Optimal Pricing and Energy Scheduling for Hybrid Energy Trading Market in Future Smart Grid

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***Abstract*—Future smart grid (SG) has been considered a complex and advanced power system, where energy consumers are connected not only to the traditional energy retailers (e.g., the utility companies), but also to some local energy networks for bidirectional energy trading opportunities. This paper aims to investigate a hybrid energy trading market that is comprised of an external utility company and a local trading market managed by a local trading center (LTC). The existence of local energy market provides new opportunities for the energy consumers and the distributed energy sellers to perform the local energy trading in a cooperative manner such that they all can benefit. This paper first quantifies the respective benefits of the energy consumers and the sellers from the local trading and then investigates how they can optimize their benefits by controlling their energy scheduling in response to the LTC's pricing. Two different types of the LTC are considered: 1) the nonprofit-oriented LTC, which solely aims at benefiting the energy consumers and the sellers; and 2) the profit-oriented LTC, which aims at maximizing its own profit while guaranteeing the required benefit for each consumer and seller. For each type of the LTC, the optimal trading problem is formulated and the associated algorithm is further proposed to efficiently find the LTC's optimal price, as well as the optimal energy scheduling for each consumer and seller. Numerical results are provided to validate the benefits of the hybrid energy trading market and the performance of the proposed algorithms.**

***Index Terms*—Demand response management, energy trading, hybrid market, monotonic optimization, optimal pricing.**

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I. Introduction

SMART GRID (SG) has been considered as a complex yet advanced power system capable of meeting the growing energy demand in a reliable, sustainable, and economical manner [1]. Facilitated by its advanced two-way communication infrastructure, future SG is featured by its efficient demand response mechanism (DRM), based on which the energy consumers and providers can flexibly schedule their energy consumption and energy supply, respectively, for achieving better performance than in the traditional power grid. The role of DRM is expected to be even more important, as the paradigm of bidirectional energy trading prevails in future SG. In particular, the wide exploitation of distributed energy sources and the deployment of smart metering systems in future SG enable the bidirectional energy trading, which thus requires a careful design of control mechanism for both the bidirectional energy scheduling and the associated economic reward optimization.

At present, there have been several related works investigating the energy trading mechanism and the associated DRM. In general, these works can be categorized into two main streams. The first stream of works focused on investigating the optimal energy consumption scheduling of energy consumers in response to the pricing of retail market. Specifically, the authors in [2] proposed a residential energy consumption scheduling framework for each household, which traded off between minimizing the electricity payment and the disutility of its appliances due to the delayed operations. The authors in [3] focused on optimizing an end-user's energy consumption scheduling for minimizing its energy acquisition cost while meeting the delay constraints for its appliances. Considering the coexistence of a large number of energy consumers, the authors in [4] and [5] proposed noncooperative games for a group of energy consumers who selfishly optimized his/her own energy consumption scheduling to minimize the individual electricity bill. The authors in [6] considered a special group of energy consumers, who have the energy storage devices and can supply the stored energy back to the grid, and proposed the corresponding load scheduling algorithms for those consumers to optimize their payoffs.

Based on the consumers' price-dependent energy consumption scheduling, the second stream of works further investigated the pricing strategy of energy retailers (e.g., the utility companies) for optimizing certain objectives. In [7], the authors first derived the energy users' price-dependent consumption scheduling to optimize their own payoffs, and then proposed an algorithm based on simulated annealing to optimize the pricing of the monopolistic utility company for reducing the peak power load and its variation. In [8], the authors further considered multiple utility companies and energy users, and modeled their interactions as a two-level game. Specifically, in the bottom level, the energy users chose their demands based on an evolutionary game, in response to the pricing of the utility companies. Meanwhile, in the top level, the utility companies competitively chose their energy prices as a noncooperative game based on the users' demands. Similar to the two-level approach, in [9], the authors proposed a Stackelberg game, a type of the two-stage dynamic games between multiple utility companies and multiple energy users, to optimize the revenue of each utility company and the payoff of each user. The authors in [10] also used the Stackelberg game to model the interaction between the energy users and the utility company and took the uncertainty of the users' demand into account. In [11], the authors considered using the electric vehicles to provide the frequency regulation in a vehicle-to-grid market, and proposed a smart pricing policy to motivate the electric vehicles to participate in the frequency regulation. The authors in [12] used the auction theory to optimize the energy exchange between the electric vehicles and the SG. In addition, the authors in [13] adopted the network utility optimization theory to design the demand response for achieving supply–demand balance.

The aforementioned works mainly considered the energy trading between the energy consumers and the external retail market. In fact, future SG is envisioned to be a hybrid system, in which energy consumers are connected not only to the external retailer but also to some local energy networks (i.e., the so-called local *ElectriNets*) for new energy trading opportunities [16], [17]. Specifically, the local energy networks facilitate the local energy transactions between the energy consumers and the distributed energy sellers (ESs) in an economical and efficient manner, and thus are expected to benefit all the participants. A typical paradigm of such local networks is the microgrid [1], in which the residential homes and/or the businesses are equipped with on-site energy supplies (e.g., the distributed energy generations and the distributed storages such as electric vehicles) and can perform the local energy exchanges with each other, as a supplement to the energy supply from the main grid. However, so far, there still lacks a knowledge about how and how much the energy consumers and the distributed ESs can benefit from the local energy trading compared to trading with the external retail market only, by taking into account both the potential local energy transmission loss and the compulsory demand–supply balance constraint in the local market. Therefore, this paper focuses on providing answers to these questions.

The main contributions of this paper are summarized as follows. This paper first models a hybrid energy trading market that comprises an external utility company (EUC) and a local trading market managed by a local trading center (LTC). Based on this market model, the energy consumers' and sellers' respective benefits from the local trading are quantified, and then the associated optimization problem is formulated to model how the energy consumers and sellers optimize their respective benefits by controlling their energy scheduling, in response to the LTC's pricing in the local market. To account for the influence of the LTC, two different types of LTC are

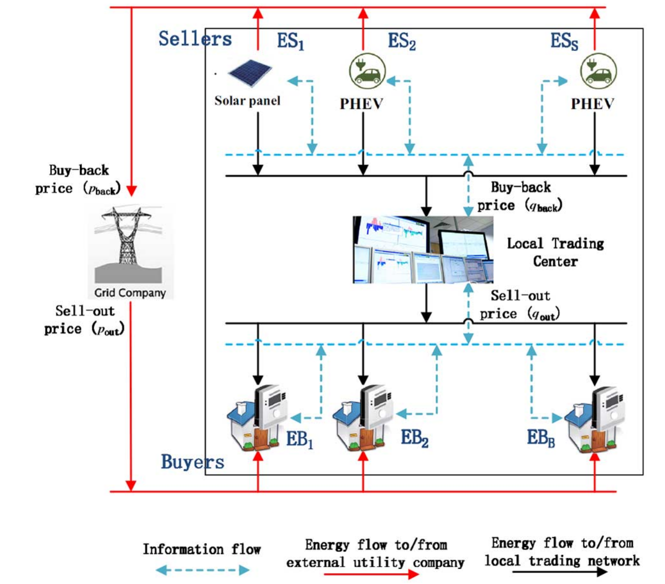

Fig. 1. Hybrid energy trading market comprising an EUC and an LTC. To meet its energy demand, each energy buyer (EB) can buy energy from both the EUC and the LTC. Each ES can sell its energy surplus to both the EUC and the LTC.

considered: 1) the nonprofit-oriented LTC, which solely aims at benefiting the energy consumers and sellers; and 2) the profit-oriented LTC, which aims at maximizing its own profit while guaranteeing the required net-gain for each energy consumer and seller. For each type of the LTC, the associated optimal trading problem is analyzed to explore its special property. Based on these properties, algorithms are proposed to efficiently find the LTC's optimal prices as well as the energy consumers' and sellers' optimal energy scheduling.

This paper is organized as follows. The hybrid trading market is described in Section II. Then, the energy trading managed by the nonprofit-oriented LTC and the profit-oriented LTC are investigated in Sections III and IV, respectively. For the sake of clear organization, the numerical results for the model of nonprofit-oriented LTC are presented in Section III-C and those for the profit-oriented LTC are presented in Section IV-D. Section V concludes this paper finally.

II. Hybrid Energy Trading Market and Model of EBs and Sellers

A. Hybrid Energy Trading Market

This section presents the hybrid energy trading market, whose illustrative example is shown in Fig. 1. Specifically, there are two types of energy users in this hybrid market: 1) a set of energy consumers or equivalent EBs, which are denoted by $\Omega_b = \{1, 2, \ldots, B\}$; and 2) a set of local ESs, which are denoted by $\Omega_s = \{1, 2, \ldots, S\}$ (in Fig. 1, the solar power panel and the electric vehicle are used as two illustrative examples for representing the possible kinds of ESs, and they are treated as the same in terms of modeling the ESs' behaviors in Section II-C). This paper focuses on a fixed period of interest (e.g., 1 h), in which each EB $i \in \Omega_b$ acquires a certain amount of energy to meet its energy demand denoted by d_i, and each

ES $j \in \Omega_s$ has a certain amount of energy surplus denoted by $\tilde{d}_j$ to sell in this hour for economic reward. Notice that similar model of one fixed period also appeared in [8], [9], and [12].

