

13.3 - Arc Length in Space

Arc Length Along a Space Curve

Length of a curve $r(t) = x(t)i + y(t)j + z(t)k$, $a \leq t \leq b$, that is traced exactly once as t increases from $t = a$ to $t = b$ is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

$$L = \int_a^b |v| dt$$

Speed on a Smooth Curve

$$\frac{ds}{dt} = |v(t)|$$

Unit Tangent Vector

$$T = \frac{v}{|v|}$$