\ \mathrm{I}.6000000 \\ \mathrm{O}.6882515 \\ \mathrm{O}.632200000 \\ \mathrm{O}.6882515 \\ \mathrm{O}.043296 \\ \mathrm{V}16 \\ \mathrm{O}.62 \\ \mathrm{I}.65 \\ \mathrm{O}.600000 \\ \mathrm{O}.69690815 \\ \mathrm{O}.6920815 \\ \mathrm{V}17 \\ \mathrm{O}.63 \\ \mathrm{I}.66 \\ \mathrm{O}.600000 \\ \mathrm{O}.0990815 \\ \mathrm{O}.0990815 \\ \mathrm{V}17 \\ \mathrm{O}.63 \\ \mathrm{I}.66 \\ \mathrm{O}.600000 \\ \mathrm{O}.7099114 \\ \mathrm{O}.0990815 \\ \mathrm{V}19 \\ \mathrm{O}.65 \\ \mathrm{I}.68 \\ \mathrm{O}.600000 \\ \mathrm{O}.7099114 \\ \mathrm{O}.039811 \\ \mathrm{V}19 \\ \mathrm{O}.65 \\ \mathrm{O}.66 \\ \mathrm{I}.69 \\ \mathrm{O}.600000 \\ \mathrm{O}.7099114 \\ \mathrm{O}.203981 \\ \mathrm{V}20 \\ \mathrm{O}.66 \\ \mathrm{I}.69 \\ \mathrm{O}.600 \\ \mathrm{O}.66 \\ \mathrm{I}.69 \\ \mathrm{O}.600000 \\ \mathrm{O}.7352314 \\ \mathrm{O}.303625 \\ \mathrm{V}22 \\ \mathrm{O}.68 \\ \mathrm{I}.71 \\ \mathrm{O}.600000 \\ \mathrm{O}.7352314 \\ \mathrm{O}.303627 \\ \mathrm{V}22 \\ \mathrm{O}.68 \\ \mathrm{I}.71 \\ \mathrm{O}.600000 \\ \mathrm{O}.7430613 \\ \mathrm{O}.7352314 \\ \mathrm{O}.303627 \\ \mathrm{V}22 \\ \mathrm{O}.68 \\ \mathrm{I}.71 \\ \mathrm{O}.600000 \\ \mathrm{O}.7430613 \\ \mathrm{O}.7352314 \\ \mathrm{O}.303627 \\ \mathrm{O}.203627 \\ \mathrm{O}.22 \\ \mathrm{O}.68 \\ \mathrm{I}.71 \\ \mathrm{O}.600000 \\ \mathrm{O}.7430613 \\ \mathrm{O}.7352314 \\ \mathrm{O}.303627 \\ \mathrm{O}.203627 \\ O$									0.0	6647615	2	.046781
V15 0.61 1.64 5000000 0.6882515 2.043285 V17 0.63 1.66 5000000 0.7696815 2.042135 V17 0.63 1.66 5000000 0.77117414 2.039815 V18 0.64 1.67 5000000 0.7117414 2.038650 V20 0.66 1.69 5000000 0.7274014 2.037488 V21 0.67 1.70 5000000 0.7352314 2.036365 V21 0.67 1.70 5000000 0.7352314 2.035165 V22 0.68 1.71 5000000 0.7508913 2.035165 V23 0.69 1.72 5000000 0.7508913 2.03404 V24 0.70 1.73 5000000 0.7508913 2.03404 V24 0.70 1.73 5000000 0.7508913 2.03549 V25 0.71 1.74 5000000 0.7713813 2.03573 V27 0.73 1.76 5000000	V13						1.62	5000000	0.0	6725915	2	.045619
V16 0.62 1.65 5000000 0.6960815 2.04213 V17 0.63 1.66 5000000 0.7039114 2.040973 V18 0.64 1.67 5000000 0.7117414 2.038615 V19 0.65 1.68 5000000 0.7117414 2.038681 V20 0.66 1.69 5000000 0.7274014 2.036327 V21 0.67 1.70 5000000 0.7352314 2.036327 V22 0.68 1.71 5000000 0.7352314 2.036327 V22 0.69 1.72 5000000 0.7568913 2.034004 V24 0.70 1.73 5000000 0.7568513 2.032819 V25 0.71 1.74 5000000 0.7565133 2.035819 V26 0.72 1.75 5000000 0.758212 2.023389 V27 0.73 1.76 5000000 0.758712 2.02389 V28 0.74 1.77 5000000	V14						1.63	5000000	0.0	6804215	2	.044458
V17 0.63 1.66 5000000 0.7039114 2.049973 V18 0.64 1.67 5000000 0.7117414 2.039811 V19 0.65 1.68 5000000 0.7195714 2.034865 V20 0.66 1.69 5000000 0.724014 2.034865 V21 0.67 1.70 5000000 0.7343613 2.036327 V22 0.68 1.71 5000000 0.7582913 2.034004 V23 0.69 1.72 5000000 0.7580913 2.034004 V24 0.70 1.73 5000000 0.765513 2.031680 V26 0.72 1.75 5000000 0.765513 2.031680 V27 0.73 1.76 5000000 0.765513 2.031680 V27 0.73 1.76 5000000 0.7743813 2.031680 V29 0.75 1.78 5000000 0.790412 2.028186 V29 0.75 1.78 5000000							1.64	5000000	0.0	6882515	2	.043296