In the hybrid market, each EB and ES are connected to both an external retailer, which is represented by the EUC, and a local trading market, which is managed by the LTC. Specifically, to meet its energy demand, each EB can buy energy from both the EUC, whose energy sell-out price is p_{out}, and the LTC, whose energy sell-out price is q_{out}. Meanwhile, each ES can sell its energy surplus to both the EUC, whose energy buy-back price is p_{back}, and the LTC, whose buy-back price is q_{back}.

B. Modeling of the EBs

This section describes the model of each EB. Specifically, for each EB $i \in \Omega_b$, let x_i denote the amount of energy it buys from the EUC, and let y_i denote the amount of energy it buys from the LTC. To meet its energy demand d_i, each EB i requires

$$x_i + (y_i - f_i(y_i)) = d_i \quad \forall i \in \Omega_b \tag{1}$$

to hold, where function $f_i(y_i)$ accounts for the energy transmission loss when EB i acquires y_i from the LTC. Recall that the EUC and the LTC use p_{out} and q_{out} to charge EB i for the energy supply, respectively. Thus, each EB i's total energy acquisition cost is given by[1]

$$C_i(x_i, y_i) = x_i p_{\text{out}} + y_i q_{\text{out}} \quad \forall i \in \Omega_b. \tag{2}$$

Meanwhile, consider *a benchmark case in which each EB is not allowed to trade with the LTC*. In this benchmark case, the cost of EB i is simply given by

$$C_i(d_i, 0) = d_i p_{\text{out}} \quad \forall i \in \Omega_b.$$

Compared to (2), the net-gain (i.e., the reduced cost) of each EB i when it buys energy from the LTC is given by

$$\begin{aligned} G_i(x_i, y_i) &= C_i(d_i, 0) - C_i(x_i, y_i) \\ &= p_{\text{out}}(d_i - x_i) - q_{\text{out}} y_i \quad \forall i \in \Omega_b. \end{aligned} \tag{3}$$

Notice that each EB i expects to obtain a nonnegative net-gain from the local energy trading, i.e., each EB i requires

$$G_i(x_i, y_i) \geq 0 \quad \forall i \in \Omega_b. \tag{4}$$

According to [14] and [15], the energy transmission loss in a local energy network mainly stems from the resistance effect of the transmission line, and this loss can be quantified by a quadratic function $f_i(y_i) = a_i(y_i)^2 + b_i y_i$ as proposed in [8], [14], and [15], where a_i and b_i are two fixed parameters. In this work, the same quadratic function is used to take into account the energy transmission loss when the EBs and ESs perform the local energy trading with the LTC.

[1] In this model, the EBs and ESs are assumed to bear the cost of the energy transmission loss when performing the energy trading with the LTC. This assumption is reasonable because 1) the EBs and ESs benefit from the local trading and thus need to afford to the consequent costs; and 2) the LTC, which manages the local energy trading, could be nonprofit-oriented and solely aims at benefiting the EBs and ESs (i.e., the nonprofit-oriented LTC considered in Section III). In this case, the EBs and ESs should bear the energy transmission loss.

C. Modeling of the ESs

This section describes the model of each ES. For each ES $j \in \Omega_s$, let $\tilde{x}_j$ denote the amount of energy sold to the EUC, and let $\tilde{y}_j$ denote the amount of energy sold to the LTC. Recall that each EU j has a fixed energy surplus $\tilde{d}_j$, which limits its total energy to sell. To meet this constraint, each ES j requires

$$\tilde{x}_j + (\tilde{y}_j + \tilde{f}_j(\tilde{y}_j)) \leq \tilde{d}_j \quad \forall j \in \Omega_s. \tag{5}$$

In (5), function $\tilde{f}_j(\tilde{y}_j)$ accounts for the energy transmission loss when ES j sells its energy to the LTC with the amount equal to $\tilde{y}_j$. Recall that the EUC and the LTC use the buy-back prices p_{back} and q_{back}, respectively, to pay ES j for its energy supply. Thus, each ES j's total income is given by

$$I_j(\tilde{x}_j, \tilde{y}_j) = p_{\text{back}} \tilde{x}_j + q_{\text{back}} \tilde{y}_j \quad \forall j \in \Omega_s. \tag{6}$$

Again, consider *a benchmark case in which each ES is not allowed to trade with the LTC*. In this benchmark case, the maximum income of each ES j is given by

$$I_j(\tilde{d}_j, 0) = p_{\text{back}} \tilde{d}_j \quad \forall j \in \Omega_s.$$

Compared to (6), the net-gain (i.e., the increased income) of each ES j when it sells energy to the LTC is given by

$$\begin{aligned} \tilde{G}_j(\tilde{x}_j, \tilde{y}_j) &= I_j(\tilde{x}_j, \tilde{y}_j) - I_j(\tilde{d}_j, 0) \\ &= p_{\text{back}}(\tilde{x}_j - \tilde{d}_j) + q_{\text{back}} \tilde{y}_j \quad \forall j \in \Omega_s. \end{aligned} \tag{7}$$

Notice that each ES j expects to receive a nonnegative net-gain, i.e., each ES j requires

$$\tilde{G}_j(\tilde{x}_i, \tilde{y}_j) \geq 0 \quad \forall j \in \Omega_s. \tag{8}$$

D. A Brief Introduction About Different Types of LTC

The LTC controls the energy trading with the EBs and ESs in the local market by adjusting its prices $(q_{\text{back}}, q_{\text{out}})$. Meanwhile, in response to the LTC's prices $(q_{\text{back}}, q_{\text{out}})$ as well as the EUC's external prices $(p_{\text{back}}, p_{\text{out}})$, each EB i determines its energy demand decisions $\{x_i, y_i\}$, and each ES j determines its energy supply decisions $\{\tilde{x}_i, \tilde{y}_j\}$ for optimizing their respective net-gains. In particular, as a first step to understand the hybrid energy trading market, this work focuses on investigating how the LTC can adjust its prices $(q_{\text{back}}, q_{\text{out}})$ to benefit the EBs and ESs by taking into account the energy trading responses of the EBs and ESs, while treating the EUC's external prices $(p_{\text{back}}, p_{\text{out}})$ as fixed.

More specifically, to account for the influence of the LTC's pricing, two different types of the LTC are investigated in the rest of this work. Specifically, Section III considers the nonprofit-oriented LTC, while Section IV considers the profit-oriented LTC. The details are presented in the next two sections.

III. Local Energy Trading Managed by the Nonprofit-Oriented LTC

A. Problem Formulation

This section considers the nonprofit-oriented LTC, which does not aim at making any profit from the local trading. In other words, the nonprofit-oriented LTC controls its $(q_{\text{back}}, q_{\text{out}})$ to benefit the EBs and ESs solely. Nevertheless, from the LTC's point of view, its $(q_{\text{back}}, q_{\text{out}})$ should at least guarantee that it does not suffer from any economic loss, i.e., the LTC requires

$$q_{\text{out}} \sum_{i\in\Omega_b} y_i \geq q_{\text{back}} \sum_{j\in\Omega_s} \tilde{y}_j. \tag{9}$$

Given the LTC's $(q_{\text{back}}, q_{\text{out}})$, each EB i determines its energy demand decisions $\{x_i, y_i\}$, and each ES j determines its energy supply decisions $\{\tilde{x}_j, \tilde{y}_j\}$ for optimizing their respective net-gains. In particular, their energy scheduling decisions should meet the demand–supply balance at the LTC as follows:

$$\sum_{i\in\Omega_b} y_i = \sum_{j\in\Omega_s} \tilde{y}_j. \tag{10}$$

Incorporating (9) and (10) and the aforementioned constraints in Section II, the following optimal energy trading problem is formulated when the LTC is nonprofit-oriented (the capital letters "NPO" stand for "nonprofit-oriented")

$$\text{(NPO): } \max \sum_{i\in\Omega_b} U_i(G_i(x_i, y_i)) + \sum_{j\in\Omega_s} \tilde{U}_j(\tilde{G}_j(\tilde{x}_i, \tilde{y}_j))$$

$$\text{subject to: Constraints } (1), (4), (5), (8), (9), (10)$$

$$p_{\text{out}} \geq q_{\text{out}} \tag{11}$$

$$q_{\text{back}} \geq p_{\text{back}} \tag{12}$$

$$\text{variables: } (q_{\text{back}}, q_{\text{out}}), \{x_i, y_i\}_{i\in\Omega_b}, \{\tilde{x}_j, \tilde{y}_j\}_{j\in\Omega_s}.$$

