发动机内流场数值分析基础

大作业报告

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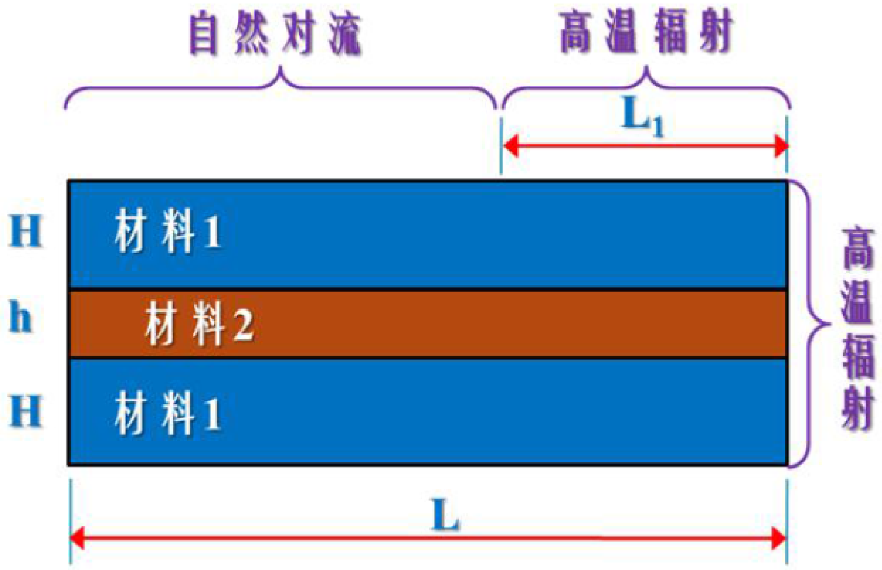
一、 问题分析



求解不同时刻多层结构温度场

材料 1 和材料 2 组成三明治结构，右端处于 923 k 的燃气中，下边界和左边界为绝热边界，上边界左段处于大气中，大气温度为 293 k。求某一时刻计算域内温度分布。

二、 参数确定

1. 系统发射率

1) 燃气发射率

气体辐射和吸收与固体和液体不同，固体和液体的辐射和吸收具有在表面上进行的特点，而气体的辐射和吸收则是在整个容积中进行的，因此必须说明气体所处容积的形状和容积的大小。我们完善本题中的物理模型如下图。

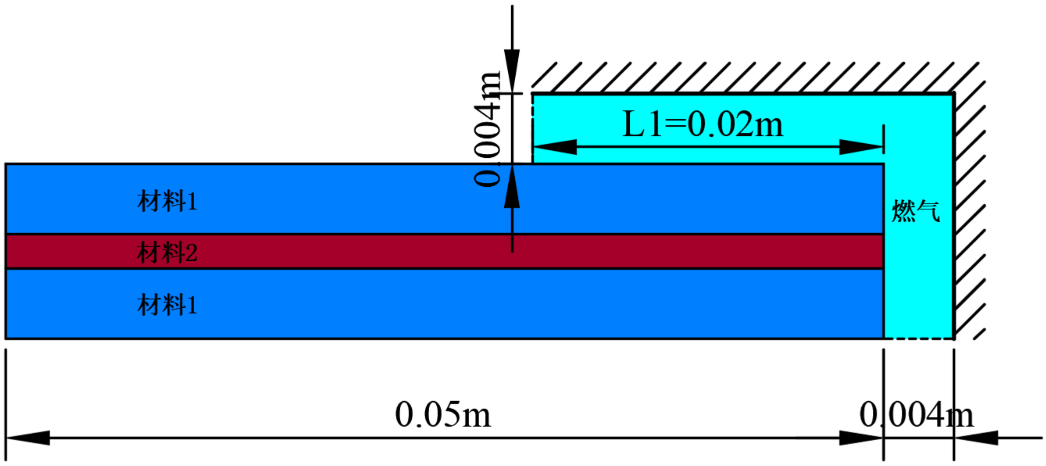


图 1 气体容积物理模型

假设燃气分布在距离材料 $d=0.004m$ 的区域内，本题中的辐射只考虑右边界和右上边界，可简化为燃气分布在上图绿色区域。查阅文献【1】，右边界和右上边界的气体容积均选取平行平板模型，则平均射线程长为 $s=1.8d$ 。

在火箭发动机燃烧产物的辐射中，起着主要作用的气体组分是 H_2O, CO_2, NO, HF ，其中三原子气体比起双原子气体辐射更为强烈。查阅文献【2】，对于含 C, H, O, N 的推进剂燃烧产物，热辐射主要取决于其中三原子水蒸气和二氧化碳的含量，燃气发射率的计算公式为

$$\varepsilon_g=\varepsilon_{H_2O}+\varepsilon_{CO_2}-\Delta\varepsilon$$

这里取 $\Delta\varepsilon=\varepsilon_{H_2O}\bullet\varepsilon_{CO_2}$

定义燃气参数：燃气温度 $T_g=923k$ ， $P_{总}=10MPa$ ，选取 85 系列推进剂热力计算得水蒸气和二氧化碳分压

$$P_{H_2O}=0.2404\times10MPa=24atm,\ P_{CO_2}=0.08367\times10MPa=8.3atm$$

$$P_{CO_2}\bullet s=8.3\times1.8\times0.004=0.05976\ atm\bullet m$$

$$P_{H_2O}\bullet s=24\times1.8\times0.004=0.1728\ atm\bullet m$$

查文献【1】p427 中气体发射率经验确定曲线得

$$\begin{cases}\varepsilon_{CO_2}{}^{*}=0.1\\ C_{CO_2}=1.2\\ \varepsilon_{CO_2}=C_{CO_2}\bullet\varepsilon_{CO_2}{}^{*}=0.12\end{cases}$$

$$\begin{cases} \varepsilon_{0,H_2O} = 0.18 \\ n = 1 + K_{H_2O}P_{H_2O} = 8 \\ \varepsilon_{H_2O} = 1-(1-\varepsilon_{0,H_2O})^n = 0.7956 \end{cases}$$

由上，可得燃气发射率

$$\varepsilon_g = \varepsilon_{H_2O} + \varepsilon_{CO_2} - \Delta\varepsilon = 0.82$$

2) 系统发射率

材料 1 的密度为 $7830kg/m^3$，与钢接近，查阅【1】得 $\varepsilon_{wall1} = \varepsilon_{钢}=0.82$；材料 2 为推进剂，大部分非金属材料的发射率都很高，一般在 0.85~0.95 之间，且与表面状况关系不大，在缺乏资料时，近似取 $\varepsilon_{wall2} = 0.90$。

因此，燃气与材料 1 之间的系统发射率为

$$\varepsilon_1 = \frac{1}{\frac{1}{\varepsilon_{wall1}} + \frac{1}{\varepsilon_g} - 1} = 0.6949$$

燃气与材料 2 之间的系统发射率为

$$\varepsilon_2 = \frac{1}{\frac{1}{\varepsilon_{wall2}} + \frac{1}{\varepsilon_g} - 1} = 0.7515$$

2. 对流换热系数

对流换热发生在左上边界，且其数值与壁面和大气温差有关。考虑到本题左上壁面温度随时间变化，故推导对流换热系数与壁面温度函数关系。选用文献【3】中均匀壁温边界条件的大空间自然对流实验关联式（水平热面向上的情形），则对流换热系数表达式如下，

$$h = Nu\frac{K_{air}}{l},\ Nu = 0.54(Gr\cdot \mathrm{Pr})^{0.25},\ Gr = \frac{ga_v(T_{wall}-T_{air})l^3}{v^2}$$

取定性温度 $t_m = \frac{t_{wall}+t_{air}}{2}(°\mathrm{C})$，特征长度 $l = 0.03m$， $g = 9.8m/s^2$，

$a_v = \frac{1}{273+t_m}$ ，

应用常压下干空气的热物理性质表拟合出 $20°C \sim 180°C$ 范围内空气的普朗克数 Pr 和运动粘度 a_v 随定性温度 t_m 的变化关系，

$$\mathrm{Pr} = 4\times10^{-14}{t_m}^6 - 3\times10^{-11}{t_m}^5 + 6\times10^{-9}{t_m}^4 - 7\times10^{-7}{t_m}^3 + 4\times10^{-5}{t_m}^2 - 0.0012t_m + 0.7163$$

$$v = 9\times10^{-11}{t_m}^2 + 9.13\times10^{-8}t_m + 1.3175\times10^{-5}$$

$$Gr = \frac{ga_v(T_{wall}-293)\times0.03^3}{v^2}$$

$$Nu = 0.54(Gr\cdot\mathrm{Pr})^{0.25} \qquad 10^4 \le Gr\cdot\mathrm{Pr} \le 10^7$$

利用以上各式求出努塞尔数，代人下式求出对流换热系数表达式，用于编程计算和 UDF 函数编写，

$$h = Nu\frac{K_{air}}{l} = Nu\times\frac{2.59}{3}$$

三、 公式推导

1. 控制方程

$$\rho c\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+S_u$$

初始条件：$T|_{t=0}=293\ \mathrm{K}$

边界条件：W：$\frac{\partial T}{\partial x}|_{x=0}=0$

$$\mathrm{S}:\frac{\partial T}{\partial y}|_{y=0}=0$$

$$\mathrm{NL}:q_{NL}=h\left(T_{air}-T\right)=k\frac{\partial T}{\partial y}|_{y=2H+h,0\le x\le L-L_1}$$

$$\mathrm{NR}:q_{NR}=\varepsilon\sigma\left(T_{gas}^4-T^4\right)=k\frac{\partial T}{\partial y}|_{y=2H+h,L-L_1\le x\le L}$$

$$\mathrm{E}:q_E=\varepsilon\sigma\left(T_{gas}^4-T^4\right)=k\frac{\partial T}{\partial x}|_{x=L}$$

2. 生成离散网格

总共分为 25 种情况，包括 4 个角点，左上侧自然对流边界，右上侧辐射换热边界，右侧辐射换热边界（材料 1 内部、材料 2 内部、材料 12 交界处、材料 21 交界处共 6 种），左侧绝热边界（材料 1 内部、材料 2 内部、材料 12 交界处、材料 21 交界处共 6 种），下侧绝热边界，内部节点（材料 1 内部、材料 2 内部、材料 12 交界处、材料 21 交界处共 6 种）。

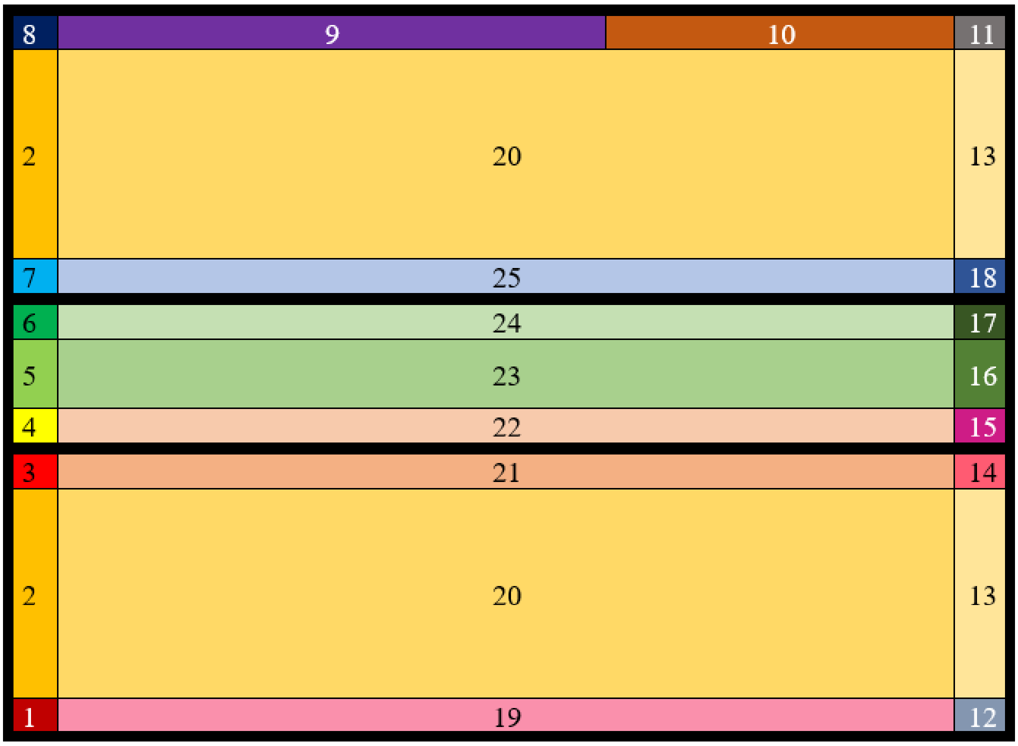


图 2 离散网格的 25 种情况

3.　构造离散方程

$$\rho \mathrm{c}\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+S_u$$

两侧同时对控制体及时间积分，

$$\int_{\Delta V}[\int_t^{t+\Delta t}\rho \mathrm{c}\frac{\partial T}{\partial t}dt]dV=\int_{\Delta V}[\int_t^{t+\Delta t}\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dt]dV+\int_{\Delta V}[\int_t^{t+\Delta t}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dt]dV+\int_{\Delta V}[\int_t^{t+\Delta t}S_u dt]dV$$

$$\int_s^n\int_w^e\int_t^{t+\Delta t}\rho c\frac{\partial T}{\partial t}dxdydt=\int_s^n\int_w^e\int_t^{t+\Delta t}\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dxdydt+\int_s^n\int_w^e\int_t^{t+\Delta t}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dxdydt+\int_t^{t+\Delta t}\overline{S}\cdot\Delta V dt$$

对时间后差，对空间中心差分，同时对温度加权处理，有

$$\frac{\partial T}{\partial t}=\frac{T_p-T_p^0}{\Delta t}$$

$$\left(\frac{\partial T}{\partial x}\right)_e=\frac{T_E-T_P}{\delta x_{EP}},\quad \left(\frac{\partial T}{\partial x}\right)_w=\frac{T_P-T_W}{\delta x_{PE}},\quad \left(\frac{\partial T}{\partial y}\right)_n=\frac{T_N-T_P}{\delta y_{NP}},\quad \left(\frac{\partial T}{\partial y}\right)_s=\frac{T_P-T_S}{\delta y_{PS}}$$

$$T=\theta T+\left(1-\theta\right)T^0$$

则可得

$$\begin{aligned}\rho c\left(T_P-T_P^0\right)\frac{\Delta x\Delta y}{\Delta t}=&\ \theta\left[\left(k_e\frac{T_E-T_P}{\delta x_{EP}}\right)-\left(k_w\frac{T_P-T_W}{\delta x_{PW}}\right)\right]\Delta y+\left(1-\theta\right)\left[\left(k_e\frac{T_E^0-T_P^0}{\delta x_{EP}}\right)-\left(k_w\frac{T_P^0-T_W^0}{\delta x_{PW}}\right)\right]\Delta y\\&+\theta\left[\left(k_n\frac{T_N-T_P}{\delta y_{NP}}\right)-\left(k_s\frac{T_P-T_S}{\delta y_{PS}}\right)\right]\Delta x+\left(1-\theta\right)\left[\left(k_n\frac{T_N^0-T_P^0}{\delta y_{NP}}\right)-\left(k_s\frac{T_P^0-T_S^0}{\delta y_{PS}}\right)\right]\Delta x\\&+S_u+S_p\left[\theta T_P+\left(1-\theta\right)T_P^0\right]\end{aligned}$$

按节点整理系数，则有：

$$\begin{aligned}a_PT_P=&\ a_E\left[\theta T_E+\left(1-\theta\right)T_E^0\right]+a_W\left[\theta T_W+\left(1-\theta\right)T_W^0\right]+a_N\left[\theta T_N+\left(1-\theta\right)T_N^0\right]+a_S\left[\theta T_S+\left(1-\theta\right)T_S^0\right]\\&+\left[a_P^0-\left(1-\theta\right)a_E-\left(1-\theta\right)a_W-\left(1-\theta\right)a_N-\left(1-\theta\right)a_S+\left(1-\theta\right)S_p\right]T_P^0+S_u\end{aligned}$$

其中，

$$a_E=\frac{k_e\Delta y}{\delta x_{EP}},\ a_W=\frac{k_w\Delta y}{\delta x_{PW}},\ a_N=\frac{k_n\Delta x}{\delta y_{NP}},\ a_S=\frac{k_s\Delta x}{\delta y_{PS}}$$

$$a_P=a_P^0+\theta\left(a_E+a_W+a_N+a_S-S_p\right)$$

依据界面上热流密度连续，可得

$$\frac{\delta y_{NP}}{k_n}=\frac{\delta y_{nP}}{k_P}+\frac{\delta y_{Nn}}{k_N}$$

对于两种材料交界处，若 $\delta y_{nP}=\delta y_{Nn}=\frac{1}{2}\delta y_{NP}$，则有 $k_n=\frac{2k_Pk_N}{k_P+k_N}$

同理，有 $k_s=\frac{2k_Pk_S}{k_P+k_S}$。

下面以显式格式（θ=0）为例。

1)　左下角节点

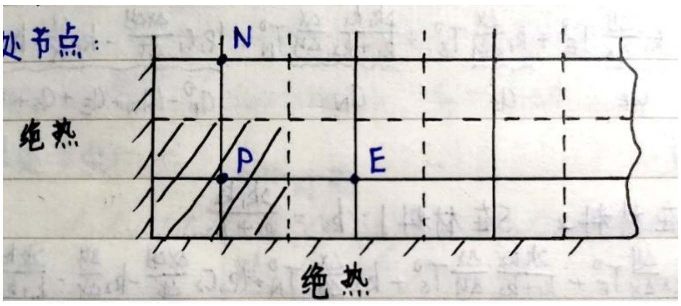


图 3 左下角节点处空间离散

绝热边界： $q_W = k_w\dfrac{T_P^0 - T_W^0}{\Delta x} = 0$， $q_S = k_s\dfrac{T_P^0 - T_S^0}{\Delta y} = 0$

$$\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}T_P}_{a_P} = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{0}_{a_S}\cdot T_S^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \underbrace{\left(\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y}\right)}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

W 边界处节点

绝热边界： $q_W = k_w\dfrac{T_P^0 - T_W^0}{\Delta x} = 0$

$$\rho c\frac{\Delta x\Delta y}{\Delta t}T_P = 0\cdot T_W^0 + k_e\frac{\Delta y}{\Delta x}T_E^0 + k_s\frac{\Delta x}{\Delta y}T_S^0 + k_n\frac{\Delta x}{\Delta y}T_N^0 + \left(\rho c\frac{\Delta x\Delta y}{\Delta t} - k_e\frac{\Delta y}{\Delta x} - k_s\frac{\Delta x}{\Delta y} - k_n\frac{\Delta x}{\Delta y}\right)T_P^0$$

2) W 边界处节点，对材料 1 内部：

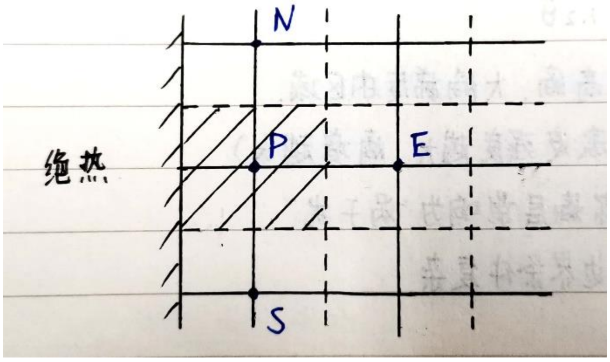


图 4 W 边界材料内部空间离散

$$\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}T_P}_{a_P} = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \underbrace{\left(\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y} - k_1\frac{\Delta x}{\Delta y}\right)}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

3) W 边界处节点，对材料 2 内部：

空间离散如图 4。

$$\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \left(\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t} - k_2\frac{\Delta y}{\Delta x} - k_2\frac{\Delta x}{\Delta y} - k_2\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P = a_P^0 = \rho_2 c_2\frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

4)　W 边界处节点，对材料 1、2 交界处，情况一：

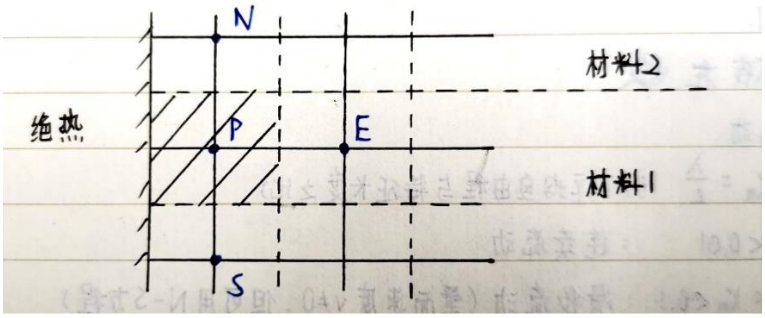


图 5 W 边界材料 1、2 交界处情况一

P、E、S 点在材料 1，N 点在材料 2，$k_n = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \left(\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

5)　W 边界处节点，对材料 1、2 交界处，情况二：

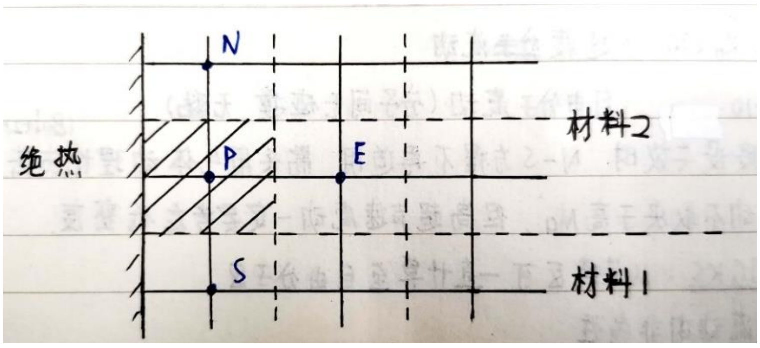


图 6 W 边界材料 1、2 交界处情况二

S 点在材料 1，N、P、E 点在材料 2，　$k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \left(\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t} - k_2\frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_2\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

6)　W 边界处节点，对材料 2、1 交界处，情况一：

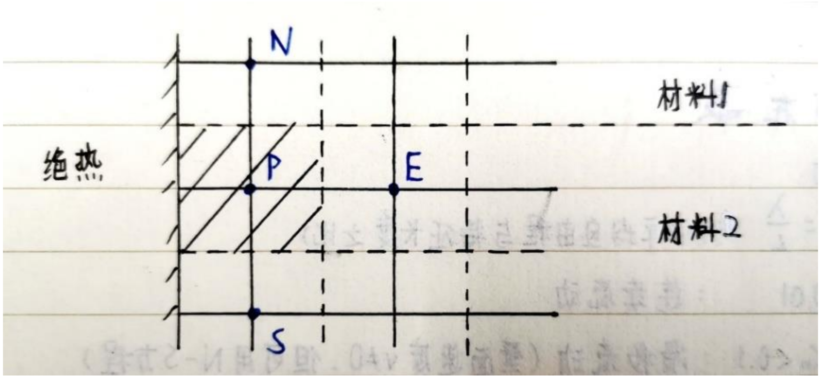


图 7 W 边界材料 2、1 交界处情况一

P、E、S 点在材料 2，N 点在材料 1， $k_n = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \left(\underbrace{\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t} - k_2\frac{\Delta y}{\Delta x} - k_2\frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

7)　W 边界处节点，对材料 2、1 交界处，情况二：

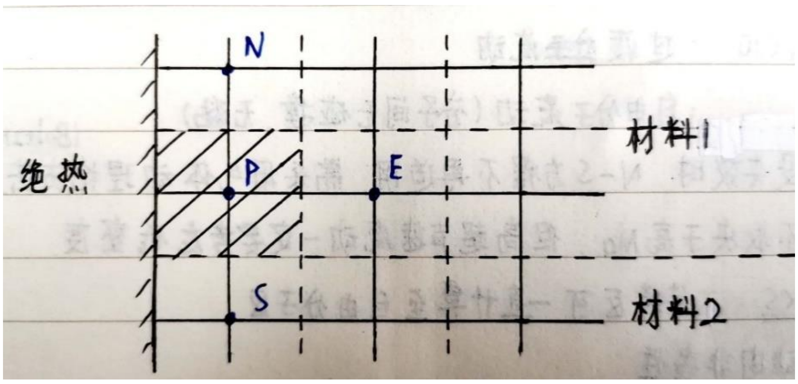


图 8 W 边界材料 2、1 交界处情况二

S 点在材料 2，N、P、E 点在材料 1， $k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P = \underbrace{0}_{a_W}\cdot T_W^0 + \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E}T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \left(\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_1\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}, S_u = 0, S_p = 0$$

8)　左上角处节点

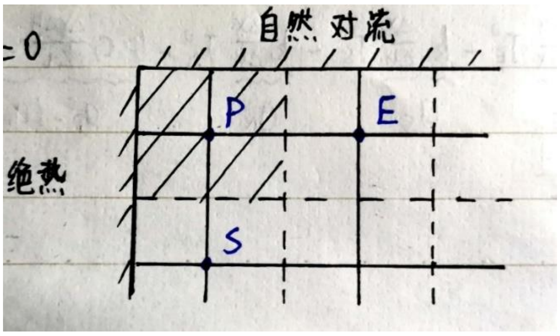


图 9 左上角处节点空间离散

绝热边界： $q_W = k_w \dfrac{T_P^0 - T_W^0}{\Delta x} = 0$

自然对流边界：$q_{NL} = h\left(T_{air} - T_P^0\right) = k_n \dfrac{T_N^0 - T_P^0}{\Delta y}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{0}_{a_W} \cdot T_W^0 + \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{0}_{a_N} T_N^0 + \Bigg(\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y} - h\Delta x}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)}\Bigg) T_P^0 + \underbrace{h\Delta x T_{air}}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = h\Delta x T_{air}, S_p = -h\Delta x$$