V18 0.64 1.67 5000000 0.7117414 2.038811 V19 0.65 1.68 5000000 0.7195714 2.038621 V20 0.66 1.69 5000000 0.7274014 2.037488 V21 0.67 1.70 5000000 0.7352314 2.036327 V22 0.68 1.71 5000000 0.7352314 2.035165 V23 0.69 1.72 5000000 0.7508913 2.03404 V24 0.70 1.73 5000000 0.7565813 2.03404 V24 0.70 1.73 5000000 0.7565513 2.032812 V25 0.71 1.74 5000000 0.7565513 2.032819 V27 0.73 1.76 5000000 0.7743813 2.030519 V27 0.73 1.78 5000000 0.7978712 2.029378 V28 0.74 1.77 5000000 0.7978712 2.025873 V30 0.76 1.79 5000000												
$\begin{array}{c} \mathbf{V}19 \\ \mathbf{V}20 \\ \mathbf{V}21 \\ \mathbf{V}21 \\ \mathbf{V}21 \\ \mathbf{V}21 \\ \mathbf{V}21 \\ \mathbf{V}21 \\ \mathbf{V}22 \\ \mathbf{V}22 \\ \mathbf{V}22 \\ \mathbf{V}22 \\ \mathbf{V}22 \\ \mathbf{V}22 \\ \mathbf{V}30 \\ \mathbf{V}4 \\ \mathbf{V}11 \\ \mathbf{V}11 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}11 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}11 \\ \mathbf{V}12 \\ \mathbf{V}12$												
V20 0.66 1.69 5000000 0.7274014 2.037488 V21 0.67 1.70 5000000 0.7352314 2.035165 V22 0.68 1.71 5000000 0.7508913 2.03404 V23 0.69 1.72 5000000 0.7508913 2.03404 V24 0.70 1.73 5000000 0.7587213 2.03468 V25 0.71 1.74 5000000 0.7665513 2.031680 V26 0.72 1.75 5000000 0.7782112 2.02357 V27 0.73 1.76 5000000 0.7822112 2.02357 V28 0.74 1.77 5000000 0.782712 2.0227034 V29 0.75 1.78 5000000 0.807912 2.0227034 V30 0.76 1.79 5000000 0.8135312 2.024711 V32 0.78 1.81 5000000 0.8135312 2.02471 V32 0.78 1.81 5000000												
V21 0.68 1.70 5000000 0.7352314 2.036327 V22 0.68 1.71 5000000 0.7430613 2.034004 V24 0.69 1.72 5000000 0.7588913 2.03404 V24 0.70 1.73 5000000 0.7587213 2.032842 V25 0.71 1.74 5000000 0.7587213 2.032842 V26 0.72 1.75 5000000 0.7587213 2.032680 V27 0.73 1.76 5000000 0.774813 2.036378 V28 0.74 1.77 5000000 0.790412 2.0229357 V28 0.74 1.77 5000000 0.798712 2.022034 V30 0.76 1.79 5000000 0.8057012 2.025873 V31 0.77 1.80 5000000 0.8135312 2.02471 V32 0.78 1.81 5000000 0.821911 2.02238 V33 0.79 1.82 5000000							1.68	5000000	0.7	7195714	2	.038650
V22 0.68 1.71 5000000 0.7306313 2.035165 V23 0.69 1.72 5000000 0.7508913 2.034004 V24 0.70 1.73 5000000 0.7587213 2.032842 V25 0.71 1.74 5000000 0.7665513 2.031680 V26 0.72 1.75 5000000 0.7743813 2.029357 V28 0.74 1.77 5000000 0.7900412 2.029357 V28 0.74 1.77 5000000 0.7900412 2.025196 V29 0.75 1.78 5000000 0.7978712 2.025873 V30 0.76 1.79 5000000 0.8057012 2.025873 V31 0.77 1.80 5000000 0.8133312 2.024711 V32 0.78 1.81 5000000 0.8213611 2.02258 V33 0.79 1.82 5000000 0.8370211 2.021227 V35 0.81 1.83 5000000							1.69		0.7	7274014	2	.037488