In Problem (NPO), function $U_i(\cdot)$ denotes the utility (or preference) of EB i associated with its net-gain $G_i(x_i, y_i)$, and function $\tilde{U}_j(\cdot)$ denotes the utility (or preference) of ES j associated with its net-gain $\tilde{G}_j(\tilde{x}_i, \tilde{y}_j)$. Constraint (11) means that the LTC's sell-out price cannot exceed the EUC's sell-out price such that each EB is willing to trade with the LTC. Similarly, Constraint (12) means that the LTC's buy-back price cannot be lower than the EUC's buy-back price such that each ES is willing to trade with the LTC. The decision variables in Problem (NPO) include three groups, i.e., 1) the LTC's prices $(q_{\text{back}}, q_{\text{out}})$; 2) each EB i's energy demand decisions $\{x_i, y_i\}$; and 3) each ES j's energy supply decisions $\{\tilde{x}_j, \tilde{y}_j\}$. In the rest of this work, $(q^*_{\text{back}}, q^*_{\text{out}})$, $\{x^*_i, y^*_i\}_{\forall i\in\Omega_b}$, and $\{\tilde{x}^*_j, \tilde{y}^*_j\}_{\forall j\in\Omega_s}$ are used to denote the optimal solutions for Problem (NPO), with the superscript "$*$" denoting the optimality. In particular, we emphasize that Problem (NPO), which involves the three groups of decisions variables aforementioned, is a typical nonconvex optimization and thus is difficult to solve directly. Nevertheless, Problem (NPO) is always feasible, because all the constraints in Problem (NPO) can be met by setting each EB i's $y_i = 0$, and each ES j's $\tilde{y}_j = 0$, i.e., without invoking any local trading.

Remark 1: This work assumes that for each EB i (or ES j), its preference function $U_i(\cdot)$ [or $\tilde{U}_j(\cdot)$] is continuously increasing and concave, with $U_i(0) = 0$ [or $\tilde{U}_j(0) = 0$]. The increasing property follows the rationale that each EB (or each ES) is happier when receiving a greater net-gain. The concave property means that each EB's (or each ES's) marginal preference in getting a greater net-gain gradually decreases. Notice that [2], [4], [6]–[9], [11], and [13] also adopted the similar preference functions.

Before leaving this section, it is worth pointing out that the nonprofit-oriented LTC in fact corresponds to the most positive case that the EBs and ESs can expect, and the corresponding optimal total welfare represents a performance upper bound (or a benchmark) that the EBs and ESs can potentially achieve by performing the local energy trading.

B. Property of Problem (NPO) and Proposed Algorithm for Achieving the Optimal Solutions

This section first identifies the important properties of Problem (NPO) and then proposes an efficient algorithm to solve Problem (NPO) based on these properties. Specifically, the following two properties hold for Problem (NPO).

Property 1: At the optimum of Problem (NPO), Constraint (5) is strictly binding.

Property 2: At the optimum of Problem (NPO), $q^*_{\text{out}} = q^*_{\text{back}}$ always holds.

Properties 1 and 2 can be proved via showing contradiction, and their details are skipped due to space limitation. In particular, Property 2 means that the nonprofit-oriented LTC earns nothing from the local trading, which is consistent with the fact that the nonprofit-oriented LTC only aims at benefiting the EBs and ESs.

Using Property 1, each ES j's $\tilde{x}_j$ can be substituted by $\tilde{y}_j$ to simplify the net-gain of ES j as follows:

$$\tilde{G}_j(y_j) = (q_{\text{back}} - p_{\text{back}})\tilde{y}_j - p_{\text{back}}\tilde{f}_j(\tilde{y}_j). \tag{13}$$

Similarly, each EB i's x_i can be substituted by y_i to simplify the net-gain of EB i as follows:

$$G_i(y_i) = (p_{\text{out}} - q_{\text{out}})y_i - p_{\text{out}}f_i(y_i). \tag{14}$$

Using (13), (14), and Property 2, Problem (NPO) can be equivalently transformed into the following optimization problem (the capital letter "E" standards for "Equivalence")

$$\text{(NPO-E): } \max \sum_{i\in\Omega_b} U_i(z_i) + \sum_{j\in\Omega_s} \tilde{U}_j(\tilde{z}_j)$$

$$\text{subject to: } (p_{\text{out}} - q)y_i - p_{\text{out}}f_i(y_i) \geq z_i \geq 0 \quad \forall i \in \Omega_b \tag{15}$$

$$(q - p_{\text{back}})\tilde{y}_j - p_{\text{back}}\tilde{f}_j(\tilde{y}_j) \geq \tilde{z}_j \geq 0 \quad \forall j \in \Omega_s \tag{16}$$

$$d_i \geq y_i - f_i(y_i) \geq 0 \quad \forall i \in \Omega_b \tag{17}$$

$$\tilde{d}_j \geq \tilde{y}_j + \tilde{f}_j(\tilde{y}_j) \geq 0 \quad \forall j \in \Omega_s \tag{18}$$

$$\sum_{i\in\Omega_b} y_i = \sum_{j\in\Omega_s} \tilde{y}_j \tag{19}$$

$$p_{\text{out}} \geq q \geq p_{\text{back}} \tag{20}$$

$$\text{variables: } q, \{y_i, z_i\}_{i\in\Omega_b}, \{\tilde{y}_j, \tilde{z}_i\}_{j\in\Omega_s}.$$

In Problem (NPO-E), the decision variables z_i and $\tilde{z}_j$ represent the achieved net-gains of EB i and ES j, respectively.

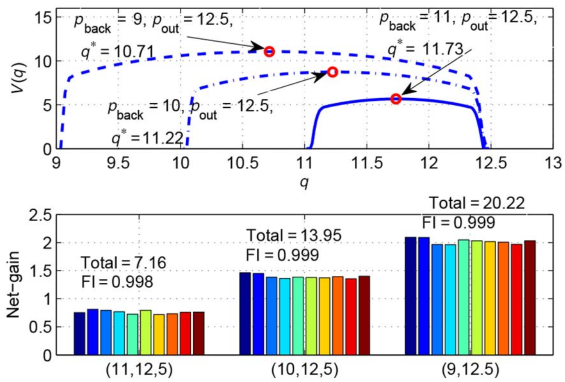

Fig. 2. Top: $V(q)$ versus the LTC's price q. Bottom: the achieved net-gains of each EB and each ES when the LTC uses q^*.

The new price q represents both $q_{\rm out}$ and $q_{\rm back}$ based on Property 2. Constraints (17) and (18) guarantee that each EB i's x_i and each ES j's $\tilde{x}_j$ are nonnegative, respectively. Constraint (20) stems from the previous (11) and (12).

The LTC's price q plays an important role in influencing the energy scheduling of the EBs and ESs. Specifically, a too small q seems to help the EBs get great net-gains, which thus encourages them to buy more energy from the LTC. However, a too small q is adverse to the ESs' net-gains, which discourages them to sell their energy to the LTC. Therefore, constrained by the supply–demand balance (19), the EBs' net-gains eventually suffer, since there is little energy available at the LTC. In summary, a too small q is adverse to both the EBs and ESs, and so is a too large q due to the similar reason (Fig. 2 verifies the effect of q). Therefore, the nonprofit-oriented LTC needs to set its q carefully.

However, finding the optimal price q^* for the LTC is not easy, since Problem (NPO-E) is a typical nonconvex optimization problem [21], which jointly involves the decision variables q, $\{y_i, z_i\}_{i\in\Omega_b}$, and $\{\tilde{y}_j, \tilde{z}_j\}_{j\in\Omega_s}$. To solve Problem (NPO-E), its important property is provided as follows.

Proposition 1: Problem (NPO-E) has the exactly same set of the optimal solutions as the following one:

$$\text{(NPO-E}'\text{): } \max \sum_{i\in\Omega_b} U_i(z_i) + \sum_{j\in\Omega_s} \tilde{U}_j(\tilde{z}_j)$$

$$\text{subject to: Constraints } (15), (16), (18), (19), (20)$$

$$d_i \geq y_i \geq 0 \quad \forall i \in \Omega_b \qquad (21)$$

$$\text{variables: } q, \{y_i, z_i\}_{i\in\Omega_b}, \text{ and } \{\tilde{y}_j, \tilde{z}_j\}_{j\in\Omega_s}$$

if $d_i \geq y_i^*$ holds for each EB $i \in \Omega_b$.

Proof: Refer to Appendix I. ■

Recall that y_i^* is EB i's optimal demand decision for Problem (NPO-E). Proposition 1 means that Problem (NPO-E) can be solved by solving Problem (NPO-E′), if each EB has a relatively large demand such that it is optimal for each EB to buy energy from both the LTC and the EUC simultaneously, which in fact is the most general and interesting case when investigating the local energy trading. Based on Proposition 1, the rest of this section focuses on solving Problem (NPO-E′) by considering that each EB has a relatively large energy demand.

