Solutions for Chapter 1

Exercise 1.1

You have a 5k resistor and a 10k resistor. What is their combined resistance (a) in series and (b) in parallel?

(a) $R = 5\text{k} + 10\text{k} = \boxed{\mathbf{15k\Omega}}$

(b) $R = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5\text{k} \cdot 10\text{k}}{5\text{k} + 10\text{k}} = \boxed{\mathbf{3.33k\Omega}}$

Exercise 1.2

If you place a 1 ohm resistor across a 12 volt car battery, how much power will it dissipate?

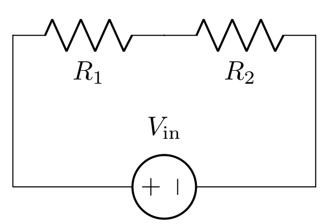
$$P = IV = \left(\frac{V}{R}\right) V = \frac{(12\text{V})^2}{1\Omega} = \boxed{\mathbf{144}\text{W}}$$

Exercise 1.3

Prove the formulas for (a) series and (b) parallel resistors.

(a) Series resistors:

Figure 1: A basic series circuit.



$V_{in} = V_1 + V_2$ (Kirchoff's voltage law)

$$R = \frac{V_{in}}{I} = \frac{(V_1 + V_2)}{I} = \frac{V_1}{I} + \frac{V_2}{I}$$

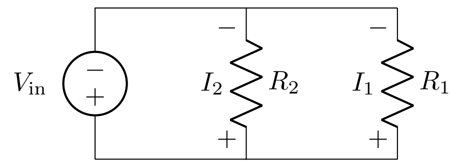
$$R_1 = \frac{V_1}{I}$$

$$R_2 = \frac{V_2}{I}$$

$$R = R_1 + R_2 \checkmark$$

(b) Parallel resistors:

Figure 2: A basic parallel circuit.



$I = I_1 + I_2$ (Kirchoff's current law)

$V_{in} = V_1 = V_2$ (Kirchoff's voltage law)

$$R = \frac{V_{in}}{I} = \frac{V_{in}}{I_1 + I_2} = \frac{1}{\frac{I_1 + I_2}{V_{in}}} = \frac{1}{\frac{I_1}{V_{in}} + \frac{I_2}{V_{in}}}$$

$$\frac{1}{R_1} = \frac{I_1}{V_{in}}$$

$$\frac{1}{R_2} = \frac{I_2}{V_{in}}$$

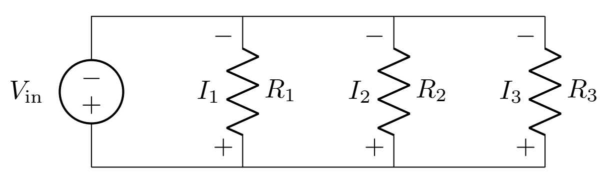
$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \checkmark$$

Exercise 1.4

Show that several resistors in parallel have resistance: $R = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...}$

Parallel resistors:

Figure 3: A multiple-resistor parallel circuit.



$I = I_1 + I_2 + I_3$ (Kirchoff's current law)

$V_{in} = V_1 = V_2 = V_3$ (Kirchoff's voltage law)

$$R = \frac{V_{in}}{I} = \frac{V_{in}}{I_1 + I_2 + I_3} = \frac{1}{\frac{I_1 + I_2 + I_3}{V_{in}}} = \frac{1}{\frac{I_1}{V_{in}} + \frac{I_2}{V_{in}} + \frac{I_3}{V_{in}}}$$

$$\frac{1}{R_1} = \frac{I_1}{V_{in}}$$
$$\frac{1}{R_2} = \frac{I_2}{V_{in}}$$
$$\frac{1}{R_3} = \frac{I_3}{V_{in}}$$
$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \checkmark$$

We can use the same formula for any number of resistors in parallel.

Exercise 1.5

Show that it is not possible to exceed the power rating of a 1/4 watt resistor of resistance greater than 1k, no matter how you connect it, in a circuit operating from a 15 volt battery.

We know that the maximum voltage we can achieve is 15V and the smallest resistance we can have across the resistor in question is 1kΩ. Therefore, the maximum amount of power dissipated can be given by

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R} = \frac{(15\text{V})^2}{1\text{k}\Omega} = \boxed{\mathbf{0.225}\text{W}}$$

This is less than the 1/4W power rating.

