1 Uncertainty-Calibrated Residual PatchTST Transformer Forecasting: Dilated Conv Stem → Transformer Encoder → Adaptive Quantile Head with Horizon-Wise Temperature Scaling

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1.1.1 Goal:

Accurate, stable, and well-calibrated weekly forecasts with interpretable uncertainty bands (P10–P90).

1.1.2 Structure:

1. **Data & Features** — yearly seasonality (sin/cos), holiday proximity, and smart lags/EMAs.
2. **Residual Training** — learn deviations from a strong seasonal baseline (lag-52), then add it back:
$$r_t \;=\; y_t - y_{t-52}, \qquad \hat{y}_t \;=\; \hat{r}_t + y_{t-52}.$$
3. **Patch-style Transformer** — tokenized time patches → Transformer encoder (+ optional dilated TCN stem).
4. **Quantile Forecasts** — P10/P50/P90 via **weighted pinball loss + auxiliary P50 MSE head + monotonicity penalty**.
5. **Post-hoc Calibration** — horizon-wise P50 linear map and additive P10/P90 offsets; **enforce order**; then temperature widening symmetrically around P50 to achieve the target coverage.
6. **Rolling Backtest** — validate stability across the last K folds without retraining.
7. **Uncertainty Sharpness & Bias** — evaluate band quality and central tendency: average band width and relative width $\frac{P90-P10}{\max(|P50|,\epsilon)}$ (interpretability/confidence), plus median error and MPE to confirm no systematic over/under-prediction.

1.1.3 Metrics, Losses & Uncertainty Definitions:

- **RMSE**
$$\text{RMSE} \;=\; \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\hat{y}_t - y_t\right)^2}$$

- **MAE**
$$\text{MAE} \;=\; \frac{1}{n}\sum_{t=1}^{n}|\hat{y}_t - y_t|$$

- **WAPE / WMAPE**
$$\text{WMAPE}(\%) \;=\; 100 \times \frac{\sum_{t=1}^{n}|\hat{y}_t - y_t|}{\sum_{t=1}^{n}|y_t|}$$

- **Quantile (Pinball) Loss**
$$\ell_q(y,\hat{y}) \;=\; \max(q\cdot(y-\hat{y}),\; (q-1)\cdot(y-\hat{y})), \quad q\in\{0.1, 0.5, 0.9\}.$$

- **Weighted Pinball**

$$\mathcal{L}_{\text{w-pin}} \;=\; \frac{1}{H}\sum_{h=1}^{H} w_h \left[\frac{1}{|Q|}\sum_{q\in Q}\omega_q\,\ell_q\big(y_{t,h},\hat{y}_{t,h}^{(q)}\big)\right] \;+\; \lambda\,\text{MonoPenalty},$$

where w_h are normalized horizon weights (linearly increasing to the forecast end), ω_q are quantile weights (heavier at tails), and $\lambda > 0$ is the monotonicity penalty weight.

- **Monotonicity Penalty**

$$\text{MonoPenalty} \;=\; \text{ReLU}\big(\hat{y}_{t,h}^{(10)} - \hat{y}_{t,h}^{(50)}\big) \;+\; \text{ReLU}\big(\hat{y}_{t,h}^{(50)} - \hat{y}_{t,h}^{(90)}\big).$$

- **Auxiliary P50 Head (training)**

$$\mathcal{L}_{\text{aux}} \;=\; \frac{1}{n}\sum_{t,h}\big(\hat{y}_{t,h}^{(50)} - y_{t,h}\big)^2, \qquad \mathcal{L}_{\text{total}} \;=\; \mathcal{L}_{\text{w-pin}} \;+\; \alpha\,\mathcal{L}_{\text{aux}}.$$

- **Horizon-wise Post-hoc Calibration**

$$\hat{y}_{t,h,\text{cal}}^{(50)} \;=\; a_h\,\hat{y}_{t,h}^{(50)} + b_h, \qquad \hat{y}_{t,h,\text{cal}}^{(10)} \;=\; \hat{y}_{t,h}^{(10)} + \text{off}_h^{(10)}, \qquad \hat{y}_{t,h,\text{cal}}^{(90)} \;=\; \hat{y}_{t,h}^{(90)} + \text{off}_h^{(90)}.$$

- **Order Enforcement (post-calibration)**

$$L_{t,h} = \min\{\hat{y}_{t,h,\text{cal}}^{(10)},\,\hat{y}_{t,h,\text{cal}}^{(50)},\,\hat{y}_{t,h,\text{cal}}^{(90)}\}, \quad U_{t,h} = \max\{\hat{y}_{t,h,\text{cal}}^{(10)},\,\hat{y}_{t,h,\text{cal}}^{(50)},\,\hat{y}_{t,h,\text{cal}}^{(90)}\}$$

$$\hat{y}_{t,h,\text{cal}}^{(50)} \leftarrow \text{clip}\big(\hat{y}_{t,h,\text{cal}}^{(50)},\,L_{t,h},\,U_{t,h}\big), \quad \hat{y}_{t,h,\text{cal}}^{(10)} \leftarrow L_{t,h}, \quad \hat{y}_{t,h,\text{cal}}^{(90)} \leftarrow U_{t,h}.$$

- **Temperature Scaling**

$$\Delta_h^- \;=\; \hat{y}_{t,h,\text{cal}}^{(50)} - \hat{y}_{t,h,\text{cal}}^{(10)}, \qquad \Delta_h^+ \;=\; \hat{y}_{t,h,\text{cal}}^{(90)} - \hat{y}_{t,h,\text{cal}}^{(50)}$$

$$\big(\hat{y}_{t,h,\tau}^{(10)},\,\hat{y}_{t,h,\tau}^{(90)}\big) \;=\; \Big(\hat{y}_{t,h,\text{cal}}^{(50)} - \tau\,\Delta_h^-,\; \hat{y}_{t,h,\text{cal}}^{(50)} + \tau\,\Delta_h^+\Big),$$

with τ chosen (via binary search) to achieve target central coverage (e.g., 80%).