9) NL 边界处节点

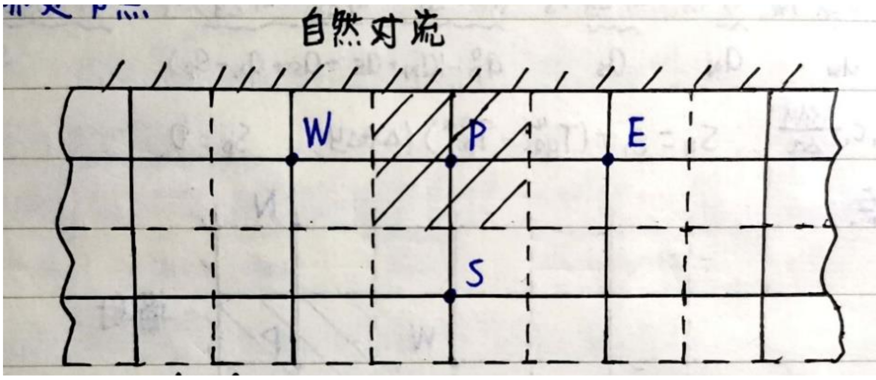


图 10 NL 边界处空间离散

自然对流边界：$q_{NL} = h\left(T_{air} - T_P^0\right) = k_n \dfrac{T_N^0 - T_P^0}{\Delta y}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} \cdot T_W^0 + \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{0}_{a_N} \cdot T_N^0 + \Bigg(\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y} - h\Delta x}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)}\Bigg) T_P^0 + \underbrace{h\Delta x T_{air}}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = h\Delta x T_{air}, S_p = -h\Delta x$$

10) NL 边界处节点

空间离散如图 10。

辐射边界： $q_{NR} = k_n \dfrac{T_N^0 - T_P^0}{\Delta y} = \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)$

$$\underbrace{\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_W}\cdot T_W^0 + \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{0}_{a_N}\cdot T_N^0 + \left(\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0 + \underbrace{\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta x}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = \varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta x, S_p = 0$$

11) 右上角处节点

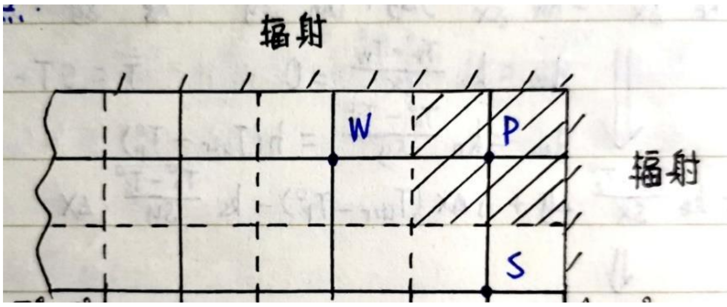


图 11 右上角处空间离散

辐射边界：$q_{NR} = k_n\dfrac{T_N^0 - T_P^0}{\Delta y} = \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)$， $q_E = k_e\dfrac{T_E^0 - T_P^0}{\Delta x} = \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)$

$$\underbrace{\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_W}\cdot T_W^0 + \underbrace{0}_{a_E}\cdot T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{0}_{a_N}\cdot T_N^0 + \left(\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0 + \underbrace{\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\left(\Delta x+\Delta y\right)}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = \varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\left(\Delta x+\Delta y\right), S_p = 0$$

12) 右下角处节点

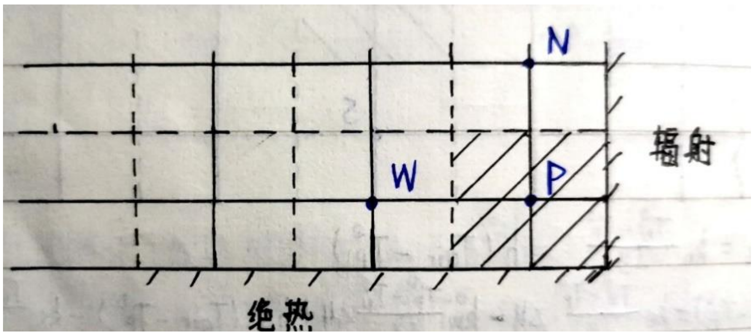


图 12 右下角处空间离散

绝热边界：$q_S = k_s\dfrac{T_P^0 - T_S^0}{\Delta x} = 0$

辐射边界：$q_E = k_e\dfrac{T_E^0 - T_P^0}{\Delta x} = \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)$

$$\underbrace{\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{0}_{a_E}\cdot T_E^0 + \underbrace{0}_{a_S}\cdot T_S^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \left(\underbrace{\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0 + \underbrace{\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = \varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y, S_p = 0$$

E 边界处节点

辐射边界：$q_E = k_e \frac{T_E^0 - T_P^0}{\Delta x} = \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)$

$$\rho c \frac{\Delta x\Delta y}{\Delta t} T_P = k_w \frac{\Delta y}{\Delta x}\cdot T_W^0 + 0\cdot T_E^0 + k_s\frac{\Delta x}{\Delta y}T_S^0 + k_n\frac{\Delta x}{\Delta y}T_N^0 + \left(\rho c\frac{\Delta x\Delta y}{\Delta t} - k_w\frac{\Delta y}{\Delta x} - k_s\frac{\Delta x}{\Delta y} - k_n\frac{\Delta x}{\Delta y}\right)T_P^0 + \varepsilon\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$$

13) E 边界处节点，对材料 1 内部：

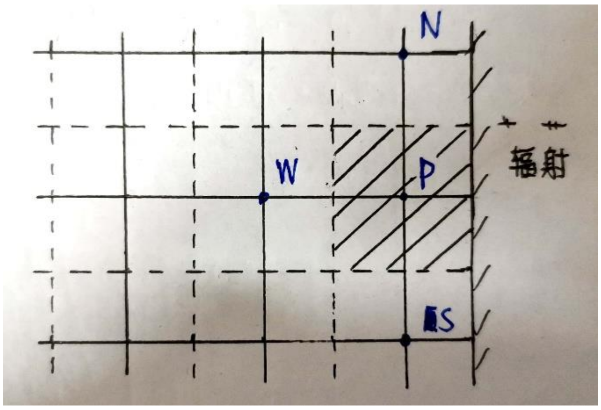


图 13 E 边界材料内部空间离散

$$\underbrace{\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}T_P}_{a_P} = \underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_W}T_W^0 + \underbrace{0}_{a_E}\cdot T_E^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \underbrace{\left(\rho_1 c_1\frac{\Delta x\Delta y}{\Delta t} - k_1\frac{\Delta y}{\Delta x} - k_1\frac{\Delta x}{\Delta y} - k_1\frac{\Delta x}{\Delta y}\right)}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}T_P^0 + \underbrace{\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1\frac{\Delta x\Delta y}{\Delta t}, S_u = \varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y, S_p = 0$$

14) E 边界处节点，对材料 2 内部：

空间离散如图 13。

$$\underbrace{\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}T_P}_{a_P} = \underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_W}T_W^0 + \underbrace{0}_{a_E}\cdot T_E^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0 + \underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_N}T_N^0 + \underbrace{\left(\rho_2 c_2\frac{\Delta x\Delta y}{\Delta t} - k_2\frac{\Delta y}{\Delta x} - k_2\frac{\Delta x}{\Delta y} - k_2\frac{\Delta x}{\Delta y}\right)}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}T_P^0 + \underbrace{\varepsilon_2\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_2 c_2\frac{\Delta x\Delta y}{\Delta t}, S_u = \varepsilon_2\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y, S_p = 0$$

15) E 边界处节点，对材料 1、2 交界处，情况一：

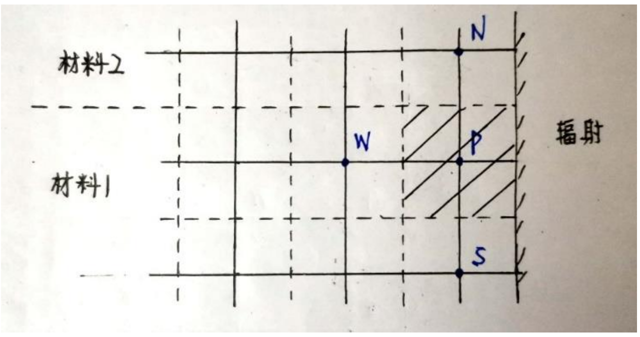


图 14 E 边界材料 1、2 交界处情况一

P、W、S 点在材料 1，N 点在材料 2，$k_n = \frac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{0}_{a_E} \cdot T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0 + \underbrace{\varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = \varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta y, S_p = 0$$

16) E 边界处节点，对材料 1、2 交界处，情况二：

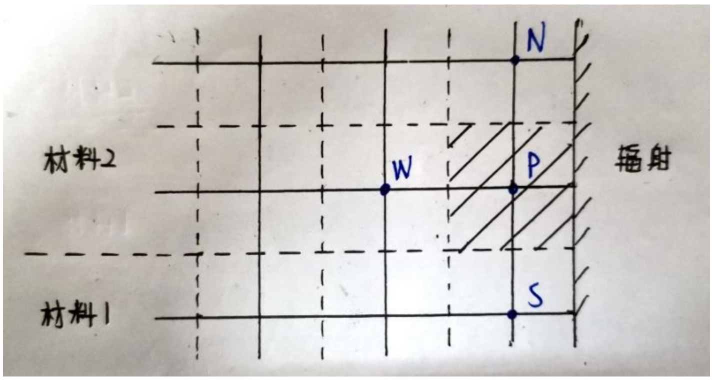


图 15 E 边界材料 1、2 交界处情况二

S 点在材料 1，N、P、W 点在材料 2， $k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{0}_{a_E} \cdot T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{k_2 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t} - k_2 \frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_2 \frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0 + \underbrace{\varepsilon_2 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}, S_u = \varepsilon_2 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta y, S_p = 0$$

17) E 边界处节点，对材料 2、1 交界处，情况一：

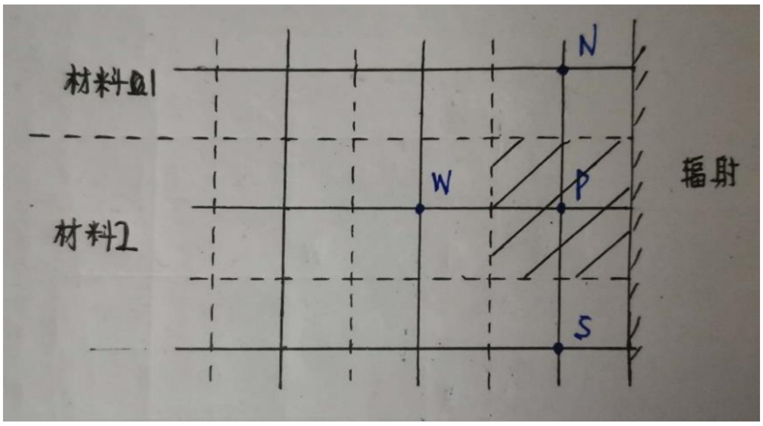


图 16 E 边界材料 2、1 交界处情况一

P、W、S 点在材料 2，N 点在材料 1， $k_n = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{0}_{a_E} \cdot T_E^0 + \underbrace{k_2 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t} - k_2 \frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_2 \frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0 + \underbrace{\varepsilon_2 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}, S_u = \varepsilon_2 \sigma \left(T_{gas}^4 - T_P^{04} \right) \Delta y, S_p = 0$$

18) E 边界处节点，对材料 2、1 交界处，情况二：

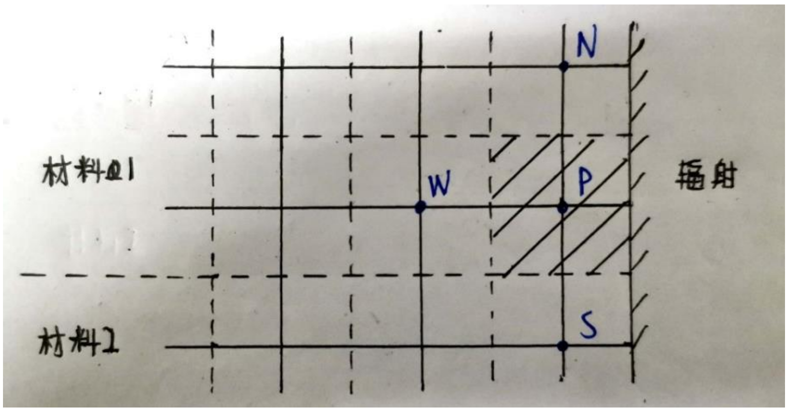


图 17 E 边界材料 2、1 交界处情况二

S 点在材料 2，N、P、W 点在材料 1， $k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{0}_{a_E} \cdot T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2} \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \Bigg(\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2} \frac{\Delta x}{\Delta y}}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} \Bigg) T_P^0 + \underbrace{\varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04} \right) \Delta y}_{S_u}$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = \varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04} \right) \Delta y, S_p = 0$$

19) S 边界处节点

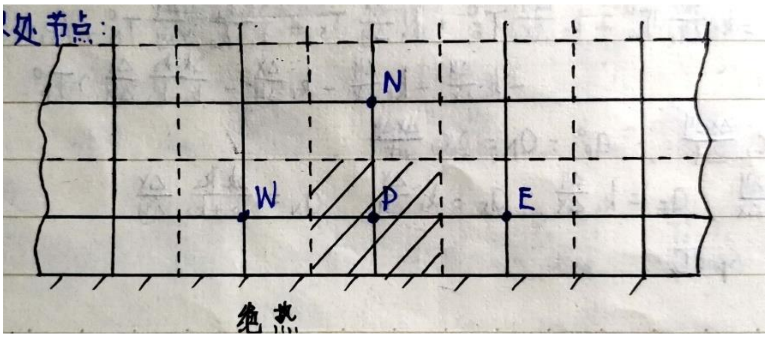


图 18 S 边界处空间离散

绝热边界： $q_S = k_s \dfrac{T_P^0 - T_S^0}{\Delta x} = 0$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{0}_{a_S} \cdot T_S^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \Bigg(\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y}}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} \Bigg) T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = 0, S_p = 0$$

内部节点

$$\rho c\frac{\Delta x\Delta y}{\Delta t}T_P=k_w\frac{\Delta y}{\Delta x}T_W^0+k_e\frac{\Delta y}{\Delta x}T_E^0+k_s\frac{\Delta x}{\Delta y}T_S^0+k_n\frac{\Delta x}{\Delta y}T_N^0+\left(\rho c\frac{\Delta x\Delta y}{\Delta t}-k_w\frac{\Delta y}{\Delta x}-k_e\frac{\Delta y}{\Delta x}-k_s\frac{\Delta x}{\Delta y}-k_n\frac{\Delta x}{\Delta y}\right)T_P^0$$

20) 内部节点，对材料 1 内部：

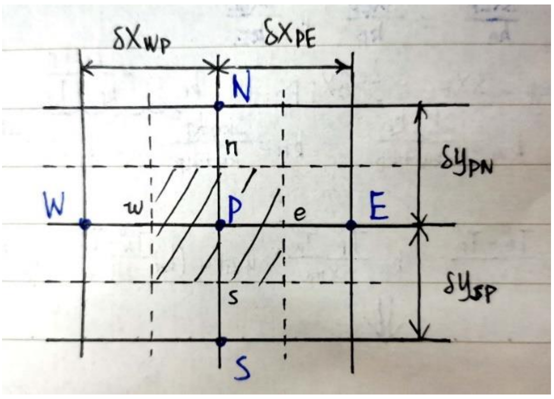


图 19 内部节点空间离散

$$\underbrace{\rho_1c_1\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P=\underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_W}T_W^0+\underbrace{k_1\frac{\Delta y}{\Delta x}}_{a_E}T_E^0+\underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0+\underbrace{k_1\frac{\Delta x}{\Delta y}}_{a_N}T_N^0+\left(\underbrace{\rho_1c_1\frac{\Delta x\Delta y}{\Delta t}-k_1\frac{\Delta y}{\Delta x}-k_1\frac{\Delta y}{\Delta x}-k_1\frac{\Delta x}{\Delta y}-k_1\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P=a_P^0=\rho_1c_1\frac{\Delta x\Delta y}{\Delta t},S_u=0,S_p=0$$

21) 内部节点，对材料 2 内部：

空间离散如图 19。

$$\underbrace{\rho_2c_2\frac{\Delta x\Delta y}{\Delta t}}_{a_P}T_P=\underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_W}T_W^0+\underbrace{k_2\frac{\Delta y}{\Delta x}}_{a_E}T_E^0+\underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_S}\cdot T_S^0+\underbrace{k_2\frac{\Delta x}{\Delta y}}_{a_N}T_N^0+\left(\underbrace{\rho_2c_2\frac{\Delta x\Delta y}{\Delta t}-k_2\frac{\Delta y}{\Delta x}-k_2\frac{\Delta y}{\Delta x}-k_2\frac{\Delta x}{\Delta y}-k_2\frac{\Delta x}{\Delta y}}_{a_P^0-\left(a_W+a_E+a_S+a_N-S_p\right)}\right)T_P^0$$

$$a_P=a_P^0=\rho_2c_2\frac{\Delta x\Delta y}{\Delta t},S_u=0,S_p=0$$

22) 内部节点，对材料 1、2 交界处，情况一：

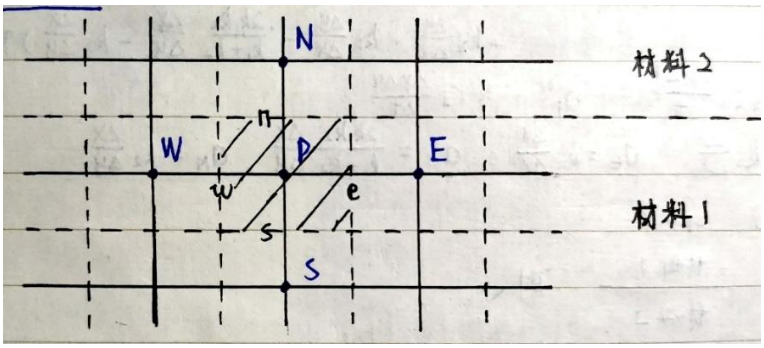


图 20 内部节点材料 1、2 交界处情况一

W、P、E、S 点在材料 1，N 点在材料 2，$k_n=\dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = 0, S_p = 0$$

23) 内部节点，对材料 1、2 交界处，情况二：

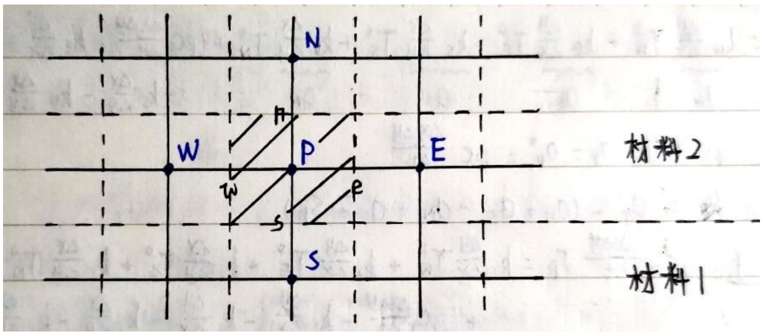


图 21 内部节点材料 1、2 交界处情况二

S 点在材料 1， N、W、P、E 点在材料 2， $k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{k_2 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t} - k_2 \frac{\Delta y}{\Delta x} - k_2 \frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_2 \frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}, S_u = 0, S_p = 0$$

24) 内部节点，对材料 2、1 交界处，情况一：

W、P、E、S 点在材料 2，N 点在材料 1， $k_n = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{k_2 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{k_2 \frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t} - k_2 \frac{\Delta y}{\Delta x} - k_2 \frac{\Delta y}{\Delta x} - k_2 \frac{\Delta x}{\Delta y} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0$$

$$a_P = a_P^0 = \rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}, S_u = 0, S_p = 0$$

25) 内部节点，对材料 2、1 交界处，情况二：

S 点在材料 2，N、W、P、E 点在材料 1， $k_s = \dfrac{2k_1k_2}{k_1+k_2}$

$$\underbrace{\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}}_{a_P} T_P = \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_W} T_W^0 + \underbrace{k_1 \frac{\Delta y}{\Delta x}}_{a_E} T_E^0 + \underbrace{\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}}_{a_S} \cdot T_S^0 + \underbrace{k_1 \frac{\Delta x}{\Delta y}}_{a_N} T_N^0 + \underbrace{\left(\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t} - k_1 \frac{\Delta y}{\Delta x} - k_1 \frac{\Delta y}{\Delta x} - \frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y} - k_1 \frac{\Delta x}{\Delta y}\right)}_{a_P^0 - \left(a_W + a_E + a_S + a_N - S_p\right)} T_P^0$$

$$a_P = a_P^0 = \rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}, S_u = 0, S_p = 0$$

4. 边界系数

1) 显式格式：

$$a_P T_P = a_E T_E^0 + a_W T_W^0 + a_N T_N^0 + a_S T_S^0 + \left[a_P^0 - \left(a_E + a_W + a_N + a_S - S_p\right)\right] T_P^0 + S_u$$

其中，$a_P = a_P^0$

2) 半隐式格式：

$$a_P T_P = a_E \frac{T_E + T_E^0}{2} + a_W \frac{T_W + T_W^0}{2} + a_N \frac{T_N + T_N^0}{2} + a_S \frac{T_S + T_S^0}{2} + \left[a_P^0 - \frac{1}{2}\left(a_E + a_W + a_N + a_S - S_p\right)\right] T_P^0 + S_u$$

其中，$a_P = a_P^0 + \frac{1}{2}\left(a_E + a_W + a_N + a_S - S_p\right)$

3) 隐式格式：

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + a_P^0 T_P^0 + S_u$$

其中，$a_P = a_P^0 + \left(a_E + a_W + a_N + a_S - S_p\right)$

表 1 显式、半隐式、隐式格式边界系数表

节点	a_P^0	a_W	a_E	a_S	a_N	S_u	S_p
1	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_1 \frac{\Delta y}{\Delta x}$	0	$k_1 \frac{\Delta x}{\Delta y}$	0	0
2	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta x}{\Delta y}$	$k_1 \frac{\Delta x}{\Delta y}$	0	0
3	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta x}{\Delta y}$	$\frac{2k_1 k_2}{k_1 + k_2} \frac{\Delta x}{\Delta y}$	0	0
4	$\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_2 \frac{\Delta y}{\Delta x}$	$\frac{2k_1 k_2}{k_1 + k_2} \frac{\Delta x}{\Delta y}$	$k_2 \frac{\Delta x}{\Delta y}$	0	0
5	$\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_2 \frac{\Delta y}{\Delta x}$	$k_2 \frac{\Delta x}{\Delta y}$	$k_2 \frac{\Delta x}{\Delta y}$	0	0
6	$\rho_2 c_2 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_2 \frac{\Delta y}{\Delta x}$	$k_2 \frac{\Delta x}{\Delta y}$	$\frac{2k_1 k_2}{k_1 + k_2} \frac{\Delta x}{\Delta y}$	0	0
7	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_1 \frac{\Delta y}{\Delta x}$	$\frac{2k_1 k_2}{k_1 + k_2} \frac{\Delta x}{\Delta y}$	$k_1 \frac{\Delta x}{\Delta y}$	0	0
8	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	0	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta x}{\Delta y}$	0	$h \Delta x T_{air}$	$-h\Delta x$
9	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta x}{\Delta y}$	0	$h \Delta x T_{air}$	$-h\Delta x$
10	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta y}{\Delta x}$	$k_1 \frac{\Delta x}{\Delta y}$	0	$\varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04}\right) \Delta x$	0
11	$\rho_1 c_1 \frac{\Delta x \Delta y}{\Delta t}$	$k_1 \frac{\Delta y}{\Delta x}$	0	$k_1 \frac{\Delta x}{\Delta y}$	0	$\varepsilon_1 \sigma \left(T_{gas}^4 - T_P^{04}\right)\left(\Delta x + \Delta y\right)$	0

