UNIVERSITY OF NEW SOUTH WALES
DEPARTMENT OF STATISTICS

MATH5856 Introduction to Statistics and Statistical Computations

Tutorial Problems week 9 (Random Samples)

1. Let $X_1, X_2, \dots, X_{25}$ be independent $N(1,4)$ variables. Let $\overline{X}$ and S^2 be the sample mean and variance.

   (a) Find $P\left(\sum_{i=1}^{25} X_i > 30\right)$.

   (b) Find $P(\overline{X} < 0)$.

   (c) Find k_1 such that $P(\overline{X} < k_1) = 0.3$.

   (d) Find k_2 such that $P\left(\frac{\overline{X}-1}{S} < k_2\right) = 0.99$.

2. Consider a random sample of size 500 from a normal distribution with mean 30 and variance 16. Show that one should expect about 494 observations to be between 20 and 40.

3. Using R or find from tables

   (a) $P(Q < 3.05)$ where $Q \sim \chi^2_{11}$,

   (b) $P(Q < 24.77)$ where $Q \sim \chi^2_{17}$,

   (c) $P(Q > 0.484)$ where $Q \sim \chi^2_{4}$,

   (d) k such that $P(Q < k) = 0.95$ where $Q \sim \chi^2_{15}$.

4. With independent standard normal variables Z_i,

(a) find the value k such that $\sum_{i=1}^{6} Z_i^2$ exceeds k 5% of the time,

(b) calculate, as accurately as tables permit, the probability that $\sum_{i=1}^{6} Z_i^2$ exceeds 16.81.

5. If $X_1, X_2, \ldots, X_{10}$ constitute a random sample from a $N(2, 25)$ distribution, find the values a and b such that with probabilities 0.01 $\frac{1}{10}\sum_{i=1}^{10}(X_i - 2)^2 > a$ and $\frac{1}{9}\sum_{i=1}^{10}(X_i - \overline{X})^2 > b$.

6. Let $X_1, X_2, \ldots, X_{25}$ be independent $N(2, 4)$ random variables with sample mean $\overline{X}$ and sample variance S^2. Calculate $P(\overline{X} > 1)$ and find the constant k so that $P(\overline{X} < kS + 2) = 0.95$.

7. Suppose $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are two independent random samples of independent $N(0, 1)$ random variables. Determine the distributions of

   (a) $\sum_{i=1}^{n} X_i^2$,

   (b) $\frac{\sum_{i=1}^{n} X_i}{\sqrt{\sum_{i=1}^{n} Y_i^2}}$,

   (c) $\frac{nX_1^2}{\sum_{i=1}^{n} Y_i^2}$.

8. Suppose Z_1, Z_2, Z_3, Z_4 are independent $N(0,1)$ random variables. Determine the constants a and b such that

$$P\left(\sum_{i=1}^{4} Z_i^2 < a\right) = 0.9 \text{ and } P(Z_1^2 + Z_2^2 < b(Z_3^2 + Z_4^2)) = 0.95.$$

9. Suppose $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples with $X_i \sim N(\mu, 1)$ and $Y_i \sim N(0,1)$. Determine the distributions of

$$\frac{1}{n}\sum_{i=1}^{n} X_i \ , \ \sum_{i=1}^{n}(X_i - \mu)^2 + \sum_{i=1}^{n} Y_i^2 \text{ and } \sum_{i=1}^{n}(X_i - \overline{X})^2.$$