13.4 - Curvature and Normal Vectors of a Curve

As a particle moves along a smooth curve, $T = \frac{dr}{dt}$ turns as the curve bends
The rate at which T turns is the curvature, where s is arc length, which is given by:

$$\kappa = \left|\frac{dT}{ds}\right|$$
$$\kappa = \frac{1}{|v|}\left|\frac{dT}{dt}\right|$$

A straight line has a curvature of 0.

Principal Unit Normal

The unit vector N orthogonal to T is the principal unit normal. Because $|T| = 1$, it's derivative is orthogonal to T. Dividing $\frac{dT}{ds}$ by its length κ gives N

$$N = \frac{1}{\kappa}\frac{dT}{ds}$$
$$N = \frac{dT/ds}{|dT/ds|}$$

Circle of Curvature

The circle of curvature at point P on a plane curve where $\kappa \neq 0$ is the circle in the plane of the curve that:

1. is tangent to the curve at P
2. has the same curvature that the curve has at P
3. has center that lies on the concave side of the curve
   The radius of curvature of the curve is the radius of the circle of curvature

   $$\text{Radius of curvature } = \rho = \frac{1}{\kappa}$$

   The center of curvature is at the center of the circle of curvature