Exercise 1.6

New York City requires about 10^{10} watts of electrical power, at 115 volts (this is plausible: 10 million people averaging 1 kilowatt each). A heavy power cable might be an inch in diameter. Let's calculate what will happen if we try to supply the power through a cable 1 foot in length made of pure copper. Its resistance is $0.05\mu\Omega(5 \times 10^{-8}\Omega)$ per foot. Calculate (a) the power lost per foot from "I^2R losses", (b) the length of cable over which you will lose all 10^{10} watts, and (c) how hot the cable will get, if you know the physics involved ($\sigma = 6 \times 10^{-12}\frac{W}{K^4cm^2}$). If you have done your computations correcty, the results should seem preposterous. What is the solution to this puzzle?

(a) We can use the voltage and the necessary power to calculate the current that should be flowing.

$$P = IV$$

$$I = \frac{P}{V} = \frac{10^{10}W}{115V} = 86956521A$$

From that, we can calculate the power lost per foor by using the resistance per foot.

$$P = I^2R$$

$$P = (86956521A)^2(5 \times 10^{-8}\Omega) = 378071833.6W = 3.780718336 \times 10^8W$$

(b) If we know how much power is dissipated per foot, it is easy enough to calculate how many feet it would take to dissipate all the power.

$$\frac{10^{10}W}{3.780718336 \times 10^{8}W} = 26.45ft$$

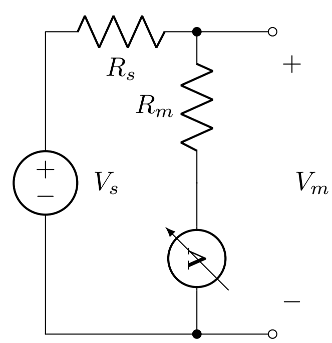
(c) **TODO: solve**

Exercise 1.7

What will a 20000 Ω/V meter read on its 1V scale, when attached to a 1V source with an internal resistance of 10k? What will it read when attached to a 10k-10k voltage divider driven by a "stiff" (zero source resistance) 1V source?

(a) We can use our knowledge of voltage dividers to solve this equation with s standing for source and m for multimeter.

Figure 4: Measuring diagram



$V_s = 1V$

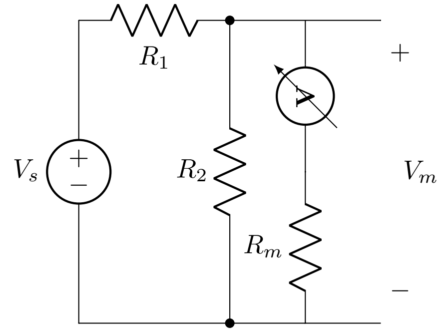
$R_s = 10000\Omega$

$R_m = 20000\Omega$

$$V_m = V_s\frac{R_m}{R_s + R_m} = \frac{2}{3} = 0.66$$

(b) Next up we use a 10k-10k voltage divider with resistances R_1 and R_2. We need to first find the common resistance of multimeter and one of the resistors, then use that common resistance to calculate read voltage via the voltage divider equation.

Figure 5: Measuring diagram for a 10k-10k voltage divider



$V_s = 1V$

$R_s = 0\Omega$

$R_1 = R_2 = 10000\Omega$

$R_m = 20000\Omega$

$$R_{com} = \frac{R_1 R_m}{R_1 + R_m} = 6666.6\Omega$$

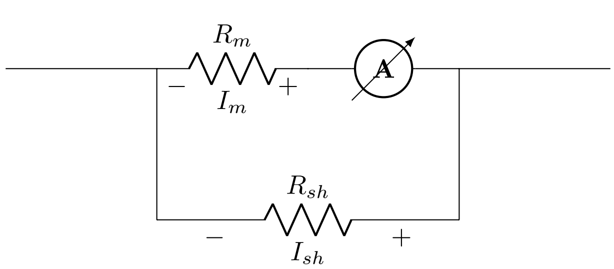
$$V_m = V_s \frac{R_{com}}{R_1 + R_{com}} = 0.4V$$

Exercise 1.8

A 50μA meter movement has an internal resistance of 5k. What shunt resistance is needed to convert it to a 0-1A meter? What series resistance will convert it to a 0-10V meter?

(a) We put a shunt resistor in parallel to the measuring device and all we need is Kirchoff's voltage law to solve this one.