12	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	0	0	$k_1\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
13	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	0	$k_1\frac{\Delta x}{\Delta y}$	$k_1\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
14	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	0	$k_1\frac{\Delta x}{\Delta y}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
15	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	0	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$k_2\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
16	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	0	$k_2\frac{\Delta x}{\Delta y}$	$k_2\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
17	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	0	$k_2\frac{\Delta x}{\Delta y}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
18	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	0	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$k_1\frac{\Delta x}{\Delta y}$	$\varepsilon_1\sigma\left(T_{gas}^4 - T_P^{04}\right)\Delta y$	0
19	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta y}{\Delta x}$	0	$k_1\frac{\Delta x}{\Delta y}$	0	0
20	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta x}{\Delta y}$	$k_1\frac{\Delta x}{\Delta y}$	0	0
21	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta x}{\Delta y}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	0	0
22	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	$k_2\frac{\Delta y}{\Delta x}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$k_2\frac{\Delta x}{\Delta y}$	0	0
23	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	$k_2\frac{\Delta y}{\Delta x}$	$k_2\frac{\Delta x}{\Delta y}$	$k_2\frac{\Delta x}{\Delta y}$	0	0
24	$\rho_2 c_2 \frac{\Delta x\Delta y}{\Delta t}$	$k_2\frac{\Delta y}{\Delta x}$	$k_2\frac{\Delta y}{\Delta x}$	$k_2\frac{\Delta x}{\Delta y}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	0	0
25	$\rho_1 c_1 \frac{\Delta x\Delta y}{\Delta t}$	$k_1\frac{\Delta y}{\Delta x}$	$k_1\frac{\Delta y}{\Delta x}$	$\frac{2k_1k_2}{k_1+k_2}\frac{\Delta x}{\Delta y}$	$k_1\frac{\Delta x}{\Delta y}$	0	0

5. 求解方程组

$$\begin{pmatrix} a_P[0][0] & -a_E[0][0] & 0 & 0 & \cdots & -a_N[0][0] & 0 & 0 & \cdots \\ -a_W[0][1] & a_P[0][1] & -a_E[0][1] & 0 & \cdots & 0 & -a_N[0][1] & 0 & \cdots \\ 0 & -a_W[0][2] & a_P[0][2] & -a_E[0][2] & \cdots & 0 & 0 & -a_N[0][2] & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -a_S[1][0] & 0 & 0 & 0 & \cdots & a_P[1][0] & -a_E[1][0] & 0 & \cdots \\ 0 & -a_S[1][1] & 0 & 0 & \cdots & -a_W[1][1] & a_P[1][1] & -a_E[1][1] & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} T[0][0] \\ T[0][1] \\ T[0][2] \\ \cdots \\ T[1][0] \\ T[1][1] \\ \cdots \end{pmatrix} = \begin{pmatrix} a_P^0[0][0]T_P^0[0][0] + Su[0][0] \\ a_P^0[0][1]T_P^0[0][1] + Su[0][1] \\ a_P^0[0][2]T_P^0[0][2] + Su[0][2] \\ \cdots \\ a_P^0[1][0]T_P^0[1][0] + Su[1][0] \\ a_P^0[1][1]T_P^0[1][1] + Su[1][1] \\ \cdots \end{pmatrix}$$

四、 编程计算思路

1. 时间步长与空间步长的关系

1) 时间步长选取原则：

为进行计算机数值求解，需要求解矩阵的系数能够满足有界性条件，即方程的各项系数大于 0。

2) 显式格式：

网格数目为 20×20 时 Δx=0.0025 m , $\Delta y = 0.0005m$

由有界性条件：$a_P^0 = a_P - \left(a_W + a_E + a_S + a_N - S_P\right) > 0$

以及：$a_E = k_e \dfrac{\Delta y}{\delta x_{EP}} \quad a_W = k_w \dfrac{\Delta y}{\delta x_{PW}} \quad a_N = k_n \dfrac{\Delta x}{\delta y_{NP}} \quad a_S = k_s \dfrac{\Delta x}{\delta y_{PS}} \quad a_P = \rho C \dfrac{\Delta x \Delta y}{\Delta t}$

得到：

$$\Delta t_{\max} = \left(\rho C\right)_{\min} \frac{\Delta x \Delta y}{k_{\max}\dfrac{\Delta y}{\delta x_{EP}} + k_{\max}\dfrac{\Delta y}{\delta x_{PW}} + k_{\max}\dfrac{\Delta x}{\delta x_{NP}} + k_{\max}\dfrac{\Delta x}{\delta x_{PS}}}$$

$$= 1500 \times 1465 \times \frac{0.0025 \times 0.0005}{53.6 \times \dfrac{0.0005}{0.0025} \times 2 + 53.6 \times \dfrac{0.0025}{0.0005} \times 2} = 0.004928$$

实际计算时取：$\Delta t_{\max} = 0.004$

显式格式的计算时间步长的最大值给出了一个相当严格的限制。此外 Δt 与成 $\Delta x \Delta y$ 正比关系，若将网格缩小，时间步长会大幅度减小，导致实际计算时为提高计算精度需花费巨大的代价。但是，显式格式的线性方程组每行仅含一个未知量，在求解线性方程组时计算量相对隐式更少，这是一个优点。

3) 半隐格式（C-N 格式）：

网格数目为 20×20 时 Δx=0.0025 m , $\Delta y = 0.0005m$

由有界性条件：$a_P^0 = a_P - 1/2 \times \left(a_W + a_E + a_S + a_N - S_P\right) > 0$

以及：$a_E = k_e \dfrac{\Delta y}{\delta x_{EP}} \quad a_W = k_w \dfrac{\Delta y}{\delta x_{PW}} \quad a_N = k_n \dfrac{\Delta x}{\delta y_{NP}} \quad a_S = k_s \dfrac{\Delta x}{\delta y_{PS}} \quad a_P = \rho C \dfrac{\Delta x \Delta y}{\Delta t}$

得到：

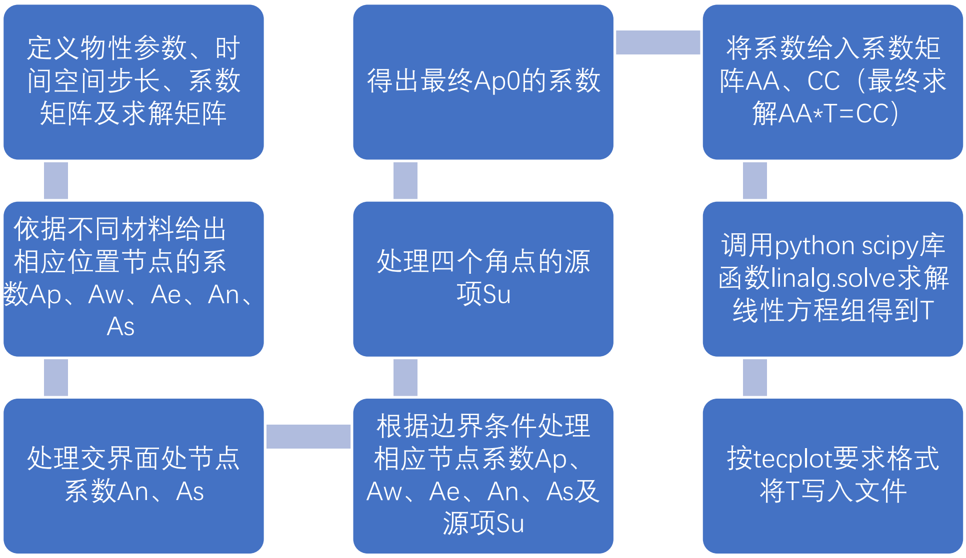
$$\Delta t_{\max} = 2 \times \left(\rho C\right)_{\min} \frac{\Delta x \Delta y}{k_{\max}\dfrac{\Delta y}{\delta x_{EP}} + k_{\max}\dfrac{\Delta y}{\delta x_{PW}} + k_{\max}\dfrac{\Delta x}{\delta x_{NP}} + k_{\max}\dfrac{\Delta x}{\delta x_{PS}}}$$

$$= 0.009856$$

4) 隐式格式：

有界性条件自动满足，无最大时间步长的要求，但时间步长的选取关乎计算结果的精确度。当时间步长取 0.02s 时能得到较好结果，当时间步长取 0.2s 时计算结果偏差较大。

2. 编程整体思路（以显式为例）



3. 迭代求解方法（半隐式）

采用亚松弛迭代的方法，以半隐式方法为例，迭代公式为$T_{i,j}=T_{i,j}^0+m\cdot\left\{\left(a_e\cdot\left(\frac{T_{i+1,j}^0+T_{i+1,j}}{2}\right)+a_w\cdot\left(\frac{T_{i-1,j}^0+T_{i-1,j}}{2}\right)+a_s\cdot\left(\frac{T_{i,j-1}^0+T_{i,j-1}}{2}\right)+a_n\cdot\left(\frac{T_{i,j+1}^0+T_{i,j+1}}{2}\right)+\left[a_{p0}-\frac{1}{2}\cdot\left(a_e+a_w+a_s+a_n-s_p\right)\right]\cdot T_{i,j}^0+s_u\right)\cdot\frac{1}{a_p}-T_{i,j}^0\right\}$，其中 m 为松弛因子。以计算网格数为 Nx=100、Ny=20（Nx 表示 x 方向的网格数量，Ny 同理）例，当计算时间 t=30 秒时，当松弛因子分别为 0.1、0.5、0.9 时，计算得到的云图分别为：

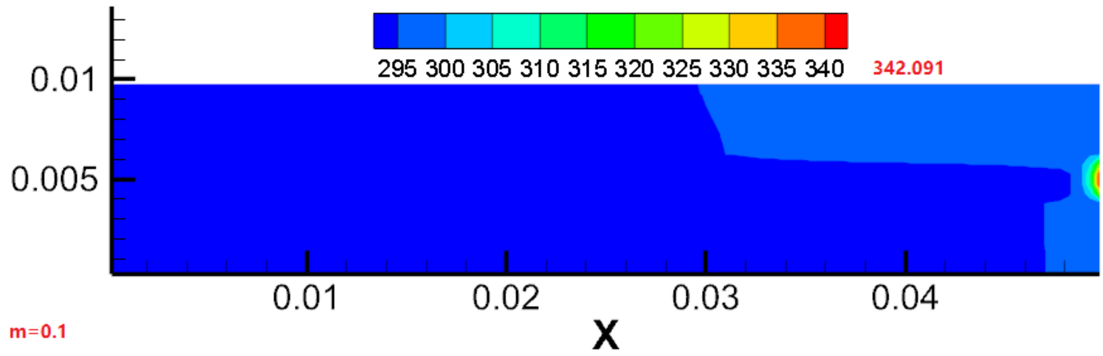


图 22 松弛因子=0.1

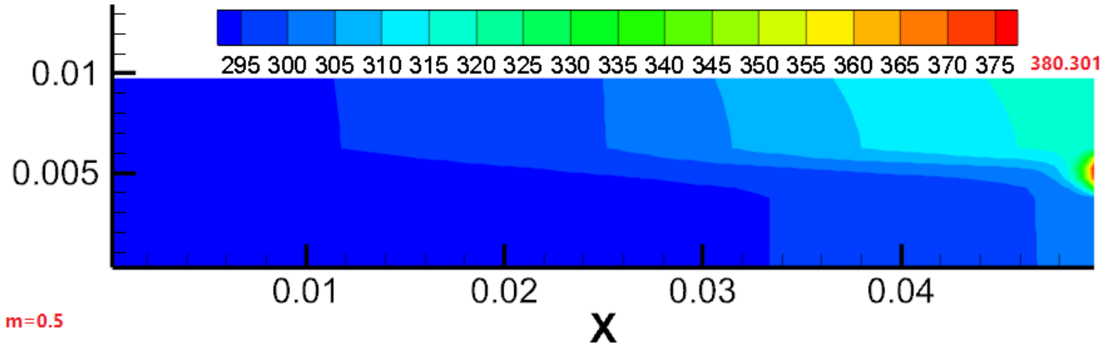


图 23 松弛因子=0.5

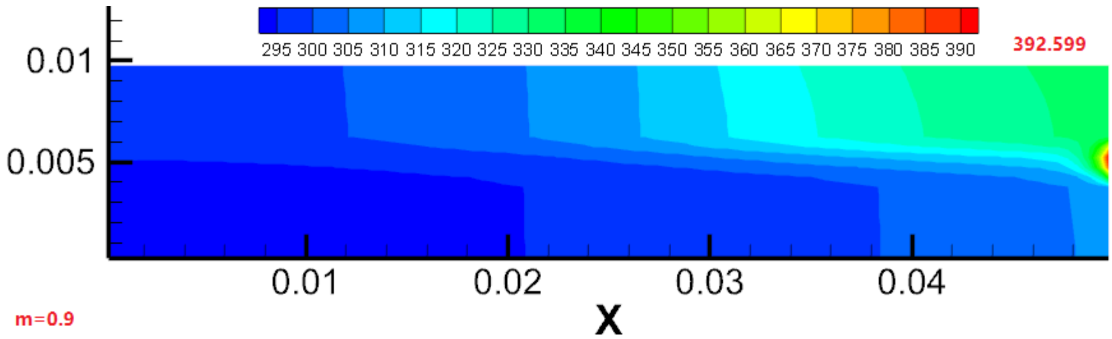


图 24 松弛因子=0.9

分析可以得到，结果逐渐收敛，最终通过计算得到最终的松弛因子为0.93，最终得到的云图为：

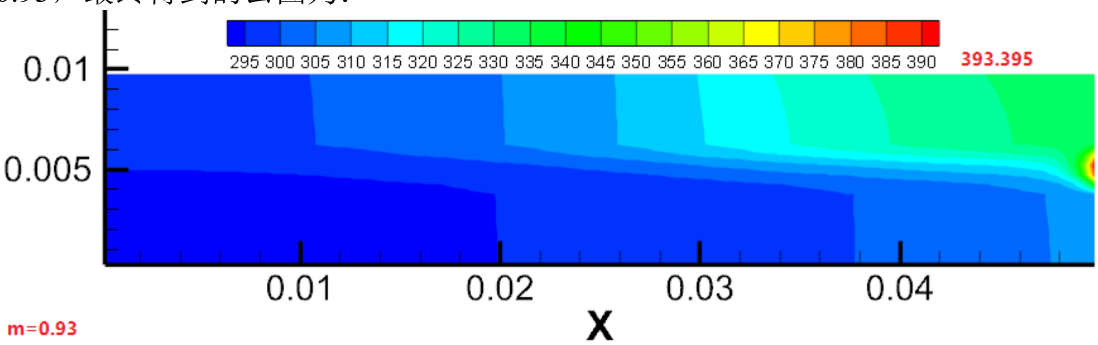


图 25 松弛因子=0.93

通过这种计算和分析方法可以得到最终收敛稳定的不同网格数量、不同时刻的数据。

4. 隐式求解方法

在 C++中引入 Eigen 库，对系数矩阵进行 QR 分解，求解每步时间步长时空间各点温度。但由于运算速度较慢，因此没有尝试 100×20 网格的计算。

五、 Fluent 仿真

1. SpaceClaim 建模

建模时要注意：

1.由于 Fluent 中有两个计算域，几何模型要划分成两个表面；

2.由于上边界水平线要分段处理边界条件，为便于在划分网格时分块与关联，水平方向的线都要分段画出，突出交点。

图 26 几何模型

2. ICEM 划分网格

1) 创建 part B1～B6，方便 Fluent 设置边界条件

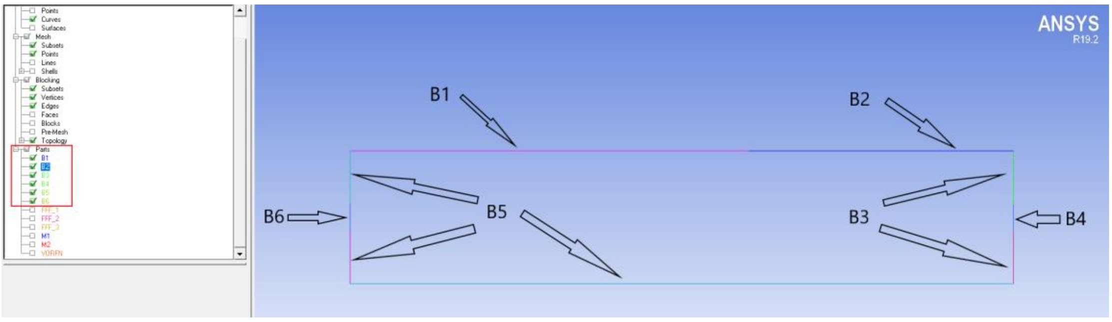


图 27 创建边界 part

2) 创建块 block，并进行分割

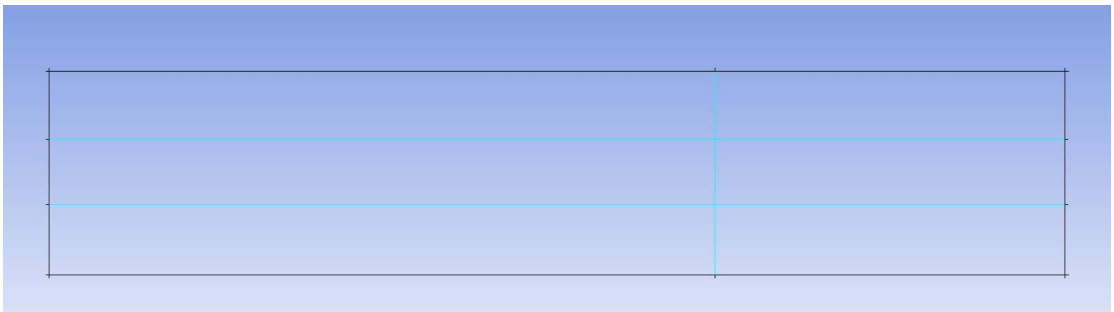


图 28 块的分割

3) 将块与几何模型进行点与线的关联

图 29 关联效果图

4) 设置各边节点数

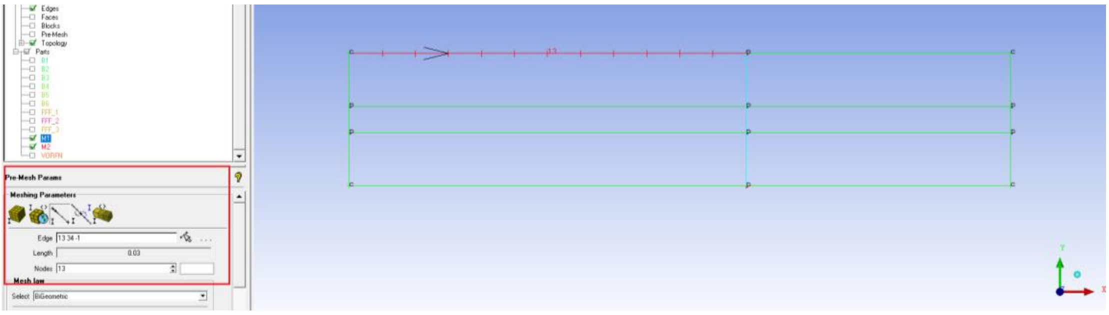


图 30 设置各边节点数目

5) 创建 part M1 与 M2，将块分成两个计算域

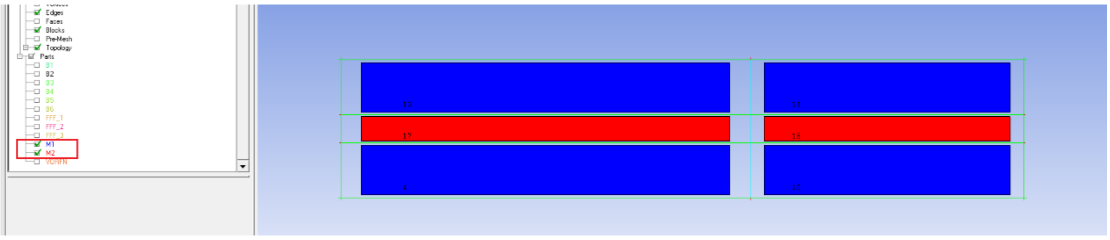


图 31 划分计算域

6) Pre—Mesh，转成非结构网格，导入 Fluent

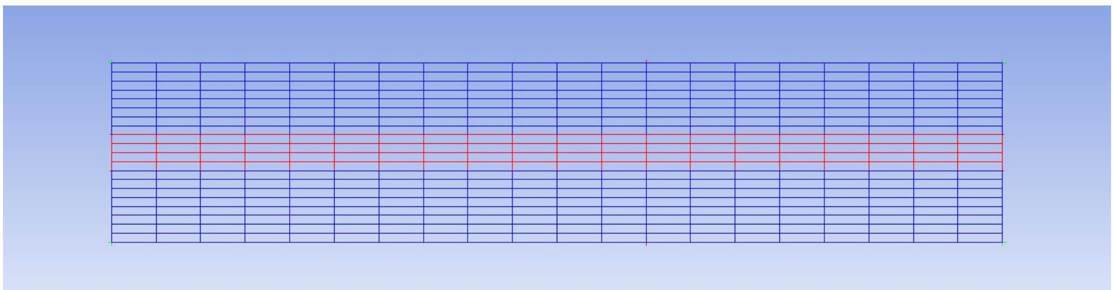


图 32 网格（20×20）

3. Fluent 处理

1) 模型选择

General 中选择瞬态计算；由于计算域内为固体的热量传递问题，因此只需要开一个能量方程。

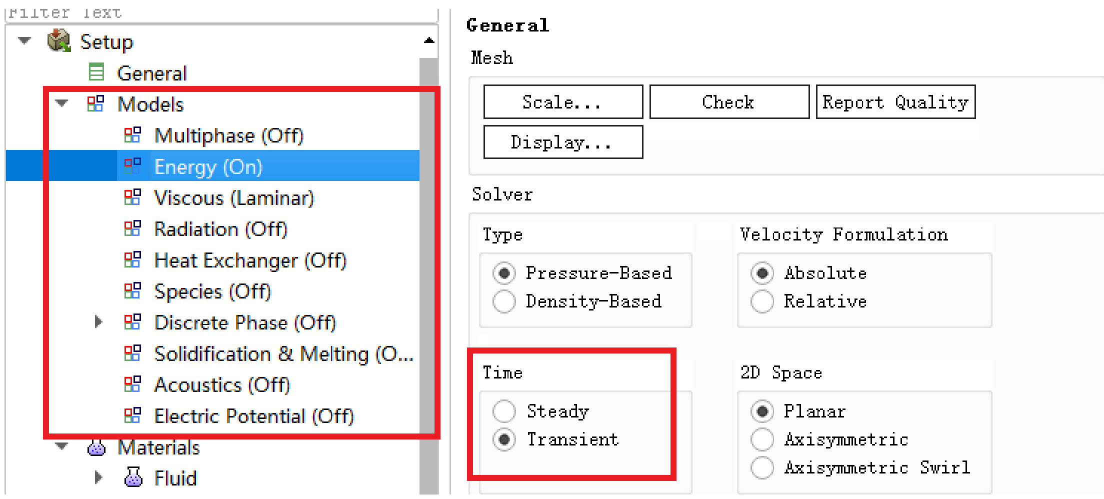


图 33 模型选择

2) 设置材料参数

按题目所给条件，给定两个计算域的材料及参数：

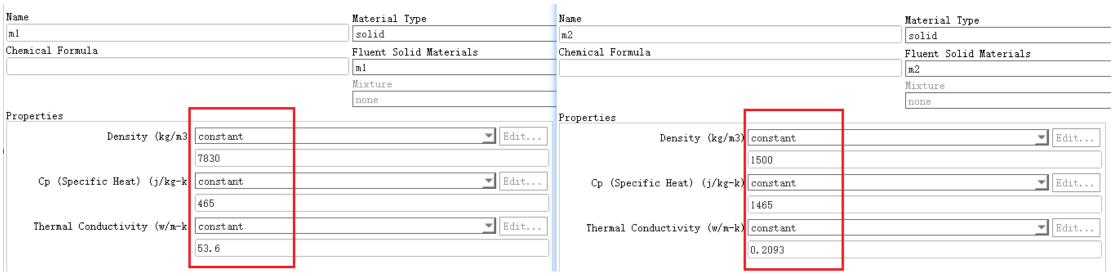


图 34 材料参数设置

3) 边界条件处理

由题目中物理模型及参数，设置各边对流、辐射或绝热等边界条件。

要注意的是，b1 边界对流换热系数需要满足实验关联式，因此需要编写 UDF 进行处理。

```c
#include "udf.h"
DEFINE_PROFILE(user_h,thread,pos)
{
double tem,tm,beta,Pr,v,Gr,Nu,h;
face_t f;
begin_f_loop(f, thread)
{
tem=F_T(f,thread);
tm=(tem-253)/2;
beta=2/(tem+293);
Pr=4*(1E-14)*pow(tm,6)-3*(1E-11)*pow(tm,5)+6*(1E-9)*pow(tm,4)-7*(1E-7)*pow(tm,3)+4*(1E-5)*pow(tm,2)-0.0012*tm+0.7163;
v=9*(1E-11)*pow(tm,2)+9*(1E-8)*tm+1.3175*(1E-5);
Gr=9.8*beta*fabs(tem-293)*pow(0.03,3)/pow(v,2);
Nu=0.54*pow(Gr*Pr,0.25);
h=Nu*2.59/3;
F_PROFILE(f,thread,pos)=h;
}
end_f_loop(f,thread)
}
```