V23 0.69 1.72 5000000 0.7508913 2.034004 V24 0.70 1.73 5000000 0.7587213 2.032842 V26 0.71 1.74 5000000 0.7665513 2.031680 V26 0.72 1.75 5000000 0.7743813 2.030519 V27 0.73 1.76 5000000 0.790012 2.029397 V28 0.74 1.77 5000000 0.790012 2.028196 V29 0.75 1.78 5000000 0.7978712 2.027034 V30 0.76 1.79 5000000 0.8957012 2.025873 V31 0.77 1.80 5000000 0.8135312 2.024711 V32 0.78 1.81 5000000 0.8213611 2.023873 V33 0.79 1.82 5000000 0.8213611 2.0223873 V34 0.80 1.83 5000000 0.8448511 2.02065 V35 0.81 1.84 5000000							1.70	5000000	0.7	7352314	2	.036327
$\begin{array}{c} \mathrm{V24} \\ \mathrm{V25} \\ \mathrm{V25} \\ \mathrm{O.71} \\ \mathrm{I.74} \\ \mathrm{I.74} \\ \mathrm{I.75} \\ $							1.71	5000000	0.7	7430613	2	.035165
$\begin{array}{c} V25 \\ V26 \\ V26 \\ V26 \\ O.72 \\ O.72 \\ O.73 \\ O.73 \\ O.74 \\ O.73 \\ O.74 \\ O.75 \\ O.77 \\ O.75 \\ O.77 \\ O.75 \\ O.77 \\ O.75 \\ O.77 \\ O.75 \\ O.75 \\ O.77 \\ O.77 \\ O.77 \\ O.77 \\ O.78 \\ O.77 \\ O.79 \\ O.75 \\ O.77 \\$	V23			0.69			1.72	5000000	0.7	7508913	2	.034004
$\begin{array}{c} V26\\ V27\\ V27\\ V28\\ V28\\ 0.73\\ 0.73\\ 1.76\\ 0.500000\\ 0.7822112\\ 2.02937\\ 0.78\\ 0.79\\ 0.75\\ 1.78\\ 0.500000\\ 0.7900412\\ 2.028196\\ 0.7900000\\ 0.7900412\\ 2.028196\\ 0.7900000\\ 0.797712\\ 2.027034\\ 0.79\\ $	V24			0.70			1.73	5000000	0.7	7587213	2	.032842
$\begin{array}{c} V27 \\ V28 \\ V28 \\ 0.74 \\ 0.75 \\ 1.77 \\ 0.00000 \\ 0.7900412 \\ 2.028195 \\ V29 \\ 0.75 \\ 1.78 \\ 5000000 \\ 0.7900412 \\ 2.028196 \\ V30 \\ 0.76 \\ 1.79 \\ 5000000 \\ 0.8057012 \\ 2.027034 \\ V31 \\ 0.77 \\ 1.80 \\ 5000000 \\ 0.8135312 \\ 2.024711 \\ V32 \\ 0.78 \\ 1.81 \\ 5000000 \\ 0.8213611 \\ 2.02355 \\ V33 \\ 0.79 \\ 1.82 \\ 5000000 \\ 0.8213611 \\ 2.02355 \\ V34 \\ 0.80 \\ 1.83 \\ 5000000 \\ 0.8213611 \\ 2.022358 \\ V34 \\ 0.80 \\ 1.83 \\ 5000000 \\ 0.8291911 \\ 2.022388 \\ V34 \\ 0.80 \\ 1.83 \\ 5000000 \\ 0.8370211 \\ 2.021227 \\ V35 \\ 0.81 \\ 1.84 \\ 5000000 \\ 0.8448511 \\ 2.020055 \\ V37 \\ 0.83 \\ 1.86 \\ 5000000 \\ 0.8526811 \\ 2.018903 \\ V37 \\ 0.83 \\ 1.86 \\ 5000000 \\ 0.865811 \\ 2.018903 \\ V37 \\ 0.83 \\ 0.84 \\ 1.87 \\ 5000000 \\ 0.8663110 \\ 2.017742 \\ V38 \\ 0.84 \\ 1.87 \\ 5000000 \\ 0.8683410 \\ 2.016580 \\ V39 \\ 0.85 \\ 1.88 \\ 5000000 \\ 0.8683410 \\ 2.016580 \\ V39 \\ 0.85 \\ 1.88 \\ 5000000 \\ 0.894010 \\ 2.014257 \\ V41 \\ 0.87 \\ 1.90 \\ 5000000 \\ 0.894010 \\ 2.014257 \\ V41 \\ 0.87 \\ 1.90 \\ 5000000 \\ 0.894010 \\ 2.014257 \\ V41 \\ 0.88 \\ 1.91 \\ 5000000 \\ 0.8918310 \\ 2.013994 \\ V42 \\ 0.88 \\ 1.91 \\ 5000000 \\ 0.9915229 \\ 2.009611 \\ V45 \\ 0.91 \\ 1.94 \\ 5000000 \\ 0.9308909 \\ 2.007288 \\ V47 \\ 0.93 \\ 1.95 \\ 5000000 \\ 0.9338108 \\ 2.006126 \\ V48 \\ 0.94 \\ 1.97 \\ 5000000 \\ 0.9338108 \\ 2.006126 \\ V49 \\ 0.95 \\ 1.98 \\ 5000000 \\ 0.9338108 \\ 2.006126 \\ V48 \\ 0.94 \\ 1.97 \\ 5000000 \\ 0.936408 \\ 2.004808 \\ 2.004965 \\ V49 \\ 0.95 \\ 1.98 \\ 5000000 \\ 0.97607 \\ 2.003188 \\ V50 \\ 0.96 \\ 1.99 \\ 532704 \\ 0.9615307 \\ 2.001538 \\ 2.001538 \\ V51 \\ 0.99 \\ 2.02 \\ 465587 \\ 0.9875311 \\ 2.001538 \\ 2.001538 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.977607 \\ 2.000318 