Figure 6: Current measuring with shunt



$I_m = 50\mu A$

$I_{max} = I_m + I_{sh} = 1A$

$I_{sh} = I_{max} - I_m \approx I_{max}$

$R_m = 5k\Omega$

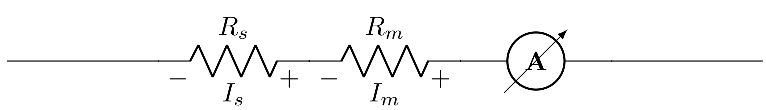
$V_m = V_{sh}$

$I_m R_m = I_{sh} R_{sh}$

$R_{sh} = \dfrac{I_m R_m}{I_{sh}} = 0.25\Omega$

(b) Another resistor is put in series.

Figure 7: Changing voltage range



$V_{max} = I_m(R_s + R_m)$

$R_s = \dfrac{V_{max}}{I_m} - R_m = 195k\Omega$

Exercise 1.10

For the circuit shown in Figure 1.12, with $V_{in} = 30V$ and $R_1 = R_2 = 10k\Omega$, find (a) the output voltage with no load attached (the open-circuit voltage); (b) the output voltage with a 10k load (treat as a voltage divider, with $R_2 and R_{load}$ combined into a single resistor); (c) the Thevenin equivalent circuit; (d) the same as in part (b), but using the Thevenin equivalent circuit (again, you wind up with a voltage divider; the answer should agree with the result in part (b)); (e) the power dissipated in each of the resistors.

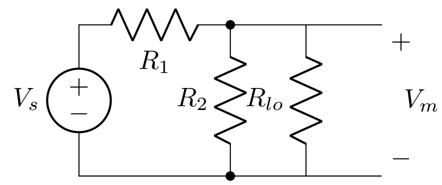
(a) With two equal-value resistors, the output voltage is half the input voltage.

$$V_{out} = \frac{1}{2}V_{in} = \frac{30\text{V}}{2} = \boxed{\mathbf{15}\text{V}}$$

(b) To treat R_2 and R_{load} as a single resistor, combine the two resistors which are in parallel to find that the combined (equivalent) resistance is 5kΩ. Now, we have a simple voltage divider with a 10kΩ resistor in series with the 5kΩ equivalent resistor. The output voltage is across this equivalent resistance. The output voltage is given by

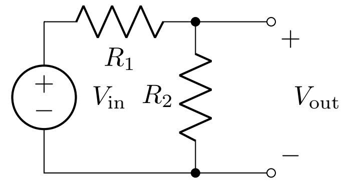
$$V_{out} = V_{in}\frac{5\text{k}\Omega}{10\text{k}\Omega + 5\text{k}\Omega} = \frac{30\text{V}}{3} = \boxed{\mathbf{10}\text{V}}$$

Figure 8: Measuring diagram for a 10k-10k voltage divider with load



(c) We can redraw the voltage divider circuit to make the "port" clearer.

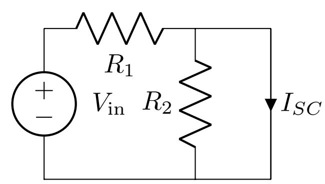
Figure 9: Voltage divider with port shown.



We can find V_{Th} by leaving the ports open (open circuit) and measuring V_{out}, the voltage across R_2. This comes out to be half the input voltage when $R_1 = R_2$, so $V_{\text{out}} = 15\text{V}$. Thus $V_{\text{Th}} = \boxed{\mathbf{15}\text{V}}$.

To find the Thévinen resistance, we need to find the short circuit current, I_{SC}. We short circuit the port and measure the current flowing through it.

Figure 10: Voltage divider with short circuit on the output.



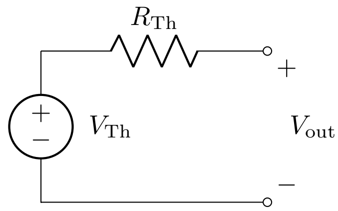
In this circuit, no current flows through R_2, flowing through the short instead. Thus we have $I_{SC} = \dfrac{V_{\text{in}}}{R_1}$.

From this, we can find R_{Th} from $R_{\text{Th}} = \dfrac{V_{\text{Th}}}{I_{SC}}$. This gives us

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{I_{SC}} = \frac{V_{\text{Th}}}{V_{\text{in}}/R_1} = \frac{15\text{V}}{30\text{V}/10\text{k}\Omega} = \boxed{\mathbf{5k\Omega}}$$

The Thévenin equivalent circuit takes the form shown below.