图 35 UDF 代码

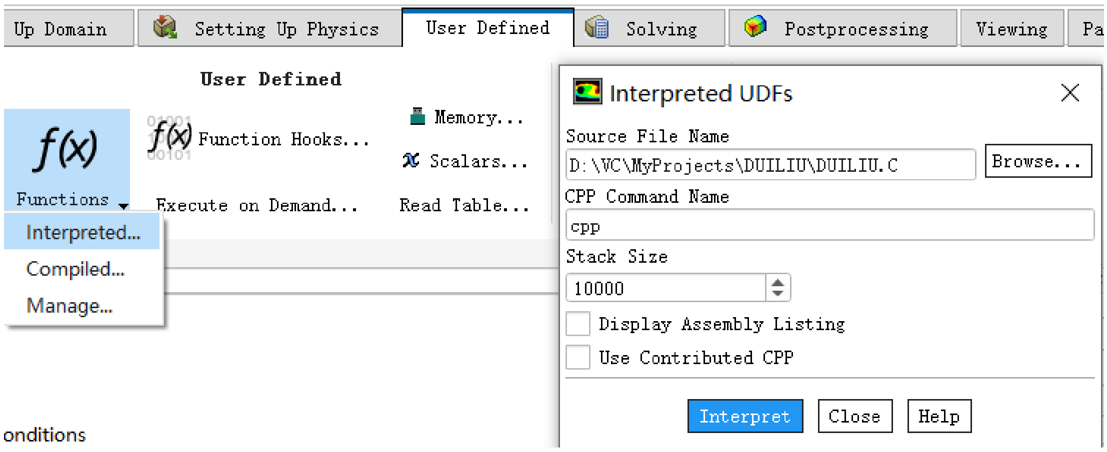


图 36 解释 UDF

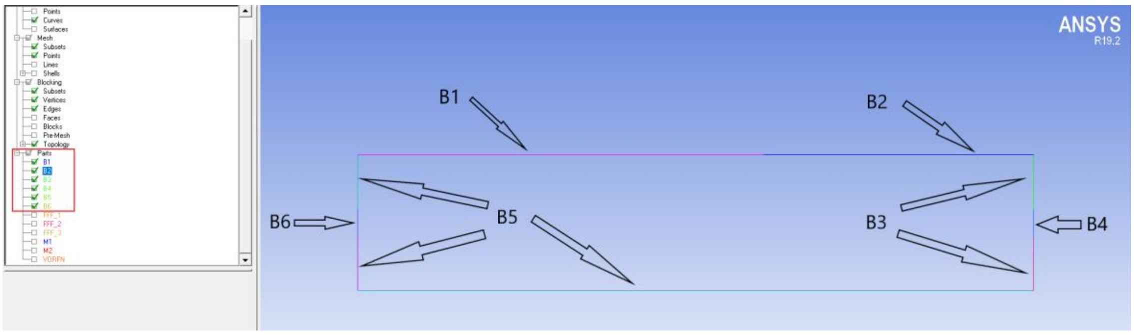


图 37 边界代号含义

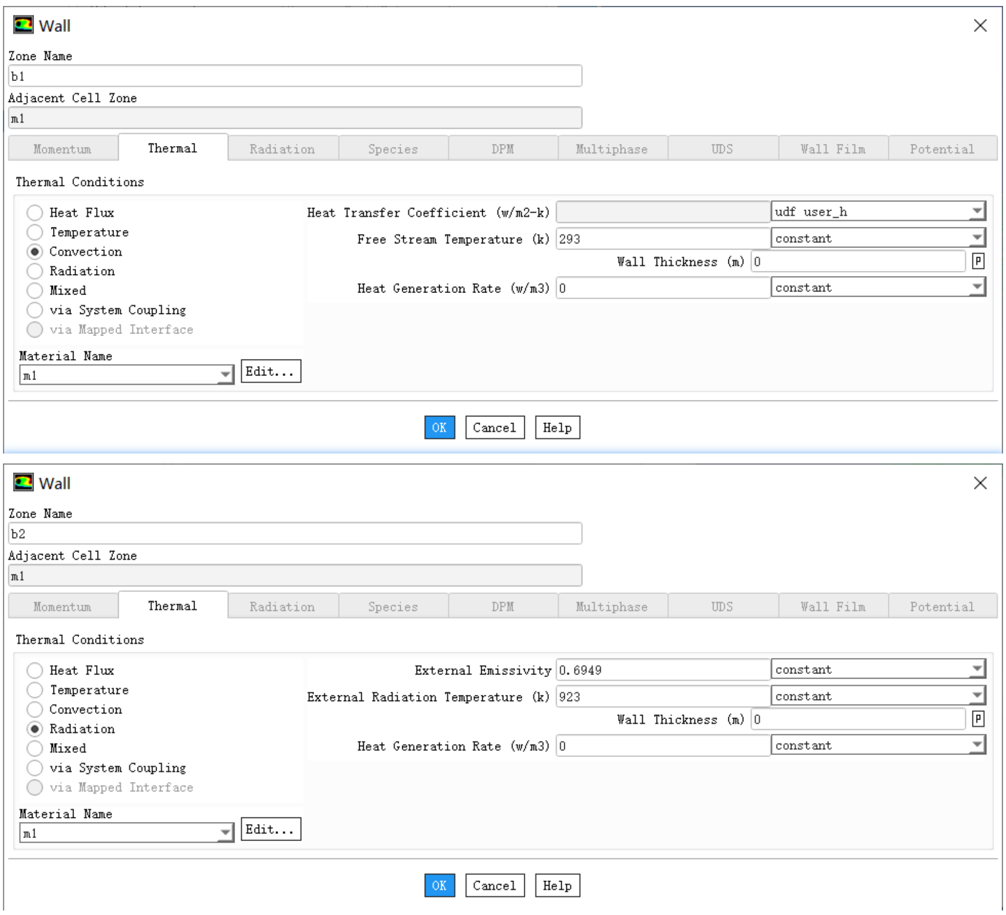


图 38 上边界条件

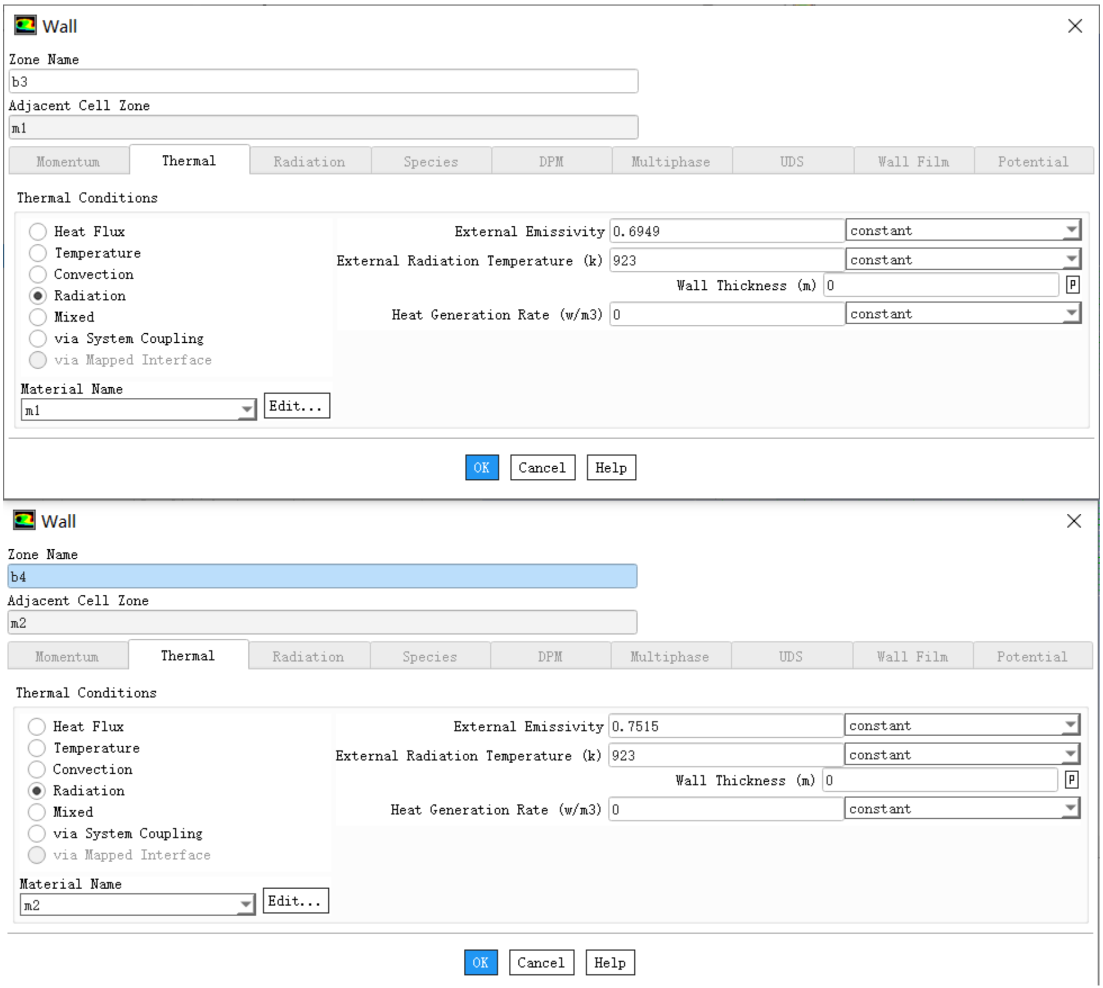
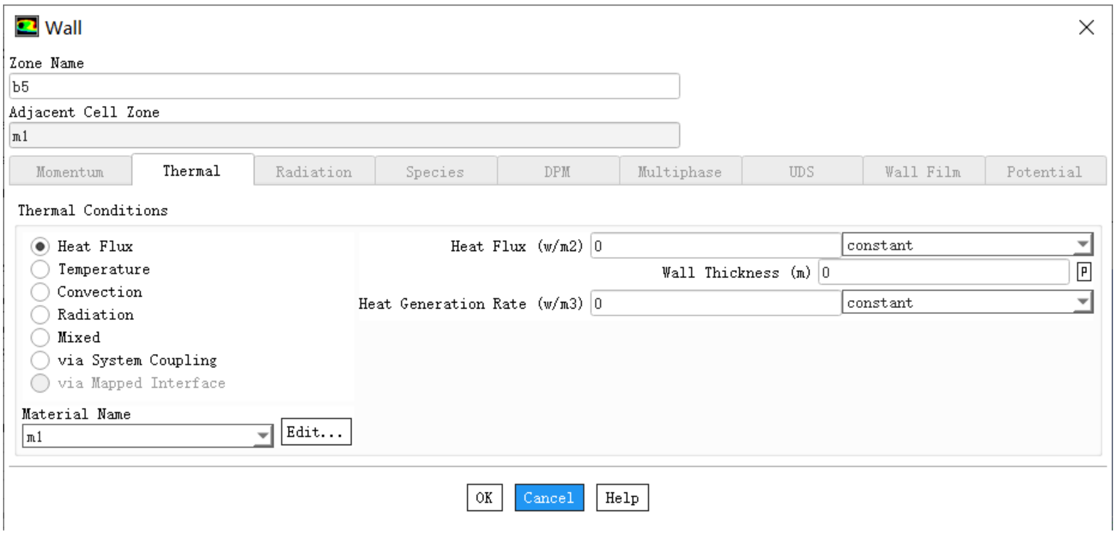

Wall
Zone Name
b3
Adjacent Cell Zone
m1
Momentum
Thermal
Radiation
Species
DPM
Multiphase
UDS
Wall Film
Potential
Thermal Conditions
Heat Flux
Temperature
Convection
Radiation
Mixed
via System Coupling
via Mapped Interface
External Emissivity 0.6949 constant
External Radiation Temperature (k) 923 constant
Wall Thickness (m) 0
Heat Generation Rate (w/m3) 0 constant
Material Name
m1
Edit...
OK
Cancel
Help
Wall
Zone Name
b4
Adjacent Cell Zone
m2
Momentum
Thermal
Radiation
Species
DPM
Multiphase
UDS
Wall Film
Potential
Thermal Conditions
Heat Flux
Temperature
Convection
Radiation
Mixed
via System Coupling
via Mapped Interface
External Emissivity 0.7515 constant
External Radiation Temperature (k) 923 constant
Wall Thickness (m) 0
Heat Generation Rate (w/m3) 0 constant
Material Name
m2
Edit...
OK
Cancel
Help

图 39 右边界条件


Wall
Zone Name
b5
Adjacent Cell Zone
m1
Momentum
Thermal
Radiation
Species
DPM
Multiphase
UDS
Wall Film
Potential
Thermal Conditions
Heat Flux
Temperature
Convection
Radiation
Mixed
via System Coupling
via Mapped Interface
Heat Flux (w/m2) 0 constant
Wall Thickness (m) 0
Heat Generation Rate (w/m3) 0 constant
Material Name
m1
Edit...
OK
Cancel
Help

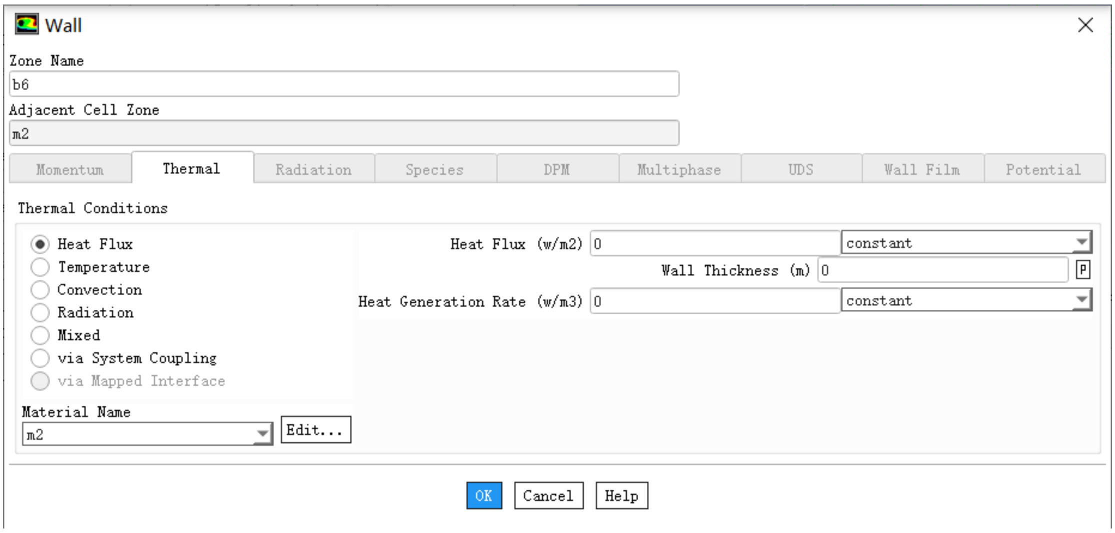


图 40 下边界及左边界条件

4) 进行初始化，设置时间步长并计算

初始化计算从全域开始，初温为 293K；时间步长与隐式格式相一致，取 0.02 s，计算 1500 个时间步，总计 30 s。

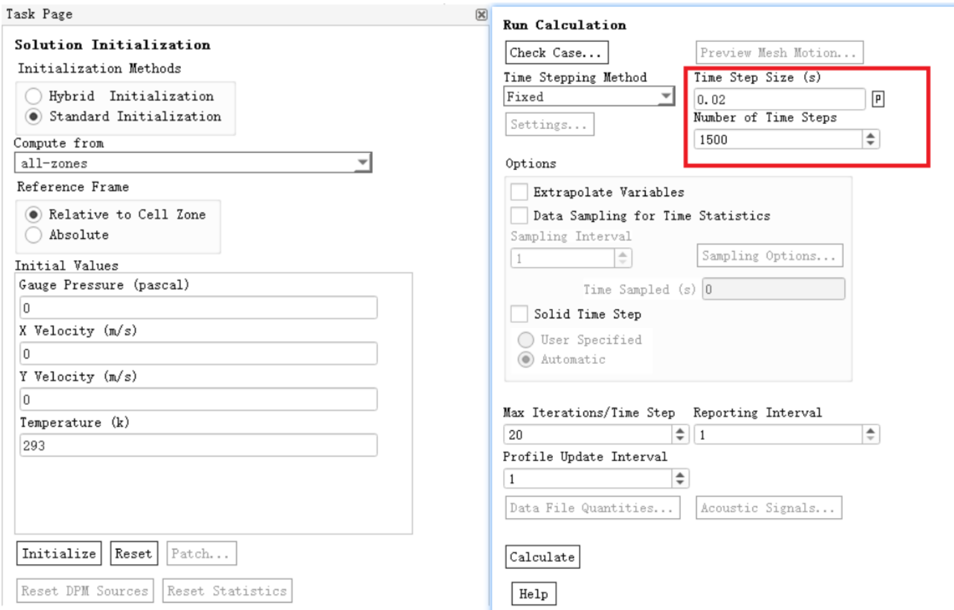


图 41 初始化及时间步长

六、 结果分析

1. 温度分布

由题目所给参数，材料一的密度大、定压比热容大、导热系数大；材料二的密度小、定压比热容小、导热系数小。两种材料对热量的吸收和热量传导会有较大的差别。由于 5s 时间太短，所以选取 30s 的情况下 fluent 计算结果进行分析。

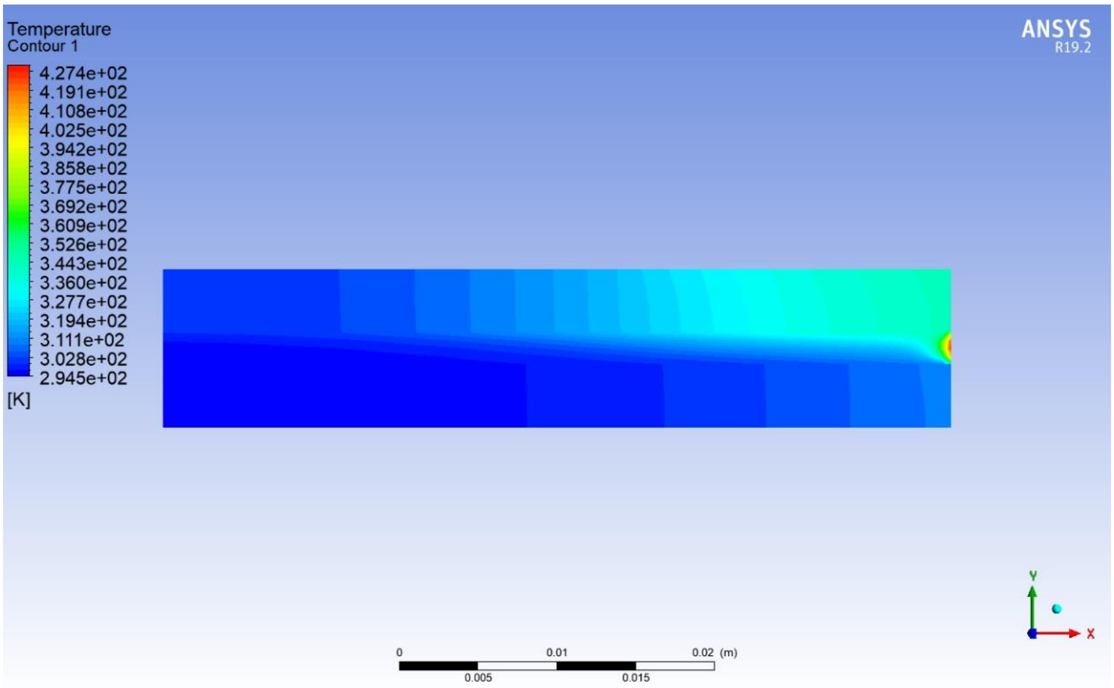


图 42 30sFluent 云图

在右端和右上部分受高温辐射的情况下，材料 2 右侧出现温度集中区，材料 1 的温度分比较均匀。计算两材料的热扩散系数：

$$a_1 = \frac{k_1}{\rho_1 c_{p1}} = 1.47 \times 10^{-5} \qquad a_2 = \frac{k_2}{\rho_2 c_{p2}} = 9.52 \times 10^{-8}$$

从计算结果，材料 1 的热扩散系数大，内部热量传递快，温度分布较均匀，材料 2 热导热系数极低，使得热量聚集在右端。材料 2 的定压比热容大，在相同时间内吸收的热量多，同时材料 2 的导热系数极低，使得热量无法快速向左侧低温区传递。

由于材料 2 所处区域受右侧及上侧高温区影响，且材料 2 热扩散系数小，扯平内部温度的能力较小，所以出现了向左倾斜的温度等值线。

由于材料热量都来自与右侧和右上部分，材料 1 左侧上部分为自然对流，散热量非常少，由此材料 1 中的左侧温度梯度小，而右侧温度梯度较大。

2. 不同求解方式的对比

1) 网格为 20×20

时间为 5s

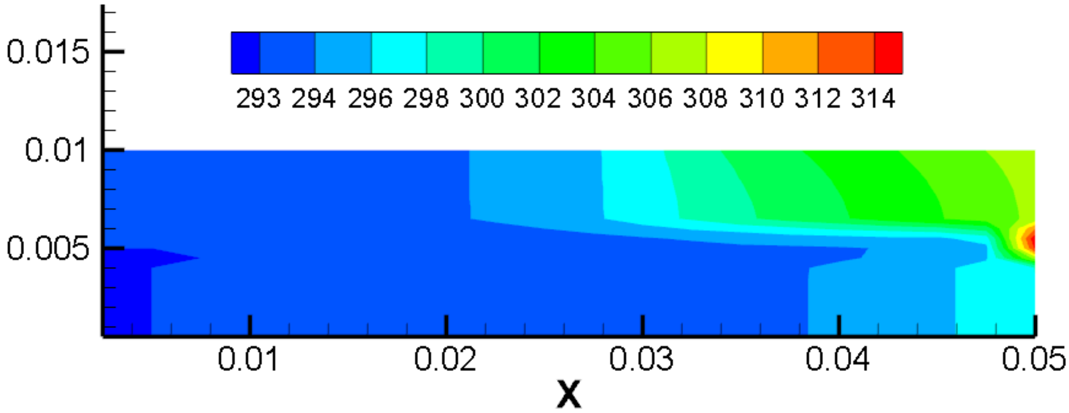


图 43 显式 5s

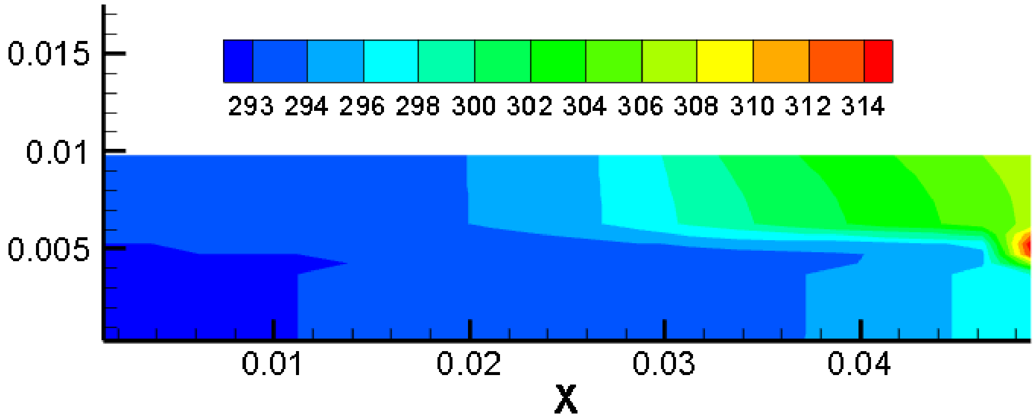


图 44 隐式 5s

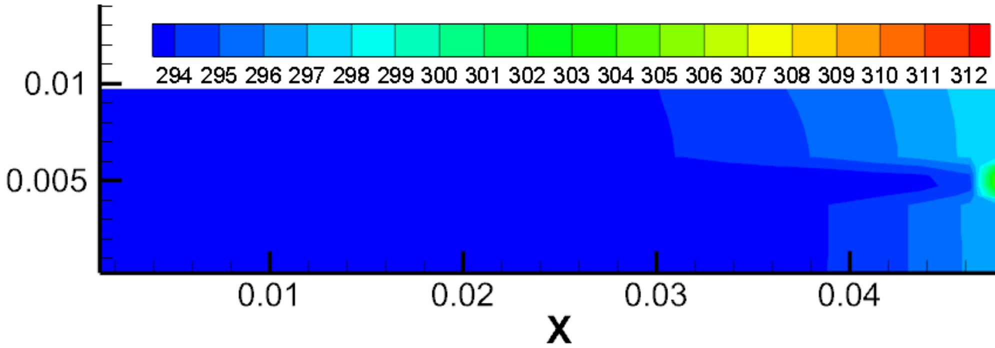


图 45 半隐式 5s

图 46 Fluent 5s

分析：显式和隐式的求解结果接近，最高温在 314K 左右，半隐式和 Fluent 的求解结果存在较大的偏差，其中 Fluent 的云图中，最高温达到了 480K，热量集中在材料 2 右端无法向右端低温区传导。

