

# Pursuing the wrong options? Adjustment costs and the relationship between uncertainty and capital accumulation

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## Abstract

Abel and Eberly (1999) prove that uncertainty has an ambiguous effect on long run capital accumulation in a real options model. We show that, with adjustment costs quadratic in investment, more uncertainty reduces capital and this effect may be large.

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**Key words:** Uncertainty, real options, adjustment costs, capital accumulation.

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# 1 Introduction

The recent literature on uncertainty and capital accumulation has focused on the effects of ‘real options’ associated with irreversible investment (e.g. Dixit and Pindyck, 1994; Abel and Eberly, 1996). The focus has been primarily on short run mechanisms. For example it has been shown that, at a higher level of uncertainty, risk neutral firms are less likely to invest in response to a positive demand shock. Long run effects have been analyzed by Abel and Eberly (1999; henceforth AE). They prove that, in a dynamic model with complete irreversibility, the sign of the effect of demand uncertainty on expected long run capital stock is theoretically ambiguous. In this paper we study the effects of uncertainty on long run capital accumulation in a model in which adjustment costs are quadratic in the investment rate, a framework that has been the bedrock of investment modelling for a long time (e.g. Summers, 1981; Hayashi and Inoue, 1991; Abel, 2002). We find that a higher level of uncertainty tends to reduce the expected long run capital stock in the presence of quadratic adjustment costs, and that this effect may be substantial.

# 2 Investment Model

Our model is a discrete version of that presented in AE, modified to allow for quadratic adjustment costs. The firm produces output using capital and labor according to a Cobb-Douglas production function with capital share  $\beta$  and labor share  $(1 - \beta)$ . Output is sold in an imperfectly competitive market with constant price elasticity of demand  $-\eta$ . Factor markets are perfectly competitive and factor prices constant. Optimal investment is defined as the solution to a dynamic

optimization problem defined by the stochastic Bellman equation

$$\begin{aligned} V(X_t, K_t) &= \max_{I_t} \left\{ \frac{h}{1-\gamma} X_t^\gamma (K_t + I_t)^{1-\gamma} - I_t - G(I_t, K_t) \right. \\ &\quad \left. + \frac{1}{1+r} E_t [V(X_{t+1}, K_{t+1})] \right\}, \end{aligned} \quad (1)$$

where  $V(\cdot)$  is the value function,  $X_t$  is a demand shock parameter,  $K_t$  is capital,  $t$  is time,  $h$  is a positive constant,  $\gamma = (1 + \beta(\eta - 1))^{-1}$ ,  $I_t$  is investment,  $G(I_t, K_t)$  is an adjustment cost function,  $r$  is the discount rate and  $E_t$  is the expectations operator.<sup>1</sup> Following AE we assume that capital does not depreciate<sup>2</sup>, and that the demand parameter evolves according to a geometric Brownian motion, written in discrete time as  $\ln X_t = \ln X_{t-1} + \tilde{\mu} + \varepsilon_t$ , with  $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$  and  $\ln X_0 = 0$ . The parameter  $\sigma^2$  measures the level of demand uncertainty and we follow AE in specifying  $\tilde{\mu} = \mu - 0.5\sigma^2 > 0$ , so that the expected level of demand  $E[X_t] = X_0 e^{\mu t}$  does not depend on the level of uncertainty.<sup>3</sup> Defining  $K_t^R$  as optimal capital under reversible investments and no adjustment costs, the outcome variable of interest is

$$\kappa(t) = \frac{E_0 \{K_t | X_0 = 1\}}{E_0 \{K_t^R | X_0 = 1\}},$$

where  $E_0 \{K_t | X_0 = 1\}$  is the expected capital stock under either irreversibility or quadratic adjustment costs.<sup>4</sup> We study the effect of changes in uncertainty on  $\kappa(T)$  as  $T \rightarrow \infty$ . Since the expected level of demand is kept constant,  $E \{K_t^R | X_0 = 1\}$  is invariant to  $\sigma$ . The sign of  $d\kappa(T)/d\sigma$  therefore signs the effect of a mean-preserving change in uncertainty on the expected long run capital level under

<sup>1</sup>Labor is a flexible input that is optimized out. The purchase price of capital is normalized to 1. The parameter  $h$  depends on  $\eta$ ,  $\gamma$  and the wage rate. See Abel and Eberly (1999) for further details.

<sup>2</sup>Allowing for positive depreciation would be straightforward, but we don't pursue this in order to keep our model comparable to that of AE.

<sup>3</sup>The condition  $\mu - 0.5\sigma^2 > 0$  needs to hold for the marginal revenue product of capital to have a nondegenerate ergodic distribution. All our results reported below are based on models satisfying this condition. See Abel and Eberly (1999) for further discussion.

