Chapter 4. Hypothesis Testing.

4.1 What's hypothesis?

4.3 Uniform & powerful test. (ump).

4.3.1 Monotone likelihood Ratio.

Def:

Definition *The family of densities* $\{f(x;\theta),\ \theta \in \Theta \subseteq \mathbb{R}\}$ *with real scalar parameter* θ *is said to be of* monotone likelihood ratio *(MLR for short) if there exists a function* $t(x)$ *such that the likelihood ratio*

$$\frac{f(x;\theta_2)}{f(x;\theta_1)}$$

Likelihood Function

is a non-decreasing function of $t(x)$ *whenever* $\theta_1 \leq \theta_2$.

NoTE: Step1 就是找到 $f(x,\theta)$ 的 likelihood function, 并将 θ_1, θ_2 代入. 小在下, 大在上.

Ex:

$$\frac{L(X,\theta_b)}{L(X,\theta_a)} = \frac{\cancel{2^n}\cdot\theta_b^{2n}}{\cancel{\prod_{i=1}^{n} X_i^3}} I_{(0,X_{(1)})}(\theta_b) \cdot \frac{1}{\frac{\cancel{2^n}\cdot\theta_a^{2n}}{\cancel{\prod_{i=1}^{n} X_i^3}} I_{(0,X_{(1)})}(\theta_a)}$$

$$= \frac{\theta_b^{2n} I_{(0,X_{(1)})}(\theta_b)}{\theta_a^{2n} I_{(0,X_{(1)})}(\theta_a)} \quad (1)$$

We have three Range for $X_{(1)}$ to lay in, We need to prove it's Increasing.

$(0,\theta_a)$, Formula (1) is undefined.

$[\theta_a, \theta_b)$ Formula (1) is 0.

$[\theta_b, \infty)$ Formula (1) is larger than 0

So $T = X_{(1)}$ is a non-decreasing Function.