时间为 30s

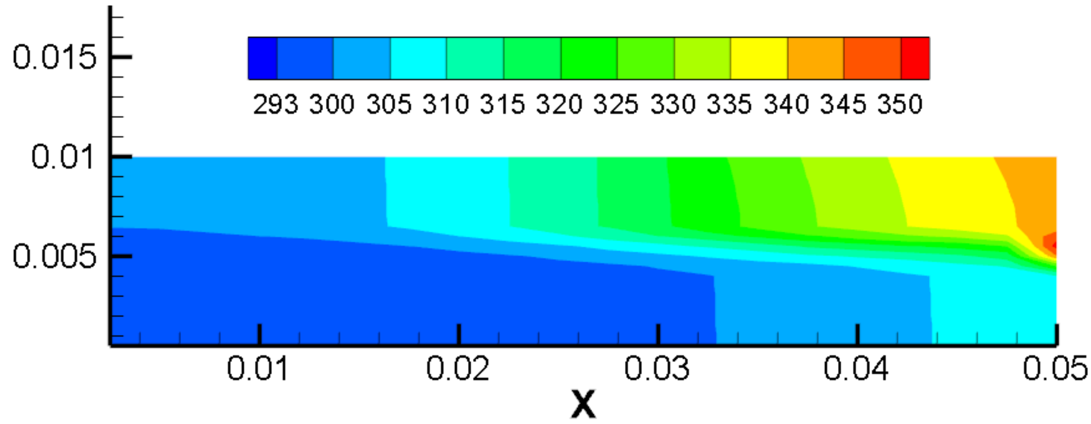


图 47 显式 30s

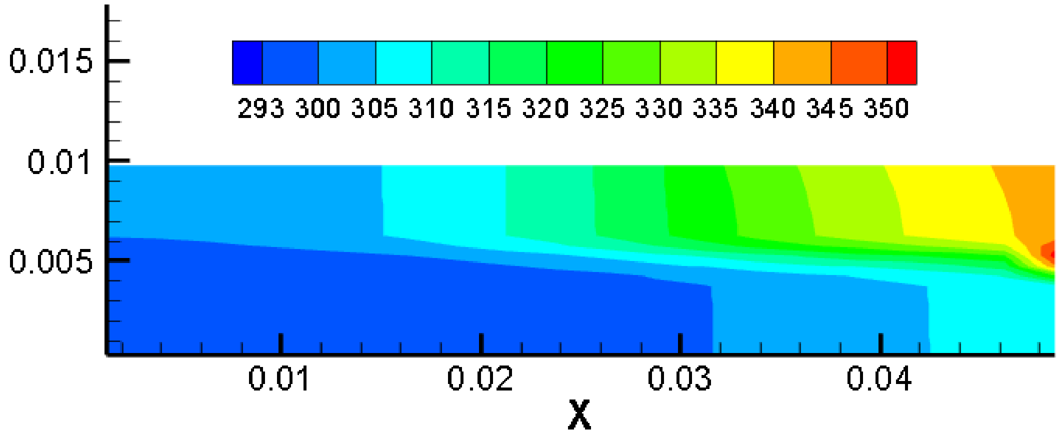


图 48 隐式 30s

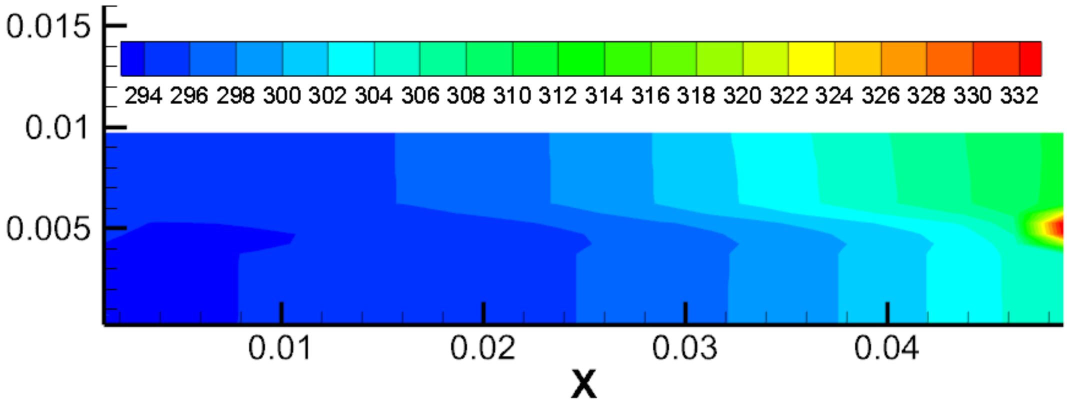


图 49 半隐式 30s

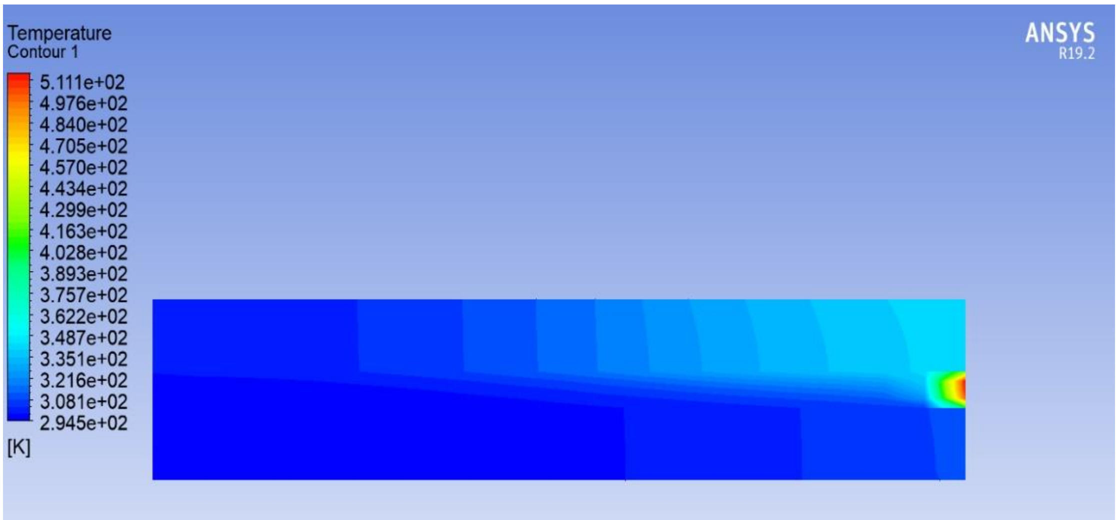


图 50 Fluent 30s

分析：显式与隐式的求解结果相差不大，最高温在 350K 左右，高温集中区向右上角移动，Fluent 的求解结果中高温集中区依然在右端材料二的位置，且最高温达到了 510K。

2) 网格为 100×20

时间为 5s

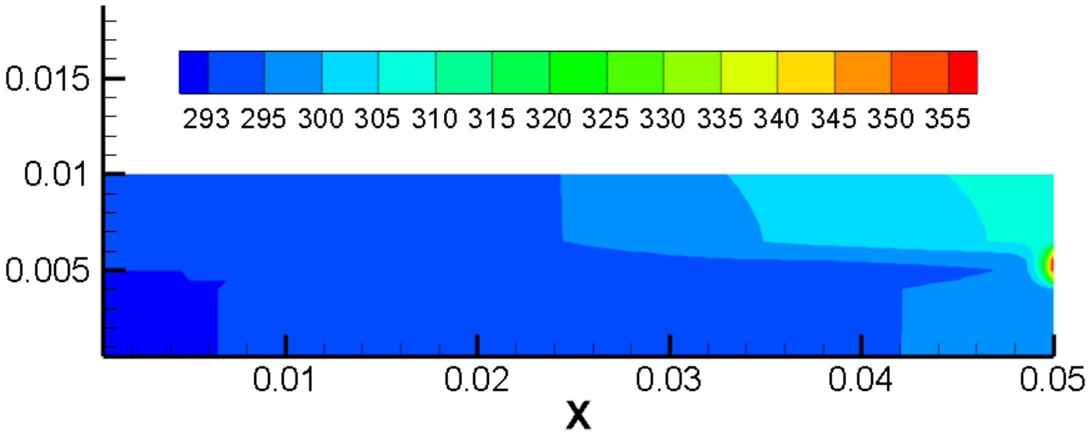


图 51 显式 5s

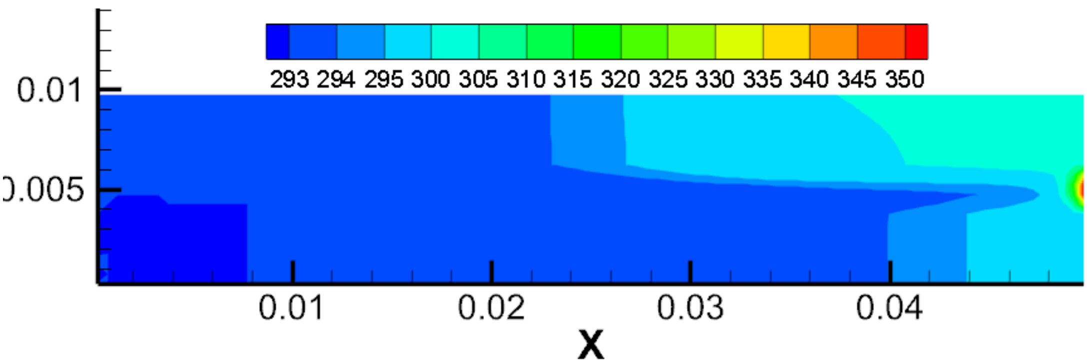


图 52 半隐式 5s

图 53 Fluent 5s

图 54 Mechanical 5s

分析：三种求解方式下，显式情况下的最高温在 355K 左右，半隐式情况下的最高温在 350K 左右，Fluent 的情况下最高温 390K 左右，三种情况的求解温度更加接近。材料 1 中的温度梯度分布接近。

时间为 30s

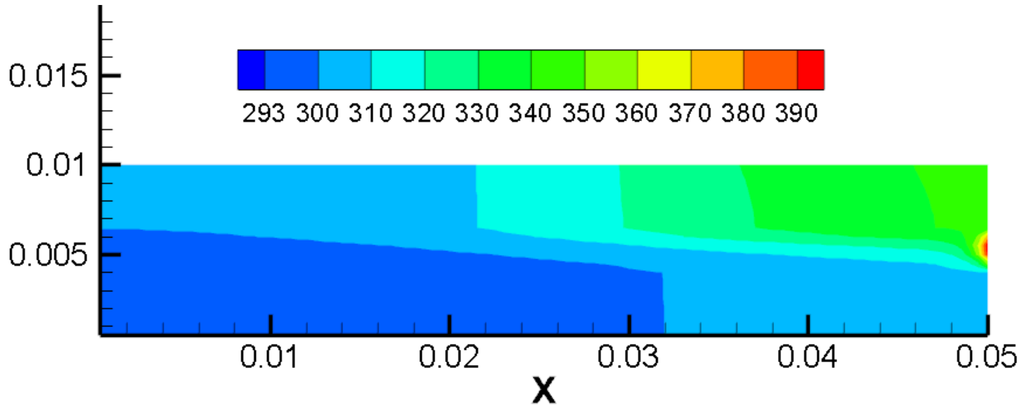


图 55 显式 30s

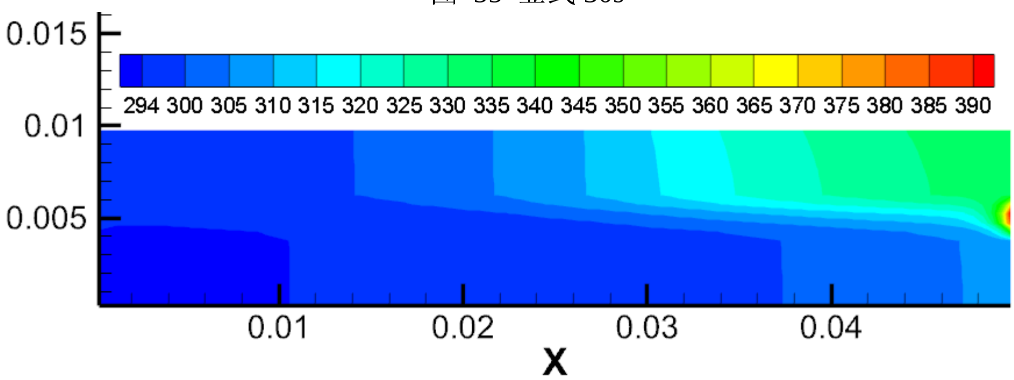


图 56 半隐式 30s

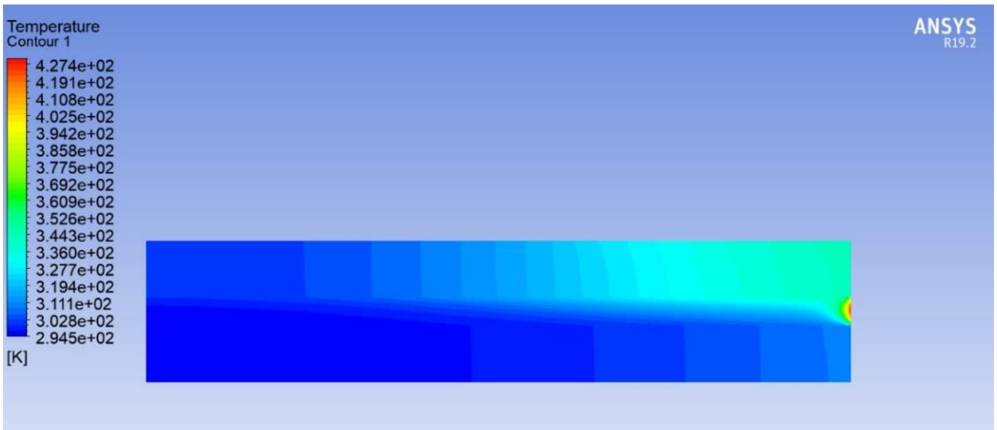


图 57 Fluent 30s

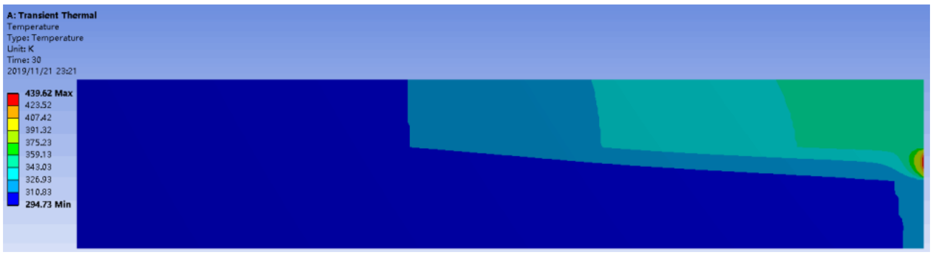


图 58 Mechanical 30s

分析：显式与半隐式求解结果最高温为 390K，材料中的温度梯度分布相差不大，Fluent 的求解结果中最高温为 430K，与显式与半隐式的计算结果差距缩小。

3. 空间温度分布对比（轴线 y=0.005m，网格数 20×20）

我们取材料 2 的中轴线即 $y = 0.005m$ 处温度分布进行对比，在网格数目均为 20×20 情况下应用 5 种方法对中轴线温度场进行求解，结果如下

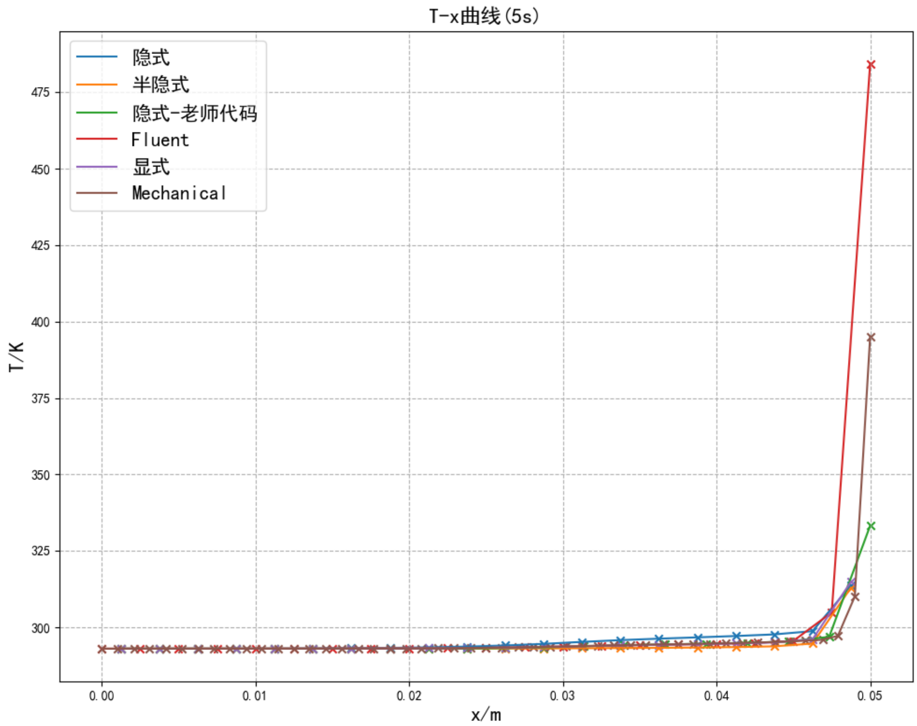


图 59 5s 时不同解法得到的曲线对比

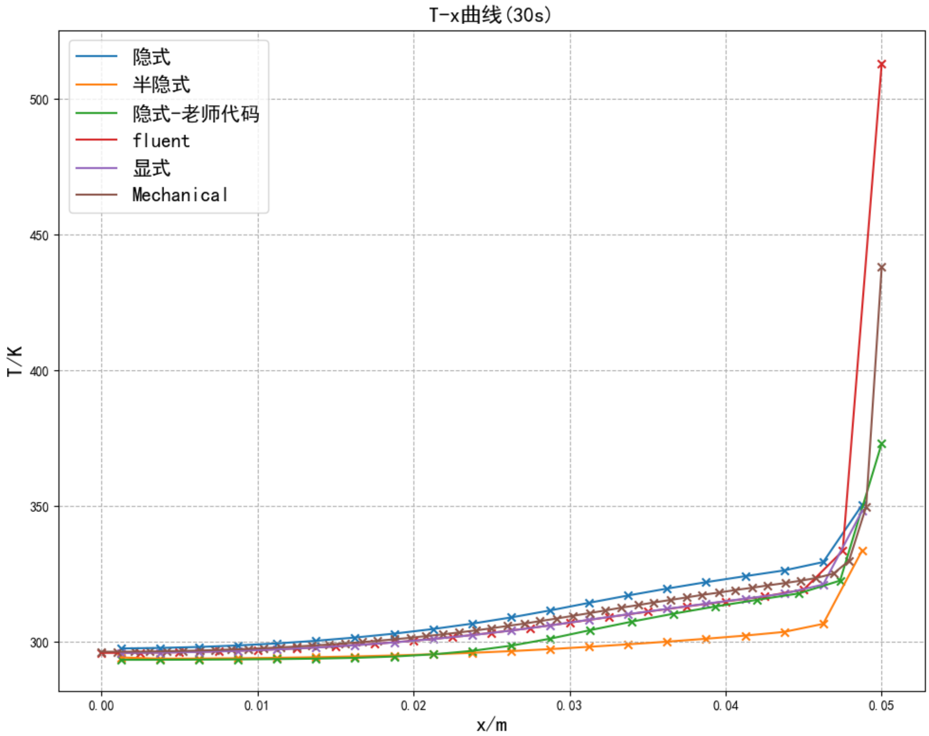


图 60 30s 时不同解法得到的曲线对比

由图分析，商业软件计算结果在 $x = 0.045$ 之前与代码计算结果吻合良好，但是在接近高温辐射壁面处（右壁面）出现较大误差。由模型所处的物理情形可知，在右壁面附件温度梯度较大，出现此种误差可能是因为网格 Δx 过大。

经验证，发现当网格数逐渐增大时，商业软件计算出的右壁面温度逐渐变小，代码计算结果则逐渐变大，两者最终都接近 $420k \sim 430k$ 。因此认为出现上图原因是网格数目过少。

4. 网格无关性分析（时间 t=30s）

1) 显式格式

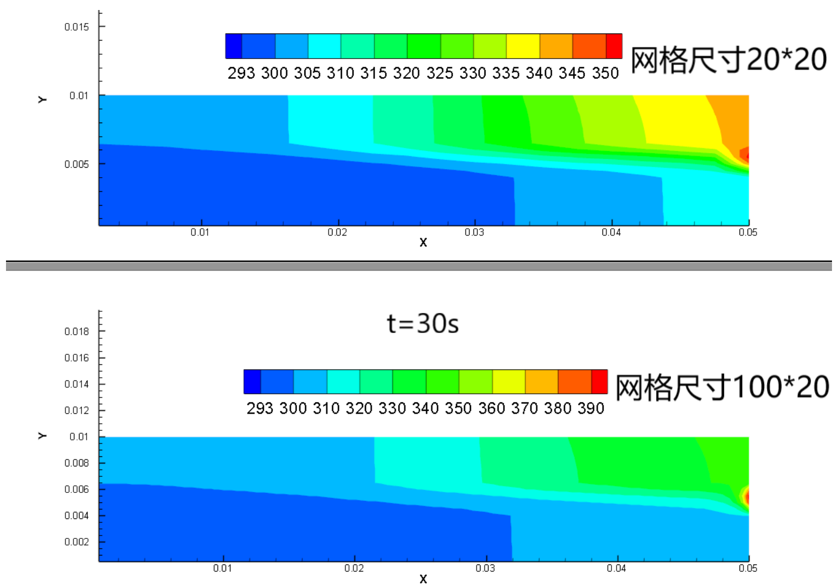


图 61 网格 20×20 和 100×20 结果对比

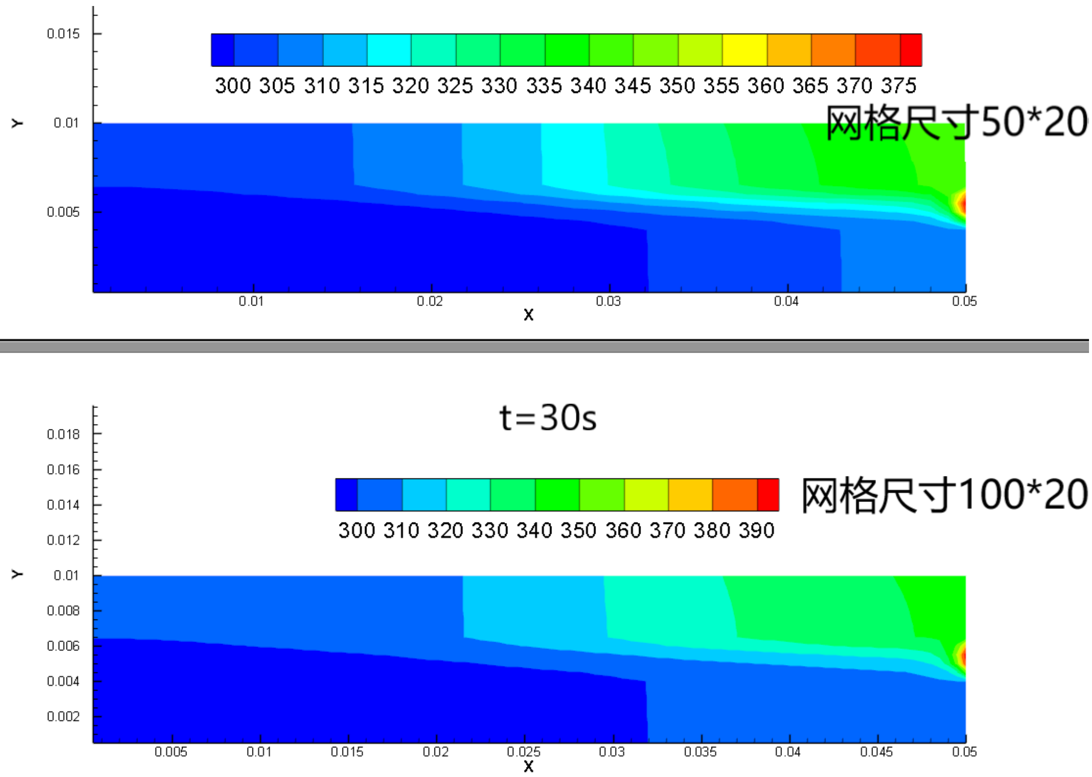


图 62 网格 50×20 和 100×20 结果对比

由网格尺寸 20×20、50×50、100×20 的对比云图可知，x 方向网格尺寸会影响最后的温度分布，其中最突出的是，随着 x 方向网格变密，最高温度会不断上升，网格尺寸 100×20 的最高温度比 20×20 的情况高 40℃，因此，计算结果与网格尺寸有关，经过验证主要与 x 方向的疏密程度有关。