<sup>4</sup>It is straightforward to show that  $E_0 \{K_t^R | X_0 = 1\} = [h(1+r)/r]^{(1/\gamma)} e^{\mu t}$ .

irreversibility or adjustment costs.

AE focus on the case where investment is completely irreversible. In the model above this is equivalent to specifying the adjustment cost as  $G = 1_{[I_t < 0]} |I_t|$ , where  $1_{[I_t < 0]}$  is a dummy variable equal to one if investment is negative and zero otherwise. Negative investment thus always generates zero resale revenues, hence the firm will never reduce its capital stock. Using the analytical formulae derived by AE, we show in the upper panel of Figure 1 (which replicates Figure 1 in AE) how  $\kappa(T)$  varies with  $\sigma$  for  $\mu = 0.029$ ,  $\eta = 10$ ,  $\beta = 0.33$ , and  $r = 0.05$ , which we refer to as our benchmark calibration. The effect of uncertainty on expected capital is small and the sign is ambiguous.<sup>5</sup>

## 2.1 Quadratic Adjustment Costs

We specify the quadratic adjustment cost function as

$$G(I_t, K_t) = 0.5b_q (I_t/K_t - c)^2 K_t,$$

where  $b_q$  measures the slope of marginal adjustment costs and  $c$  denotes the rate of investment associated with zero adjustment costs. With this adjustment cost function we cannot obtain an analytical solution for optimal investment, or an analytical expression for  $\kappa(T)$ , based on (1). We therefore use numerical dynamic programming to solve (1), and construct a simulated counterpart to Figure 1 in AE based on simulated data on long run capital stocks for hypothetical firms.<sup>6</sup> The lower panel in Figure 1 plots simulated  $\kappa(T)$  at different values of the uncertainty parameter  $\sigma$  for  $b_q = 0.5$  and  $b_q = 3.0$  and benchmark values of the

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<sup>5</sup>Larger effects can be obtained by altering the parameter values (see footnote 18 in AE). We return to this issue below.

<sup>6</sup>For the simulations we use large enough samples so as to result in very low standard errors. For details on the procedure, and additional results, see the Online Appendix available at [www.soderbom.net/appendix\\_bsw\\_el11.pdf](http://www.soderbom.net/appendix_bsw_el11.pdf). This appendix also contains results showing that we can replicate Figure 1 very closely using our numerical methods. This suggests there are no major errors in our code.

other parameters in the model.<sup>7</sup> For reference, we also show in this graph  $\kappa(T)$  under irreversibility. With quadratic adjustment costs, the expected capital stock falls monotonically as we consider higher levels of demand uncertainty. With the current calibration, uncertainty matters much more with quadratic costs than under irreversibility. For example, evaluated at  $\sigma = 0.15$ , in the model with low quadratic costs expected capital is 7% lower than in the absence of uncertainty; for high quadratic costs the difference is 20%. These results stand in sharp contrast to those generated by the real options model, where capital is 1% *higher* at  $\sigma = 0.15$  than under certainty.

The relationship between  $\kappa(T)$  and  $\sigma$  is not independent of the other parameters in the model. Indeed, larger effects can be obtained for the real options model, and the result obtained above that higher levels of uncertainty has larger negative effects on expected long run capital with quadratic adjustment costs than under irreversibility is certainly not general.<sup>8</sup> Given how influential the real options framework has been in shaping current ideas as to how uncertainty impacts capital accumulation, it is of interest to compare the magnitude of the uncertainty effects in this framework to those obtained with quadratic costs across a reasonably wide range of parameter values. We therefore vary  $\sigma$  between 0.06 and 0.24,  $\mu$  between 0.02 and 0.047,  $\eta$  between 5 and 20, and  $r$  between 0.03 and 0.07 and, for each set of parameter values, compute  $\kappa(T)$  for the investment models introduced above.<sup>9</sup> Spatial constraints prevent us from reporting all the results, these

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<sup>7</sup>Estimates of the quadratic adjustment cost parameter  $b_q$  vary substantially in the literature. Investment-Q regressions typically imply very high values of  $b_q$  (usually in excess of 10; see e.g. Hayashi and Inoue, 1991), while estimates based on simulated moments are much lower (between 0.4 and 3.9 according to Eberly, Rebelo and Vincent, 2008; close to zero according to Cooper and Haltiwanger, 2006, and Bloom, 2009). Our selected values of  $b_q$  thus fall somewhere in between. The parameter  $c$  is set to  $\exp(\mu) - 1$ , which is equal to the optimal investment rate under certainty. Hence  $\kappa(\infty) = 1$  whenever  $\sigma = 0$ .

