14.1 - Functions of Several Variables

Domain and Range

Define a set of n-tuples in $\mathbb{R}^n$: $D = (x_1, x_2, x_3, \ldots x_n)$
A real-valued function f on D has w such that $w = f(x_1, x_2, x_3, \ldots x_n)$
In this case, D is the domain, and w is the range

Inputs that lead to a divide by zero of imaginary numbers are excluded from the domain

(a) These are functions of two variables. Note the restrictions that apply to their domains in order to obtain a real value for the dependent variable z.

Function	Domain	Range
$z = \sqrt{y - x^2}$	$y \geq x^2$	$[0, \infty)$
$z = \dfrac{1}{xy}$	$xy \neq 0$	$(-\infty, 0) \cup (0, \infty)$
$z = \sin xy$	Entire plane	$[-1, 1]$

(b) These are functions of three variables with restrictions on some of their domains.

Function	Domain	Range
$w = \sqrt{x^2 + y^2 + z^2}$	Entire space	$[0, \infty)$
$w = \dfrac{1}{x^2 + y^2 + z^2}$	$(x, y, z) \neq (0, 0, 0)$	$(0, \infty)$
$w = xy \ln z$	Half-space $z > 0$	$(-\infty, \infty)$

Boundary point : on the boundary of a set
Interior point : fully contained in the set
A region is **bounded** if it lies inside a disk of finite radius. A region is **unbounded** if it is not bounded.

Graphs and Level Curves

The set of points where a function $f(x, y)$ has a constant value c is a **level** curve of f
The set of all points $(x, y, f(x, y))$ in space for the domain of f is the graph of f
Graph is also called the surface, $z = f(x, y)$

For functions of 2 variables: can't graph them effectively, but can see how they behave by looking at their level surfaces