2) 半隐格式

以 30 秒为例，分析各种数量网格计算出的结果之间的差异，分析对比的内容包括云图、轴线 T-x 曲线，之后分析 30s 内特征点 T-t 曲线。

云图对比

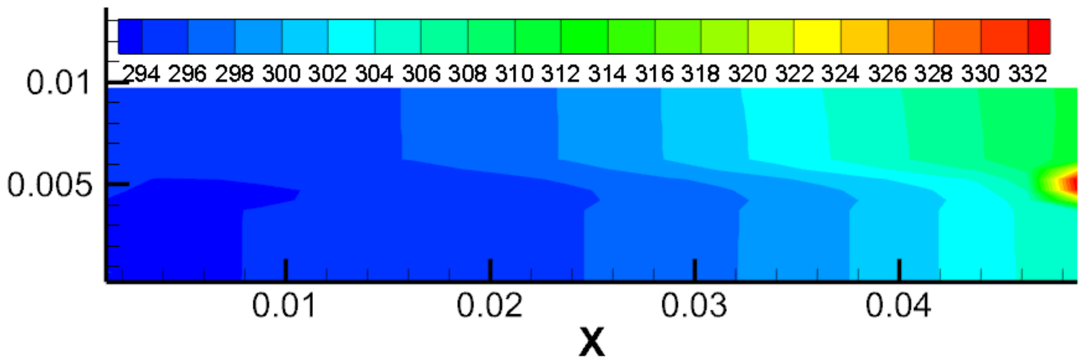


图 63 20×20

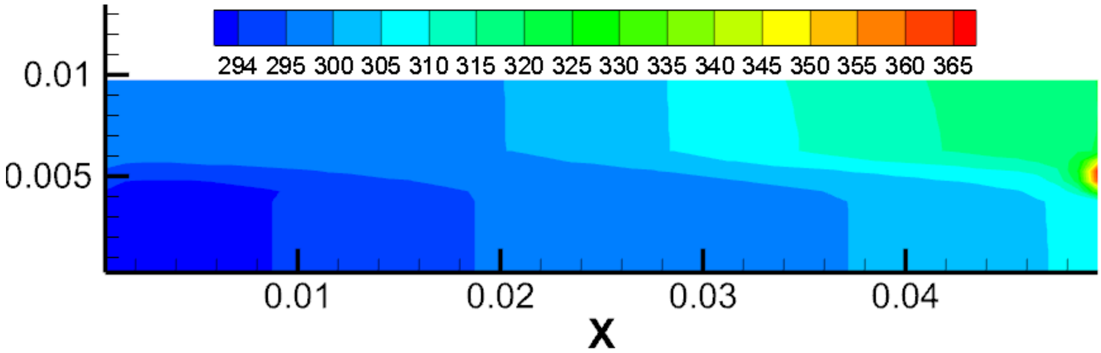


图 64 50×20

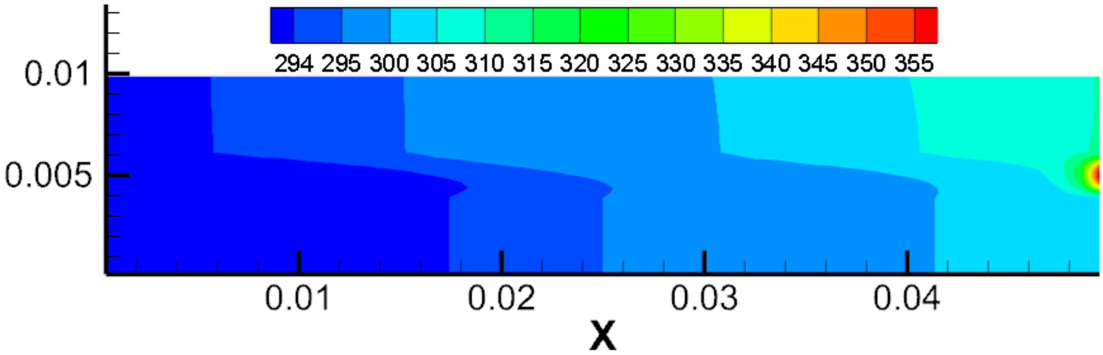


图 65 50×40

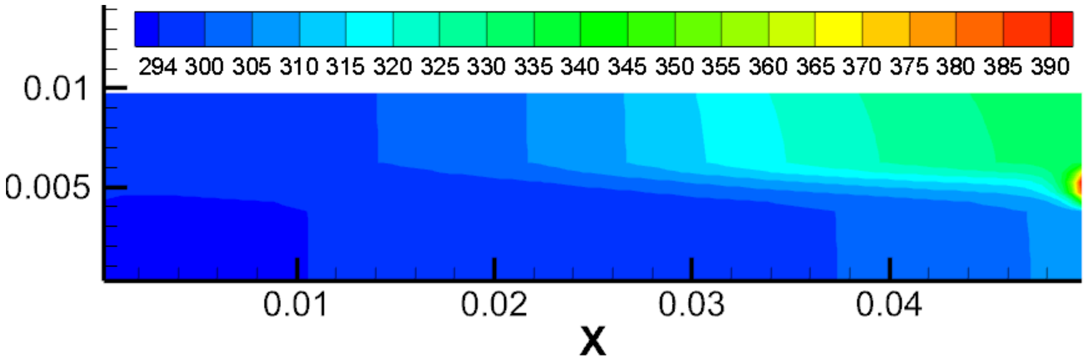


图 66 100×20

由以上四张云图可得，随着 X 方向网格数的增加，最高温度逐渐增大，294K 等温线所包围的左下区域的面积逐渐减小，并且当网格数增加到一定程度时，这些差异逐渐减小，故当网格 X 方向网格数 100 时，结果逐渐趋于稳定，计算结果与网格数量无关。但是对比网格分别为50×20与50×40的云图可以发现，Y 方向网格数目增加时，最高温度值减小，294K 等温线所包围的区域的面积表达，故 X 方向的网格数目对结果的影响有待验证。

T-x 曲线对比

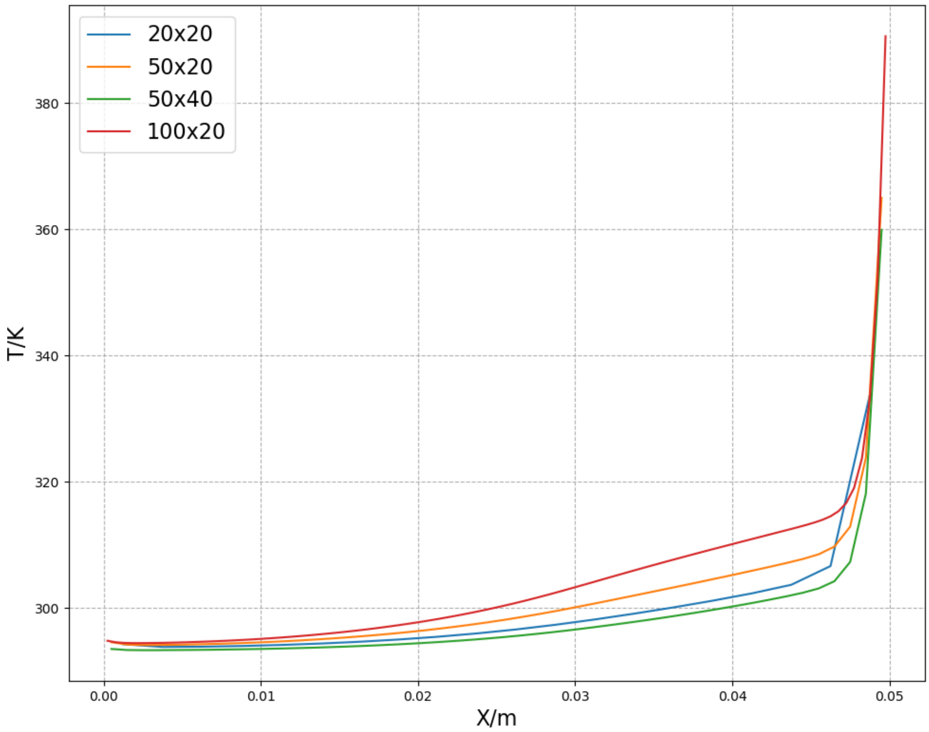


图 67 T-x 曲线

由$T-x$曲线图可得，网格越密，中轴线上的温度均比其余网格数量都要高，这与云图分析出来的结果类似。

T-t 曲线对比

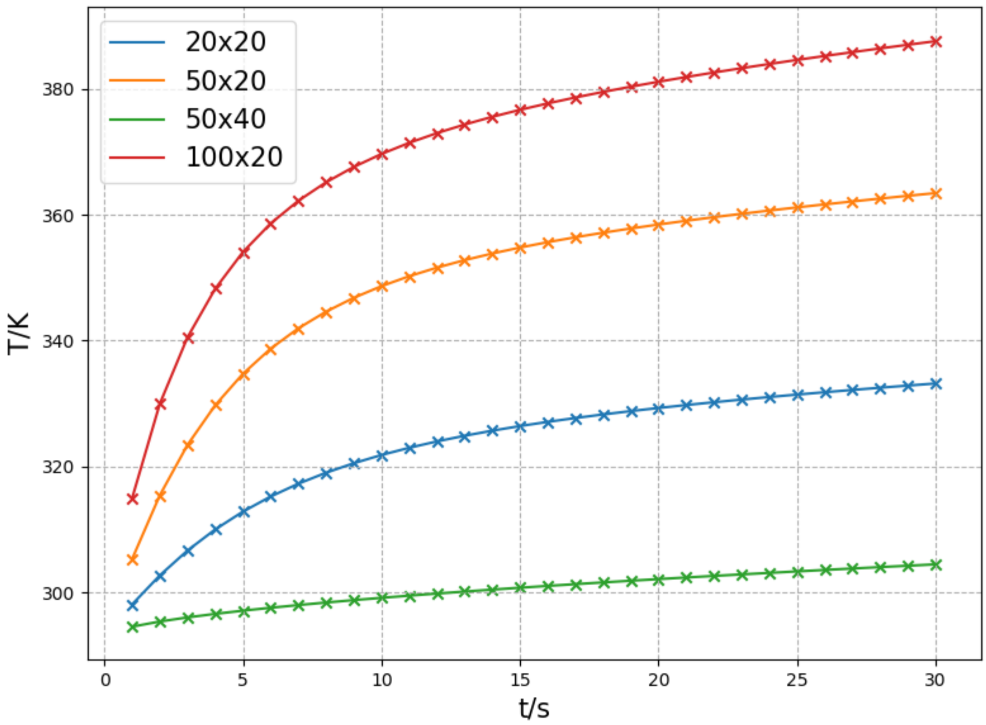


图 68 T-t 曲线

特征点取最右边中轴线靠上的点，具体坐标与网格数量有关，在此不给

出。

分析$T-t$曲线可得，对于该点而言其平均大小顺序为：$100\times20>50\times20>20\times20$，而$50\times40$的温度平均值最小，与云图分析类似，Y 方向的网格数目对于考察某点温度的时间变化较为重要，需要进行进一步验证；而 X 方向的网格数量的增加对于温度的影响逐渐减小，最终趋于稳定，即当 X 方向的网格加密到一定数量时，再增大网格数量，其不再对温度产生，即验证了计算结果与网格无关。

3) Fluent 仿真

云图对比

（1）5 s 时不同网格规格温度对比

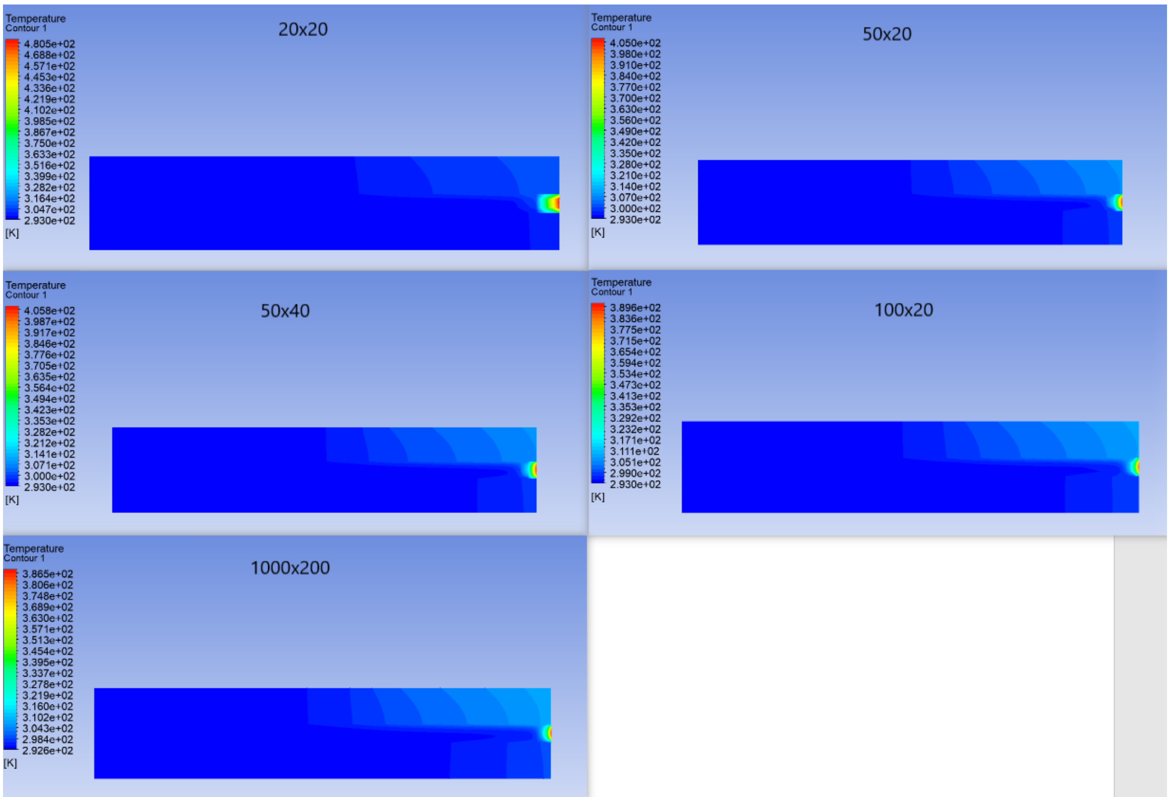
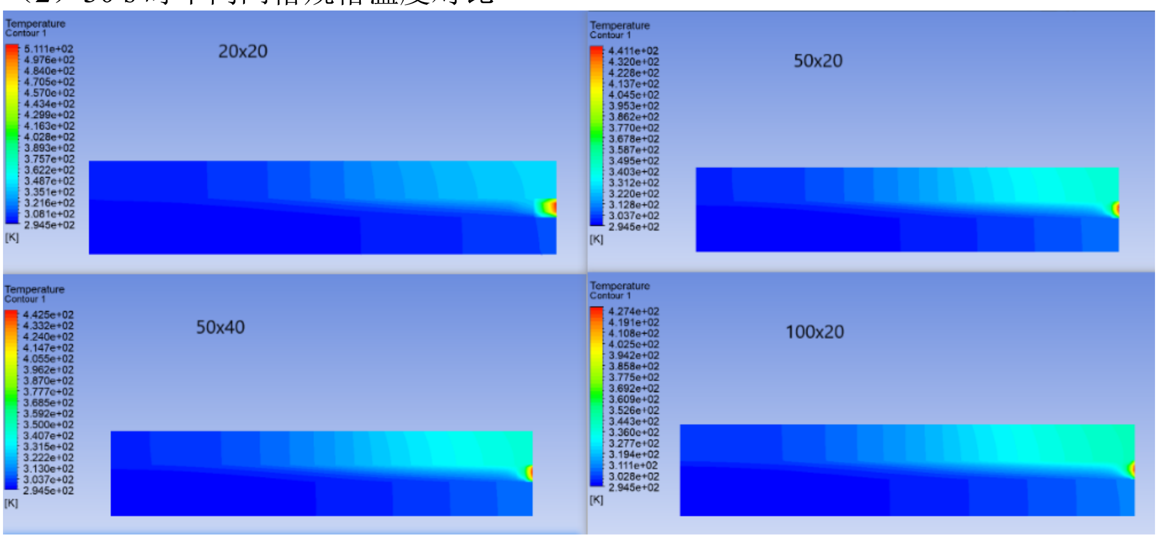


图 69 5 s 时温度对比

（2）30 s 时不同网格规格温度对比



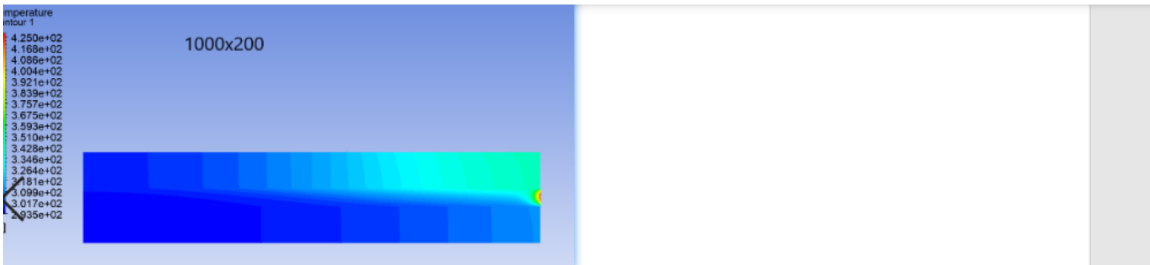


图 70 30 s 时温度对比

由 5 s 和 30 s 的云图可以明显看出不同网格规格最高温度的差别，为了更直观具体的表示，列出不同规格的辐射边界中点（即温度最高点）的温度。

表 2 不同网格规格温度最大值对比

网格规格	**20×20**	**50×20**	**50×40**	**100×20**	**1000×200**
5 s 时温度	484.41 K	407.38 K	408.11 K	391.6 K	388.45 K
30 s 时温度	513.23 K	444.20 K	444.57 K	430.14 K	426.99 K

由上表反映的规律，我猜想是，随网格数量增加，最高温度逐渐降低，结果趋于 427 K；但要注意，50×40 网格数量是 50×20 的 2 倍，但与其温度十分接近，且略有增大，说明 x 方向网格数的增多才是温度降低的关键。

轴线温度对比

单靠最高温度点的数据缺乏说服力，因此选取膏体发动机轴线上各点的温度进行对比。

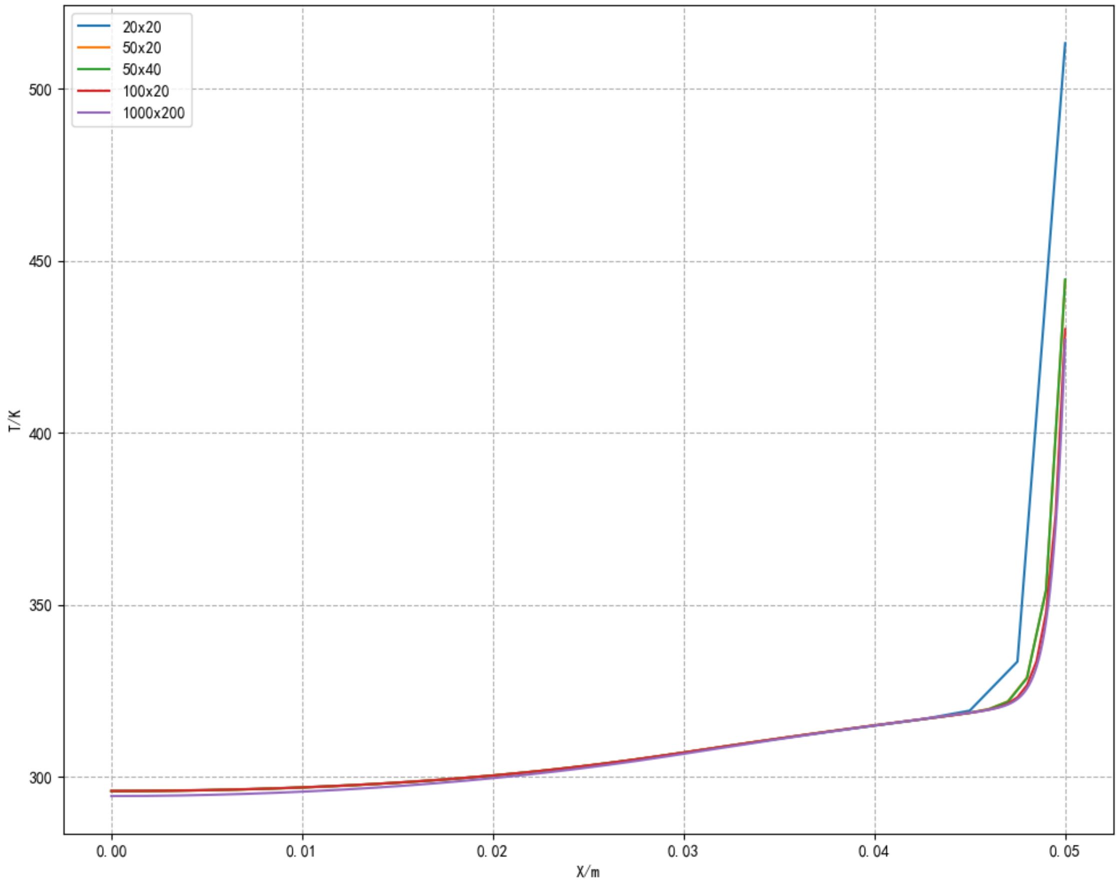


图 71 30 s 时不同网格规格轴线温度对比

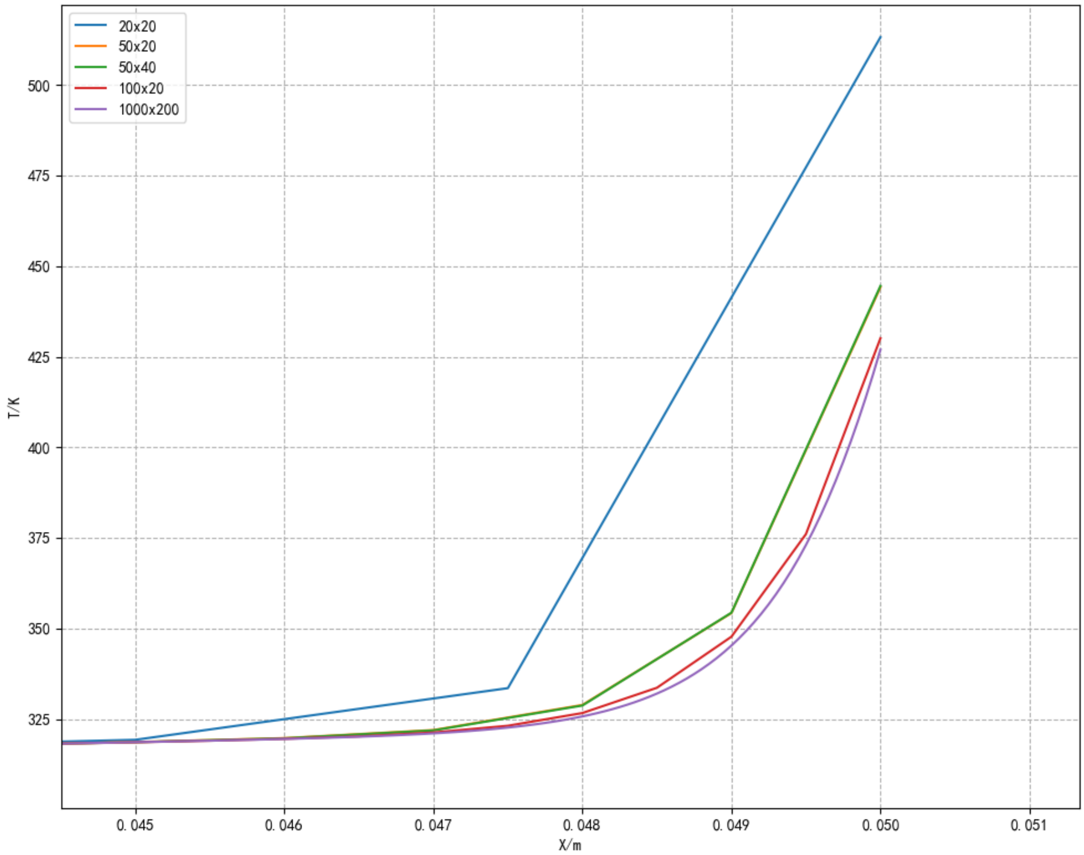


图 72 30 s 时不同网格规格轴线温度对比细节图

随网格数量增加，最高温度逐渐降低，结果趋于 427 K；但要注意，50×40 与 50×20 的温度十分接近，说明 x 方向网格数的增多才是温度降低的关键。轴线数据较好的吻合了上述猜想。

5. 最终结果

随着网格的加密，计算结果渐渐趋于稳定，100×20 与 1000×200 结果已经相当接近，计算结果较为可靠，所以最终结果取 100×20 网格的数据。

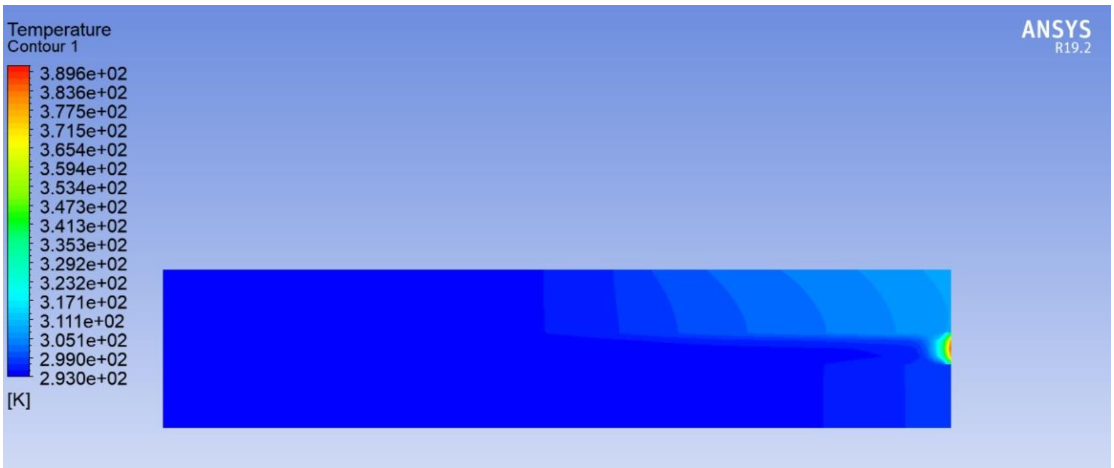


图 73 5sFluent 仿真

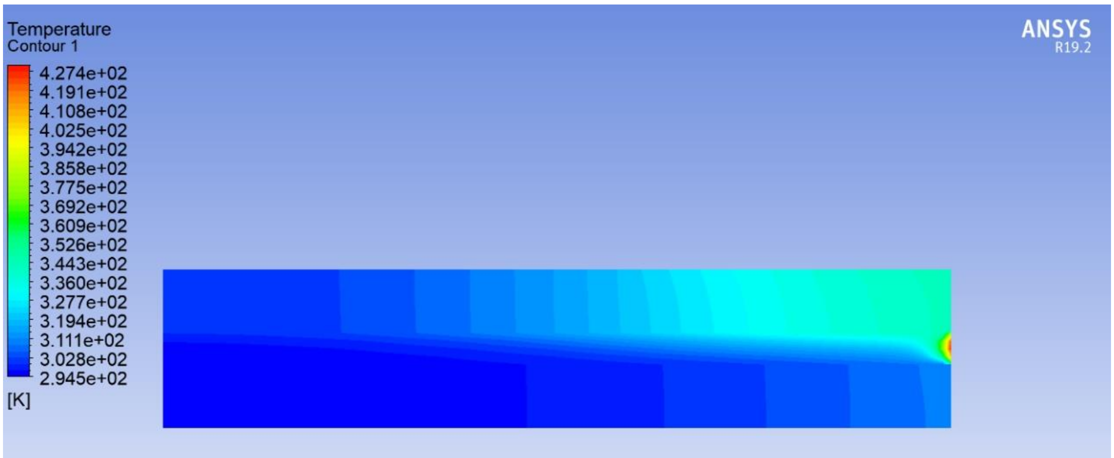


图 74 30sFluent 仿真

七、 参考文献 2

[1]杨世铭、陶文铨. (2006).*传热学*-第 4 版.
[2]郑亚. (2006). 固体火箭发动机传热学. 北京航空航天大学出版社等.
[3]杨世铭. 细长竖圆柱外及竖圆管内的自然对流传热[J]. 西安交通大学学报, 1980(03):119-135.