To solve Problem (NPO-E′) [and thus Problem (NPO)], its subproblem with a fixed LTC's price $q \in [q_{\rm back}, q_{\rm out}]$ is first characterized below

$$\text{(NPO-E}'\text{-Sub): } V(q) = \max \sum_{i\in\Omega_b} U_i(z_i) + \sum_{j\in\Omega_s} \tilde{U}_j(\tilde{z}_j)$$

$$\text{subject to: Constraints } (15), (16), (18), (19), (21)$$

$$\text{variables: } \{y_i, z_i\}_{i\in\Omega_b}, \{\tilde{y}_j, \tilde{z}_j\}_{j\in\Omega_s}.$$

Furthermore, the following proposition characterizes the convexity of Problem (NPO-E′-Sub):

Proposition 2: For each given $q \in [q_{\rm back}, q_{\rm out}]$, Problem (NPO-E′-Sub) is a convex optimization problem with respect to the decision variables $\{y_i, z_i\}_{i\in\Omega_b}$ and $\{\tilde{y}_j, \tilde{z}_j\}_{j\in\Omega_s}$.

Proof: Given q, the constraints in Problem (NPO-E′-Sub) yield a convex feasible set. Thus, according to [21], the convex feasible set of Problem (NPO-E′-Sub) along with its concave objective function guarantees that it is a convex optimization problem. ■

Convexity of Problem (NPO-E′-Sub) implies that it can be solved efficiently using the optimization software package CVX [20], which is based on the convex optimization [21]. Thus, the value of $V(q)$, i.e., the output of subproblem (NPO-E'-Sub), can be efficiently calculated for each $q \in [p_{\rm back}, p_{\rm out}]$.

Algorithm (A1). to Solve Problem (NPO-E')

1: Set a small step-size δ_1. Set $q = p_{\rm back}$. Set τ_1 as a very small number.
2: **while** $q \leq p_{\rm out}$ **do**
3: Solve Problem (NPO-E'-Sub) under the given q. Denote the correspondingly optimal solution of Problem (NPO-E'-Sub) as $\{y_i^o, z_i^o\}_{i\in\Omega_b}$, and $\{\tilde{y}_j^o, \tilde{z}_j^o\}_{j\in\Omega_s}$.
4: If $V(q) > \tau_1$, set $q^* = q$, $\{y_i^*, z_i^*\}_{i\in\Omega_b} = \{y_i^o, z_i^o\}_{i\in\Omega_b}$ and $\{\tilde{y}_j^*, \tilde{z}_j^*\}_{j\in\Omega_s} = \{\tilde{y}_j^o, \tilde{z}_j^o\}_{j\in\Omega_s}$, and set $\tau_1 = V(q)$.
5: Set $q = q + \delta_1$.
6: **end while**
7: Output q^*, $\{y_i^*, z_i^*\}_{i\in\Omega_b}$, and $\{\tilde{y}_j^*, \tilde{z}_j^*\}_{j\in\Omega_s}$.

Based on Proposition 2, Algorithm (A1) is proposed to solve Problem (NPO-E′) [as well as Problem (NPO-E)] completely. Algorithm (A1) consists of a one-dimensional (1-D) linear search on $q \in [q_{\rm back}, q_{\rm out}]$ (i.e., the WHILE-loop from lines 2–6). For each q, Algorithm (A1) evaluates $V(q)$ by solving Problem (NPO-E′-Sub) (i.e., line 3), which is a convex optimization problem according to Proposition 2 and thus is ready to be solved using the optimization package CVX [20].[2] Based on the outcome of line 3, Algorithm (A1) updates the current best solution in line 4. Finally, after getting $\{y_i^*\}_{i\in\Omega_b}$ and $\{\tilde{y}_j^*\}_{j\in\Omega_s}$ from the output of Algorithm (A1), each EB i's x_i^* is determined from (1), and each ES j's $\tilde{x}_j^*$ is determined from

[2]Thanks to its convexity (i.e., Proposition 2), Problem (NPO-E'-Sub) can be solved in a distributed manner by relaxing constraint (19) with the Lagrangian dual decomposition. In the distributed manner, each EB (or each ES) can individually determine its energy trading decision in response to the LTC's pricing while guaranteeing to achieve the optimality of Problem (NPO-E's-Sub) and satisfying (19) after convergence. Due to the space limitation, we skip the details of this distributed manner without compromising the completeness of this work.

(5), according to Property 1. Meanwhile, the LTC's optimal prices are $q^*_{\text{back}} = q^*_{\text{out}} = q^*$. In summary, Problem (NPO-E) is completely solved.

C. Numerical Results for the Type of Nonprofit-Oriented LTC

Numerical results are presented to show the performance of the local energy trading managed by the nonprofit-oriented LTC. The numerical experiments are carried out using MATLAB on the PC with intel(R) Core(TM) i7-4770 CPU at 3.4 GHz and 8 GB RAM. In particular, the data sets from the U.S. energy information administration are used [23]. According to [23], the expected residential electricity retail price in 2014 will be 12.5 cents/kWh, i.e., $p_{\text{out}} = 12.5$ cents. Meanwhile, the national annual residential energy consumption is $1\,391\,090 \times 10^6$ kWh, and the number of residential customers is 126 832 343, which corresponds to that the energy consumption of each EB is approximately 1.25 kWh/h.

First, the top figure of Fig. 2 illustrates the effect of the LTC's price q, and the bottom figure shows the optimized net-gains of each EB and ES when the LTC uses the optimal price q^*. For the sake of a clear presentation, this numerical example uses five EBs and five ESs. To make them different, the parameters $\{a_i\}_{i\in\Omega_b}$ and $\{a_j\}_{j\in\Omega_s}$ in their energy loss functions are randomly chosen according to the uniform distribution within $[0.0025, 0075]$, and the parameters $b_i = b_j = 0.005\ \forall i, j$. Meanwhile, each EB i's energy demand (and each ES j's energy surplus) is set as $d_i = \tilde{d}_j = 1.25$ kWh. In addition, the preference function for each EB i is set as $U_i(z_i) = \ln(1 + z_i)$ (according to Remark 1), and so does each ES j. The top figure shows the value of $V(q)$, which is the output of Problem (NPO-E′-Sub), versus different LTC's price q. Specifically, three different cases of $(p_{\text{back}}, p_{\text{out}})$, $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5), (10, 12.5)$, and $(11, 12.5)$ cents, are tested. For each tested case, the corresponding optimal price q^* is marked out by the circle. The results in the top figure verify that both the small and the large q are adverse to the total preference of all the EBs and ESs, and the LTC needs to carefully choose its q such that the EBs and ESs can benefit most. In addition, the results show that a wider range of the EUC's external prices $(p_{\text{back}}, p_{\text{out}})$ can make the EBs and ESs benefit more from the local energy trading. This is correct, since a wider range of external prices $(p_{\text{back}}, p_{\text{out}})$ provides the LTC a greater freedom in choosing its price to benefit the EBs and ESs, and thus the EBs and ESs can benefit more. Corresponding to the above three cases tested, the bottom figure of Fig. 2 further shows the optimized net-gains of each EB and each ES, when the LTC uses q^*. For each case, 1) the total net-gain of all the EBs and ESs is marked out, which is given by $\text{Total} = \sum_{i\in\Omega_b} z^*_i + \sum_{j\in\Omega_s} \tilde{z}^*_j$; and 2) the fairness index [22] of the achieved net-gains of all the EBs and ESs is also marked out, which is given by $\text{FI} = \frac{1}{|\Omega_b|+|\Omega_s|} \frac{(\sum_{i\in\Omega_b} z^*_i + \sum_{j\in\Omega_s} \tilde{z}^*_j)^2}{\sum_{i\in\Omega_b}(z^*_i)^2 + \sum_{j\in\Omega_s}(\tilde{z}^*_j)^2}$. As shown in the bottom figure, for each tested case, the fairness index is always very close to 1, meaning that the EBs and ESs achieve their respective net-gains in a very fair manner.

Fig. 3 shows the performance of local energy trading managed by the nonprofit-oriented LTC versus different numbers of EBs (or ESs). The top figure shows the total net-gain of all the EBs and ESs,[3] which increases as the number of the EBs (or ESs) increases. For each tested case, the average net-gain of individual EB (or ES), which is given by $\frac{1}{|\Omega_b|+|\Omega_s|}\left(\sum_{i\in\Omega_b} z^*_i + \sum_{j\in\Omega_s} \tilde{z}^*_j\right)$, is also marked out. The results show that the average net-gain of individual EB (or ES) always keeps at a stable level. The bottom figure of Fig. 3 shows the corresponding average energy transmission loss ratio versus different numbers of EBs (or ESs). Specifically, the average energy loss ratio is given by $\frac{1}{|\Omega_b|+|\Omega_s|}\left(\sum_{i\in\Omega_b} \frac{f_i(y^*_i)}{y^*_i} + \sum_{j\in\Omega_s} \frac{\tilde{f}_j(\tilde{y}^*_j)}{\tilde{y}^*_j}\right)$. The results show that the average energy loss ratio is always very small as the number of EBs (or ESs) increases, which is a desirable property for the local energy trading involving a large number of the EBs and ESs. Finally, the comparisons between the two different sets of EUC's external prices (i.e., $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ and $(10, 12.5)$ cents) again show that the wider range of $(p_{\text{back}}, p_{\text{out}})$ yields a greater net-gain for the EBs and ESs (in the top figure), and the external prices have little influence on the consequent energy transmission loss ratio (in the bottom figure).

