MATH5905Inference MLE

Week7 Likelihood Inference

5.1 What is asymptotics

> We want to observe the performance of MLE's when $n \to \infty$ by having hugh amount of information

5.5.2 Delta Method

Definition: delta method

$$\sqrt{\pi}(h(\hat{\theta}_{MLE}) - h(\theta_0)) \to N(0, [\frac{\partial h(\theta_0)}{\partial \theta}]^2 I^{-1}(\theta_0))$$

Distribution of $h(\hat{\theta}_{mle})$

$$h(\hat{\theta}_{mle}) \approx N(h(\theta_0), \frac{1}{n}[\frac{\partial h(\theta_0)}{\partial \theta}])^2 I^{-1}(\theta_0)$$

常見的三大類型統計理論

5.5.4 deviance

$$D(\theta) = -2log\left(\frac{L(X,\theta)}{L(X,\hat{\theta}_{mle})}\right)$$

> deviance is a **quality-of-fit statistic** for a model that is often used for statistical hypothesis testing. (wikipedia)

Confusion

1. **What's the motivaton of trying to find the distribution of $D(\mu_0)$?**
   Answer:

   $$\Lambda = \frac{L(X_1, \theta_0)}{L(X_1, \hat{\theta}_{mle})} \in (0, 1)$$

   If the value is closer to 0. it shows the data can't support us to find the true value θ_0
   So we can carry out hypothesis testing based on setting a value, let's say ξ, if $\theta_0 < \xi$, we reject, otherwise, we accept.But people may try many times with altering the value of ξ.
   In order to amplify, we use log to the original ratio:

   $$-2ln\Lambda = -2ln\ \frac{L(X_1, \theta_0)}{L(X_1, \hat{\theta}_{mle})} \in (0, \infty)$$

   Now (η, ∞) becomes reject region and $(0, \eta)$ is accept region.
   Once we have the distribution of deviance, we can easily construct our test.
2. **I try to understand concept of deviance, as variance describes the extent of data spreading, what does deviance try to depict?**