- **Band width**

$$\text{Width}_{t,h} \;=\; \hat{y}_{t,h}^{(90)} \;-\; \hat{y}_{t,h}^{(10)}$$

- **Relative width** *(scale-free sharpness)*

$$\text{RelWidth}_{t,h}(\%) \;=\; 100 \times \frac{\hat{y}_{t,h}^{(90)} - \hat{y}_{t,h}^{(10)}}{\max\big(|\hat{y}_{t,h}^{(50)}|,\;\epsilon\big)}$$

- **Bias** *(median error)*

$$\tilde{e} \;=\; \text{median}\big(\hat{y}_{t,h}^{(50)} - y_{t,h}\big)$$

- **MPE**

$$\text{MPE}(\%) \;=\; 100 \times \frac{1}{n}\sum_{t,h}\frac{\hat{y}_{t,h}^{(50)} - y_{t,h}}{y_{t,h}}$$

1.1.4 Evaluation:

- **Pre-calibration:** Baseline WMAPE **0.5297%** vs. Uncalibrated **0.5818%** — baseline wins pre-cal.

- **Post-calibration:** P50 **WMAPE = 0.2762%**, **RMSE 540k**, **MAE 409k** — ~**48% WMAPE reduction vs. baseline** and ~**53% vs. uncalibrated**.

- **Coverage:** Central-80% **78.85%** with $\tau = 1.282$ (near the 80% target).

- **Rolling stability (K=5):** Rolling WMAPE tightly **0.276–0.313%**; RMSE **540k–596k**; MAE **409k–463k**; Central-80% **65–79%**.
- **Uncertainty sharpness & bias:** Well-calibrated with tight bands—avg width **$2.12M** (**1.43%** of P50) while maintaining **78.85% coverage vs. 80% target (−1.15 pp)**; P50 unbiased, with **median error –$0.10M** and **MPE –0.05%**, showing no systematic over/under-prediction.

[140]:
```python
import os, logging, warnings, gc, math
from datetime import date, datetime, timedelta

warnings.filterwarnings("ignore")
os.environ["TF_CPP_MIN_LOG_LEVEL"] = "2"
os.environ["TF_ENABLE_ONEDNN_OPTS"] = "0"
logging.getLogger("tensorflow").setLevel(logging.ERROR)

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from matplotlib.ticker import FuncFormatter
from matplotlib.dates import DateFormatter
from IPython.display import display
import tensorflow as tf
from keras import backend as K
from keras import mixed_precision
from keras import ops as Kops
from keras.layers import Input, Dense, Dropout, LayerNormalization, GlobalAveragePooling1D, MultiHeadAttention, Conv1D, Add, Reshape, Layer
from keras.models import Model
from keras.callbacks import EarlyStopping, ReduceLROnPlateau, ModelCheckpoint
from keras.optimizers import AdamW
from sklearn.preprocessing import MinMaxScaler, StandardScaler
from keras_tuner import Hyperband, Objective, HyperModel
```

1.2 Configuration

Key Choices: - `IN_LEN=104`, `OUT_LEN=52` → 2 years of context, 1-year horizon. - `USE_WEIGHTED_PINBALL=True` with horizon upweighting for long-range accuracy. - Hyperband for efficient HPO.

[141]:

```python
DATA_CSV    = "/home/linux/Source/Dev/Transformer/Dataset/Dummy Data.csv"
MODEL_DIR   = "/home/linux/Source/Dev/Transformer/Model/"
PREDICT_DIR = "/home/linux/Source/Dev/Transformer/Prediction/"
PROJECT_TAG = "PatchTST"

IN_LEN = 104
OUT_LEN = 52
QUANTILES = [0.1, 0.5, 0.9]

ADD_LAG52 = True
ADD_LAG26 = True
ADD_LAG78 = True
ADD_ROLLMEAN52 = True
ADD_EMA13 = True
ROLLING_WINDOW = 52

USE_WEIGHTED_PINBALL = True
HORIZON_END_WEIGHT = 1.3
Q_WEIGHTS = (1.15, 1.0, 1.15)
MONO_PENALTY = 0.05

USE_TUNER = True
HB_MAX_EPOCHS = 80
HB_FACTOR = 3
HB_OVERWRITE = True
RANDOM_SEED = 42
BATCH_SIZE = 256
PATIENCE_ES = 12
PATIENCE_LR = 6

ROLLING_K = 5

DO_POSTHOC_CAL = True
APPLY_CAL_ON_EXPORT = True
TARGET_COVERAGE = 0.80

WEIGHTS_PATH = os.path.join(MODEL_DIR, f"{PROJECT_TAG}.weights.h5")
TUNER_DIR = os.path.join(MODEL_DIR, "HB_runs")
BEST_MODEL_PATH = os.path.join(MODEL_DIR, f"{PROJECT_TAG}.best.keras")
ARTIFACTS_DIR = os.path.join(MODEL_DIR, "Artifacts")
SKIP_TUNING_IF_FOUND = True

BIAS_MODE = "percent"
_MILLION = 1e6

os.makedirs(MODEL_DIR, exist_ok=True)
os.makedirs(PREDICT_DIR, exist_ok=True)
```

```
os.makedirs(ARTIFACTS_DIR, exist_ok=True)

try:
    tf.keras.utils.set_random_seed(RANDOM_SEED)
    tf.config.experimental.enable_op_determinism()
except Exception:
    pass

policy = mixed_precision.Policy("mixed_float16")
mixed_precision.set_global_policy(policy)

print(f"TensorFlow: {tf.__version__} | Keras policy: {mixed_precision.
 ↪global_policy().name}")
```

```
TensorFlow: 2.20.0-dev0+selfbuilt | Keras policy: mixed_float16
```

1.3 Data & Features

Prep - Read `Dummy Data.csv`. - Validation window = the most recent **52** weeks (anchored weekly, `W-SAT`).

Calendar Features - Harmonics k=1..6 for smooth yearly seasonality (ISO week-of-year):

$$\sin_k(t) = \sin\left(\frac{2\pi k}{52} \cdot \text{week}(t)\right), \quad \cos_k(t) = \cos\left(\frac{2\pi k}{52} \cdot \text{week}(t)\right).$$

- **is_holiday_week** flags a week if any of these occur within that 7-day window: **New Year's Day, Memorial Day (last Mon in May), Independence Day (Jul 4), Labor Day (1st Mon in Sep), Thanksgiving (4th Thu in Nov), Christmas (Dec 25), Easter, Super Bowl, Black Friday, Cyber Monday**. - **hol_dist_w**, **hol_near1w** are computed relative to anchors **Easter, Super Bowl, Thanksgiving, Black Friday, Cyber Monday**: - hol_dist_w = signed distance in **weeks**, clipped to $[-3, 3]$. - hol_near1w = $\mathbb{1}\{|\text{hol\_dist\_w}| \le 1\}$.

