

$$M(f, \{x_i\}) = \max_{i=1, \dots, n-1} \left| a_i f(x_{i+1}) + b_i f(x_i) + c_i f(x_{i-1}) \right|$$

$$\boxed{D} \leq \|f''\|_\infty \leq C(h) \max_i \{f(t_i)\} + \{t_i\}$$

$$h = \max_i t_{i+1} - t_i$$

$$\leq C(\) M(f, \{x_i\})$$

Approx.
 $f = ?$
 $\min_{a \leq x \leq b} f(x) = ?$
 $\|f''\|_\infty \leq \frac{n^2}{b-a} \left\| f' - \frac{f(b) - f(a)}{b-a} \right\|_\infty \rightarrow$
 Optimization
 global synthesis

- global adaption J. Comp. 14
- local adaption Yizhi thesis

$O\left(\frac{n^2 \ f''\ _\infty}{b-a}\right)$	✓ Works	global
✓ Comput. cost	✓ cost	✓ works
✓ Lower bd. complexity	✗ lower bd. complexity	✗ complexity

Integration
 $\int_a^b f(x) dx = ?$
 $\text{Var}(f') \leq \frac{n^2}{b-a} \left\| f' - \frac{f(b) - f(a)}{b-a} \right\|_\infty$
 global ✓
 J. Comp. 14 ✓
 $\text{Var}(f) \leq C(h) \text{Var}(f', \{t_i\})$
 $O\left(\sqrt{\frac{\text{Var}(f')}{b-a}} + n^2\right) + \{t_i\}$
 $h = \max_i t_{i+1} - t_i$
 global ✓
 Rejected ✓
 $\text{Var}(f'') \leq \text{Var}(f', \{t_i\})$
 Yizhi thesis
 Simpson's