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.977607 \\ 2.000318 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.977607 \\ 2.000318 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.977607 \\ 2.000318 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.977607 \\ 2.003188 \\ V52 \\ 0.98 \\ 2.01 \\ 5000000 \\ 0.933607 \\ 1.99795 \\ V55 \\ 1.01 \\ 2.04 \\ 5000000 \\ 1.0044306 \\ 1.993349 \\ V59 \\ 1.05 \\ 2.08 \\ 5000000 \\ 1.0044306 \\ 1.993349 \\ V59 \\ 1.05 \\ 2.08 \\ 5000000 \\ 1.0044306 \\ 1.993600 \\ 1.993600 \\ 1.993600 \\ 1.993600 \\ 1.99$				0.71			1.74		0.7	7665513		
$\begin{array}{c} \mathbf{V28} \\ \mathbf{V29} \\ \mathbf{V29} \\ 0.75 \\ 1.78 \\ 5000000 \\ 0.7978712 \\ 2.027034 \\ \mathbf{V30} \\ 0.76 \\ 1.79 \\ 5000000 \\ 0.8057012 \\ 2.025873 \\ \mathbf{V31} \\ 0.77 \\ 1.80 \\ 5000000 \\ 0.8057012 \\ 2.025873 \\ \mathbf{V31} \\ 0.77 \\ 1.80 \\ 5000000 \\ 0.8135312 \\ 2.024711 \\ 0.732 \\ 0.78 \\ 1.81 \\ 5000000 \\ 0.8213611 \\ 2.023583 \\ 0.79 \\ 1.82 \\ 5000000 \\ 0.8213611 \\ 2.022338 \\ 0.79 \\ 1.82 \\ 5000000 \\ 0.82313611 \\ 2.022338 \\ 0.79 \\ 1.82 \\ 5000000 \\ 0.82313611 \\ 2.022338 \\ 0.84 \\ 0.80 \\ 1.83 \\ 5000000 \\ 0.8370211 \\ 2.021227 \\ 0.735 \\ 0.81 \\ 1.84 \\ 5000000 \\ 0.8448511 \\ 2.02065 \\ 0.82 \\ 1.85 \\ 5000000 \\ 0.800000 \\ 0.8526811 \\ 2.018903 \\ 0.85 \\ 0.84 \\ 1.87 \\ 5000000 \\ 0.8663110 \\ 2.017742 \\ 0.88 \\ 0.84 \\ 1.87 \\ 5000000 \\ 0.8663410 \\ 0.86 \\ 1.89 \\ 5000000 \\ 0.80863410 \\ 2.016580 \\ 0.82 \\ 0.88 \\ 1.91 \\ 5000000 \\ 0.8918310 \\ 0.10396 \\ 0.10396 \\ 0.8918310 \\ 2.01399 \\ 0.85 \\ 1.88 \\ 1.91 \\ 5000000 \\ 0.8918310 \\ 2.01399 \\ 0.10372 \\ 0.10722 \\ 0.88 \\ 1.91 \\ 5000000 \\ 0.8918310 \\ 0.10399 \\ 0.10728 \\ 0.92 \\ 0.193 \\ 0.96 \\ 0.92 \\ 1.95 \\ 5000000 \\ 0.9913509 \\ 0.90611 \\ 0.900000 \\ 0.9130809 \\ 0.901308 \\ 2.007288 \\ 0.94 \\ 0.95 \\ 1.98 \\ 5000000 \\ 0.938108 \\ 0.003000 \\ 0.913080 \\ 0.901308 \\ 0.003000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.90300000 \\ 0.9030000 \\ 0.9030000 \\ 0.9030000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.90300000 \\ 0.903000$							1.75		0.7	7743813	2	.030519
$\begin{array}{c} V29 \\ V30 \\ V30 \\ O.76 \\ O.76 \\ O.76 \\ O.76 \\ O.76 \\ O.77 \\ O.76 \\ O.77 \\ O.76 \\ O.77 \\ O.78 \\ O.78 \\ O.77 \\ O.77 \\ O.77 \\ O.78 \\ O.79 $				0.73			1.76	5000000	0.7	7822112	2	0.029357
$\begin{array}{c} \textbf{V}30 & 0.76 & 1.79 & 5000000 & 0.8057012 & 2.025873 \\ \textbf{V}31 & 0.77 & 1.80 & 5000000 & 0.8135312 & 2.024711 \\ \textbf{V}32 & 0.78 & 1.81 & 5000000 & 0.8213611 & 2.023550 \\ \textbf{V}33 & 0.79 & 1.82 & 5000000 & 0.8291911 & 2.0223850 \\ \textbf{V}34 & 0.80 & 1.83 & 5000000 & 0.8370211 & 2.021227 \\ \textbf{V}35 & 0.81 & 1.84 & 5000000 & 0.8448511 & 2.020065 \\ \textbf{V}36 & 0.82 & 1.85 & 5000000 & 0.8566811 & 2.018903 \\ \textbf{V}37 & 0.83 & 1.86 & 5000000 & 0.8605110 & 2.017742 \\ \textbf{V}38 & 0.84 & 1.87 & 5000000 & 0.8605110 & 2.017680 \\ \textbf{V}40 & 0.86 & 1.89 & 5000000 & 0.8761710 & 2.015419 \\ \textbf{V}41 & 0.87 & 1.90 & 5000000 & 0.8918310 & 2.014257 \\ \textbf{V}42 & 0.88 & 1.91 & 5000000 & 0.8918310 & 2.013096 \\ \textbf{V}43 & 0.89 & 1.92 & 5000000 & 0.896609 & 2.011934 \\ \textbf{V}44 & 0.90 & 1.93 & 5000000 & 0.9074909 & 2.010772 \\ \textbf{V}44 & 0.90 & 1.93 & 5000000 & 0.9153209 & 2.009611 \\ \textbf{V}45 & 0.91 & 1.94 & 5000000 & 0.9388108 & 2.006126 \\ \textbf{V}48 & 0.94 & 1.97 & 5000000 & 0.9388108 & 2.006126 \\ \textbf{V}48 & 0.94 & 1.97 & 5000000 & 0.9388108 & 2.006126 \\ \textbf{V}48 & 0.94 & 1.97 & 5000000 & 0.9388108 & 2.006126 \\ \textbf{V}48 & 0.94 & 1.97 & 5000000 & 0.934708 & 2.003408 \\ \textbf{V}50 & 0.96 & 1.99 & 532704 & 0.9615307 & 2.001563 \\ \textbf{V}51 & 0.97 & 2.00 & 5000000 & 0.970409 & 2.01053 \\ \textbf{V}52 & 0.98 & 2.01 & 5000000 & 0.974007 & 2.000163 \\ \textbf{V}55 & 0.99 & 2.02 & 465587 & 0.9875311 & 2.01563 \\ \textbf{V}55 & 1.01 & 2.04 & 5000000 & 0.9936207 & 1.997995 \\ \textbf{V}55 & 1.01 & 2.04 & 5000000 & 1.0014507 & 1.996834 \\ \textbf{V}56 & 1.02 & 2.05 & 5000000 & 1.0014507 & 1.996834 \\ \textbf{V}59 & 1.05 & 2.08 & 5000000 & 1.00249406 & 1.993349 \\ \textbf{V}59 & 1.05 & 2.08 & 5000000 & 1.0249406 & 1.993349 \\ \textbf{V}60 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.07 & 2.10 & 5000000 & 1.0249406 & 1.993849 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0481959 & 2.001563 \\ \textbf{V}61 & 1.06 & 2.09 & 483537 & 1.0$	V28			0.74			1.77	5000000	0.7	7900412	2	.028196
V31 0.77 1.80 5000000 0.8135312 2.024711 V32 0.78 1.81 5000000 0.8213611 2.02358 V33 0.79 1.82 5000000 0.8291911 2.022388 V34 0.80 1.83 5000000 0.8370211 2.021227 V35 0.81 1.84 5000000 0.8448511 2.02065 V36 0.82 1.85 5000000 0.8508811 2.018903 V37 0.83 1.86 5000000 0.8605110 2.017742 V38 0.84 1.87 5000000 0.8683410 2.016580 V39 0.85 1.88 5000000 0.8761710 2.015419 V40 0.86 1.89 5000000 0.8840010 2.014257 V41 0.87 1.90 5000000 0.8996609 2.011934 V42 0.88 1.91 5000000 0.8996609 2.011934 V43 0.89 1.92 5000000	V29			0.75			1.78	5000000	0.7	7978712	2	.027034
$\begin{array}{c} \mathbf{V}32\\ \mathbf{V}33\\ 0.79\\ 1.82\\ 0.80\\ 0.80\\ 1.83\\ 0.79\\ 0.82\\ 1.82\\ 0.500000\\ 0.8291911\\ 1.2022388\\ 0.84\\ 0.80\\ 0.83\\ 0.81\\ 0.81\\ 1.84\\ 5000000\\ 0.8348511\\ 1.2021027\\ 0.83\\ 0.81\\ 0.82\\ 1.85\\ 5000000\\ 0.8448511\\ 1.2020065\\ 0.82\\ 0.83\\ 0.84\\ 1.87\\ 5000000\\ 0.8668110\\ 0.2017742\\ 0.83\\ 0.84\\ 1.87\\ 5000000\\ 0.8668110\\ 0.2017742\\ 0.88\\ 0.84\\ 1.87\\ 5000000\\ 0.86683410\\ 0.2016580\\ 0.868410\\ 0.2016580\\ 0.868410\\ 0.2016580\\ 0.866\\ 1.89\\ 5000000\\ 0.8761710\\ 0.86410\\ 0.2015419\\ 0.86\\ 0.89\\ 0.85\\ 0.89\\ 0.99\\ $	V30			0.76			1.79	5000000	0.0	8057012	2	.025873