八、 程序代码

1. 显式代码

```python
# 二维非稳态导热问题的有限体积数值解法#
from scipy import linalg
import os
# 定义求解函数，形参为网格数目,求解时间,eff1,eff2
def solve(N_x_grid, N_y_grid ,t,eff1,eff2):
    # 物性参数：发动机长度，高温辐射区域长度，两层材料的宽度、热传导系数、密度、比热容，燃气温度，初始温度,玻尔兹曼常数
    L = 0.05
    L1 = 0.02
    HH = 0.004
    hh = 0.002
    k1 = 53.6
    k2 = 0.2093
    k0 = 0.41697
    rou1 = 7830
    rou2 = 1500
    Cp1 = 465
    Cp2 = 1465
    Tair= 293
    Tgas = 923
    T0 = 293  # 初始温度
    sigma = 5.67 * 10 ** -8

    # 网格尺寸：
    delta_x = L / N_x_grid  # 20 个网格为 0.0025
    delta_y = (2 * HH + hh) / N_y_grid  # 20 个网格为 0.0005

    # 时间步长及迭代次数：
    delta_t = 0.004*(20/N_x_grid)*(20/N_y_grid)
    cal_num = int(t / delta_t)

    # 定义求解线性方程组 AA*T=CC 的动态数组 AA、CC：
    AA = np.zeros((N_x_grid * N_y_grid, N_x_grid * N_y_grid))
    CC = np.zeros((N_x_grid * N_y_grid))

    # 定义初始 0 矩阵:
    a_w = np.zeros((N_x_grid, N_y_grid))
    a_e = np.zeros((N_x_grid, N_y_grid))
    a_n = np.zeros((N_x_grid, N_y_grid))
    a_s = np.zeros((N_x_grid, N_y_grid))
    sp = np.zeros((N_x_grid, N_y_grid))
```

```
    su = np.zeros((N_x_grid, N_y_grid))
    a_p = np.zeros((N_x_grid, N_y_grid))
    a_p0 = np.zeros((N_x_grid, N_y_grid))
    T = np.zeros((N_x_grid, N_y_grid))
    X = np.zeros((N_x_grid, N_y_grid))
    Y = np.zeros((N_x_grid, N_y_grid))

    # 离散方程系数计算
    for k in range(0, cal_num):
        # 给定初始温度场
        if k== 0:
            for i in range(0, N_x_grid):
                for j in range(0, N_y_grid):
                    T[i][j] = T0
                    Y[i][j] = delta_y * (j+1)
                    X[i][j] = delta_x * (i+1)

        else:
            for i in range(0, N_x_grid):
                for j in range(0, N_y_grid):
                    # 结点位置：
                    Y[i][j] = delta_y * (j + 1)
                    X[i][j] = delta_x * (i + 1)

                    #对流换热系数计算：
                    tm=(T[i][j]-253)/2
                    beta=2/(T[i][j]+293)
                    Pr=4*10**(-14)*tm**6-3*10**(-11)*tm**5+6*10**(-9)*tm**4-7*10**(-7)*tm**3+4*10**(-5)*tm**2-0.0012*tm+0.7163
                    v=9*10**(-11)*tm**2+9.13*10**(-8)*tm+1.3175*10**(-5)
                    Gr=9.8*beta*(T[i][j]-293)*0.03**3/v/v
                    if Gr<0:
                        print(i,j,Gr,T[i][j],Tair)
                        break
                    Nu=0.54*(Gr*Pr)**0.25
                    h=2.59/3*Nu

                    # 1.判断结点属于哪个材料，给出 a_e,a_w,a_s,a_n,a_p 相应系数
                    if j <= 0.4 * N_y_grid - 1 or j >= 0.6 * N_y_grid:  # 20 个网格对应 0—7，12—19
                        a_w[i][j] = k1 / delta_x * delta_y  # 10.72
                        a_e[i][j] = k1 / delta_x * delta_y  # 10.72
                        a_s[i][j] = k1 / delta_y * delta_x  # 268
```

```
                a_n[i][j] = k1 / delta_y * delta_x  # 268
                a_p[i][j] = rou1 * Cp1 * delta_x * delta_y / delta_t
            else:  # 20 个网格对应 8—11
                a_w[i][j] = k2 / delta_x * delta_y
                a_e[i][j] = k2 / delta_x * delta_y
                a_s[i][j] = k2 / delta_y * delta_x
                a_n[i][j] = k2 / delta_y * delta_x
                a_p[i][j] = rou2 * Cp2 * delta_x * delta_y / delta_t

            # 2.交界面处系数
            if j == 0.6 * N_y_grid - 1 or j == 0.4 * N_y_grid - 1:  # 7, 11
                a_n[i][j] = k0 / delta_y * delta_x
            if j == 0.6 * N_y_grid or j == 0.4 * N_y_grid:  # 8, 12
                a_s[i][j] = k0 / delta_y * delta_x

            # 3.边界条件
            if i == 0:  # 左边界为绝热边界
                a_w[i][j] = 0.0
                 su[i][j] = 0.0
                 sp[i][j] = 0.0
             if i == N_x_grid - 1:  # 右边界为辐射换热边界
                 a_e[i][j] = 0.0
                 if j <= 0.4 * N_y_grid - 1 or j >= 0.6 * N_y_grid: #材料 1 辐射换热系数 eff1
                     su[i][j] = eff1 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_y
                 else : #材料 2 辐射换热系数 eff2
                     su[i][j] = eff2 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_y
                 sp[i][j] = 0.0
             if j == N_y_grid - 1:  # 上边界为混合边界
                 if i <= N_x_grid * 0.6 - 1:  # 上边界左侧为对流换热
                     a_n[i][j] = 0.0
                     su[i][j] = h * (Tair - T[i][j]) * delta_x
                     sp[i][j] = 0
                 else:  #改动 1:  上边界右侧为辐射换热
                     a_n[i][j] = 0.0
                     su[i][j] = eff1 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_x
                     sp[i][j] = 0.0
```

```
                if j == 0:  # 下边界为绝热边界
                    a_s[i][j] = 0.0
                    su[i][j] = 0.0
                    sp[i][j] = 0.0

                # 4.四个角点系数
                if i == 0 and j == 0:  # 左下角点
                    su[i][j] = 0.0
                if i == 0 and j == N_y_grid - 1:  # 左上角点
                    su[i][j] = h * (Tair - T[i][j]) * delta_x
                if i == N_x_grid - 1 and j == N_y_grid - 1:  # 右上角点
                    su[i][j] = eff1 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_x + eff1 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_y
                if i == N_x_grid - 1 and j == 0:  # 右下角点
                    su[i][j] = eff1 * sigma * (Tgas ** 4 - (T[i][j]) ** 4) * delta_y

                # 5.最终 a_p0 系数
                a_p0[i][j] = a_p[i][j] - (a_w[i][j] + a_e[i][j] + a_s[i][j] + a_n[i][j] - sp[i][j])

        # 将系数给入系数矩阵 AA,CC
                AA[i * N_y_grid + j][i * N_y_grid + j] = a_p[i][j]
                CC[i * N_y_grid + j] = su[i][j] + a_p0[i][j] * T[i][j]

                if j != 0:
                    CC[i * N_y_grid + j] = CC[i * N_y_grid + j] + a_s[i][j] * T[i][j - 1]
                if j != N_y_grid - 1:
                    CC[i * N_y_grid + j] = CC[i * N_y_grid + j] + a_n[i][j] * T[i][j + 1]
                if i != 0:
                    CC[i * N_y_grid + j] = CC[i * N_y_grid + j] + a_w[i][j] * T[i - 1][j]
                if i != N_x_grid - 1:
                    CC[i * N_y_grid + j] = CC[i * N_y_grid + j] + a_e[i][j] * T[i + 1][j]

        #调用 scipy 库求解线性方程组
        result=linalg.solve(AA, CC)
        for i in range(0, N_x_grid):
            for j in range(0, N_y_grid):
                if result[i * N_y_grid + j]>293:
```

```
                        T[i][j] = result[i * N_y_grid + j]
            if k*delta_t==5 or k*delta_t==29.96:
                for i in range(0,N_x_grid):
                    print((T[i][9]+T[i][10])/2)

###################################################################
    #建立文件夹及写入温度计算数据，导入 tecplot 进行后处理
        # 判断是否存在文件夹，不存在则建立
        if not os.path.exists('G:\\python project\\python project1\\内流场大作业代码上边界对流 eff={0}-{1}-n_x={2}-n_y={3}' .format(eff1,eff2,N_x_grid,N_y_grid)):
            os.mkdir('G:\\python project\\python project1\\内流场大作业代码上边界对流 eff={0}-{1}-n_x={2}-n_y={3}'.format(eff1,eff2,N_x_grid,N_y_grid))

        if k%(0.02/delta_t)==0 or k==cal_num  :#每隔 0.02 秒输出一个 plt 文件，避免文件太多导入 tecplot 太慢
            with open('G:\\python project\\python project1\\内流场大作业代码上边界对流 eff={3}-{4}-n_x={5}-n_y={6}\\T-2d-Nx={0}-Ny={1}-t={2}.plt'.format(N_x_grid, N_y_grid, k  * delta_t,eff1,eff2,N_x_grid, N_y_grid), 'w', encoding='UTF-8') as fp1:
                fp1.write("VARIABLES = X, Y, T, ap,an,as,aw,ae,sp,su\n")  # 按计算节点数输出结果
                fp1.write("ZONE I=%d,J=%d, F=POINT,t=\"%.3f\"\n" % (N_x_grid, N_y_grid,k*delta_t))
                for j in range(N_y_grid - 1, -1, -1):
                    for i in range(0, N_x_grid):
                        fp1.write(
                            "%6.5Lf    %6.5Lf    %6.3Lf    %6.3Lf    %6.3Lf    %6.3Lf    %6.3Lf    %6.3Lf    %6.3Lf    %6.3Lf\n" % (
                                X[i][j], Y[i][j], T[i][j], a_p[i][j], a_n[i][j], a_s[i][j], a_w[i][j],
                                a_e[i][j], sp[i][j], su[i][j]))


# 调用函数的主程序
import numpy as np
#N_x_grid,N_y_grid,t,eff1,eff2=input("请输入 x 方向网格数量，y 方向网格数量，计算时间，材料 1 辐射率，材料 2 辐射率").split()
#solve(int(N_x_grid),int(N_y_grid),int(t),float(eff1),float(eff2))#N_xgrid=20,N_ygrid=20,t=50,eff1=0.6949,eff2=0.7515
solve(20,20,30,0.6949,0.7515)#N_xgrid=20,N_ygrid=20,t=50,eff1=0.6949,eff2=0.7515
```

2. 半隐式代码

```python
"""author:gaoyi"""
import numpy as np
from time import time,sleep

##########################################################################################################################
"""参数"""
rho1 = 7830  #材料1的密度
rho2 = 1500  #材料2的密度
c1 = 465          #材料1的比热
c2 = 1465.4       #材料2的比热
k1 = 53.6         #材料1的导热系数
k2 = 0.2093       #材料2的导热系数
k3 = 2*k1*k2/(k1+k2)   #导热系数 -> 交界面
T_air = 293    #大气温度,K
T_gas = 923    #燃气温度,K

"""辐射边界相关参数"""
sigma = 5.67*10**-8
# epsilon_eff = 0.7

"""几何参数"""
H1 = 0.004
H2 = 0.002
H3 = 0.004
H = H1+H2+H3

NX = 50   #x方向单元体数量
NY = 40    #y方向单元体数量

NY_1 = NY*(H1/H)
NY_2 = NY*(H2/H)
NY_3 = NY*(H3/H)

L = 0.05

"""初始化矩阵"""
a_e = np.zeros((NX, NY))
a_w = np.zeros((NX, NY))
a_n = np.zeros((NX, NY))
a_s = np.zeros((NX, NY))
a_p = np.zeros((NX, NY))
```

```
CN = np.zeros((NX, NY))
T_0 = np.zeros((NX+2, NY+2))
T = np.zeros((NX+2, NY+2))

delta_x = L / NX
delta_y = (H1 + H2 + H3) / NY

s_u = np.zeros((NX, NY))
s_p = np.zeros((NX, NY))

m = float(input("请输入迭代系数:"))   #迭代系数
print("迭代系数为:{}".format(m))

ap0 = np.zeros((NX, NY))

# delta_t1 = rho1*c1/k1*(delta_y**2)
# delta_t2 = rho2*c2/k2*(delta_y**2)
# delta_t3 = rho1*c1/k1*(delta_x**2)
# delta_t4 = rho2*c2/k2*(delta_x**2)

delta_t = 2*min(rho1*c1, rho2*c2)/max(k1, k2)*delta_x*delta_y/(2*delta_y/delta_x+2*delta_x/delta_y)

# delta_t = "%.6f" % min(delta_t1, delta_t2, delta_t3, delta_t4)   #时间步长
# delta_t = float(delta_t)
print("时间步长为:{}".format(delta_t))
delta_t = float(input("请输入手动修正后的时间步长:\n"))

sec = 30   #计算秒数

########################################################################################################################
def cal_h(T):
    """计算自然对流换热系数"""
    if T > T_air:
        tm = (T-253)/2
        beta = 2/(T+293)
        Pr = 4*10**-14*tm**6-3*10**-11*tm**5+6*10**-9*tm**-4-7*10**-7*tm**3+4*10**-5*tm**2-0.0012*tm+0.7163
        v = 9*10**-11*tm**2+9.13*10**-8*tm+1.3175*10**-5
        Gr = 9.8*beta*(T-293)*0.03**2/(v**2)
        Nu = 0.54*(Gr*Pr)**(1/4)
        h = Nu*2.59/3
```

```
    else:
        h = 0
    return h


###########################################################################################################################
def cal_epsilon(j):
    if 0 <= j <= NY_1-1:
        e = 0.6949
    elif NY_1 <= j <= NY_1+NY_2-1:
        e = 0.7515
    elif NY_1+NY_2 <= j <= NY-1:
        e = 0.6949

    return e


###########################################################################################################################
def cal_property(i, j):
    """计算节点的物性参数"""
    """初始化变量"""
    k = 0
    ks = 0
    kn = 0
    rho = 0
    c = 0

    """判断节点位置并赋值"""
    if j == NY_1-1:
        k = k1
        ks = k1
        kn = k3
        rho = rho1
        c = c1


    elif j == NY_1:
        k = k2
        ks = k3
        kn = k2
        rho = rho2
        c = c2
```

```python
    elif j == NY_1+NY_2-1:
        k = k2
        ks = k2
        kn = k3
        rho = rho2
        c = c2
    elif j == NY_1+NY_2:
        k = k1
        ks = k3
        kn = k1
        rho = rho1
        c = c1
    elif 0 <= j <= (NY_1-2) or (NY_1+NY_2+1) <= j <= (NY-1):
        k = k1
        ks = k1
        kn = k1
        rho = rho1
        c = c1
    elif (NY_1+1) <= j <= (NY_1+NY_2-2):
        k = k2
        ks = k2
        kn = k2
        rho = rho2
        c = c2
    else:
        print("error，无法计算物性参数！！！")

    return [k, ks, kn, rho, c]


########################################################################################################################
"""计算系数矩阵"""
def cal_matrix(i, j):

    h = cal_h(T[i,j])
    [k,ks,kn,rho,c] = cal_property(i,j)
    epsilon_eff = cal_epsilon(j)

    if j == (NY-1) and i != 0 and i != (NX-1):#上边界 - 自然对流+辐射
        if 0 <= i <= (NX*3/5-1):#自然对流部分

            a_e[i,j] = k*delta_y/delta_x
            a_w[i,j] = k*delta_y/delta_x
```

```
                a_s[i,j] = ks*delta_x/delta_y
                a_n[i,j] = 0

                s_u[i,j] = h*delta_x*T_air
                s_p[i,j] = -h*delta_x

                ap0[i,j] = rho*c*delta_x*delta_y/delta_t
                a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

            elif (NX*3/5) <= i <= (NX-1):#辐射部分

                a_e[i,j] = k*delta_y/delta_x
                a_w[i,j] = k*delta_y/delta_x
                a_s[i,j] = ks*delta_x/delta_y
                a_n[i,j] = 0

                s_u[i,j] = epsilon_eff*sigma*(T_gas**4-T_0[i,j]**4)*delta_y
                s_p[i,j] = 0

                ap0[i,j] = rho*c*delta_x*delta_y/delta_t
                a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

        elif i == (NX-1) and j != 0 and j != (NY-1):#右边界 - 辐射

            a_e[i,j] = 0
            a_w[i,j] = k*delta_y/delta_x
            a_s[i,j] = ks*delta_x/delta_y
            a_n[i,j] = kn*delta_x/delta_y

            s_u[i,j] = epsilon_eff*sigma*(T_gas**4-T_0[i,j]**4)*delta_y
            s_p[i,j] = 0

            ap0[i,j] = rho*c*delta_x*delta_y/delta_t
            a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

        elif j == 0 and i != 0 and i != (NX-1):#下边界 - 绝热

            a_e[i,j] = k*delta_y/delta_x
            a_w[i,j] = k*delta_y/delta_x
            a_s[i,j] = 0
            a_n[i,j] = kn*delta_x/delta_y
```

```
            s_u[i,j] = 0
            s_p[i,j] = 0

            ap0[i,j] = rho*c*delta_x*delta_y/delta_t
            a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

        elif i == 0 and j != 0 and j != (NY-1):#左边界 - 绝热

            a_e[i,j] = k*delta_y/delta_x
            a_w[i,j] = 0
            a_s[i,j] = ks*delta_x/delta_y
            a_n[i,j] = kn*delta_x/delta_y

            s_u[i,j] = 0
            s_p[i,j] = 0

            ap0[i,j] = rho*c*delta_x*delta_y*delta_t
            a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

        elif i == 0 and j == 0:#左下点

            a_e[i,j] = k*delta_y/delta_x
            a_w[i,j] = 0
            a_s[i,j] = 0
            a_n[i,j] = kn*delta_x/delta_y

            s_u[i,j] = 0
            s_p[i,j] = 0

            ap0[i,j] = rho*c*delta_x*delta_y/delta_t
            a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

        elif i == (NX-1) and j == 0:#右下点

            a_e[i,j] = 0
            a_w[i,j] = k*delta_y/delta_x
            a_s[i,j] = 0
            a_n[i,j] = kn*delta_x/delta_y

            s_u[i,j] = epsilon_eff*sigma*(T_gas**4-T_0[i,j]**4)*delta_y
```

```python
        s_p[i,j] = 0

        ap0[i,j] = rho*c*delta_x*delta_y/delta_t
        a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

    elif i == (NX-1) and j == (NY-1):#右上点

        a_e[i,j] = 0
        a_w[i,j] = k*delta_y/delta_x
        a_s[i,j] = ks*delta_x/delta_y
        a_n[i,j] = 0

        s_u[i,j] = epsilon_eff*sigma*(T_gas**4-T_0[i,j]**4)*delta_y
        s_p[i,j] = 0

        ap0[i,j] = rho*c*delta_x*delta_y/delta_t
        a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

    elif i == 0 and j == (NY-1):#左上点

        a_e[i,j] = k*delta_y/delta_x
        a_w[i,j] = 0
        a_s[i,j] = ks*delta_x/delta_y
        a_n[i,j] = 0

        s_u[i,j] = h*delta_x*T_air
        s_p[i,j] = -h*delta_x

        ap0[i,j] = rho*c*delta_x*delta_y/delta_t
        a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])

    else:#内部

        a_e[i,j] = k*delta_y/delta_x
        a_w[i,j] = k*delta_y/delta_x
        a_s[i,j] = ks*delta_x/delta_y
        a_n[i,j] = kn*delta_x/delta_y

        s_u[i,j] = 0
        s_p[i,j] = 0

```

```
        ap0[i,j] = rho*c*delta_x*delta_y/delta_t
        a_p[i,j] = ap0[i,j]+ 1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j])


###########################################################################################################################
def solve():
    """迭代法求解一步的温度"""
    err = 0#误差

    """初始化温度场"""
    for i in range(NX+2):
        for j in range(NY+2):
            T_0[i,j] = T[i,j]

    while err < 0.05:
        err0 = err
        err = 0

        """计算系数矩阵"""
        for i in range(NX):
            for j in range(NY):
                cal_matrix(i,j)

        for i in range(NX):
            for j in range(NY):
                n = T_0[i,j]+m*((
                        a_e[i,j]*(T_0[i+1,j]+T[i+1,j])/2+
                        a_w[i,j]*(T_0[i-1,j]+T[i-1,j])/2+
                        a_s[i,j]*(T_0[i,j-1]+T[i,j-1])/2+
                        a_n[i,j]*(T_0[i,j+1]+T[i,j+1])/2+
                        (ap0[i,j]-1/2*(a_e[i,j]+a_w[i,j]+a_s[i,j]+a_n[i,j]-s_p[i,j]))*T_0[i,j]
                        +s_u[i,j])/a_p[i,j]-T_0[i,j])

                err = err+abs(T_0[i,j]-n)
                #print("%.8f"%(abs(T_0[i,j]-n)))
                T_0[i,j] = n

        #print("err:{}\n".format("%.8f"%err))

    for i in range(NX+2):
        for j in range(NY+2):
```

```
            T[i, j] = T_0[i, j]


########################################################################################################################
t1=time()

for i in range(NX+2):
    for j in range(NY+2):
        T[i,j] = T_air

        if i == 0 or i == NX+1 or j == 0 or j == NY+1:
            T[i,j] = T_air

for i in range(1, sec+1):
    for j in range(1, int(1/delta_t+1)):
        solve()
        t2 = time()
        print("正在迭代计算第{}秒的温度场，请稍等...{}min".format("%02d" % i, "%.4f" % ((t2-t1)/60)))

    file_path = r"D:\python_project\college_assignment\data_half\data_t={}s.plt".format(i)
    with open(file=file_path, mode="w") as f:
        # f.write('TITLE ="m={}\n"'.format("%.10f" % m))
        f.write("VARIABLES = X,Y,T\n")
        f.write('ZONE T="ZONE 1",I={},J={},F=POINT,t="{}"\n'.format(NX,NY,i))

        for k in range(NY):
            yy = delta_y/2+delta_y*k
            for l in range(NX):

                xx = delta_x/2+delta_x*l
                f.write("{} {} {}\n".format(
                    "%.8f"%xx,
                    "%.8f"%yy,
                    "%.4f"%T[l, k],
                ))
```