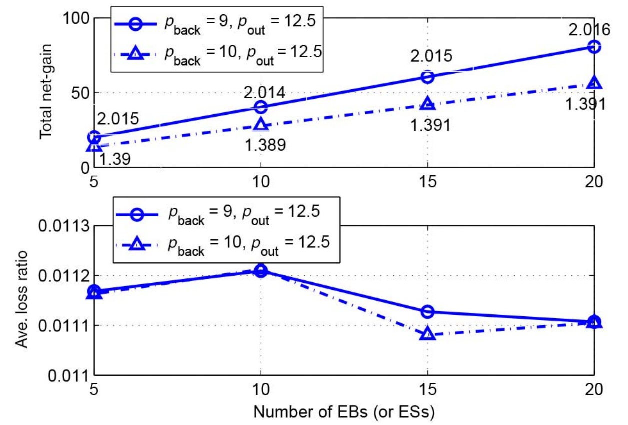

Fig. 3. Performance of the local energy trading versus different numbers of EBs (or ESs). Top: the total net-gain of all the EBs and ESs. The average net-gain of individual EB (or ES) is also marked out. Bottom: the average energy loss ratio.

Fig. 4 further shows the performance of local energy trading versus different energy demands of each EB (or energy surpluses of each ES). The top figure shows that by trading with the LTC, the EBs and ESs can obtain a greater total net-gain as their energy demands (or their energy surpluses) increase, which is consistent with the intuition very well. Notice that the average net-gain of individual EB (or ES) is also marked out in the top figure, which also keeps on increasing. However, as shown in the bottom figure, the downside is that the average energy loss ratio increases slightly as the energy demand (or surplus) increases. This result essentially stems from the quadratic energy loss function, which has an increasing margin. Thus, the more local energy trading via the LTC, the greater energy loss results in. This phenomenon implies the importance of optimizing the EBs' and ESs' energy scheduling to avoid

[3]To obtain the result shown in Fig. 3, 30 different sets of $\{a_i\}_{\forall i\in\Omega_b}$ and $\{a_j\}_{\forall j\in\Omega_s}$ are randomly and independently generated first. Then, Algorithm (A1) is used to obtain the optimal solution for each of these data sets. Fig. 3 shows the average results based on the 30 different sets of the optimal solutions obtained.

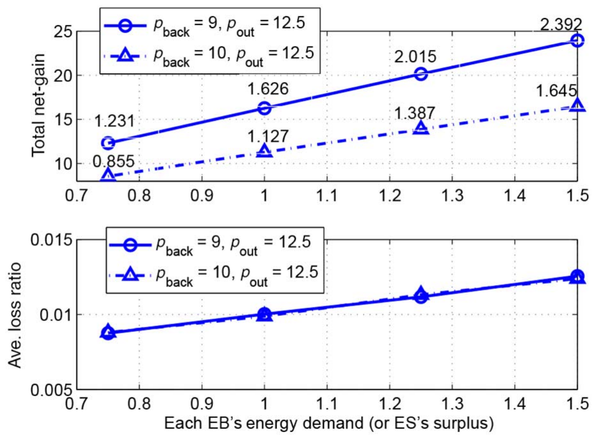

Fig. 4. Performance of the local trading versus different energy demands of each EB (or energy surpluses of each ES). Top: the total net-gain of all the EBs and ESs. Bottom: the average energy loss ratio.

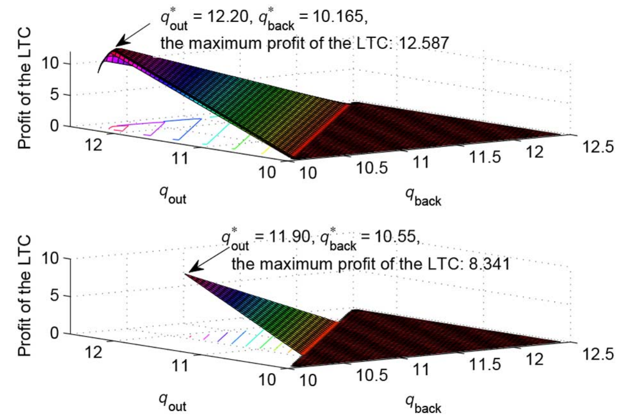

Fig. 5. Profit-oriented LTC's maximum profit versus its prices $(q_{\text{back}}, q_{\text{out}})$. Top: the required net-gain $\Delta_i = \tilde{\Delta}_j = 0.02$ for each EB and ES. Bottom: the required net-gain $\Delta_i = \tilde{\Delta}_j = 0.5$ for each EB and ES.

excessive energy loss. Finally, in the bottom figure of Fig. 4, the comparisons between the two sets of EUC's external prices show that they have a negligible influence on the consequent energy transmission loss ratio.

IV. Local Energy Trading Managed by the Profit-Oriented LTC

A. Problem Formulation

This section considers the profit-oriented LTC, which controls its prices $(q_{\text{back}}, q_{\text{out}})$ for maximizing its own profit, while guaranteeing the required net-gain for each EB and each ES. Based on this objective, the optimal trading problem managed by the profit-oriented LTC can be constructed by two *subproblems*, namely, the *top problem* for the LTC's profit maximization and the *bottom problem* for the EBs' and ESs' optimal energy scheduling. Specifically, the top problem is as follows (the capital letters "PO" stand for "profit-oriented"):

$$\text{(PO-Top):} \max_{q_{\text{back}}, q_{\text{out}}} q_{\text{out}} \sum_{i \in \Omega_b} y_i^r - q_{\text{back}} \sum_{j \in \Omega_s} \tilde{y}_j^r$$
$$\text{subject to: } p_{\text{out}} \geq q_{\text{out}} \geq q_{\text{back}} \geq p_{\text{back}} \quad (22)$$

where the EBs' demand decisions $\{y_i^r\}_{i \in \Omega_b}$ and the ESs' supply decisions $\{\tilde{y}_j^r\}_{j \in \Omega_s}$ are the *best responses* to the LTC's prices $(q_{\text{back}}, q_{\text{out}})$, i.e., the optimal solutions of the bottom problem as

$$\text{(PO-bottom): } \left\{\{y_i^r, z_i^r\}_{i \in \Omega_b}, \{\tilde{y}_j^r, \tilde{z}_j^r\}_{j \in \Omega_s}\right\} = \arg\max \sum_{i \in \Omega_b} U_i(z_i) + \sum_{j \in \Omega_s} \tilde{U}_j(\tilde{z}_j)$$
$$\text{subject to: } (p_{\text{out}} - q_{\text{out}})y_i - p_{\text{out}} f_i(y_i) \geq z_i \geq \Delta_i \quad \forall i \in \Omega_b \quad (23)$$
$$(q_{\text{back}} - p_{\text{back}})\tilde{y}_j - p_{\text{back}} \tilde{f}_j(\tilde{y}_j) \geq \tilde{z}_j \geq \tilde{\Delta}_j \quad \forall j \in \Omega_s \quad (24)$$
$$\text{and Constraints } (18), (19), (21)$$
$$\text{variables: } \{y_i, z_i\}_{i \in \Omega_b}, \{\tilde{y}_j, \tilde{z}_j\}_{j \in \Omega_s}.$$

In Problem (PO-bottom), under the given $(p_{\text{back}}, p_{\text{out}})$, each EB i determines its $\{y_i, z_i\}$, and each ES j determines its $\{\tilde{y}_j, \tilde{z}_j\}$ to optimize their total preference while guaranteeing the required net-gain Δ_i for each EB i and the required net-gain $\tilde{\Delta}_j$ for each ES j. Notice that Δ_i (or $\tilde{\Delta}_j$) can be considered the *threaten point* beyond which EB i (or ES j) expects to obtain from trading with the LTC. Otherwise, it will not choose to perform the local energy trading. It is worth emphasizing that the feasibility of Problem (PO-bottom), taking $\{\Delta_i\}_{i \in \Omega_b}$ and $\{\tilde{\Delta}_j\}_{j \in \Omega_s}$ into account, can also be checked in an easy manner (the details are skipped here due to space limitation). To solve Problem (PO-Top), the next section focuses on solving Problem (PO-Bottom) first.