V33 0.79 1.82 5000000 0.8291911 2.022388 V34 0.80 1.83 5000000 0.8370211 2.021227 V35 0.81 1.84 5000000 0.8448511 2.02065 V36 0.82 1.85 5000000 0.8526811 2.018903 V37 0.83 1.86 5000000 0.8605110 2.017742 V38 0.84 1.87 5000000 0.8683410 2.016588 V39 0.85 1.88 5000000 0.8761710 2.015419 V40 0.86 1.89 5000000 0.8840010 2.015419 V41 0.87 1.90 5000000 0.8840010 2.014257 V41 0.87 1.90 5000000 0.8918310 2.01396 V42 0.88 1.91 5000000 0.8918310 2.011934 V43 0.89 1.92 5000000 0.9974909 2.01974 V44 0.90 1.93 5000000	V31			0.77			1.80	5000000	0.0	8135312	2	.024711
$\begin{array}{c} \mathbf{V}34 \\ \mathbf{V}35 \\ \mathbf{V}35 \\ \mathbf{V}36 \\ \mathbf{V}38 \\ \mathbf{V}37 \\ \mathbf{V}38 \\ \mathbf{V}39 \\ \mathbf{V}39 \\ \mathbf{V}39 \\ \mathbf{V}36 \\ \mathbf{V}39 \\ \mathbf{V}39 \\ \mathbf{V}36 \\ \mathbf{V}39 \\ \mathbf{V}40 \\ \mathbf{V}38 \\ \mathbf{V}40 \\ \mathbf{V}38 \\ \mathbf{V}40 \\ \mathbf{V}38 \\ \mathbf{V}41 \\ \mathbf{V}40 \\ \mathbf{V}41 \\ \mathbf{V}42 \\ \mathbf{V}43 \\ \mathbf{V}43 \\ \mathbf{V}43 \\ \mathbf{V}43 \\ \mathbf{V}43 \\ \mathbf{V}49 \\ \mathbf{V}44 \\ \mathbf{V}90 \\ \mathbf{V}90 \\ \mathbf{V}44 \\ \mathbf{V}90 \\ \mathbf{V}91 \\ \mathbf{V}44 \\ \mathbf{V}90 \\ \mathbf{V}91 \\ \mathbf{V}45 \\ \mathbf{V}49 \\ \mathbf{V}91 \\ \mathbf{V}45 \\ \mathbf{V}49 \\ \mathbf{V}91 \\ \mathbf{V}95 \\ \mathbf{V}49 \\ \mathbf{V}95 \\ \mathbf{V}99 \\ \mathbf{V}95 \\ \mathbf{V}99 \\ \mathbf{V}95 \\ \mathbf{V}99 \\ \mathbf{V}96 \\ \mathbf{V}99 \\ \mathbf{V}99 \\ \mathbf{V}90 \\ \mathbf{V}95 \\ \mathbf{V}99 \\ \mathbf{V}90 \\ \mathbf{V}95 \\ \mathbf{V}99 \\ \mathbf{V}90 \\ \mathbf{V}95 \\ \mathbf{V}90 \\ \mathbf{V}9$	V32			0.78			1.81	5000000	0.0	8213611	2	.023550
$\begin{array}{c} V35\\ V36\\ V36\\ O.82\\ O.82\\ O.83\\ O.83\\ O.83\\ O.84\\ O.84\\ O.84\\ O.85\\ O.85\\ O.85\\ O.85\\ O.85\\ O.86\\ O.85\\ O.85\\ O.86\\ O.85\\ O.86\\ O.85\\ O.85\\ O.86\\ O.85\\ O.86\\ O.85\\ O.86\\ O.85\\ O.86\\ O.86\\ O.87\\ O.86\\ O.86\\ O.87\\ O.86\\ O.88\\ O.88\\ O.88\\ O.89\\ O.$				0.79			1.82	5000000	0.0	8291911	2	.022388
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	V34			0.80			1.83	5000000	0.0	8370211	2	.021227
$\begin{array}{c} \textbf{V}37\\ \textbf{V}38\\ \textbf{V}38\\ \textbf{O}.84\\ \textbf{0}.84\\ \textbf{0}.84\\ \textbf{1}.87\\ \textbf{5}000000\\ \textbf{0}.8683410\\ \textbf{0}.2016580\\ \textbf{V}39\\ \textbf{0}.85\\ \textbf{1}.88\\ \textbf{5}000000\\ \textbf{0}.8761710\\ \textbf{2}.015419\\ \textbf{V}40\\ \textbf{0}.86\\ \textbf{1}.89\\ \textbf{5}000000\\ \textbf{0}.8840010\\ \textbf{0}.8840010\\ \textbf{2}.014257\\ \textbf{V}41\\ \textbf{0}.87\\ \textbf{1}.90\\ \textbf{5}000000\\ \textbf{0}.8918310\\ \textbf{2}.013096\\ \textbf{V}42\\ \textbf{0}.88\\ \textbf{1}.91\\ \textbf{5}000000\\ \textbf{0}.8996609\\ \textbf{2}.011934\\ \textbf{V}43\\ \textbf{0}.89\\ \textbf{1}.92\\ \textbf{5}000000\\ \textbf{0}.9974909\\ \textbf{2}.010772\\ \textbf{V}44\\ \textbf{0}.90\\ \textbf{0}.90\\ \textbf{1}.93\\ \textbf{5}000000\\ \textbf{0}.9153209\\ \textbf{2}.009611\\ \textbf{V}45\\ \textbf{0}.91\\ \textbf{1}.94\\ \textbf{5}000000\\ \textbf{0}.938109\\ \textbf{2}.008449\\ \textbf{V}46\\ \textbf{0}.92\\ \textbf{1}.95\\ \textbf{5}000000\\ \textbf{0}.9388108\\ \textbf{2}.007288\\ \textbf{V}47\\ \textbf{0}.93\\ \textbf{1}.96\\ \textbf{5}000000\\ \textbf{0}.9388108\\ \textbf{2}.006126\\ \textbf{V}48\\ \textbf{0}.94\\ \textbf{1}.97\\ \textbf{5}000000\\ \textbf{0}.9388108\\ \textbf{2}.006126\\ \textbf{V}49\\ \textbf{0}.95\\ \textbf{1}.98\\ \textbf{5}000000\\ \textbf{0}.97466408\\ \textbf{2}.004965\\ \textbf{V}49\\ \textbf{0}.95\\ \textbf{1}.98\\ \textbf{5}000000\\ \textbf{0}.9388108\\ \textbf{2}.006126\\ \textbf{V}49\\ \textbf{0}.95\\ \textbf{1}.98\\ \textbf{5}000000\\ \textbf{0}.9746408\\ \textbf{2}.004965\\ \textbf{V}50\\ \textbf{0}.96\\ \textbf{1}.99\\ \textbf{5}32704\\ \textbf{0}.9615307\\ \textbf{2}.001563\\ \textbf{V}51\\ \textbf{0}.97\\ \textbf{2}.00\\ \textbf{5}000000\\ \textbf{0}.9779607\\ \textbf{2}.0001563\\ \textbf{V}52\\ \textbf{0}.98\\ \textbf{2}.01\\ \textbf{5}000000\\ \textbf{0}.9779607\\ \textbf{2}.000188\\ \textbf{2}.001480\\ \textbf{V}52\\ \textbf{0}.98\\ \textbf{2}.01\\ \textbf{5}000000\\ \textbf{0}.9779607\\ \textbf{2}.000318\\ \textbf{V}53\\ \textbf{0}.99\\ \textbf{2}.02\\ \textbf{4}6587\\ \textbf{0}.9875311\\ \textbf{2}.001563\\ \textbf{V}55\\ \textbf{1}.01\\ \textbf{2}.04\\ \textbf{5}000000\\ \textbf{0}.9779607\\ \textbf{2}.000318\\ \textbf{V}55\\ \textbf{1}.01\\ \textbf{2}.04\\ \textbf{5}000000\\ \textbf{0}.9936207\\ \textbf{1}.997995\\ \textbf{V}55\\ \textbf{1}.01\\ \textbf{2}.04\\ \textbf{5}000000\\ \textbf{1}.0014507\\ \textbf{1}.996834\\ \textbf{V}56\\ \textbf{1}.02\\ \textbf{2}.05\\ \textbf{5}000000\\ \textbf{1}.0044507\\ \textbf{1}.996834\\ \textbf{V}59\\ \textbf{1}.03\\ \textbf{2}.06\\ \textbf{2}.09\\ \textbf{4}83537\\ \textbf{1}.0481959\\ \textbf{2}.001563\\ \textbf{V}59\\ \textbf{1}.05\\ \textbf{2}.08\\ \textbf{5}000000\\ \textbf{1}.0484306\\ \textbf{1}.993349\\ \textbf{V}62\\ \textbf{1}.08\\ \textbf{2}.11\\ \textbf{5}000000\\ \textbf{1}.0484306\\ \textbf{1}.989864\\ \textbf{V}62\\ \textbf{1}.08\\ \textbf{2}.11\\ \textbf{5}000000\\ \textbf{1}.0484306\\ \textbf{1}.989863\\ \textbf{1}.988703\\ \textbf{1}.988703\\$	V35			0.81			1.84	5000000	0.0	8448511	2	.020065
$\begin{array}{c} \mathbf{V}38 \\ \mathbf{V}39 \\ \mathbf{V}39 \\ \mathbf{V}39 \\ \mathbf{V}40 \\ \mathbf{V}40 \\ \mathbf{V}40 \\ \mathbf{V}40 \\ \mathbf{V}40 \\ \mathbf{V}41 \\ \mathbf{V}41 \\ \mathbf{V}41 \\ \mathbf{V}41 \\ \mathbf{V}41 \\ \mathbf{V}42 \\ \mathbf{V}42 \\ \mathbf{V}42 \\ \mathbf{V}42 \\ \mathbf{V}43 \\ \mathbf{V}42 \\ \mathbf{V}43 \\ \mathbf{V}43 \\ \mathbf{V}44 \\ \mathbf{V}44 \\ \mathbf{V}44 \\ \mathbf{V}44 \\ \mathbf{V}44 \\ \mathbf{V}44 \\ \mathbf{V}45 \\ \mathbf{V}44 \\ \mathbf{V}45 \\ \mathbf{V}45 \\ \mathbf{V}45 \\ \mathbf{V}45 \\ \mathbf{V}45 \\ \mathbf{V}46 \\ \mathbf{V}47 \\ \mathbf{V}49 \\ \mathbf{V}46 \\ \mathbf{V}49 \\ \mathbf{V}46 \\ \mathbf{V}49 \\ \mathbf{V}4$	V36			0.82			1.85	5000000	0.0	8526811	2	.018903
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	V37			0.83			1.86	5000000	0.0	8605110	2	.017742