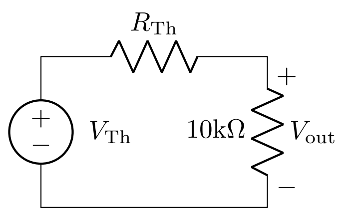
Figure 11: Thévenin equivalent circuit.



In terms of behavior at the ports, this circuit is equivalent to the circuit in Figure 9.

(d) We connect the 10kΩ load to the port of the Thévenin equivalent circuit in Figure 11 to get the following circuit.

Figure 12: Thévenin equivalent circuit with 10kΩ load.



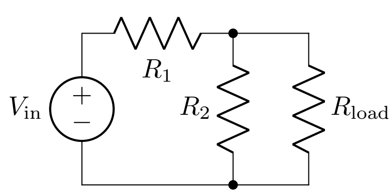
From here, we can find V_{out}, treating this circuit as a voltage divider.

$$V_{\text{out}} = \frac{10\text{k}\Omega}{R_{\text{Th}} + 10\text{k}\Omega} V_{\text{Th}} = \frac{10\text{k}\Omega}{5\text{k}\Omega + 10\text{k}\Omega} \cdot 15\text{V} = \boxed{\mathbf{10}\text{V}}$$

This is the same answer we got in part (b).

(e) To find the power dissipated in each resistor, we return to the original three-resistor circuit.

Figure 13: Original voltage divider with 10kΩ load attached.



From part (d), we know that the output voltage is 10V and that this is the voltage across the load resistor. Since $P = IV = \frac{V^2}{R}$, we find that the power through R_{load} is

$$P_{\text{load}} = \frac{V^2}{R_{\text{load}}} = \frac{(10\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{10}\text{mW}}$$

Similarly, we know that the power across R_2 is the same since the voltage across R_2 is the same as the voltage across R_{load}. Thus we have

$$P_2 = \boxed{\mathbf{10}\text{mW}}$$

To find the power dissipated in R_1, we first have to find the voltage across it. From Kirchoff's loop rule, we know that the voltage around any closed loop in the circuit must be zero. We can choose the loop going through the voltage source, R_1, and R_2. The voltage supplied by the source is 30V. The voltage dropped across R_2 is 10V as discussed before. Thus the voltage dropped across R_1 must be $30\text{V} - 10\text{V} - 20\text{V}$. Now we know the voltage across and the resistance of R_1. We use the same formula as before to find the power dissipated.

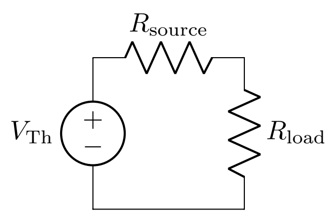
$$P_1 = \frac{V^2}{R_1} = \frac{(20\text{V})^2}{10\text{k}\Omega} = \boxed{\mathbf{40}\text{mW}}$$

Exercise 1.11

Show that $R_{load} = R_{source}$ maximizes the power in the load for a given source resistance.

Consider the following Thévenin circuit where R_{source} is just another name for the Thévenin resistance, R_{Th}.

Figure 14: Standard Thévenin circuit with attached load.



We will first calculate the power dissipated in the load and then maximize it with calculus. We can find the power through a resistor using current and resistence since $P = IV = I(IR) = I^2R$. To find the total current flowing through the resistors, we find the equivalent resistance which is $R_{\text{source}} + R_{\text{load}}$. Thus the total current flowing is $I = \frac{V_{\text{Th}}}{R_{\text{source}}+R_{\text{load}}}$. The power dissipated in R_{load} is thus

$$P_{\text{load}} = I^2 R_{\text{load}} = \frac{V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{source}} + R_{\text{load}})^2}$$

To maximize this function, we take the derivative and set it equal to 0.

$$\begin{aligned}
\frac{dP_{\text{load}}}{dR_{\text{load}}} &= V_{\text{Th}} \frac{(R_{\text{source}} + R_{\text{load}})^2 - 2R_{\text{load}}(R_{\text{source}} + R_{\text{load}})}{(R_{\text{source}} + R_{\text{load}})^4} = 0 \\
&\implies R_{\text{source}} + R_{\text{load}} = 2R_{\text{load}} \\
&\implies R_{\text{source}} = R_{\text{load}}
\end{aligned}$$

Exercise 1.12

Determine the voltage and power ratios for a pair of signals with the following decibel ratios: (a) 3 dB, (b) 6 dB, (c) 10 dB, (d) 20dB.