3. 隐式代码

```
#include <iostream>
#include <math.h>
```

```
#include <stdio.h>
#include <stdlib.h>
#include<cstring>
#include <algorithm>
#include <Eigen/Dense>

#define rou1 7830.                             //物性参数
#define rou2 1500.0
#define c1 465.0
#define c2 1465.4
#define k1 53.6
#define k2 0.2093
#define Tair 293.0
#define Tgas 923.0
//#define h 8
#define epsilon1 0.6949
#define epsilon2 0.7515
#define sigma 5.67*pow(10,-8)
#define NX 20                                  //网格尺寸
#define NY1 8
#define NY2 4
#define NY3 8
#define NY (NY1+NY2+NY3)
#define L 0.05                                 //模型尺寸
#define L1 0.02
#define L2 0.03
#define H1 0.004
#define H2 0.002
#define H3 0.004
#define del_x (L/NX)                           // 0.0025
#define del_y (H1/NY1)                         // 0.0005
#define del_t 0.02

using namespace Eigen;                         // 改成这样亦可 using Eigen::MatrixXd;
using namespace std;

double AP[NX + 1][NY + 1], AP0[NX + 1][NY + 1],
AW[NX + 1][NY + 1], AE[NX + 1][NY + 1], AN[NX + 1][NY + 1], AS[NX + 1][NY + 1],
SU[NX + 1][NY + 1], SP[NX + 1][NY + 1],
T0[NX + 2][NY + 2], T[NX + 2][NY + 2],
A[(NX + 1)*(NY + 1)][(NX + 1)*(NY + 1)], B[(NX + 1)*(NY + 1)],
x[NX + 2], y[NX + 2];
```

```
void boundary_matrix(int i,int j)                //AP,AP0,AW,AE,AN,AS,SU,SP 的确定
{
    double h = 0.0, tm = 0.0, beta = 0.0, Pr = 0.0, niu = 0.0, Gr = 0.0, Nu = 0.0;
    if (i == 0 && j == 0)                //左下角：1
    {
        AW[i][j] = 0;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = 0;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == 0 && ((i > 0 && i < (NY1 - 1)) || (i > (NY1 + NY2) && i < (NY - 1))))      //材料 1 左边界：2
    {
        AW[i][j] = 0;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == 0 && i == (NY1 - 1))                //材料 1、2 交界处左边界：3
    {
        AW[i][j] = 0;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == 0 && i == NY1)                //材料 1、2 交界处左边界：4
    {
        AW[i][j] = 0;
        AE[i][j] = k2 * del_y / del_x;
```

```
        AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
        AN[i][j] = k2 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
    }
    if (j == 0 && i > NY1 && i < (NY1 + NY2 - 1))
          //材料 2 左边界：5
    {
        AW[i][j] = 0;
        AE[i][j] = k2 * del_y / del_x;
        AS[i][j] = k2 * del_x / del_y;
        AN[i][j] = k2 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
    }
    if (j == 0 && i == (NY1 + NY2 - 1))
            //材料 2、1 交界处左边界：6
     {
         AW[i][j] = 0;
         AE[i][j] = k2 * del_y / del_x;
         AS[i][j] = k2 * del_x / del_y;
         AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
         SU[i][j] = 0;
         SP[i][j] = 0;
         AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
     }
     if (j == 0 && i == (NY1+ NY2))
             //材料 1、2 交界处左边界：7
     {
         AW[i][j] = 0;
         AE[i][j] = k1 * del_y / del_x;
         AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
         AN[i][j] = k1 * del_x / del_y;
         SU[i][j] = 0;
         SP[i][j] = 0;
         AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
     }
     if (j == 0 && i == (NY - 1))
             //左上角：8
     {
         tm = (T0[i][j] - 253.0) / 2.0;
         beta = 2.0 / (T0[i][j] + 293.0);
```

```
        Pr = 4 * pow(10, -14)*pow(tm, 6) - 3 * pow(10, -11)*pow(tm, 5) + 6 * pow(10, -9)*pow(tm, 4) - 7 * pow(10, -7)*pow(tm, 3) + 4 * pow(10, -5)*pow(tm, 2) - 0.0012*tm + 0.7163;
        niu = 9 * pow(10, -11)*pow(tm, 2) + 9.13*pow(10, -8)*tm + 1.3175*pow(10, -5);
        Gr = 9.8*beta*fabs(T0[i][j] - 293.0)*pow(0.03, 3) / pow(niu, 2);
        Nu = 0.54*pow(Gr*Pr, 0.25);
        h = 2.59 *Nu / 3.0;
        AW[i][j] = 0;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 0;
        SU[i][j] = h * del_x*Tair;
        SP[i][j] = -h * del_x;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (i == (NY - 1) && j > 0 && j <= ((L2 / del_x) - 1))          //上边界左半部分：9
    {
        tm = (T0[i][j] - 253.0) / 2.0;
        beta = 2.0 / (T0[i][j] + 293.0);
        Pr = 4 * pow(10, -14)*pow(tm, 6) - 3 * pow(10, -11)*pow(tm, 5) + 6 * pow(10, -9)*pow(tm, 4) - 7 * pow(10, -7)*pow(tm, 3) + 4 * pow(10, -5)*pow(tm, 2) - 0.0012*tm + 0.7163;
        niu = 9 * pow(10, -11)*pow(tm, 2) + 9.13*pow(10, -8)*tm + 1.3175*pow(10, -5);
        Gr = 9.8*beta*fabs(T0[i][j] - 293.0)*pow(0.03, 3) / pow(niu, 2);
        Nu = 0.54*pow(Gr*Pr, 0.25);
        h = 2.59 *Nu / 3.0;
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 0;
        SU[i][j] = h * del_x*Tair;
        SP[i][j] = -h * del_x;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (i == (NY - 1) && j > ((L2 / del_x) - 1) && j < (NX - 1))                //上边界右半部分：10
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 0;
```

```
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_x;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (i == (NY - 1) && j == (NX - 1))                    //右上角：11
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 0;
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* (del_x + del_y);
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (i == 0 && j == (NX - 1))                    //右下角：12
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = 0;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == (NX - 1) && ((i > 0 && i < (NY1 - 1)) || (i > (NY1 + NY2) && i < (NY - 1))))//材料1右边界：13
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == (NX - 1) && i == (NY1 - 1))                    //材料1、2交界处右边界：14
    {
```

```
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j == (NX - 1) && i == NY1)                //材料 1、2 交界处右边界：15
    {
        AW[i][j] = k2 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
        AN[i][j] = k2 * del_x / del_y;
        SU[i][j] = epsilon2 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
    }
    if (j == (NX - 1) && i > NY1 && i < (NY1 + NY2 - 1))        //材料 2 右边界：16
    {
        AW[i][j] = k2 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = k2 * del_x / del_y;
        AN[i][j] = k2 * del_x / del_y;
        SU[i][j] = epsilon2 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
    }
    if (j == (NX - 1) && i == (NY1 + NY2 - 1))                //材料 2、1 交界处右边界：17
    {
        AW[i][j] = k2 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = k2 * del_x / del_y;
        AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
        SU[i][j] = epsilon2 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
```

```
    }
    if (j == (NX - 1) && i == (NY1 + NY2))          //材料 1、2 交界处右边界：18
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = 0;
        AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = epsilon1 * sigma*(pow(Tgas, 4) - pow(T0[i][j], 4))* del_y;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (i == 0 && j > 0 && j < (NX - 1))          //下边界：19
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = 0;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j > 0 && j < (NX - 1) && ((i > 0 && i < (NY1 - 1)) || (i > (NY1 + NY2) && i < (NY - 1))))//材料 1 内部：20
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = k1 * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    if (j > 0 && j < (NX - 1) && i == (NY1 - 1))          //材料 1、2 交界处内部：21
    {
        AW[i][j] = k1 * del_y / del_x;
        AE[i][j] = k1 * del_y / del_x;
        AS[i][j] = k1 * del_x / del_y;
        AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
        SU[i][j] = 0;
        SP[i][j] = 0;
```

```
            AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
        }
        if (j > 0 && j < (NX - 1) && i == NY1)
                    //材料 1、2 交界处内部：22
        {
            AW[i][j] = k2 * del_y / del_x;
            AE[i][j] = k2 * del_y / del_x;
            AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
            AN[i][j] = k2 * del_x / del_y;
            SU[i][j] = 0;
            SP[i][j] = 0;
            AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
        }
        if (j > 0 && j < (NX - 1) && i > NY1 && i < (NY1 + NY2 - 1))
                  //材料 2 内部：23
        {
            AW[i][j] = k2 * del_y / del_x;
            AE[i][j] = k2 * del_y / del_x;
            AS[i][j] = k2 * del_x / del_y;
            AN[i][j] = k2 * del_x / del_y;
            SU[i][j] = 0;
            SP[i][j] = 0;
            AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
        }
        if (j > 0 && j < (NX - 1) && i == (NY1 + NY2 - 1))
                    //材料 2、1 交界处内部：24
        {
            AW[i][j] = k2 * del_y / del_x;
            AE[i][j] = k2 * del_y / del_x;
            AS[i][j] = k2 * del_x / del_y;
            AN[i][j] = 2.0*k1*k2 / (k1 + k2) * del_x / del_y;
            SU[i][j] = 0;
            SP[i][j] = 0;
            AP0[i][j] = rou2 * c2* del_x *del_y / del_t;
        }
        if (j > 0 && j < (NX - 1) && i == (NY1 + NY2))
                    //材料 1、2 交界处内部：25
        {
            AW[i][j] = k1 * del_y / del_x;
            AE[i][j] = k1 * del_y / del_x;
            AS[i][j] = 2.0*k1*k2 / (k1 + k2)  * del_x / del_y;
            AN[i][j] = k1 * del_x / del_y;
            SU[i][j] = 0;
            SP[i][j] = 0;
```

```
        AP0[i][j] = rou1 * c1* del_x *del_y / del_t;
    }
    AP[i][j] = AP0[i][j] + AW[i][j] + AE[i][j] + AS[i][j] + AN[i][j] - SP[i][j];
}

void mesh()
{
    int i;
    FILE *fp;
                //导出数据
    errno_t err_mesh;
    err_mesh = fopen_s(&fp, "mesh.dat", "w");
    x[0] = 0.0;
    fprintf(fp, "    i  dx[i]      xu[i]:     \n");
    fprintf(fp, "    0 %5.5f %5.5f\n", 0.0, x[0]);
    for (i = 1; i <= NX; i++)
    {
        x[i] = x[i - 1] + del_x;
        fprintf(fp, "%5d %5.5f %5.5f\n", i, del_x, x[i]);
    }
    fprintf(fp, "\n");

    y[0] = 0.0;
    fprintf(fp, "    j  dy[j]       yv[j]:     \n");
    fprintf(fp, "    0 %5.5f %5.5f\n", 0.0, y[0]);
    for (i = 1; i <= NY; i++) {
        y[i] = y[i - 1] + del_y;
        fprintf(fp, "%5d %5.5f %5.5f\n", i, del_y, y[i]);
    }
    fprintf(fp, "\n");
    fclose(fp);
}

int main(int argc, char** argv)
{
    int i, j;
    double k;
    MatrixXd coefficient_matrix(((NX + 1)*(NY + 1)), ((NX + 1)*(NY + 1)));

    coefficient_matrix = MatrixXd::Zero(((NX + 1)*(NY + 1)), ((NX + 1)*(NY + 1)));
    mesh();
    VectorXd Tv_vector((NX + 1)*(NY + 1));
```

```
    VectorXd B_vector((NX + 1)*(NY + 1));
    for (i = 0; i <= (NY - 1); i++)
    {
        for (j = 0; j <= (NX - 1); j++)
        {
            T0[i][j] = 293;                //初始条件
        }
    }

    for (k = 0; k <= 30; k = k + 0.02)
    {
        for (i = 0; i <= (NY - 1); i++)

        {
            for (j = 0; j <= (NX - 1); j++)
            {
                boundary_matrix(i,j);            //AP,AP0,AW,AE,AN,AS,SU,SP 的确定
                coefficient_matrix(i*NX + j, i*NX + j) = AP[i][j];            //求解系数矩阵//AP
                Tv_vector(i*NX + j) = T[i][j];
                B_vector(i*NX + j) = AP0[i][j] * T0[i][j] + SU[i][j];
                if (i != 0)
                    coefficient_matrix(i*NX + j, (i - 1)*NX + j) = -AS[i][j];            //AS
                if (i != (NY - 1))
                    coefficient_matrix(i*NX + j, (i + 1)*NX + j) = -AN[i][j];            //AN
                if (j != 0)
                    coefficient_matrix(i*NX + j, i*NX + j - 1) = -AW[i][j];             //AW
                if (j != (NX - 1))
                    coefficient_matrix(i*NX + j, i*NX + j + 1) = -AE[i][j];             //AE
            }
        }

        //Tv_vector = coefficient_matrix.householderQr().solve(B_vector);
        Tv_vector = coefficient_matrix.colPivHouseholderQr().solve(B_vector);            //QR 分解
        cout << "k =" << endl << k << endl;
        cout << " " << endl;
        //cout << "Tv_vector =" << endl << Tv_vector << endl;
```

```
        //cout <<" "<< endl;
        FILE *fp;
            //导出数据
        errno_t err_T;
        err_T = fopen_s(&fp, "T(t=30)(100,20)", "w");
        fprintf(
            fp,
            "TITLE = \"2D Plot temperature\"\nVARIABLES = X, Y, Temp\nZONE T=\"ZONE 1\", I=%d, J=%d, F=POINT, t=\"%2.2f\"\n",
            NX, NY, k);

        for (i = 0; i <= (NY - 1); i++)
        {
            for (j = 0; j <= (NX - 1); j++)
            {
                T0[i][j] = Tv_vector(i*NX + j);
            //把向量再转化为数组
                fprintf(fp, "%4.8f %4.8f %4.2f   \n", 0.5 * (x[j] + x[j + 1]), 0.5 * (y[i] + y[i + 1]), T0[i][j]);
            }
        }
        fclose(fp);
    }
    system("pause");
    return 0;
}
```

4. T-x 曲线绘制代码

```
"""author:gaoyi"""
import matplotlib.pylab as plt

fff = 16

X1=[]
T1=[]
file_1 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\半隐网格无关性验证\T-x\20x20.txt"
with open(file_1, "r") as f1:
    for i in f1:
        X1.append(float("%.6f" % float(i.split()[0])))
        T1.append(float("%.6f" % float(i.split()[1])))

X2=[]
```

```
T2=[]
file_2 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\半隐网格无关性验证\T-x\50x20.txt"
with open(file_2, "r") as f2:
    for i in f2:
        X2.append(float("%.6f" % float(i.split()[0])))
        T2.append(float("%.6f" % float(i.split()[1])))

X3=[]
T3=[]
file_3 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\半隐网格无关性验证\T-x\50x40.txt"
with open(file_3, "r") as f3:
    for i in f3:
        X3.append(float("%.6f" % float(i.split()[0])))
        T3.append(float("%.6f" % float(i.split()[1])))

X4=[]
T4=[]
file_4 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\半隐网格无关性验证\T-x\100x20.txt"
with open(file_4, "r") as f4:
    for i in f4:
        X4.append(float("%.6f" % float(i.split()[0])))
        T4.append(float("%.6f" % float(i.split()[1])))

plt.figure(figsize=(10, 8))

plt.plot(X1, T1, label="20x20")
# plt.scatter(X1, T1, marker="x")

plt.plot(X2, T2, label="50x20")
# plt.scatter(X2, T2, marker="x")

plt.plot(X3, T3, label="50x40")
# plt.scatter(X3, T3, marker="x")

plt.plot(X4, T4, label="100x20")
# plt.scatter(X4, T4, marker="x")

plt.xlabel("X/m", fontsize=fff)
plt.ylabel("T/K", fontsize=fff)
plt.grid(linestyle="--")
plt.legend(loc=2, fontsize=fff)
```

```
plt.tight_layout()
plt.show()
```

5. T-t 曲线绘制代码

```
"""author:gaoyi"""
import matplotlib.pyplot as plt
import numpy as np

fff = 15

t = np.arange(1, 31, 1)

T1 = []
for i in t:
    file_1 = r"D:\python_project\college_assignment\半隐式网格无关性验证\半隐式-数据\20x20_m=0.95\data_t={}s.plt".format(i)
    with open(file_1, "r") as f:
        k=1
        for i in f:
            if k == 202:
                T1.append(float("%.6f" % float(i.split()[2])))
            k = k+1

T2 = []
for i in t:
    file_2 = r"D:\python_project\college_assignment\半隐式网格无关性验证\半隐式-数据\50x20_m=0.95\data_t={}s.plt".format(i)
    with open(file_2, "r") as f_1:
        k1=1
        for i in f_1:
            if k1 == 502:
                T2.append(float("%.6f" % float(i.split()[2])))
            k1 = k1+1


T3 = []
for i in t:
    file_3 = r"D:\python_project\college_assignment\半隐式网格无关性验证\半隐式-数据\50x40_m=0.95\data_t={}s.plt".format(i)
    with open(file_3, "r") as f2:
        k3=1
        for i in f2:
            if k3 == 502:
```

```python
                T3.append(float("%.6f" % float(i.split()[2])))
            k3 = k3+1


T4 = []
for i in t:
    file_4 = r"D:\python_project\college_assignment\半隐式网格无关性验证\半隐式-数据\100x20_m=0.95\data_t={}s.plt".format(i)
    with open(file_4, "r") as f3:
        k4=1
        for i in f3:
            if k4 == 1002:
                T4.append(float("%.6f" % float(i.split()[2])))
            k4 = k4+1
plt.figure(figsize=(8, 6))

plt.plot(t, T1, label="20x20")
plt.scatter(t, T1, marker="x")

plt.plot(t, T2, label="50x20")
plt.scatter(t, T2, marker="x")

plt.plot(t, T3, label="50x40")
plt.scatter(t, T3, marker="x")

plt.plot(t, T4, label="100x20")
plt.scatter(t, T4, marker="x")

plt.xlabel("t/s", fontsize=fff)
plt.ylabel("T/K", fontsize=fff)
plt.grid(linestyle="--")
plt.legend(loc=2, fontsize=fff)
plt.tight_layout()
plt.show()
```

6. UDF 代码

```c
#include "udf.h"
DEFINE_PROFILE(user_h,thread,pos)
{
double tem,tm,beta,Pr,v,Gr,Nu,h;
face_t f;
begin_f_loop(f, thread)
{
```

```
tem=F_T(f,thread);
tm=(tem-253)/2;
beta=2/(tem+293);
Pr=4*(1E-14)*pow(tm,6)-3*(1E-11)*pow(tm,5)+6*(1E-9)*pow(tm,4)-7*(1E-7)*pow(tm,3)+4*(1E-5)*pow(tm,2)-0.0012*tm+0.7163;
v=9*(1E-11)*pow(tm,2)+9*(1E-8)*tm+1.3175*(1E-5);
Gr=9.8*beta*fabs(tem-293)*pow(0.03,3)/pow(v,2);
Nu=0.54*pow(Gr*Pr,0.25);
h=Nu*2.59/3;
F_PROFILE(f,thread,pos)=h;
}
end_f_loop(f,thread)
}
```

7. 其余 Python 代码

```
"""author:gaoyi"""
import numpy as np
import matplotlib.pyplot as plt
"""5 秒 T-x 曲线绘制"""
"""网格数目为 20x20"""

x1 = []
T1 = []
file_path_1 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\赵明潇数据_隐式\t=5s 隐式.txt"
with open(file=file_path_1, mode="r") as f1:
    for row in f1:
        x1.append(float(row.split()[0]))
        T1.append(float(row.split()[2]))

x2=[0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02875,0.03125,
    0.03375,0.03625,0.03875,0.04125,0.04375,0.04625,0.04875]
T2=[293.0016,293.00015,293.00025,293.00055,293.0011,293.0023,293.00455,293.00895,293.01695,293.03115,293.05535
    ,293.09405,293.1481,293.20575,293.27385,293.36985,293.5191,293.7679,294.6746,312.9441]

x_ls_5s=[0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02874999,0.03133333
    ,0.03399999,0.03666666,0.03933333,0.042,0.04466666,0.04733333,0.05]
T_ls_5s=[293,293,293,293,293,293,293,293.005,293.02,293.065,293.175,293.41,293.795,294.125,294.395,294.64,294.92,295.325
```

```python
    ,296.87,333.235]

x_yy_5s = [0.0000,0.0025,0.0050,0.0075,0.0100,0.0125,0.0150,0.0175,0.0200,0.0225,0.0250,0.0275,0.0300,0.0325,0.0350,0.0375
    ,0.0400,0.0425,0.0450,0.0475,0.0500]
T_yy_5s = [293.0009,293.0013,293.0024,293.0043,293.0086,293.0169,293.0310,293.0558,293.0987,293.1678,293.2809,293.4550
    ,293.6898,293.9368,294.1571,294.3589,294.5748,294.8633,295.5243,304.8741,484.4058 ]

x_xs_5s = [0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02875,0.03125,
    0.03375,0.03625,0.03875,0.04125,0.04375,0.04625,0.04875]
T_xs_5s = [293.0009374,293.0014957,293.0028748,293.0056875,293.0110869,293.021089,293.039066,293.0704529,293.1236799
    ,293.2111965,293.3497042,293.555334,293.8187676,294.0529783,294.2554055,294.4562743,294.6950132,295.0291772
    ,296.0560942,314.8997893]

x_M_5s = []
T_M_5s = []
file_path_2 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\M 商业软件\data_5s.txt"
with open(file=file_path_2, mode="r") as f2:
    for row in f2:
        x_M_5s.append(float(row.split()[0]))
        T_M_5s.append(float(row.split()[1]))

"""绘图"""
"""解决中文显示问题"""
fff = 16    #字体大小

plt.rcParams['font.sans-serif']=['SimHei']
plt.rcParams['axes.unicode_minus'] = False

plt.figure(figsize=(10, 8))
plt.title("T-x 曲线(5s)",fontsize=fff)

plt.plot(x1, T1, label="隐式")
plt.scatter(x1, T1,marker="x")

plt.plot(x2,T2,label="半隐式")
plt.scatter(x2, T2,marker="x")

```

```
plt.plot(x_ls_5s, T_ls_5s, label="隐式-老师代码")
plt.scatter(x_ls_5s, T_ls_5s, marker="x")

plt.plot(x_yy_5s, T_yy_5s, label="Fluent")
plt.scatter(x_yy_5s, T_yy_5s, marker="x")

plt.plot(x_xs_5s, T_xs_5s, label="显式")
plt.scatter(x_xs_5s, T_xs_5s, marker="x")

plt.plot(x_M_5s, T_M_5s, label="Mechanical")
plt.scatter(x_M_5s, T_M_5s, marker="x")

plt.xlabel("x/m",fontsize=fff)
plt.ylabel("T/K",fontsize=fff)
plt.grid(linestyle="--")
plt.tight_layout()
plt.legend(loc=2,fontsize=15)
##########################################################################################################################
"""30s 曲线绘制"""
x_y_30s = [0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02875,0.03125
    ,0.03375,0.03625,0.03875,0.04125,0.04375,0.04625,0.04875]
T_y_30s = [297.6,297.785,298.145,298.705,299.455,300.42,301.61,303.05,304.75,306.73,309.02,311.605,314.41,317.12
    ,319.66,322.025,324.225,326.35,329.425,350.535]

x_by_30s = [0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02875,0.03125
    ,0.03375,0.03625,0.03875,0.04125,0.04375,0.04625,0.04875]
T_by_30s = [294.2728,293.84555,293.88235,293.99035,294.153,294.37415,294.65985,295.01775,295.45645,295.98585,296.61585
    ,297.35425,298.1974,299.1153,300.1042,301.17215,302.33455,303.6891,306.66115,333.7786]

x_ls_30s = [0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02874999,0.03133333
    ,0.03399999,0.03666666,0.03933333,0.042,0.04466666,0.04733333,0.05]
T_ls_30s = [293.42,293.44,293.49,293.57,293.685,293.86,294.14,294.62,295.42,296.71,298.605,301.19,304.355,307.495,310.405
    ,313.06,315.49,317.85,322.595,373.17]

x_yy_30s=[0.0000,0.0025,0.0050,0.0075,0.0100,0.0125,0.0150,0.0175,0.0200,0.0225,0.0250,0.0275,0.0300,0.0325,0.0350,0.0375
```

```
    ,0.0400,0.0425,0.0450,0.0475,0.0500]
T_yy_30s=[295.9475,296.0095,296.2000,296.5242,296.9882,297.6006,298.3710,299.3138,300.4441,301.7757,303.3292,305.1130,307.0928
    ,309.1440,311.1511,313.0759,314.9232,316.7640,319.2902,333.5459,513.2305]

x_xs_30s = [0.00125,0.00375,0.00625,0.00875,0.01125,0.01375,0.01625,0.01875,0.02125,0.02375,0.02625,0.02875,0.03125
    ,0.03375,0.03625,0.03875,0.04125,0.04375,0.04625,0.04875]
T_xs_30s = [295.9266267,296.0544902,296.3128458,296.7069128,297.2444243,297.9355105,298.7925357,299.8298793,301.0635943
    ,302.510614,304.1858346,306.0897203,308.1641478,310.2201699,312.198582,314.0980879,315.9360367,317.8130201,321.1519307
    ,348.4038164]

x_M_30s = []
T_M_30s = []
file_path_3 = r"D:\大学课程\大 4-上\发动机内流场数值分析基础（刘洋、胡春波）\大作业\M 商业软件\data_30s.txt"
with open(file=file_path_3, mode="r") as f3:
    for row in f3:
        x_M_30s.append(float(row.split()[0]))
        T_M_30s.append(float(row.split()[1]))

plt.figure(figsize=(10, 8))
plt.title("T-x 曲线(30s)",fontsize=fff)

plt.plot(x_y_30s, T_y_30s, label="隐式")
plt.scatter(x_y_30s, T_y_30s, marker="x")

plt.plot(x_by_30s,T_by_30s,label="半隐式")
plt.scatter(x_by_30s, T_by_30s, marker="x")

plt.plot(x_ls_30s, T_ls_30s, label="隐式-老师代码")
plt.scatter(x_ls_30s, T_ls_30s, marker="x")

plt.plot(x_yy_30s, T_yy_30s, label="fluent")
plt.scatter(x_yy_30s, T_yy_30s, marker="x")

plt.plot(x_xs_30s, T_xs_30s, label="显式")
plt.scatter(x_xs_30s, T_xs_30s, marker="x")

plt.plot(x_M_30s, T_M_30s, label="Mechanical")
plt.scatter(x_M_30s, T_M_30s, marker="x")
```

```

plt.xlabel("x/m",fontsize=fff)
plt.ylabel("T/K",fontsize=fff)
plt.grid(linestyle="--")
plt.tight_layout()
plt.legend(loc=2,fontsize=15)
plt.show()
```