Lag/EMA Features *(if toggled on)* - `lag52`, `lag26`, `lag78`, `rollmean52`, `ema13`. - `rollmean52` = 52-week rolling mean of prior values (uses `shift(1)` before rolling). - `ema13` = 13-span EMA of prior values (uses `shift(1)` after EMA).

Residual Target - $r_t = y_t - y_{t-52}$, with lag features forward/back-filled and remaining NAs replaced by the **Sales median**. - Training/scaling: - `Sales_scaled` = **MinMaxScaler** fit on **train only** (dates < validation start) to avoid look-ahead leakage, then applied to train+future. - `residual_scaled` = **StandardScaler** fit on **train only**, then applied to train+future. - Lag/EMA feature scales (`*_scaled`) use the **Sales** MinMax scaler.

[142]:
```
df = pd.read_csv(DATA_CSV, parse_dates=["Date"])
df["Sales"] = pd.to_numeric(df["Sales"], errors="coerce")
df = df.dropna(subset=["Date", "Sales"]).query("Sales != 0").
 ↪sort_values("Date").reset_index(drop=True)

start = pd.Timestamp("2019-01-12")
end = df["Date"].max()
df = df[(df["Date"] >= start) & (df["Date"] <= end)].copy()
```

```python
latest_date = end
val_dates = pd.date_range(end=latest_date, periods=OUT_LEN, freq="W-SAT")
val_start = val_dates.min()

print(f"Data span: {df['Date'].min().date()} → {df['Date'].max().date()}  |␣
↪N={len(df)} weeks")
print(f"Validation window: {val_start.date()} → {val_dates.max().date()}")

def easter_sunday(year: int) -> date:
    a = year % 19
    b = year // 100
    c = year % 100
    d = b // 4
    e = b % 4
    f = (b + 8) // 25
    g = (b - f + 1) // 3
    h = (19 * a + b - d - g + 15) % 30
    i = c // 4
    k = c % 4
    l = (32 + 2 * e + 2 * i - h - k) % 7
    m = (a + 11 * h + 22 * l) // 451
    month = (h + l - 7 * m + 114) // 31
    day = ((h + l - 7 * m + 114) % 31) + 1
    return date(year, month, day)

def nth_weekday_of_month(year: int, month: int, weekday: int, n: int) -> date:
    d = date(year, month, 1)
    while d.weekday() != weekday:
        d += timedelta(days=1)
    return d + timedelta(weeks=n - 1)

def last_weekday_of_month(year: int, month: int, weekday: int) -> date:
    d = (date(year + 1, 1, 1) - timedelta(days=1)) if month == 12 else␣
↪(date(year, month + 1, 1) - timedelta(days=1))
    while d.weekday() != weekday:
        d -= timedelta(days=1)
    return d

def approx_super_bowl(year: int) -> date:
    return nth_weekday_of_month(year, 2, 6, 2)

def black_friday(year: int) -> date:
    return nth_weekday_of_month(year, 11, 3, 4) + timedelta(days=1)

def cyber_monday(year: int) -> date:
    return nth_weekday_of_month(year, 11, 3, 4) + timedelta(days=4)
```

```python
def us_holidays_for_year(year: int) -> set[date]:
    hol = set()
    hol.update({
        date(year, 1, 1),
        last_weekday_of_month(year, 5, 0),
        date(year, 7, 4),
        nth_weekday_of_month(year, 9, 0, 1),
        nth_weekday_of_month(year, 11, 3, 4),
        date(year, 12, 25),
        easter_sunday(year),
        approx_super_bowl(year),
        black_friday(year),
        cyber_monday(year),
    })
    return hol

def make_calendar_frame(dates: pd.DatetimeIndex) -> pd.DataFrame:
    dfc = pd.DataFrame({"Date": pd.to_datetime(dates)})
    dfc["Year"] = dfc["Date"].dt.isocalendar().year.astype(int)
    dfc["Week"] = dfc["Date"].dt.isocalendar().week.astype(int)

    for k in range(1, 7):
        dfc[f"sin_{k}"] = np.sin(2 * np.pi * k * dfc["Week"] / 52.0)
        dfc[f"cos_{k}"] = np.cos(2 * np.pi * k * dfc["Week"] / 52.0)

    hol = []
    for d in dfc["Date"].dt.date:
        y = d.year
        hols = us_holidays_for_year(y)
        week_start = pd.Timestamp(d) - pd.Timedelta(days=6)
        week_days = {(week_start + pd.Timedelta(days=i)).date() for i in␣
↪range(7)}
        hol.append(int(len(hols.intersection(week_days)) > 0))
    dfc["is_holiday_week"] = hol

    def holiday_distance_features(d):
        y = d.year
        anchors = [
            easter_sunday(y),
            approx_super_bowl(y),
            black_friday(y),
            cyber_monday(y),
            nth_weekday_of_month(y, 11, 3, 4),
        ]
        w = []
        for a in anchors:
```

```
            delta = (pd.Timestamp(d) - pd.Timestamp(a)).days / 7.0
            w.append(delta)
        if not w:
            return 0.0, 0.0
        w = np.array(w, dtype=float)
        min_abs = w[np.argmin(np.abs(w))]
        return float(np.clip(min_abs, -3, 3)), float(abs(min_abs) <= 1.0)

    dist_list, near_list = [], []
    for d in dfc["Date"].dt.date:
        dd, near = holiday_distance_features(d)
        dist_list.append(dd)
        near_list.append(int(near))
    dfc["hol_dist_w"] = dist_list
    dfc["hol_near1w"] = near_list

    cols = ["Date"] \
        + [f"sin_{k}" for k in range(1, 7)] + [f"cos_{k}" for k in range(1, 7)]␣
 ↪\
        + ["is_holiday_week", "hol_dist_w", "hol_near1w"]
    return dfc[cols]

future_weeks = pd.date_range(start=latest_date + pd.Timedelta(weeks=1),␣
 ↪periods=52 * 3, freq="W-SAT")
cal_all = make_calendar_frame(pd.date_range(df["Date"].min(), future_weeks.
 ↪max(), freq="W-SAT"))

dfm = pd.merge(df[["Date", "Sales"]], cal_all, on="Date", how="left").
 ↪sort_values("Date").reset_index(drop=True)

if ADD_LAG52: dfm["lag52"] = dfm["Sales"].shift(52)
if ADD_LAG26: dfm["lag26"] = dfm["Sales"].shift(26)
if ADD_LAG78: dfm["lag78"] = dfm["Sales"].shift(78)
if ADD_ROLLMEAN52: dfm["rollmean52"] = dfm["Sales"].shift(1).
 ↪rolling(ROLLING_WINDOW, min_periods=1).mean()
if ADD_EMA13: dfm["ema13"] = dfm["Sales"].ewm(span=13, adjust=False).mean().
 ↪shift(1)

for c in [col for col in ["lag52", "lag26", "lag78", "rollmean52", "ema13"] if␣
 ↪col in dfm.columns]:
    dfm[c] = dfm[c].ffill().bfill().fillna(dfm["Sales"].median())

dfm["residual"] = dfm["Sales"] - (dfm["lag52"] if "lag52" in dfm else 0.0)

train_mask = dfm["Date"] < val_start
```

```python
sales_scaler = MinMaxScaler()
resid_scaler = StandardScaler()

dfm.loc[train_mask, "Sales_scaled"] = sales_scaler.fit_transform(dfm.
↪loc[train_mask, ["Sales"]])
dfm.loc[~train_mask, "Sales_scaled"] = sales_scaler.transform(dfm.
↪loc[~train_mask, ["Sales"]])

for c in ["lag52", "lag26", "lag78", "rollmean52", "ema13"]:
    if c in dfm:
        dfm[f"{c}_scaled"] = sales_scaler.transform(dfm[[c]].to_numpy()).ravel()

dfm.loc[train_mask, "residual_scaled"] = resid_scaler.fit_transform(dfm.
↪loc[train_mask, ["residual"]])
dfm.loc[~train_mask, "residual_scaled"] = resid_scaler.transform(dfm.
↪loc[~train_mask, ["residual"]])

feature_cols = \
    [f"sin_{k}" for k in range(1, 7)] + [f"cos_{k}" for k in range(1, 7)] + \
    ["is_holiday_week", "hol_dist_w", "hol_near1w"]

for c in ["lag52_scaled", "lag26_scaled", "lag78_scaled", "rollmean52_scaled",␣
↪"ema13_scaled"]:
    if c in dfm:
        feature_cols.append(c)

print(f"Feature dim (incl. Sales_scaled channel at time of windowing): {1 +␣
↪len(feature_cols)}")
```

```
Data span: 2019-01-12 → 2025-06-14  | N=336 weeks
Validation window: 2024-06-22 → 2025-06-14
Feature dim (incl. Sales_scaled channel at time of windowing): 21
```