<sup>8</sup>For example, there must exist an arbitrarily low value of  $b_q$  for which the uncertainty effects are smaller with quadratic costs than under irreversibility.

<sup>9</sup>The parameter  $\eta$  affects the value function (1) through  $\gamma$  and  $h$ .

can be found in the Online Appendix. Table 1 summarizes them, showing means, minima and maxima of  $\kappa(T)$  as well as summary statistics of the differences in  $\kappa(T)$  across the different models. For the real options model the average of  $\kappa(T)$  is 1.0, indicating that on average, across the different parameter values considered, the expected long run capital stock is not different from what it would be under reversible investment. The lowest value of  $\kappa(T)$  is 0.80, which is obtained for  $\sigma = 0.24$ ,  $\mu = 0.029$ ,  $\eta = 20$ ,  $r = 0.03$ . Hence, in this particular case, eliminating uncertainty altogether would increase the expected long run capital stock by 25%. For the quadratic costs models the lowest values of  $\kappa(T)$  are obtained at the same combination of parameter values as for the real options model. In this case we obtain  $\kappa(T) = 0.66$  for  $b_q = 0.5$  (the associated standard error is 0.005) and  $\kappa(T) = 0.24$  (standard error 0.0005) for  $b_q = 3.0$ . These imply much larger uncertainty effects than for the real options model. Subtracting  $\kappa(T)$  obtained under real options from  $\kappa(T)$  with quadratic costs, we find that this is always negative.<sup>10</sup> Hence, for all the parameter values that we consider, uncertainty results in lower levels of expected capital under quadratic costs than with irreversibility.

Some intuition as to why higher uncertainty leads to lower expected long run capital stock in the presence of quadratic adjustment costs can be gained by rewriting the value of the firm so as to more explicitly highlight the user cost of capital:

$$V_t = K_t + \max_{\{K_{t+1}\}} \sum_{s=0}^{\infty} \left( \frac{1}{1+r} \right)^s E_t \left\{ \frac{h}{1-\gamma} X_{t+s}^\gamma K_{t+s+1}^{1-\gamma} - u_{t+s+1} \cdot K_{t+s+1} \right\} - 0.5b_q ((K_{t+1} - K_t) / K_t - c)^2 K_t,$$

where  $u_{t+s+1} = (1+r)^{-1} [r + 0.5b_q (I_{t+s+1}/K_{t+s+1} - c)^2]$  is interpretable as the implicit rental cost of capital. All else equal, higher uncertainty will increase the variability of future investments, thereby raising expected values of  $u_{t+s+1}$ .

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<sup>10</sup>This can be inferred from the table by noting that the maximum of the difference is negative, both for  $b_q = 0.5$  and for  $b_q = 3.0$ .

Thus, for this model the negative association between uncertainty and capital accumulation documented above can be interpreted as arising due to the positive effect of uncertainty on the user cost of capital.

