Homework Report Week #2

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April 01, 2025

1 Introduction

This week's set of problems focuses on two basic search techniques: linear search and binary search. Utilizing these techniques masterfully brings tremendous aids in more convoluted algorithm concepts.

This report, along with C++ solutions, can be found over at this Github repo.

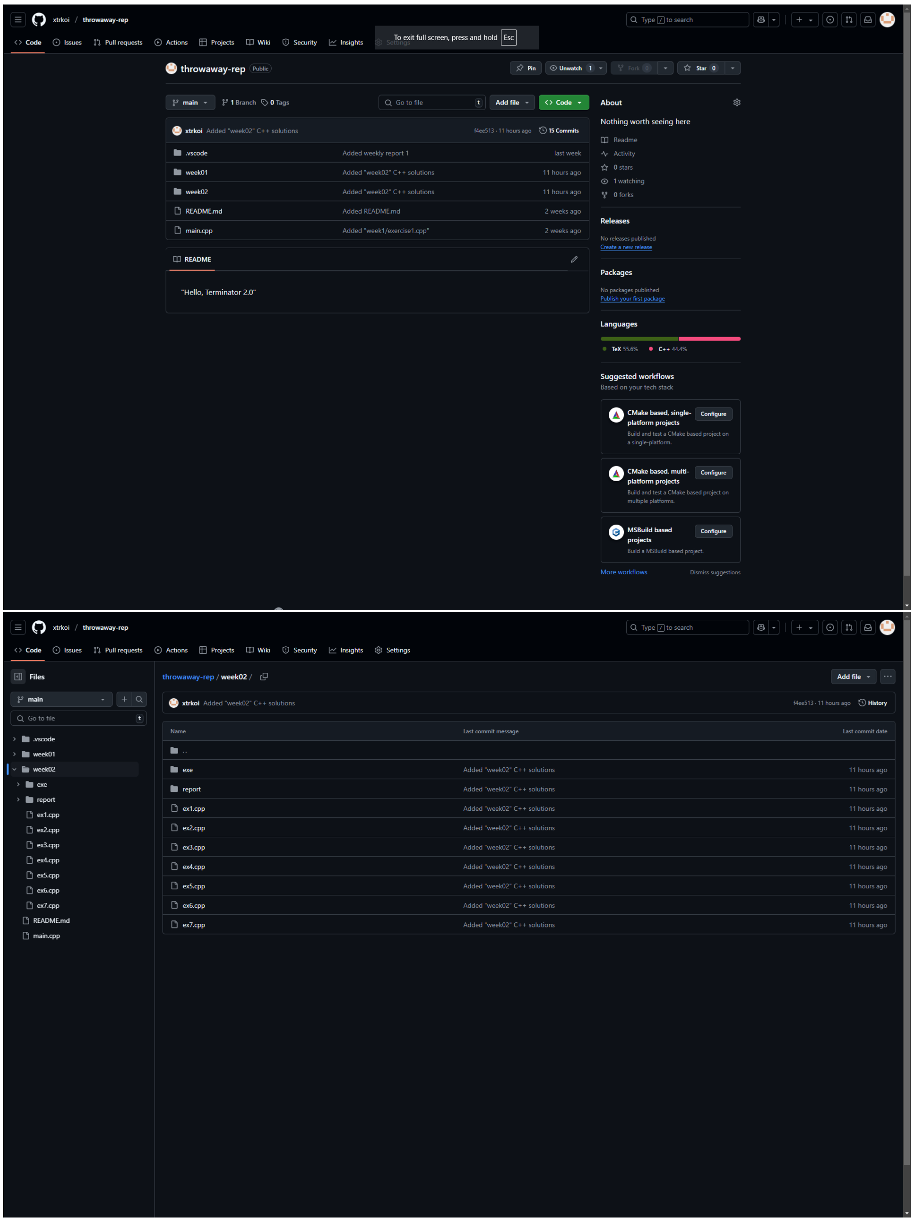

Figure 1: Online Github Repository

2 The Problems

2.1 Linear Search

Problem 2.1: Linear Search

Given an array of N integers and a target integer K, implement a function to find the first occurence of K in the array using Linear Search. If K is found, return its (0-based) index. Otherwise, return -1.

Approach. Iterate from left to right to find the first occurence, right to left to find the last. Runs in $O(n)$.

2.2 Linear Search with Sentinel

Problem 2.2: Linear Search with Sentinel

Solve Problem 2.1 using Linear Search with Sentinel.

Approach. Set the last integer as the target integer to reduce out-of-range comparison checks.

2.3 Find the minimum element in a rotated array

Problem 2.3

An array of length n, containing n unique intgers, is sorted in ascending order, rotated between 1 and n times to the right. Find the minimum integer in the array.

Approach. Define k as the last integer in the array (a_{n-1}), and a function $\phi(x)$ which accepts any integer x as input, and returns 0 if $x > k$, 1 if $x \leq k$. It can be seen that $\phi(x)$ is monotonically increasing for $x \in \{a_0, a_1, \ldots, a_{n-1}\}$. We can use binary search to find the first i so that $\phi(x) = \phi(a_i) = 1$. The answer is a_i and the program runs in $O(\log n)$ as only binary search is used.

2.4 Find the minimum capacity

Problem 2.4

There are n packages ready to be shiped in order, the *i-th* package has weight w_i. The ship has a capacity C and can carry packages with the sum of weights not exceeding C in any given day. The ship also has only D days to ship the packages. Find the minimal C that satisfy.

Approach. Define $\phi(x)$ as a function that returns 1 if all packages can be delivered in x days, 0 otherwise. It can be proved that if there exists a so $\phi(a) = 1$, then $\phi(x) = 1 \forall x \geq a$ and $\phi(x) = 0 \forall x < a$. This shows $\phi(x)$ is monotonically increasing, and by applying binary search, we can find the value a.

Since the order of the packages are given, $\phi(x)$ can be implemented by load the maximum possible number of packages in any given day, then compare the number of days needed to deliver all packages to x.

2.5 Shortest subarray with at least given sum

Problem 2.5

An array of n positive integers is given, along with a target sum T. Find the length of the shortest subarray $[a_L, a_{L+1}, \ldots, a_R]$ that has the sum $\sum_{i=L}^{R} a_i$ greater than or equal to T.

Approach. Define a partial sum array $[P_i] = [\sum_{j=0}^{i} a_i]$ for $i \in [0, n)$. Since $a_i > 0 \forall i \in [0, n)$, $[P_i]$ is monotonically increasing. The sum $\sum_{i=L}^{R} a_i$ can be calculated as

$$\begin{cases} P_R, & \text{if} \quad L = 0 \\ P_R - P_{L-1}, & \text{if} \quad L > 0 \end{cases}$$

By iterating L, this problem can be converted into finding a value $k = T - P_{L-1}$ or $k = T$ if $L = 0$ in a sorted array with binary search, which runs in $O(n \log n)$.

2.6 Find a pair with given sum

Problem 2.6

Given a sorted array of n integers and a target sum T, determine if there exists $i, j \in [0, n); i \neq j$ so $a_i + a_j = T$.

Approach. Iterate i and find $T - a_i$ with binary search.

2.7 Find all zero-sum triplets

Problem 2.7

Given an array a of n integers, find all distinct triplets $\{a_i, a_j, a_k\}, i \neq j \neq k$ so that $a_i + a_j + a_k = 0$. The triplets can be output in any order.

Approach.

3 Conclusion