CAP4612 - Homework 3

- Complete the following problems. Please hand write your solutions and once you are done please upload a PDF document containing all the pages to canvas, thanks.

- Note there are applications on your phone that you could use to take pictures and convert all these pictures into PDF. Very important do not send me individual files of each page. I wish to have one document.

- If you like you can just use this document and write your answers in the space provided.

1. Please put your name and panther ID below.

Name: _CHRISTIAN ANGUS_

ID: _6 19 18 13_

CERTIFICATION: I understand FIU's academic policies, and I certify that this work is my own and that none of it is the work of any other person.

2. Draw the Markov Chain for the following information:
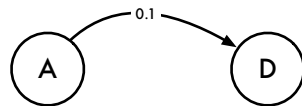
States: A, B, C, D
Transitions: A -> A with probability 0.3, A -> B with probability 0.2, A -> C with probability 0.4,
                 A -> D with probability 0.1

                 B -> A with probability 0.2, B-> B with probability 0.5, B -> C with probability 0.1,
                 B -> D with probability 0.2

                 C -> C with probability 1.0

                 D -> A with probability 0.25, D-> B with probability 0.15, D -> C with probability 0.6

Example: A -> D with probability 0.1

Draw your Draw Markov Chain below:
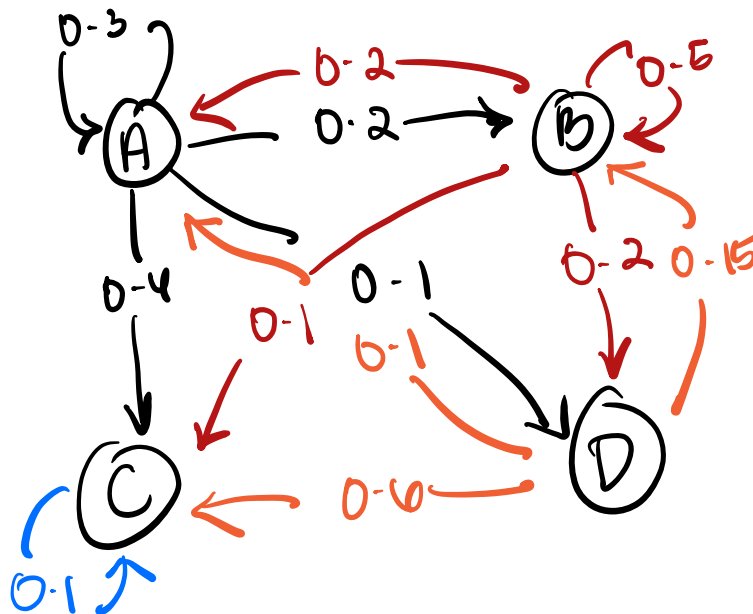
Look at your diagram and write a one or two sentence comment about State C in regard to the equilibrium of this Markov Chain.

STATE C IS RECURRENT BECAUSE THERE IS NO WAY OF
TRAVERSING OUT OF STATE C WHEN IT IS REACHED-

3. Markov Chain topics

Part A) Find the Markov Transition Matrix A for the Markov Chain shown below:

Markov Transition Matrix A:

$$\begin{bmatrix} 0.0 & 0.8 & 0.2 \\ 0.0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$$
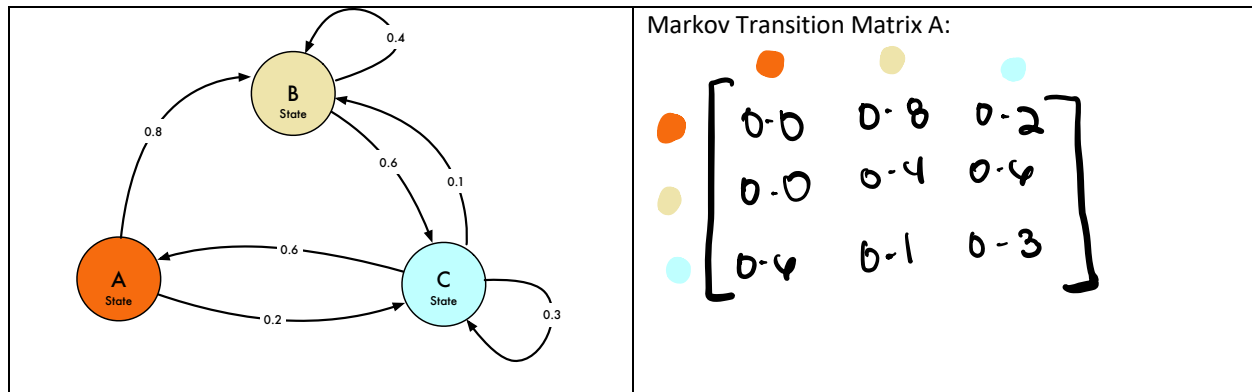
Part B) For this part you need to do the work by hand and show your work. Starting form State B, [0 1 0], find the first three transition vectors ... look at the notes for this... $s_0 A = s_1$

$$S_0 B = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0.0 & 0.8 & 0.2 \\ 0.0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.4 & 0.6 \end{bmatrix}$$

$$S_1 B = \begin{bmatrix} 0.0 & 0.4 & 0.6 \end{bmatrix} \times \begin{bmatrix} 0.0 & 0.8 & 0.2 \\ 0.0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.36 & 0.22 & 0.42 \end{bmatrix}$$

$$S_2 B = \begin{bmatrix} 0.36 & 0.22 & 0.42 \end{bmatrix} \times \begin{bmatrix} 0.0 & 0.8 & 0.2 \\ 0.0 & 0.4 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.252 & 0.418 & 0.33 \end{bmatrix}$$

Part C) You can use a computer for this part. What is the state vector that this environment settles down too? How many iterations does it take to start converging to this vector?

