12.5 - Lines & Planes in Space

Line Segment = determined by point and slope
Slope = scalar for 2d, vector for 3d+

Vector Eq. for a line

$$r(t) = r_0 + tv$$

where r is the position vector of a point $P(x, y, z)$ on L and r_0 is the position vector of $P_0(x_0, y_0, z_0)$

Detailed Vector Eq. for a Line

$$r(t) = r_0 + t|v|\frac{v}{|v|}$$

where $|v|$ is the "direction" of the line, and $\frac{v}{|v|}$ is the "speed" of the line

Distance from a point to a Line in Space

Where P is a point on the line, S is the point being tested, and v is a vector parallel of the Plane
To get $\vec{PS}$, subtract P from S

$$d = \frac{|\vec{PS} \times v|}{|v|}$$

Eq. for a Plane in Space

The plane through $P_0(x_0, y_0, z_0)$ normal to a nonzero vector $n = Ai + Bj + Ck$ has:

$$\begin{aligned}
&n \cdot \vec{P_0P} = 0 \\
&A(x - x_0) + B(y - y_0) + c(z - z_0) = 0 \\
&Ax + By + Cz = D
\end{aligned}$$

Distance from a Point to a Plane

Where P is a point on the plane, S is the point being tested, and n is the normal vector of the Plane

$$d = \left|\vec{PS} \cdot \frac{n}{|n|}\right|$$

Angle between two Planes

$$\theta = \cos^{-1}\left(\frac{n_1 \cdot n_2}{|n_1||n_2|}\right)$$

Properties of Lines

If L_1 and L_2 are parallel, $L_1 \times L_2 = 0$
If L_1 and L_2 are skew, L_1 and L_2 lie in parallel planes, so their planes have the same normal vector