To exhibit the effect of the LTC's prices, Fig. 5 enumerates the LTC's prices $(q_{\text{back}}, q_{\text{out}})$ and plots the corresponding profit achieved by the LTC (in the z-axis). The parameter setting is similar to that in Fig. 2, except using $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents. In both figures of Fig. 5, the LTC's profit is directly set as zero if $q_{\text{back}} > q_{\text{out}}$ [i.e., constraint (22) fails to hold]. Particularly, in each figure, the optimal prices $(q^*_{\text{back}}, q^*_{\text{out}})$ for Problem (PO-Top) are marked out, and they are obtained by the algorithm, which we propose in Section IV-C. Fig. 5 verifies that the LTC's profit strongly depends on its prices $(q_{\text{back}}, q_{\text{out}})$, and an inappropriate choice of $(q_{\text{back}}, q_{\text{out}})$ might result in a very low profit for the LTC, which indicates the importance of finding the optimal prices for the LTC.

B. Property of Problem (PO-Bottom) and Its Optimal Solution

To solve Problem (PO-Top), this section focuses on solving Problem (PO-Bottom) first. Specifically, the following property is identified for Problem (PO-Bottom).

Proposition 3: Given the LTC's prices $(q_{\text{back}}, q_{\text{out}})$ that meet (22), Problem (PO-Bottom) is a convex optimization with respect to the decision variables $\{y_i, z_i\}_{i \in \Omega_b}$ and $\{\tilde{y}_j, \tilde{z}_j\}_{j \in \Omega_s}$.

Proof: The proof is similar to that for Proposition 2 and thus is skipped due to space limitation. ■

The convexity of Problem (PO-Bottom) means that it can be solved efficiently, for each given $(q_{\text{back}}, q_{\text{out}})$, using the

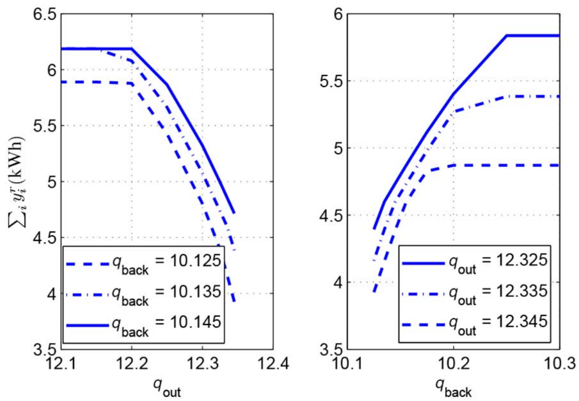

Fig. 6. Examples to verify Properties 3 and 4. *Left*: value of $\sum_{i\in\Omega_b} y_i^r$ versus different q_{out}. *Right*: value of $\sum_{j\in\Omega_s} \tilde{y}_j^r$ versus different q_{back}.

optimization package CVX [20]. Furthermore, the following properties are identified for Problem (PO-Bottom).

Property 3: For a given $q_{\text{back}} \in [p_{\text{back}}, p_{\text{out}}]$, the value of $\sum_{i\in\Omega_b} y_i^r$ (or $\sum_{j\in\Omega_s} \tilde{y}_j^r$ equivalently) is nonincreasing in q_{out}.

Property 4: For a given $q_{\text{out}} \in [p_{\text{back}}, p_{\text{out}}]$, the value of $\sum_{j\in\Omega_s} \tilde{y}_j^r$ (or $\sum_{i\in\Omega_b} y_i^r$ equivalently) is nondecreasing in q_{back}.

Property 3 is from the fact that a greater q_{out} discourages the EBs to buy energy from the LTC, and thus their total energy demand in response to q_{out} is nonincreasing. Property 4 is from the fact that a greater q_{back} encourages the ESs to sell their energy to the LTC, and thus their total energy supply in response to q_{back} is nondecreasing. The proofs for Properties 3 and 4 can be performed using the Lagrangian multiplier method, and their details are skipped here due to space limitation. To verify Properties 3 and 4, Fig. 6 shows $\sum_{i\in\Omega_b} y_i^r$ (and $\sum_{j\in\Omega_s} \tilde{y}_j^r$ equivalently) versus different $(q_{\text{back}}, q_{\text{out}})$, using the same parameter setting as the top figure of Fig. 5. The left plot shows that $\sum_{i\in\Omega_b} y_i^r$ is nonincreasing in q_{out} under the fixed q_{back}, thus verifying Property 3, and the right plot shows that $\sum_{j\in\Omega_s} \tilde{y}_j^r$ is nondecreasing in q_{back} under the fixed q_{out}, thus verifying Property 4.

Based on its convexity (according to Proposition 3), Problem (PO-bottom) can be solved for each $(q_{\text{back}}, q_{\text{out}})$. Therefore, Problem (PO-top) can at least be solved by performing a two-dimensional (2-D) exhaustive search over $(q_{\text{back}}, q_{\text{out}})$ within $[p_{\text{back}}, p_{\text{out}}]$. However, the 2-D exhaustive search consumes a heavy computational burden. To tackle with this difficulty, in the next section, the hidden monotonicity of Problem (PO-Top) is identified first, and then an efficient algorithm is proposed to solve Problem (PO-Top) completely. It is worth pointing out that the profit achieved by the LTC not only depends on the value of $q_{\text{out}} - q_{\text{back}}$ but also on the detailed values of q_{back} and q_{out} which influence the energy trading decisions of the EBs and ESs in the bottom problem (according to Properties 3 and 4 characterized before). Hence, an algorithm that can find the optimal values of q_{back} and q_{out} together for maximizing the LTC's profit is required.

C. Property of Problem (PO-Top) and Its Optimal Solution

The major difficulty in solving Problem (PO-Top) stems from the fact that its objective function cannot be derived in a closed-form expression. For the sake of clear presentation, an implicit function $O(q_{\text{out}}, q_{\text{back}})$ is first introduced as follows:

$$O(q_{\text{out}}, q_{\text{back}}) = \sum_{i\in\Omega_b} y_i^r = \sum_{j\in\Omega_s} \tilde{y}_j^r.$$

Recall again that $\{y_i^r\}_{i\in\Omega_b}$ and $\{\tilde{y}_j^r\}_{j\in\Omega_s}$ are the optimal solutions of Problem (PO-Bottom) under the given $(q_{\text{back}}, q_{\text{out}})$.

Using $O(q_{\text{out}}, q_{\text{back}})$, Problem (PO-Top) can thus be solved efficiently via a *two-step approach* as follows.

Step I: For each given $q_{\text{back}} \in [p_{\text{back}}, p_{\text{out}}]$, solve the following optimization problem with respect to the decision variable q_{out}:

Problem (PO-Top-I):

$$J(q_{\text{back}}) = \max_{q_{\text{back}} \le q_{\text{out}} \le p_{\text{out}}} (q_{\text{out}} - q_{\text{back}}) O(q_{\text{out}}, q_{\text{back}}).$$

Step II: After getting $J(q_{\text{back}})$, solve the following optimization problem with respect to the decision variable q_{back}:

$$\text{Problem (PO-Top-II): } \max_{p_{\text{back}} \le q_{\text{back}} \le p_{\text{out}}} J(q_{\text{back}}).$$

The rest of this section focuses on solving Problem (PO-Top-I) and Problem (PO-Top-II), starting with Problem (PO-Top-I) first. Specifically, an important property of *hidden monotonicity* is identified for Problem (PO-Top-I) as follows.

Proposition 4: For each given q_{back}, Problem (PO-Top-I) can be equivalently transformed into the optimization problem as follows (the letters "Mono" stand for "Monotonicity"):

$$\text{(PO-Top-I-Mono): } \max_{u,t} \ln(u) + t - \ln\left(\frac{1}{O(p_{\text{out}}, q_{\text{back}})}\right)$$
$$\text{subject to: } 0 < u < p_{\text{out}} - q_{\text{back}}$$
$$0 \le t \le \ln\left(\frac{O(q_{\text{back}}, q_{\text{back}})}{O(p_{\text{out}}, q_{\text{back}})}\right)$$
$$t + \ln\left(\frac{1}{O(q_{\text{back}} + u, q_{\text{back}})}\right) \le \ln\left(\frac{1}{O(p_{\text{out}}, q_{\text{back}})}\right)$$

which is a *standard monotonic optimization problem* with respect to the decision variables (u, t). Further let (u^o, t^o) denote the optimal solutions of Problem (PO-Top-I-Mono). Then, the optimal q_{out}^o for Problem (PO-Top-I) is given by $q_{\text{out}}^o = u^o + q_{\text{back}}$.