4. Parameter Optimization using the Envelope Theorem. You need to hand write your work and show all of it.

$$Maximize: \ U(X,Y) = 10XY \quad subject \ to \ P_x X + P_y Y - 5M$$

Part A) Find the Lagrangians:

$$L = 10XY - \lambda(P_x X + P_y Y - 5M)$$

$$L_x = Y - \lambda P_x = 0 \implies Y = \lambda P_x \implies \lambda = \frac{Y}{P_x}$$

$$L_y = X - \lambda P_y = 0 \implies X = \lambda P_y \implies \lambda = \frac{X}{P_y}$$

$$L_\lambda = P_x X + P_y Y - 5M = 0$$

$$\frac{Y}{P_x} = \lambda = \frac{X}{P_y} \implies Y = \frac{X P_x}{P_y} \implies X = \frac{Y P_y}{P_x}$$

4. Parameter Optimization using Envelope Theorem continued:
You need to hand write your work and show all of it.

Part B) Find X^*, Y^* *and* $U^*(X^*, Y^*)$

$$P_x X + P_y y - 5M = 0$$

$$P_x X + P_y \left(\frac{XP_x}{P_y}\right) - 5M = 0 \implies P_x X + XP_x - 5M = 0$$

$$2P_x X = 5M \implies \boxed{X^* = \frac{5M}{2P_x}}$$

$$y = \frac{XP_x}{P_y} \implies y = \frac{\frac{5M}{2P_x} P_x}{P_y} \implies \boxed{y^* = \frac{5M}{2P_y}}$$

$$U^*(X^*, y^*) = \left(\frac{5M}{2P_x}\right)\left(\frac{5M}{2P_y}\right) \implies \boxed{\frac{25M^2}{4P_x P_y}}$$

Part C) Find $\frac{\partial U^*(X^*, Y^*)}{\partial M}$

$$\frac{\partial U^*}{\partial M} = \frac{50M}{4P_x P_y} \implies \boxed{\frac{25M}{2P_x P_y}}$$

5. Bellman Equation

Part A) Write the Bellman Equation and explain in one to two sentences what the following elements of the Bellman Equation are:

Write the Bellman Equation:

$$V(s) = \text{MAX}_a \left(R(s, a) + \gamma V(s') \right)$$

Explain $V(s)$:

YOUR OPTIMIZATION FOR YOUR PROBLEM TODAY.

Explain $R(a, s)$:

YOUR UTILITY FUNCTION THAT ACCOUNTS FOR YOUR GROWTH RATE OVER THE PERIOD OF TIME. THE MORE YOU CONSUME THE BETTER YOU FEEL.
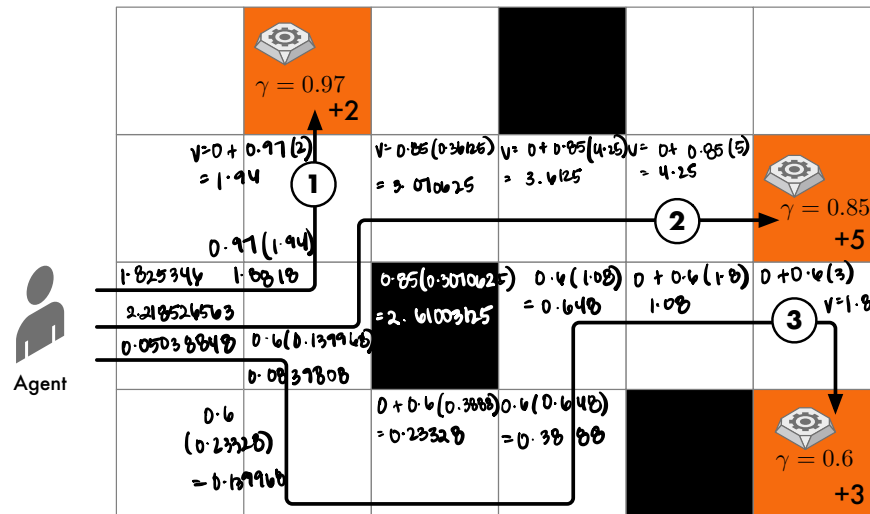
Explain γ :

YOUR DISCOUNT FACTOR, CONTROLS THE FUTURE UTILITY YOU ARE GOING TO GET.

Explain $V(s')$:

YOUR OPTIMIZATION FOR YOUR PROBLEM IN THE FUTURE.

5. Bellman Equation continued:

Part B) Calculate **V(s)** for each step of the paths shown below and state with path the agent would take. Round your answers to three digits. You must show all your work.

THE AGENT WOULD SELECT PATH #2