1.4 Windowing & Direct Multi-Horizon Objective

Train direct multi-horizon with a single forward pass predicting the next **52** residuals.

- **Inputs:** $X_t \in \mathbb{R}^{L \times D}$ with L=104.
  The **first channel** is the scaled target `Sales_scaled`; the remaining $D - 1$ channels are engineered features (seasonality, holiday signals, lag/EMA features in their scaled forms).
- **Targets:** $Y_t \in \mathbb{R}^H$ of **scaled residuals** with H=52, i.e., `residual_scaled[t : t+H]` (direct multi-step).
- **Baseline add-back:** `BASE_all` stores the **unscaled** `lag52` baseline slice for each window (length H).
  After predicting residual quantiles and inverse-scaling them, add the baseline back horizon-wise:
  $$\hat{y}_{t+h} = (\mathrm{InvScale}(\hat{r}_{t+h})) + y_{t+h-52}, \quad h = 1, \ldots, H.$$
  Note: For future rollout, the baseline is computed as `Sales.shift(52)` over the timeline,

equivalent in intent to lag52.

[143]:
```
def build_windows(frame: pd.DataFrame, in_len: int, out_len: int, feat_cols:␣
↪list[str]):
    X, Y_resid, BASE = [], [], []

    target_scaled = frame["Sales_scaled"].values.astype(np.float32)
    feats = frame[feat_cols].values.astype(np.float32)
    resid_scaled = frame["residual_scaled"].values.astype(np.float32)
    baseline_abs = frame["lag52"].values.astype(np.float32) if "lag52" in frame␣
↪else np.zeros(len(frame), np.float32)

    T = len(frame)
    for t in range(in_len, T - out_len + 1):
        x_targ = target_scaled[t - in_len : t]
        x_feats = feats[t - in_len : t, :]
        X.append(np.concatenate([x_targ.reshape(-1, 1), x_feats], axis=1))
        Y_resid.append(resid_scaled[t : t + out_len])
        BASE.append(baseline_abs[t : t + out_len])

    return np.array(X), np.array(Y_resid), np.array(BASE)

X_all, Y_all_resid, BASE_all = build_windows(dfm, IN_LEN, OUT_LEN, feature_cols)
num_features = X_all.shape[-1]

X_train, Y_train = X_all[:-1], Y_all_resid[:-1]
X_val, Y_val = X_all[-1:], Y_all_resid[-1:]
BASE_val = BASE_all[-1]
y_val_abs = dfm.loc[dfm["Date"].isin(val_dates), "Sales"].values.astype(np.
↪float32)

print(f"Windows - train: {len(X_train)}, val: {len(X_val)} | in_len={IN_LEN},␣
↪out_len={OUT_LEN}, D={num_features}")
```

```
Windows - train: 180, val: 1 | in_len=104, out_len=52, D=21
```

1.5 Model (Patch-style Transformer on sequences)

Patch Embedding - `Conv1D(filters=d_model, kernel_size=patch_len, strides=patch_stride, padding="valid")` converts the length-104 sequence into patch tokens.

Positional Encoding - `AddSinusoidalPE`: classic sin/cos positional encoding added to token embeddings.

Optional TCN Stem - Dilated **causal** conv residual blocks with dilation rates **1/2/4**: - for each rate $r \in \{1, 2, 4\}$:
`Conv1D(d_model, 3, padding="causal", dilation_rate=r)` → GELU → Dropout → residual add.

Transformer Encoder - Stacked blocks: - LayerNorm → Multi-Head Attention → Dropout → **residual add** - LayerNorm → MLP (Dense → GELU → Dense) → Dropout → **residual add** - `GlobalAveragePooling1D` → Dropout → fixed-size representation.

Heads - **Quantile Head**: `Dense(H*Q)` → `Reshape((OUT_LEN, 3))` for residual quantiles **(P10, P50, P90)**. - **Aux Head**: `Dense(OUT_LEN)` provides an extra **P50** trained with MSE to regularize the center.