(a) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{3/20} = \boxed{\mathbf{1.413}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{3/10} = \boxed{\mathbf{1.995}}$

(b) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{6/20} = \boxed{\mathbf{1.995}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{6/10} = \boxed{\mathbf{3.981}}$

(c) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{10/20} = \boxed{\mathbf{3.162}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{10/10} = \boxed{\mathbf{10}}$

(d) Voltage ratio: $\frac{V_2}{V_1} = 10^{db/20} = 10^{20/20} = \boxed{\mathbf{10}}$

Power ratio: $\frac{P_2}{P_1} = 10^{db/10} = 10^{20/10} = \boxed{\mathbf{100}}$

Exercise 1.13

We might call this amusing exercise "Desert Island dBs": in the table below we've started entering some values for power ratios corresponding to the first dozen integral dBs, using the results for parts (a) and (c) of the last exercise. Your job is to complete the table, without recourse to a calculator. A possibly helpful hint: starting at 10 dB, go down the table in steps of 3 dB, then up in a step of 10 dB, then down again. Finally, get rid of yucky numbers like 3.125 (and its near relatives) by noticing that it's charmingly close to Π *.*

There are two important facts to notice from Exericse 1.12:

1. An increase of 3dB corresponds to doubling the power
2. An increase of 10dB corresponds to 10 times the power.

Using these two facts, we can fill in the table. Start from 10dB. Fill in 7dB, 4dB, and 1dB using fact 1. Then fill in 11dB using fact 2. Then fill in 8dB, 5dB, and 2dB using fact 1 and approximating 3.125 as π.

dB	ratio(P/P_0)
0	1
1	$\boxed{\mathbf{1.25}}$
2	$\boxed{\pi/\mathbf{2}}$
3	2
4	$\boxed{\mathbf{2.5}}$
5	$\boxed{\mathbf{3.125} \approx \pi}$
6	4
7	$\boxed{\mathbf{5}}$
8	$\boxed{\mathbf{6.25}}$
9	8
10	10
11	$\boxed{\mathbf{12.5}}$

Exercise 1.14

Take the energy challenge: imagine charging up a capacitor of capacitance C, from 0V to some final voltage V_f*. If you do it right, the result won't depend on how you get there, so you don't need to assume constant current charging (though you are welcome to do so). At any instant the rate of flow of energy into the* *capacitor is VI (joules/s); so you need to integrate dU = VIdt from start to finish. Take it from there.*

Main thing to note is that energy is an integral of power over a period of time.

Recall the relationship between I, V, and C: $I = C\frac{dV}{dt}$. Now, we perform the integration:

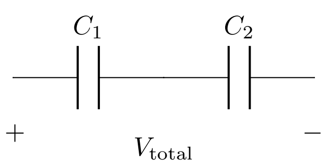
$$\begin{aligned}
\int dU &= \int_{t_0}^{t_1} VI dt \\
U &= \int_{t_0}^{t_1} CV\frac{dV}{dt}dt \\
&= C\int_0^{V_f} VdV \\
U &= \frac{1}{2}CV_f^2
\end{aligned}$$

Exercise 1.15

Derive the formula for the capcitance of two capacitors in series. Hint: because there is no external connection to the point where the two capacitors are connected together, they must have equal stored charges.

Consider the following two capacitors in series.

Figure 15: Two capacitors in series.



To prove the capacitance formula, we need to express the total capacitance of both of these capacitors in terms of the individual capacitances. From the definition of capacitance, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}}$$

Notice that V_{total} is the sum of the voltages across C_1 and C_2. We can get each of these voltages using the definition of capacitance.

$$V_{\text{total}} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

The key observation now is that because the right plate of C_1 is connected to the left plate of C_2, the charge stored on both plates must be of equal magnitude.[1]Therefore, we have $Q_1 = Q_2$. Let us call this charge stored Q (i.e. $Q = Q_1 = Q_2$). Now, we know that the total charge stored is also Q.[2]Therefore, we know that $Q_{\text{total}} = Q$. Now, we have

$$C_{\text{total}} = \frac{Q_{\text{total}}}{V_{\text{total}}} = \frac{Q}{Q_1/C_1 + Q_2/C_2} = \frac{Q}{Q/C_1 + Q/C_2} = \frac{1}{1/C_1 + 1/C_2}$$

[2]If this were not true, then there would be a net charge on these two plates and the wire between them. Because we assume that the capacitors started out with no net charge and there is no way for charge to leave the middle wire or the two plates it connects, this is impossible.