Proof: Refer to Appendix II. Notice that the superscript "o" is used to denote the optimality of Problem (PO-Top-I) and Problem (PO-Top-I-Mono). ■

A brief introduction about the monotonic optimization is as follows (interested readers refer to [18] and [19] for the details). Monotonic optimization refers to a type of mathematical programming problems that aim at maximizing a monotonic function subject to some monotonic constraints. Specifically, the monotonic structure has an important property that any level set of a monotonic function (which is a *normal set* defined in Definition 1) can be approximated by a nested sequence of sets of polyblocks. Meanwhile, the maximum of an increasing function over a polyblock is attained at one of the vertexes of the polyblock. Exploiting the aforementioned two properties, a polyblock approximation (PA) algorithm was proposed in [19] to solve the monotonic optimization problem. The PA algorithm includes two key steps in each round of iteration: 1) to construct a new polyblock (based on the previous one) that approximates the feasible set closer; and 2) to find a best vertex that maximizes the objective function under the newly constructed polyblock. The iteration process continues until the best vertex is found, which falls within the feasible region of the considered optimization problem. This best vertex can be considered the optimal solution. The PA algorithm has been shown to solve the monotonic optimization problems very efficiently [18].

Definition 1: (Normal Set) A set $\mathcal{G} \subset \mathcal{R}_+^n$ is considered to be a normal set if the following condition is met. For any two points x and $x' \in \mathcal{R}_+^n$ with $x' \leq x$. If $x \in \mathcal{G}$ holds, then $x' \in \mathcal{G}$ holds.

Based on its monotonicity, Problem (PO-Top-I-Mono) can thus be solved efficiently using the PA algorithm. Due to space limitation, the detailed description about the PA algorithm is skipped here (interested readers refer to [19] for the details). Let (u^o, t^o) denote the output of the PA algorithm, then the optimal solution for Problem (PO-Top-I) is $q_{\text{out}}^o = u^o + q_{\text{back}}$, and its objective value is given by $J(q_{\text{back}}) = (q_{\text{out}}^o - q_{\text{back}})O(q_{\text{out}}^o, q_{\text{back}})$. We thus finish solving Problem (PO-Top-I).

Using $J(q_{\text{back}})$, we continue to solve Problem (PO-Top-II)

$$\max J(q_{\text{back}}), \text{ subject to: } p_{\text{back}} \leq q_{\text{back}} \leq p_{\text{out}}.$$

Although $J(q_{\text{back}})$ cannot be derived in a closed-form expression, $J(q_{\text{back}})$ can be evaluated for each given q_{back} by solving Problem (PO-Top-I). Thus, Problem (PO-Top-II) can be solved by performing an 1-D linear search over $q_{\text{back}} \in [p_{\text{back}}, p_{\text{out}}]$.

Combining the above two-step approach, Algorithm (A2) is proposed to solve Problem (PO-Top) completely. Specifically, Algorithm (A2) includes an 1-D linear search on q_{back} with the step-size δ_2 (i.e., from lines 2 to 7). In line 3, for each given q_{back}, the PA algorithm is used to solve Problem (PO-Top-I-Mono) based on its monotonicity (according to Proposition 4). Consequently, in line 4, the output of Problem (PO-Top-I), i.e., the value of $J(q_{\text{back}})$, can be evaluated. In addition, in lines 3 and 4, for each given $(q_{\text{back}}, q_{\text{out}})$, $O(q_{\text{out}}, q_{\text{back}})$ can be evaluated using CVX to solve Problem (PO-bottom), based on its convexity (according to Proposition 3). Based on the output of line 4, Algorithm (A2) updates the current best prices in line 5. In summary, Algorithm (A2) solves Problem (PO-Top) completely. Thanks to exploiting the hidden monotonicity of Problem (PO-Top-I), Algorithm (A2) only requires an 1-D linear search to solve Problem (PO-Top), and thus it can save a significant computational time compared to the 2-D exhaustive search method described at the end of Section IV-B.

D. Numerical Results for the Type of Profit-Oriented LTC

Numerical results are presented here to show the performance of Algorithm (A2) and the performance of the local trading managed by the profit-oriented LTC.

Fig. 7 shows the performance of Algorithm (A2) using the similar parameter setting as Fig. 2 but varying the required net-gain of each EB and each ES from $\Delta_i = \tilde{\Delta}_j = 0.1$ to 0.6. The

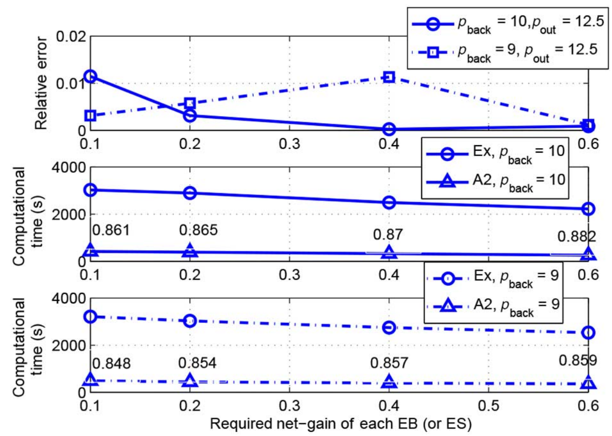

Fig. 7. Top: relative error between the maximum LTC's profit obtained by Algorithm (A2) and that obtained by the 2-D exhaustive search. Middle: computational time of Algorithm (A2) and the 2-D exhaustive search (denoted by "Ex" for brevity) when $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents. Bottom: computational time when $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ cents.

top figure shows that the relative error between the LTC's maximum profit obtained by Algorithm (A2) and that obtained by the 2-D exhaustive search. As shown by the tested cases, the maximum relative error is no greater than 1.5%, which validates the accuracy of Algorithm (A2) proposed to solve Problem

Algorithm (A2). to Solve Problem (PO-Top)

1: Set a small step-size δ_2. Set $q_{\text{back}} = p_{\text{back}}$. Set τ_2 as a very small number.
2: **while** $q_{\text{back}} \leq p_{\text{out}}$ **do**
3: Given q_{back}, use the PA algorithm to solve Problem (PO-Top-I-Mono) to obtain (u^o, t^o).
4: Set $q_{\text{out}}^o = q_{\text{back}} + u^o$. Evaluate $J(q_{\text{back}}) = (q_{\text{out}}^o - q_{\text{back}}) O(q_{\text{out}}^o, q_{\text{back}})$.
5: If $J(q_{\text{back}}) > \tau_2$, set $(q_{\text{back}}^*, q_{\text{out}}^*) = (q_{\text{back}}, q_{\text{out}}^o)$, and set $\tau_2 = J(q_{\text{back}})$.
6: Set $q_{\text{back}} = q_{\text{back}} + \delta_2$.
7: **end while**
8: Output $(q_{\text{back}}^*, q_{\text{out}}^*)$ for Problem (PO-Top).

(PO-Top). Followed by the top figure, the computational time consumed by Algorithm (A2) and the 2-D exhaustive search are shown the middle figure when $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents and in the bottom figure when $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ cents. Both the middle and the bottom figures show that Algorithm (A2) consumes a significantly less computational time than the 2-D exhaustive search. To quantify this advantage, in the middle and bottom figures, the saved computational time is marked out [i.e., those numbers above the line denoting the computational time of Algorithm (A2)]. For instance, when the required net-gain is 0.2 for each EB and ES, Algorithm (A2) can save the computational time of the 2-D exhaustive search up to 86.5% when $p_{\text{back}} = 10$ cents (in the middle figure) and 85.4% when $p_{\text{back}} = 9$ cents (in the bottom figure). Therefore, Algorithm (A2) is very efficient. Moreover, for the different cases in Fig. 7, the average fairness index (which measures the fairness for the achieved net-gains of different EBs and EBs

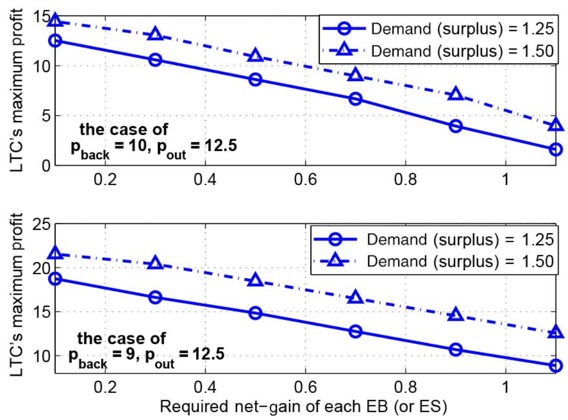

Fig. 8. Maximum profit of LTC versus the required net-gain of each EB (and ES). Top: the results when $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents. Bottom: the results when $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ cents.