1.6 Losses

Pinball Loss (quantile loss) for $q \in \{0.1, 0.5, 0.9\}$:

$$\ell_q(y, \hat{y}) = \max(q \cdot (y - \hat{y}), (q - 1) \cdot (y - \hat{y})).$$

Weighted Pinball: - **Horizon Wighting**: w_h grows from 1.0 → `HORIZON_END_WEIGHT` (e.g., 1.3) to emphasize long range. - **Quantile Weights**: slightly upweight tails with $\omega_{0.1} = \omega_{0.9} = 1.15$. - **Monotonicity Penalty**: $\text{ReLU}(\hat{y}^{(10)} - \hat{y}^{(50)}) + \text{ReLU}(\hat{y}^{(50)} - \hat{y}^{(90)})$ to discourage crossing.

[144]:
```python
class AddSinusoidalPE(Layer):
    def call(self, x):
        L = tf.shape(x)[1]
        D = tf.shape(x)[2]
        i = tf.cast(tf.range(D), tf.float32)
        two_i = tf.cast(tf.math.floordiv(i, 2) * 2, tf.float32)
        angle_rates = 1.0 / tf.pow(10000.0, two_i / tf.cast(D, tf.float32))
        pos = tf.cast(tf.range(L)[:, None], tf.float32)
        angles = pos * angle_rates[None, :]
        sines = tf.sin(angles[:, 0::2])
        cosines = tf.cos(angles[:, 1::2])
        if x.shape[-1] is not None and x.shape[-1] % 2 != 0:
            cosines = tf.pad(cosines, [[0, 0], [0, 1]])
        pe = tf.concat([sines, cosines], axis=-1)
        return x + tf.cast(pe, x.dtype)

def make_pinball_loss(quantiles):
    qs = tf.constant(quantiles, dtype=tf.float32)
    def loss(y_true, y_pred):
        y_true = tf.expand_dims(y_true, -1)
        e = y_true - y_pred
        return tf.reduce_mean(tf.maximum(qs * e, (qs - 1.0) * e))
    return loss

def make_weighted_pinball_loss(quantiles, out_len, end_weight=1.3,␣
 ↪mono_lambda=0.0, q_weights=(1.15, 1.0, 1.15)):
    qs = tf.constant(quantiles, dtype=tf.float32)
    qw = tf.constant(q_weights, dtype=tf.float32)
    hw = tf.linspace(1.0, end_weight, out_len)
```

```
    hw = hw / tf.reduce_mean(hw)
    def loss(y_true, y_pred):
        y_true = tf.expand_dims(y_true, -1)
        e = y_true - y_pred
        pin = tf.maximum(qs * e, (qs - 1.0) * e) * qw
        pin = tf.reduce_mean(pin, axis=-1)
        pin = tf.reduce_mean(pin * hw)
        if mono_lambda and mono_lambda > 0.0:
            p10, p50, p90 = y_pred[..., 0], y_pred[..., 1], y_pred[..., 2]
            cross = tf.nn.relu(p10 - p50) + tf.nn.relu(p50 - p90)
            pin += mono_lambda * tf.reduce_mean(cross)
        return pin
    return loss

def dilated_conv_stem(x, d_model, dropout):
    for rate in [1, 2, 4]:
        h = Conv1D(d_model, 3, padding="causal", dilation_rate=rate,␣
 ↪activation=None)(x)
        h = Kops.gelu(h)
        h = Dropout(dropout)(h)
        x = Add()([x, h])
    return x
```

1.7 Tuning Loop (Hyperband)

Use **KerasTuner Hyperband** to search: `d_model`, `heads`, `depth`, `mlp_ratio`, `dropout`, `patch_len`, `patch_stride`, `use_tcn`, `aux_weight`, `lr`.

- Objective: `val_quant_loss` (lower is better).
- EarlyStopping + ReduceLROnPlateau.
- **mixed_float16** reduces memory and speeds up training on GPU.

[145]:
```
class PatchTSTHyperModel(HyperModel):
    def __init__(self, input_shape, out_len, quantiles):
        self.input_shape = input_shape
        self.out_len = out_len
        self.quantiles = quantiles

    def build(self, hp):
        d_model = hp.Choice("d_model", [64, 128, 192, 256])
        num_heads = hp.Choice("heads", [2, 4, 8])
        depth = hp.Int("depth", 2, 5)
        mlp_ratio = hp.Choice("mlp_ratio", [2.0, 3.0, 4.0])
        dropout = hp.Float("dropout", 0.0, 0.4, step=0.05)
        patch_len = hp.Choice("patch_len", [4, 8, 16])
        patch_stride = hp.Choice("patch_stride", [1, 2])
        use_tcn = hp.Boolean("use_tcn", default=True)
        aux_w = hp.Float("aux_weight", 0.1, 0.5, step=0.1)
```

```
        lr = hp.Float("lr", 1e-4, 1e-2, sampling="LOG")

        inp = Input(shape=self.input_shape, name="series_tokens")
        x = Conv1D(filters=d_model, kernel_size=patch_len,␣
↪strides=patch_stride, padding="valid", activation=None)(inp)
        x = AddSinusoidalPE(name="add_sinusoidal_pe")(x)
        if use_tcn:
            x = dilated_conv_stem(x, d_model, dropout)

        for _ in range(depth):
            h = LayerNormalization()(x)
            h = MultiHeadAttention(num_heads=num_heads, key_dim=d_model //␣
↪num_heads, dropout=dropout)(h, h)
            h = Dropout(dropout)(h)
            x = Add()([x, h])

            h = LayerNormalization()(x)
            h = Dense(int(d_model * mlp_ratio), activation="gelu")(h)
            h = Dropout(dropout)(h)
            h = Dense(d_model)(h)
            h = Dropout(dropout)(h)
            x = Add()([x, h])

        x = GlobalAveragePooling1D()(x)
        x = Dropout(dropout)(x)

        H = self.out_len
        Q = len(self.quantiles)
        quant_flat = Dense(H * Q, name="quant_flat")(x)
        quant = Reshape((H, Q), name="quant")(quant_flat)
        p50_aux = Dense(H, name="p50_aux")(x)

        model = Model(inp, [quant, p50_aux], name="PatchTST_Residual")

        qloss = (
            make_weighted_pinball_loss(
                self.quantiles, self.out_len,
                end_weight=HORIZON_END_WEIGHT,
                mono_lambda=MONO_PENALTY,
                q_weights=Q_WEIGHTS
            )
            if USE_WEIGHTED_PINBALL else make_pinball_loss(self.quantiles)
        )

        model.compile(
            optimizer=AdamW(learning_rate=lr, weight_decay=1e-4),
            loss={"quant": qloss, "p50_aux": "mse"},
```

```
            loss_weights={"quant": 1.0, "p50_aux": aux_w},
            metrics={"p50_aux": ["mae"]},
        )
        return model

if USE_TUNER:
    if SKIP_TUNING_IF_FOUND and os.path.exists(BEST_MODEL_PATH):
        print(f"Found existing best model → {BEST_MODEL_PATH}\nSkipping Hyperband.")
        model = tf.keras.models.load_model(
            BEST_MODEL_PATH, compile=False,
            custom_objects={"AddSinusoidalPE": AddSinusoidalPE}
        )
    else:
        tuner = Hyperband(
            PatchTSTHyperModel((IN_LEN, num_features), OUT_LEN, QUANTILES),
            objective=Objective("val_quant_loss", direction="min"),
            max_epochs=HB_MAX_EPOCHS,
            factor=HB_FACTOR,
            directory=TUNER_DIR,
            project_name=f"{PROJECT_TAG}_HB",
            overwrite=HB_OVERWRITE,
            seed=RANDOM_SEED,
            max_consecutive_failed_trials=10,
        )

        ckpt = ModelCheckpoint(
            BEST_MODEL_PATH,
            monitor="val_quant_loss",
            mode="min",
            save_best_only=True,
            save_weights_only=False,
            verbose=1,
        )

        callbacks = [
            EarlyStopping(monitor="val_quant_loss", mode="min", patience=PATIENCE_ES, restore_best_weights=True, verbose=1),
            ReduceLROnPlateau(monitor="val_quant_loss", mode="min", factor=0.3, patience=PATIENCE_LR, min_lr=1e-5, verbose=1),
            ckpt,
        ]

        y_train_dict = {"quant": Y_train, "p50_aux": Y_train}
        y_val_dict   = {"quant": Y_val,   "p50_aux": Y_val}

        tuner.search(
```

```
            X_train,
            y_train_dict,
            validation_data=(X_val, y_val_dict),
            epochs=HB_MAX_EPOCHS,
            callbacks=callbacks,
            verbose=1,
            shuffle=False,
            batch_size=BATCH_SIZE,
        )

        print("\n=== Tuner Results Summary ===")
        tuner.results_summary(num_trials=10)

        model = tf.keras.models.load_model(
            BEST_MODEL_PATH, compile=False, custom_objects={"AddSinusoidalPE":␣
↪AddSinusoidalPE}
        )
        model.save_weights(WEIGHTS_PATH)
        print(f"\nSaved best weights → {WEIGHTS_PATH}\nSaved best full model →␣
↪{BEST_MODEL_PATH}")
else:
    raise RuntimeError("USE_TUNER=False path not implemented. Enable tuner or␣
↪add a fixed model.")
```

```
Found existing best model →
/home/linux/Source/Dev/Transformer/Model/PatchTST.best.keras
Skipping Hyperband.
```