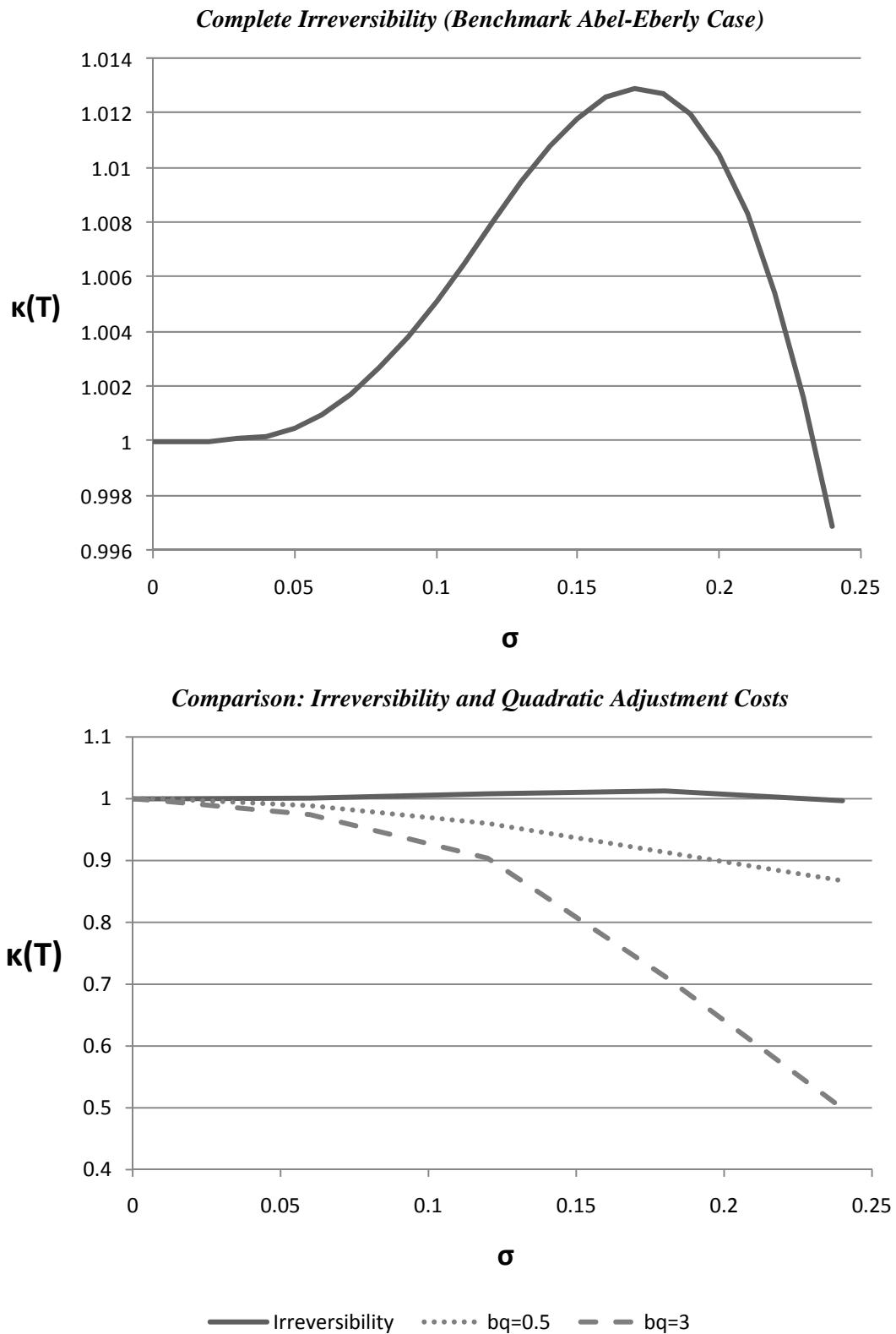
### 3 Conclusions

A widely held view amongst economists and practitioners is that uncertainty has nontrivial negative effects on capital accumulation. In tune with this, the real options model predicts that the response of investment to demand shocks is weakened by higher uncertainty. However, as established by Abel and Eberly (1999), the model does not accord with the intuition that uncertainty has a negative effect on long run capital accumulation. In this paper we have investigated the effects of uncertainty on long run capital accumulation using a quadratic adjustment costs framework. Across a wide range of parameter values we have found these effects to be consistently negative, and often much larger than in the real options model, even if the marginal adjustment cost parameter  $b_q$  is low. Quadratic adjustment costs were the bedrock of investment models in the 1980s, and still feature as an important component of investment models in recent papers (e.g. Cooper and Haltiwanger, 2006; Bloom, 2009; Eberly, Rebelo and Vincent, 2008). The evidence is clear that quadratic costs cannot account for certain patterns in micro datasets, e.g. lumpy and intermittent investment. Our analysis indicates, however, that quadratic costs may deliver predictions that square better with the notion that uncertainty hampers long run capital accumulation than the real options framework.

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**Figure 1: The Long Run Effect of Uncertainty on Capital Accumulation**




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Note: The results for the model with irreversibility are based on the analytical expressions derived in Abel and Eberly (1999). The results for the models with quadratic adjustment costs are obtained by means of numerical methods and simulations.

**Table 1. Evaluation of  $\kappa(T)$  at different levels of uncertainty, market power, demand growth and discount rate. Summary statistics.**

	Real options model	Quadratic adjustment costs		Differences compared to real options model	
		$b_q = 0.5$	$b_q = 3.0$	$b_q = 0.5$	$b_q = 3.0$
Mean	0.999	0.927	0.774	-0.072	-0.225
Minimum	0.797	0.663	0.242	-0.285	-0.689
Maximum	1.133	0.996	0.986	-0.003	-0.017
Standard Dev	0.053	0.070	0.203	0.065	0.190
Observations	81	81	81	81	81

Note:  $\sigma$  varies between 0.06 and 0.24;  $\eta$  between 5 and 20;  $\mu$  between 0.02 and 0.047;  $r$  between 0.03 and 0.07. Not all permutations of parameter values are considered, see Online Appendix for details. Cases for which  $\mu - 0.5\sigma^2 \leq 0$  (see footnote 3), or  $r \leq \mu$  (non-converging value function), are excluded from the analysis.