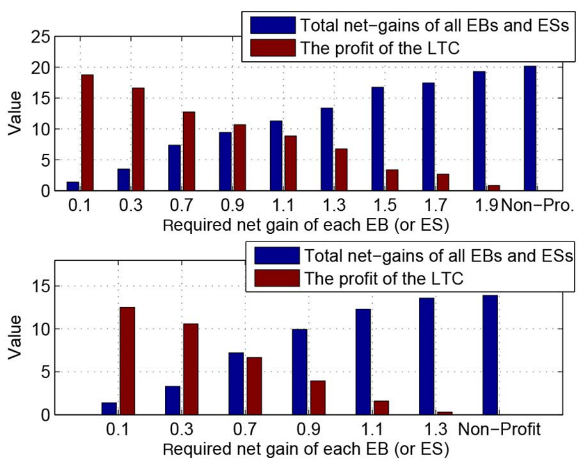

Fig. 9. Performance comparisons between the nonprofit-oriented LTC and the profit-oriented LTC. Top: $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ cents. Bottom: $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents.

as stated in Section III-C) is 0.989, meaning that the EBs and ESs achieve their respective optimized net-gains in a very fair manner. In addition, the average energy transmission loss ratio (which is also defined in Section III-C) is 0.0112, meaning that the optimized energy scheduling decisions of the EBs and ESs result in a very small energy loss.

Fig. 8 shows the maximum profit of the LTC versus the required net-gain of each EB (or each ES). The results illustrate the tradeoff between the LTC and the EBs (and ESs) in benefiting from the local energy trading, i.e., the greater the net-gain required by the EBs and ESs, the smaller the LTC's maximum profit. Again, it is observed that a greater energy demand of the EBs (or the energy surplus of the ESs) can yield a greater benefit for the LTC as well as the EBs and ESs.

Fig. 9 further compares the LTC's profit and the total net-gains of all EBs and ESs when the LTC is profit-oriented and nonprofit-oriented, respectively. Specifically, the top figure plots the results under $(p_{\text{back}}, p_{\text{out}}) = (9, 12.5)$ cents (the other parameter settings are same as those for Fig. 8), and we vary the required net-gain of each EB (or ES) from 0.1 to 1.9. Notice that, in Fig. 9, the rightmost case represents the case of nonprofit-oriented LTC. The results in the top figure again show the tradeoff between the LTC and the EBs and ESs in benefiting from the local energy trading. Specifically, the total net-gain of all EBs and ESs increases as each EB (or ES) requires to gain more, and accordingly, the profit achieved by the LTC decreases. In particular, the benchmark case occurs when the LTC is nonprofit-oriented (i.e., the rightmost case in Fig. 9). In this case, the LTC gains nothing while the EBs and ESs gain the most. Similar comparison results are also shown in the bottom figure, in which we use $(p_{\text{back}}, p_{\text{out}}) = (10, 12.5)$ cents and vary the required net-gain of each EB (or ES) from 0.1 to 1.3. In particular, the above understanding can be quantitatively explained as follows. Let us consider the case of the profit-oriented LTC. By summing up constraints (23) and (24) together, we obtain the following necessary constraint for them:

$$\left(p_{\text{out}} \sum_{i\in\Omega_b} y_i - p_{\text{back}} \sum_{j\in\Omega_s} \tilde{y}_j\right) - p_{\text{out}} \sum_{i\in\Omega_b} f_i(y_i) - p_{\text{back}} \sum_{j\in\Omega_s} \tilde{f}_j(\tilde{y}_j) \geq \sum_{i\in\Omega_b} z_i + \sum_{j\in\Omega_s} \tilde{z}_j + \left(q_{\text{out}} \sum_{i\in\Omega_b} y_i - q_{\text{back}} \sum_{j\in\Omega_s} \tilde{y}_j\right). \tag{25}$$

In particular, based on (19), the last two items in (25) together represent the profit achieved by the LTC. Thus, constraint (25) indicates that the LTC and the EBs and ESs are sharing the benefit from the local energy trading. Specifically, the left-hand side of (25) is the *difference* between the total reduced cost by performing the local trading [i.e., the first two items at the left-hand side of (25)] and the corresponding total loss due to the energy transfer [i.e., the third and fourth items at the left-hand side of (25)]. This difference in fact represents the total net benefit which the LTC and the EBs and ESs can achieve together. In this sense, (25) means that bounded by this total net benefit, the LTC and the EBs and ESs are sharing the benefit from the local energy trading, i.e., the more net-gain gained by the EBs and ESs, the less profit achieved by the LTC, which has been well verified in Fig. 9. In particular, the case of nonprofit-oriented LTC corresponds to a special case of (25) when the last two items representing the LTC's achieved profit vanish (recall that $q_{\text{back}} = q_{\text{out}}$ holds for the case of nonprofit-oriented LTC). In this special case, the LTC gains nothing while the EBs and ESs gain the most.

V. Conclusion

This paper investigates the optimal energy trading problem in the hybrid energy market comprising the external retailer and the local trading market managed by the LTC. The local market provides new opportunities for the EBs and sellers to perform the local energy trading in a cooperative manner such that they all can benefit. This paper quantifies the respective benefits of EBs and sellers from the local trading and studies their optimal energy scheduling in response to the LTC's pricing. Two different types of the LTC have been considered:

1) the nonprofit-oriented LTC and 2) the profit-oriented LTC. For each type of the LTC, the corresponding optimal energy trading problem is formulated, and the associated algorithm is proposed to find the optimal solution efficiently. Numerical results are presented to validate the benefits from the considered hybrid market and the performance of the proposed algorithms. As described earlier, this work focuses more on investigating the optimal pricing of the LTC and the associated demand response management for the local trading market. An important future work is to investigate a joint management scheme, in which the LTC and EUC competitively (or cooperatively) adjust their respective prices for benefiting the energy users as well as themselves.

APPENDIX I
PROOF OF PROPOSITION

It can be observed that Problem (NPO-E′) is similar to Problem (NPO-E) but having a smaller feasible region. Hence, if the condition that $y_i^* \le d_i$ holds for each EB $i \in \Omega_b$ [i.e., being feasible to Problem (NPO-E′)], then the set of the optimal solutions, including q^*, $\{y_i^*, z_i^*\}_{i\in\Omega_b}$, and $\{\tilde{y}_j^*, \tilde{z}_j^*\}_{j\in\Omega_s}$, for Problem (NPO) suffice to be the optimal solutions for Problem (NPO-E′).

APPENDIX II
PROOF OF PROPOSITION

It can be observed that Problem (PO-Top-I) is equivalent to

$$\max_{q_{\text{back}} \le q_{\text{out}} \le p_{\text{out}}} \ln(q_{\text{out}} - q_{\text{back}}) - \ln\left(\frac{1}{O(q_{\text{out}}, q_{\text{back}})}\right).$$

Based on Property 3, the second part of the above objective function is a nondecreasing function of q_{out}, under the given q_{back}. Thus, the objective function is structured by the difference between two nondecreasing parts of q_{out}. This property helps transform Problem (PO-Top-I) into a monotonic optimization problem as follows. We first introduce the auxiliary variable $u = q_{\text{out}} - q_{\text{back}}$, and another auxiliary variable t. Then, using (u, t), Problem (PO-Top-I) can be equivalently transformed into Problem (PO-Top-I-Mono) given in Proposition 4. Notice that, for the given q_{back}, Problem (PO-Top-I-Mono) is a standard monotonic optimization problem with respect to (u, t). The reason is as follows. First, it is observed that the objective function of Problem (PO-Top-I-Mono) is increasing. Meanwhile, the feasible set of (u, t) is the intersection of the normal set (refer to Definition 1 in Section IV-C) given by $\mathcal{G} = \Big\{(u,t)|0 \le u \le p_{\text{out}} - q_{\text{back}}, 0 \le t \le \ln\left(\frac{O(q_{\text{back}}, q_{\text{back}})}{O(p_{\text{out}}, q_{\text{back}})}\right)$, and $t + \ln\left(\frac{1}{O(q_{\text{back}}+u, q_{\text{back}})}\right) \le \ln\left(\frac{1}{O(p_{\text{out}}, q_{\text{back}})}\right)\Big\}$ and the reversed normal set (refer to Definition 2 provided at the end of this Appendix) $\mathcal{H} = \Big\{(u,t)|0 \le u \le p_{\text{out}} - q_{\text{back}}$, and $0 \le t \le \ln\left(\frac{O(q_{\text{back}},\, q_{\text{back}})}{O(p_{\text{out}},\, q_{\text{back}})}\right)\Big\}$. Thus, according to [18], Problem (PO-Top-I-Mono) is a standard monotonic optimization with respect to the decision variables (u, t).

Definition 2: (Reversed Normal Set) A set $\mathcal{H} \subset \mathcal{R}_+^n$ is considered to be a reversed normal set if the following condition is met. For any two points x and $x' \in \mathcal{R}_+^n$ with $x' \ge x$. If $x \in \mathcal{H}$ holds, then $x' \in \mathcal{H}$ holds.

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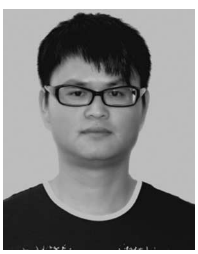

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