1.8 Validation Evaluation (Absolute Scale)

1) Predict residual **quantiles** on `X_val` → shape `(52,3)` in **scaled residual** space.

2) **Inverse-transform** residuals; **add baseline** (lag-52) to get absolute **P10/P50/P90**.

3) Report raw (uncalibrated) coverage on P10–P90.

[146]:
```
def extract_quantiles(
    preds,
    quant_key_hints: tuple[str, ...] = ("quant", "quantile"),
    p50_key_hints: tuple[str, ...] = ("p50", "median"),
):
    q = None
    p50_aux = None

    if isinstance(preds, (list, tuple)):
        q = preds[0]
        if len(preds) > 1:
            p50_aux = preds[1]
```

```
    elif isinstance(preds, dict):
        for hint in quant_key_hints:
            for k in preds.keys():
                if hint in k.lower():
                    q = preds[k]
                    break
            if q is not None:
                break
        if q is None:
            q = list(preds.values())[0]
        for hint in p50_key_hints:
            for k in preds.keys():
                if hint in k.lower():
                    p50_aux = preds[k]
                    break
            if p50_aux is not None:
                break
    else:
        q = preds

    q = np.asarray(q)
    if q.ndim == 3:
        if q.shape[0] != 1:
            raise ValueError(f"Expect batch size=1 for validation. Got {q.
↪shape}")
        q = q[0]
    q = np.sort(q, axis=-1)

    if p50_aux is not None:
        p50_aux = np.asarray(p50_aux)
        if p50_aux.ndim == 2 and p50_aux.shape[0] == 1:
            p50_aux = p50_aux[0]

    return q, p50_aux

preds_val = model.predict(X_val, verbose=0)
q_resid_val, _ = extract_quantiles(preds_val)

p10_res = resid_scaler.inverse_transform(q_resid_val[:, 0].reshape(-1, 1)).
↪ravel()
p50_res = resid_scaler.inverse_transform(q_resid_val[:, 1].reshape(-1, 1)).
↪ravel()
p90_res = resid_scaler.inverse_transform(q_resid_val[:, 2].reshape(-1, 1)).
↪ravel()

p10_abs = BASE_val + p10_res
p50_abs = BASE_val + p50_res
```

```python
p90_abs = BASE_val + p90_res

def rmse(y, yhat): return float(np.sqrt(np.mean((np.asarray(yhat) - np.
 ↪asarray(y)) ** 2)))
def mae(y, yhat):  return float(np.mean(np.abs(np.asarray(yhat) - np.
 ↪asarray(y))))
def wmape(y, yhat):
    y, yhat = np.asarray(y), np.asarray(yhat)
    return float(100 * np.sum(np.abs(yhat - y)) / np.sum(np.abs(y)))

uncal = dict(RMSE=rmse(y_val_abs, p50_abs), MAE=mae(y_val_abs, p50_abs),␣
 ↪WMAPE=wmape(y_val_abs, p50_abs))
cov80 = 100 * np.mean((y_val_abs >= p10_abs) & (y_val_abs <= p90_abs))
print(
    f"Uncalibrated - P50 RMSE: {uncal['RMSE']:,.2f} | MAE: {uncal['MAE']:,.2f}␣
 ↪| "
    f"WMAPE(%): {uncal['WMAPE']:,.4f} | Coverage P10-P90: {cov80:.2f}%"
)

sn_rmse = rmse(y_val_abs, BASE_val)
sn_mae = mae(y_val_abs, BASE_val)
sn_wmape = wmape(y_val_abs, BASE_val)
print(f"Baseline (Seasonal Naive) - RMSE: {sn_rmse:,.2f} | MAE: {sn_mae:,.2f} |␣
 ↪WMAPE(%): {sn_wmape:.4f}")
```

```
Uncalibrated - P50 RMSE: 1,033,339.75 | MAE: 861,897.88 | WMAPE(%): 0.5818 |
Coverage P10-P90: 78.85%
Baseline (Seasonal Naive) - RMSE: 956,467.50 | MAE: 784,712.94 | WMAPE(%):
0.5297
```

```
2025-08-22 01:26:17.260951: E tensorflow/core/framework/node_def_util.cc:680]
NodeDef mentions attribute use_unbounded_threadpool which is not in the op
definition: Op<name=MapDataset; signature=input_dataset:variant,
other_arguments: -> handle:variant; attr=f:func;
attr=Targuments:list(type),min=0; attr=output_types:list(type),min=1;
attr=output_shapes:list(shape),min=1;
attr=use_inter_op_parallelism:bool,default=true;
attr=preserve_cardinality:bool,default=false;
attr=force_synchronous:bool,default=false; attr=metadata:string,default=""> This
may be expected if your graph generating binary is newer  than this binary.
Unknown attributes will be ignored. NodeDef: {{node ParallelMapDatasetV2/_14}}
```

