Sum of Finite Geometric Series

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1 Statement

$$\sum_{k=0}^{n} b^k = \frac{b^{n+1}-1}{b-1}$$

2 Proof

2.1 Verification by Intuition

Written in base b, b^k is represented as a one followed by k zeroes. As such, the sum in the statement above is represented as n ones in base b.

Continuing, b^{n+1} is represented as a one followed by $n+1$ zerores. This means that $b^{n+1}-1$ is represented as n characters representing the value of $b-1$ (in the case of base ten, this character would be nine). Finally, dividing the number by $b-1$ results in n ones, which is equal to the value of the sum as shown initially.

2.2 Proof by Induction

The base case, $n=0$, is trivial; both sides evaluate to 1.

2.2.1 Inductive step: $n \geq 1$

$$\begin{aligned}
\frac{b^{n+1}-1}{b-1} &= \frac{b^n-1}{b-1} + b^n && (1)\\
&= \frac{b^n-1+(b-1)b^n}{b-1} && (2)\\
&= \frac{(1+(b-1))b^n-1}{b-1} && (3)\\
&= \frac{(b)b^n-1}{b-1} && (4)\\
&= \frac{b^{n+1}-1}{b-1} && (5)
\end{aligned}$$