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				0.84			1.87	5000000	0.0	8683410	2	.016580
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				0.85			1.88	5000000	0.0	8761710	2	.015419
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V57 1.03 2.06 5000000 1.0171106 1.994511 V58 1.04 2.07 5000000 1.0249406 1.993349 V59 1.05 2.08 5000000 1.0327706 1.992187 V60 1.06 2.09 483537 1.0481959 2.001563 V61 1.07 2.10 5000000 1.0484306 1.989864 V62 1.08 2.11 5000000 1.0562605 1.988703												
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V63 1.09 2.12 5000000 1.0640905 1.987541												
	V63			1.09			2.12	5000000	1.0)640905	1	.987541

	$\operatorname{start}_{_}$	$_{ m value}$	_theta0	$\operatorname{start}_{_}$	$_{ m value}$	$_{ m theta1}$	numOfIteratio	ons	$output_theta0$	output_thetal
V64			1.10			2.13	50000	000	1.0719205	1.986380
V65			1.11			2.14	50000	000	1.0797505	1.985218
V66			1.12			2.15	50000	000	1.0875805	1.984056
V67			1.13			2.16	50000	000	1.0954104	1.982895
V68			1.14			2.17	50000	000	1.1032404	1.981733
V69			1.15			2.18	50000	000	1.1110704	1.980572
V70			1.16			2.19	50000	000	1.1189004	1.979410
V71			1.17			2.20	50000	000	1.1267304	1.978249
V72			1.18			2.21	50000	000	1.1345603	1.977087
V73			1.19			2.22	50000	000	1.1423903	1.975926
V74			1.20			2.23	50000	000	1.1502203	1.974764
V75			1.21			2.24	50000	000	1.1580503	1.973602
V76			1.22			2.25	50000	000	1.1658803	1.972441
V77			1.23			2.26	50000	000	1.1737102	1.971279
V78			1.24			2.27	50000	000	1.1815402	1.970118
V79			1.25			2.28	50000	000	1.1893702	1.968956
V80			1.26			2.29	50000	000	1.1972002	1.967795
V81			1.27			2.30	50000	000	1.2050302	1.966633
V82			1.28			2.31	50000	000	1.2128601	1.965472
V83			1.29			2.32	50000	000	1.2206901	1.964310
V84			1.30			2.33	50000	000	1.2285201	1.963148
V85			1.31			2.34	50000	000	1.2363501	1.96198
V86			1.32			2.35	50000	000	1.2441801	1.960825
V87			1.33			2.36	50000	000	1.2520100	1.959664
V88			1.34			2.37	50000	000	1.2598400	1.958502
V89			1.35			2.38	50000	000	1.2676700	1.957341
V90			1.36			2.39	50000	000	1.2755000	1.956179
V91			1.37			2.40	50000	000	1.2833300	1.955018
V92			1.38			2.41	50000	000	1.2911599	1.953856
V93			1.39			2.42	50000	000	1.2989899	1.952694
V94			1.40			2.43	50000	000	1.3068199	1.951533
V95			1.41			2.44	50000	000	1.3146499	1.950371
V96			1.42			2.45	50000	000	1.3224799	1.949210
V97			1.43			2.46	50000	000	1.3303098	1.948048
V98			1.44			2.47	50000		1.3381398	1.94688
V99			1.45			2.48	50000		1.3459698	1.945725
V100			1.46			2.49	50000		1.3537998	1.944563

```
result3[result3[, 3] == min(result3[, 3]), ]
```

```
## start_value_theta0 start_value_theta1 numOfIterations
## 9.900000e-01 2.020000e+00 4.655870e+05
## output_theta0 output_theta1
## 9.875311e-01 2.001563e+00
```

The least number of iteration is 4.655870e+05, the starting values (theta0 = 0.99, theta1 = 2.02) are not the closest one to the true value (0.9696, 2.002). So, the closer starting values do not mean the shorter time.