[2]If you are having trouble seeing this, suppose we apply a positive voltage to the left plate of C_1 relative to the right plate

Exercise 1.16

Show that the rise time (the time required for going from 10% to 90% of its final value) of this signal is 2.2RC.

Equation 1.21 gives us the relationship between the time and the voltage (V_{out}) across the capacitor while charging. To find the rise time, subtract the time it takes to reach 10% of the final value from the time it takes to reach 90% of the final value.

$$\begin{aligned} V_{\text{out}} &= 0.1V_f = V_f(1 - e^{-t_1/RC}) \\ 0.1 &= 1 - e^{-t_1/RC} \\ t_1 &= -RC\ln(0.9) \end{aligned}$$

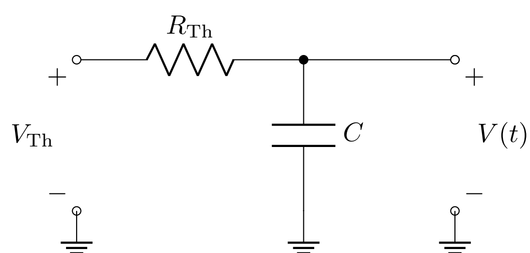
Similarly, we find that $t_2 = -RC\ln(0.1)$. Subtracting these two gives us

$$t_2 - t_1 = -RC(\ln(0.1) - \ln(0.9)) = 2.2RC$$

Exercise 1.17

The voltage divider on the left side of the circuit can be replaced with the Thévenin equivalent circuit found Exercise 1.10 (c). Recall that $V_{\text{Th}} = \frac{1}{2}V_{\text{in}}$ and $R_{\text{Th}} = 5\text{k}\Omega$. This gives us the following circuit.

Figure 16: Thévenin equivalent circuit to Figure 1.36 from the textbook.



Now we have a simple RC circuit which we can apply Equation 1.21 to. The voltage across the capacitor is given by

$$V(t) = V_{\text{final}}(1 - e^{-t/RC}) = V_{\text{Th}}(1 - e^{-t/R_{\text{Th}}C} = \boxed{\mathbf{\frac{1}{2}V_{\text{in}}(1 - e^{-t/5\times10^{-4}})}}$$

TODO: Add graph

Exercise 1.18

From the capacitor equation in the previous paragraph, we have

$$V(t) = (I/C)t = (1\text{mA}/1\mu\text{F})t = 10\text{V}$$

This gives us

$$\boxed{\mathbf{t = 0.01}\text{s}}$$

of C_2. Suppose this causes the left plate of C_1 to charge to some charge q. We now must have a charge of $-q$ on the right plate of C_1 because q units of charge are now pushed onto the left plate of C_2. Now the left of C_2 has q units of charge which causes a corresponding $-q$ charge on the right side of C_2. Thus the overall total charge separated across these two capacitors is q.

Exercise 1.19

The magnetic flux produced within the coil is proportional to the number of turns. Now, because the inductance of the inductor is proportional to the amount of magnetic flux that passes through all the coils, it is proportional to the product of the magnetic flux and the number of coils. Thus the inductance is propotional to the square of the number of turns. **TODO: Check/clarify this answer**

Exercise 1.20

We can use the formula for the full-wave rectifier ripple voltage to find the capacitance.

$$\frac{I_{\text{load}}}{2fC} = \Delta V \leq 0.1\text{V}_{\text{p-p}}$$

The maximum load current is 10mA and assuming a standard wall outlet frequency of 60 Hz, we have

$$C \geq \frac{10\text{mA}}{2 \times 120\text{Hz} \times 0.1\text{V}} = \boxed{\mathbf{417}\mu\text{F}}$$

Now we need to find the AC input voltage. The peak voltage after rectification must be 10V (per the requirements). Since each phase of the AC signal must pass through 2 diode drops, we have to add this to find out what our AC peak-to-peak voltage must be. Thus we have

$$V_{\text{in,p-p}} = 10\text{V} + 2(0.6\text{V}) = \boxed{\mathbf{11.2}\text{V}}$$

Exercise 1.30

V_{out} is simply the voltage at the output of an impedance voltage divider. We know that $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$. Thus we have

$$V_{\text{out}} = \frac{Z_C}{Z_R + Z_C} V_{\text{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{\text{in}} = \frac{1}{1 + j\omega RC} V_{\text{in}}$$

The magnitude of this expression can be found by multiplying by the complex conjugate and taking the square root.

$$sqrt V_{\text{out}} V_{\text{out}}^* = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} V_{\text{in}}$$