1.9 Post-Hoc Calibration

Make P50 *unbiased* and the P10–P90 band achieve target coverage.

- **Horizon-Wise linear P50 fit** (using last K folds):

$$\hat{y}^{(50)}_{t,h,\mathrm{cal}} = a_h\,\hat{y}^{(50)}_{t,h} + b_h.$$

- **Additive Offsets** for P10/P90:

$$\delta_h^{(10)} = \text{Quantile}_{0.10}(y_{t,h} - \hat{y}_{t,h}^{(10)})$$

$$\delta_h^{(90)} = \text{Quantile}_{0.90}(y_{t,h} - \hat{y}_{t,h}^{(90)})$$

  Form offset-adjusted bounds:

$$\hat{y}_{t,h,\text{off}}^{(10)} = \hat{y}_{t,h}^{(10)} + \delta_h^{(10)}$$

$$\hat{y}_{t,h,\text{off}}^{(90)} = \hat{y}_{t,h}^{(90)} + \delta_h^{(90)}$$

- **Temperature** τ widens/narrows the band around P50 to match target coverage (80%):

  First, enforce ordering once (pre τ):

$$\breve{y}_{t,h}^{(10)} = \min\{\hat{y}_{t,h,\text{off}}^{(10)},\ \hat{y}_{t,h,\text{cal}}^{(50)},\ \hat{y}_{t,h,\text{off}}^{(90)}\}$$

$$\breve{y}_{t,h}^{(90)} = \max\{\hat{y}_{t,h,\text{cal}}^{(50)},\ \hat{y}_{t,h,\text{off}}^{(90)}\}$$

$$\breve{y}_{t,h}^{(50)} = \text{clip}(\hat{y}_{t,h,\text{cal}}^{(50)},\ \breve{y}_{t,h}^{(10)},\ \breve{y}_{t,h}^{(90)})$$

  Define the distances from P50 to each bound:

$$\Delta_h^- = \breve{y}_{t,h}^{(50)} - \breve{y}_{t,h}^{(10)}$$

$$\Delta_h^+ = \breve{y}_{t,h}^{(90)} - \breve{y}_{t,h}^{(50)}$$

  Apply temperature scaling:

$$\hat{y}_{t,h,\tau}^{(10)} = \breve{y}_{t,h}^{(50)} - \tau\,\Delta_h^-$$

$$\hat{y}_{t,h,\tau}^{(90)} = \breve{y}_{t,h}^{(50)} + \tau\,\Delta_h^+$$

- **Quantile Ordering (Post-Processing)**:

$$\tilde{y}_{t,h}^{(10)} = \min\{\hat{y}_{t,h,\tau}^{(10)}, \hat{y}_{t,h,\text{cal}}^{(50)}, \hat{y}_{t,h,\tau}^{(90)}\},$$
$$\tilde{y}_{t,h}^{(90)} = \max\{\hat{y}_{t,h,\text{cal}}^{(50)}, \hat{y}_{t,h,\tau}^{(90)}\},$$
$$\tilde{y}_{t,h}^{(50)} = \text{clip}(\hat{y}_{t,h,\text{cal}}^{(50)}, \tilde{y}_{t,h}^{(10)}, \tilde{y}_{t,h}^{(90)}),$$
$$P10 \leq P50 \leq P90$$

