Problem 3: (25 pts)

Consider the problem of inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, and 59 into a **hash table** table of length 11 (**TableSize = 11**) using **open addressing** with the standard function **h(k) = k mod TableSize**.

Illustrate the result of inserting these keys using:

A) **Linear probing**: **h(k) = k mod TableSize**

h(10) = 10 % 11 = 10
Attempting to insert 10 at position 10.
Inserting 10 at position 10.

h(22) = 22 % 11 = 0
Attempting to insert 22 at position 0.
Inserting 22 at position 0.

h(31) = 31 % 11 = 9
Attempting to insert 31 at position 9.
Inserting 31 at position 9.

h(4) = 4 % 11 = 4
Attempting to insert 4 at position 4.
Inserting 4 at position 4.

h(15) = 15 % 11 = 4
Attempting to insert 15 at position 4. Collision
Attempting to insert 15 at position 5.
Inserting 15 at position 5.

h(28) = 28 % 11 = 6
Attempting to insert 28 at position 6.
Inserting 28 at position 6.

h(17) = 17 % 11 = 6
Attempting to insert 17 at position 6. Collision
Attempting to insert 17 at position 7.
Inserting 17 at position 7.

h(88) = 88 % 11 = 0
Attempting to insert 88 at position 0. Collision
Attempting to insert 88 at position 1.
Inserting 88 at position 1.

h(59) = 59 % 11 = 4
Attempting to insert 59 at position 4. Collision
Attempting to insert 59 at position 5. Collision
Attempting to insert 59 at position 6. Collision
Attempting to insert 59 at position 7. Collision
Attempting to insert 59 at position 8.
Inserting 59 at position 8.

0	1	2	3	4	5	6	7	8	9	10
22	88			4	15	28	17	59	31	10

B) **Quadratic probing** with quadratic probe function **c(i) = 3i^2 + i**.

h(k) = k mod TableSize
h(k, i) = (h(k) + c(i)) mod TableSize

h(k) = 10 % 11 = 10
h(10, 0) = (10 + 3(0)^2 + 0) % 11
= 10 % 11
= 10
Slot 10 is empty, so insert **10 into slot 10**

h(k) = 22 % 11 = 0
h(22, 0) = (0 + 3(0)^2 + 0) % 11
= 0 % 11
= 0
Slot 0 is empty, so insert **22 into slot 0**

h(k) = 31 % 11 = 9
h(31, 0) = (9 + 3(0)^2 + 0) % 11
= 9 % 11
= 9
Slot 9 is empty, so insert **31 into slot 9**

h(k) = 4 % 11 = 4
h(4, 0) = (4 + 3(0)^2 + 0) % 11
= 4 % 11
= 4
Slot 4 is empty, so insert **4 into slot 4**

h(k) = 15 % 11 = 4
h(15, 0) = (4 + 3(0)^2 + 0) % 11
= 4 % 11
= 4
Slot 4 is occupied, so increment i. i = 1
h(15, 1) = (4 + 3(1)^2 + 1) % 11
= 8 % 11
= 8
Slot 8 is empty, so insert **15 into slot 8**

h(k) = 28 % 11 = 6
h(28, 0) = (6 + 3(0)^2 + 0) % 11
= 6 % 11
= 6
Slot 6 is empty, so insert **28 into slot 6**

h(k) = 17 % 11 = 6
h(17, 0) = (6 + 3(0)^2 + 0) % 11
= 6 % 11
= 6
Slot 6 is occupied, so increment i. I = 1
h(17, 1) = (6 + 3(1)^2 + 1) % 11
= 10 % 11
= 10
Slot 10 is occupied, so increment i. I = 2
h(17, 2) = (6 + 3(2)^2 + 2) % 11
= 20 % 11
= 9
Slot 9 is occupied, so increment i. I = 3

h(17, 3) = (6 + 3(3)^2 + 3) % 11
= 36 % 11
= 3
Slot 3 is empty, so insert **17 into slot 3**

h(k) = 88 % 11 = 0
h(88, 0) = (0 + 3(0)^2 + 0) % 11
= 0 % 11
= 0
Slot 0 is occupied, so increment i. I = 1
h(88, 1) = (0 + 3(1)^2 + 1) % 11
= 4 % 11
= 4
Slot 4 is occupied, so increment i. I = 2
h(88, 2) = (0 + 3(2)^2 + 2) % 11
= 14 % 11
= 3
Slot 3 is occupied, so increment i. I = 3
h(88, 3) = (0 + 3(3)^2 + 3) % 11
= 30 % 11
= 8
Slot 8 is occupied, so increment i. I = 4
h(88, 4) = (0 + 3(4)^2 + 4) % 11
= 52 % 11
= 8
Slot 8 is occupied, so increment i. I = 5
h(88, 5) = (0 + 3(5)^2 + 5) % 11
= 80 % 11
= 3
Slot 3 is occupied, so increment i. I = 6
h(88, 6) = (0 + 3(6)^2 + 6) % 11
= 114 % 11
= 4
Slot 4 is occupied, so increment i. I = 7
h(88, 7) = (0 + 3(7)^2 + 7) % 11
= 154 % 11
= 0
Slot 0 is occupied, so increment i. I = 8
h(88, 8) = (0 + 3(8)^2 + 8) % 11
= 200 % 11
= 2

Slot 2 is empty, so insert **88 into slot 2**

h(k) = 59 % 11 = 4
h(59, 0) = (4 + 3(0)^2 + 0) % 11
= 4 % 11
= 4
Slot 4 is occupied, so increment i. I = 1
h(59, 1) = (4 + 3(1)^2 + 1) % 11
= 8 % 11
= 8
Slot 8 is occupied, so increment i. I = 2
h(59, 2) = (4 + 3(2)^2 + 2) % 11
= 18 % 11
= 7
Slot 7 is empty, so insert **59 into slot 7**

0	1	2	3	4	5	6	7	8	9	10
22		88	17	4		28	59	15	31	10

C) **Double hashing** with **u(k) = k** and **v(k) = 1 + (k mod(TableSize - 1))**.

u(k) = k % 11
v(k) = 1 + (k % 10)
h(k, i) = (u(k) + iv(k)) % 11

u(10) = 10 % 11 = 10
v(10) = 1 + (10 % 10) = 1
h(10,0) = (10 + 0(1)) % 11
= **10**

u(22) = 22 % 11 = 0
v(22) = 1 + (22 % 10) = 3
h(22, 0) = (0 + 0(3)) % 11
= **0**

u(31) = 31 % 11 = 9
v(31) = 1 + (31 % 10) = 31
h(31, 0) = (9 + 0(31)) % 11
= **9**

u(4) = 4 % 11 = 4
v(4) = 1 + (4 % 10) = 5
h(4, 0) = (4 + 0(5)) % 11
= **4**

u(15) = 15 % 11 = 4
v(15) = 1 + (15 % 10) = 6
h(15, 0) = (4 + 0(6)) % 11
= 4 Collision
h(15, 1) = (4 + 1(6)) % 11
= 10 Collision
h(15, 2) = (4 + 2(6)) % 11
= **5**

u(28) = 28 % 11 = 6
v(28) = 1 + (28 % 10) = 9
h(28, 0) = (6 + 0(9)) % 11
= **6**

u(17) = 17 % 11 = 6
v(17) = 1 + (17 % 10) = 8
h(17, 0) = (6 + 0(8)) % 11
= 6 Collision
h(17, 1) = (6 + 1(8)) % 11
= **3**

u(88) = 88 % 11 = 0
v(88) = 1 + (88 % 10) = 9
h(88, 0) = (0 + 0(9)) % 11
= 0 Collision
h(88, 1) = (0 + 1(9)) % 11
= 9 Collision
h(88, 2) = (0 + 2(9)) % 11
= **7**

u(59) = 59 % 11 = 4
v(59) = 1 + (59 % 10) = 10
h(59, 0) = (4 + 0(10)) % 11
= 4 Collision
h(59, 1) = (4 + 1(10)) % 11
= 3 Collision
h(59, 2) = (4 + 2(10)) % 11
= **2**

0	1	2	3	4	5	6	7	8	9	10
22		59	17	4	15	28	88		31	10