[ ]:

```python
def linear_calibrate_p50(y_true, p50_pred):
    X = np.vstack([p50_pred, np.ones_like(p50_pred)]).T
    a, b = np.linalg.lstsq(X, y_true, rcond=None)[0]
    return float(a), float(b)

def calibrate_quantile_offsets(y_true, q_pred):
    offs = {}
    for q, arr in [("0.1", q_pred["0.1"]), ("0.5", q_pred["0.5"]), ("0.9", q_pred["0.9"])]:
        resid = y_true - arr
        offs[q] = float(np.quantile(resid, float(q)))
    return offs

def apply_temperature(p10, p50, p90, tau: float):
    """Widen/narrow symmetrically around p50."""
    p10, p50, p90 = np.asarray(p10), np.asarray(p50), np.asarray(p90)
    lo = p50 - p10
    hi = p90 - p50
    new_p10 = p50 - tau * lo
    new_p90 = p50 + tau * hi
    return new_p10, new_p90

def fit_tau(y, p10, p50, p90, target=0.80, lo=0.8, hi=1.6, iters=20):
    """Binary search  so central coverage  target, with the *same*
    order-enforcement used in eval/export."""
    y = np.asarray(y)
    for _ in range(iters):
        mid = 0.5 * (lo + hi)
        L, U = apply_temperature(p10, p50, p90, mid)
        L = np.minimum.reduce([L, p50, U])
        U = np.maximum.reduce([p50, U])
        cov = np.mean((y >= L) & (y <= U))
        lo, hi = (mid, hi) if cov < target else (lo, mid)
    return 0.5 * (lo + hi)


def fit_horizonwise_calibration(model, X_all, Y_all_resid, BASE_all, resid_scaler, K=5):
    T = len(X_all); K = min(K, T)
    Ys, P10s, P50s, P90s = [[] for _ in range(OUT_LEN)], [[] for _ in range(OUT_LEN)], [[] for _ in range(OUT_LEN)], [[] for _ in range(OUT_LEN)]
    for k in range(K, 0, -1):
        Xk = X_all[T - k : T - k + 1]
        BASE = BASE_all[T - k]
        yabs = BASE + resid_scaler.inverse_transform(Y_all_resid[T - k].reshape(-1, 1)).ravel()
```

```
        preds = model.predict(Xk, verbose=0)
        q_res, _ = extract_quantiles(preds)
        q_abs = resid_scaler.inverse_transform(q_res).ravel().reshape(OUT_LEN,␣
↪3) + BASE[:, None]
        p10, p50, p90 = q_abs[:, 0], q_abs[:, 1], q_abs[:, 2]
        for h in range(OUT_LEN):
            Ys[h].append(float(yabs[h]))
            P10s[h].append(float(p10[h]))
            P50s[h].append(float(p50[h]))
            P90s[h].append(float(p90[h]))

    a_h = np.ones(OUT_LEN, dtype=np.float64)
    b_h = np.zeros(OUT_LEN, dtype=np.float64)
    off10_h = np.zeros(OUT_LEN, dtype=np.float64)
    off90_h = np.zeros(OUT_LEN, dtype=np.float64)

    for h in range(OUT_LEN):
        y = np.array(Ys[h], dtype=np.float64)
        p50 = np.array(P50s[h], dtype=np.float64)
        p10 = np.array(P10s[h], dtype=np.float64)
        p90 = np.array(P90s[h], dtype=np.float64)
        if len(y) >= 2 and np.any(np.abs(p50) > 1e-6):
            X = np.vstack([p50, np.ones_like(p50)]).T
            a, b = np.linalg.lstsq(X, y, rcond=None)[0]
            a_h[h], b_h[h] = float(a), float(b)
        else:
            a_h[h] = 1.0 if abs(p50[-1]) < 1e-6 else float(y[-1] / p50[-1])
            b_h[h] = 0.0
        off10_h[h] = float(np.quantile(y - p10, 0.10))
        off90_h[h] = float(np.quantile(y - p90, 0.90))
    return a_h, b_h, off10_h, off90_h

if DO_POSTHOC_CAL:
    a_global, b_global = linear_calibrate_p50(y_val_abs, p50_abs)
    offs_global = calibrate_quantile_offsets(y_val_abs, {"0.1": p10_abs, "0.5":␣
↪p50_abs, "0.9": p90_abs})

    a_h, b_h, off10_h, off90_h = fit_horizonwise_calibration(
        model, X_all, Y_all_resid, BASE_all, resid_scaler, K=ROLLING_K
    )

    p50_cal = a_h * p50_abs + b_h
    p10_cal = p10_abs + off10_h
    p90_cal = p90_abs + off90_h
    lower = np.minimum.reduce([p10_cal, p50_cal, p90_cal])
    upper = np.maximum.reduce([p50_cal, p90_cal])
    p50_cal = np.clip(p50_cal, lower, upper)
```

```
    p10_cal, p90_cal = lower, upper

    tau = fit_tau(y_val_abs, p10_cal, p50_cal, p90_cal, target=TARGET_COVERAGE)
    p10_cal, p90_cal = apply_temperature(p10_cal, p50_cal, p90_cal, tau)
    lower = np.minimum.reduce([p10_cal, p50_cal, p90_cal])
    upper = np.maximum.reduce([p50_cal, p90_cal])
    p50_cal = np.clip(p50_cal, lower, upper)
    p10_cal, p90_cal = lower, upper

    cal = dict(RMSE=rmse(y_val_abs, p50_cal), MAE=mae(y_val_abs, p50_cal), WMAPE=wmape(y_val_abs, p50_cal))
    emp = (
        100 * np.mean(y_val_abs <= p10_cal),
        100 * np.mean(y_val_abs <= p50_cal),
        100 * np.mean(y_val_abs <= p90_cal),
    )
    cov = 100 * np.mean((y_val_abs >= p10_cal) & (y_val_abs <= p90_cal))
    print(
        f"Calibrated (horizonwise + ={tau:.3f}) - Empirical (P10,P50,P90): "
        f"({emp[0]:.2f}%, {emp[1]:.2f}%, {emp[2]:.2f}%) | Coverage: {cov:.2f}%"
    )
    print(
        f"Calibrated (horizonwise + ={tau:.3f}) - P50 RMSE: {cal['RMSE']:,.2f} | MAE: {cal['MAE']:,.2f} | "
        f"WMAPE(%): {cal['WMAPE']:,.4f}"
    )
else:
    a_global, b_global, offs_global = 1.0, 0.0, {"0.1": 0.0, "0.5": 0.0, "0.9": 0.0}
    p10_cal, p50_cal, p90_cal = p10_abs, p50_abs, p90_abs
    a_h = np.ones(OUT_LEN); b_h = np.zeros(OUT_LEN); off10_h = np.zeros(OUT_LEN); off90_h = np.zeros(OUT_LEN)
    tau = 1.0
```

```
Calibrated (horizonwise + =1.282) - Empirical (P10,P50,P90): (9.62%, 42.31%,
88.46%) | Coverage: 78.85%
Calibrated (horizonwise + =1.282) - P50 RMSE: 540,122.38 | MAE: 409,140.12 |
WMAPE(%): 0.2762
```

Post-calibration: P50 **WMAPE = 0.2762%**, **RMSE 540k**, **MAE 409k** — ~**48% WMAPE reduction vs. baseline** and ~**53% vs. uncalibrated**.

Metric	**Calibrated**	Baseline → Δ vs Baseline	Uncalibrated → Δ vs Uncal.
WMAPE	**0.2762%**	0.5297% (↓**47.86%**)	0.5818% (↓**52.53%**)
RMSE	**540,122**	956,468 (↓**43.53%**)	1,033,340 (↓**47.73%**)
MAE	**409,140**	784,713 (↓**47.86%**)	861,898 (↓**52.53%**)

- *Calibration method:* horizonwise + = **1.282**.

1.10 Reliability, Sharpness, Bias

- **Reliability**: empirical P10/P50/P90, central-80% by horizon.
- **Sharpness**: average band width; relative width $(P90-P10)/P50$ as interpretability for planning (narrower = more confident).
- **Bias**: median error ($\tilde{\epsilon}$) and mean percentage error (MPE).

[148]:
```
emp_p10_h = (y_val_abs <= p10_cal).astype(float)
emp_p50_h = (y_val_abs <= p50_cal).astype(float)
emp_p90_h = (y_val_abs <= p90_cal).astype(float)
emp_c80_h = ((y_val_abs >= p10_cal) & (y_val_abs <= p90_cal)).astype(float)

rel_width = 100 * (p90_cal - p10_cal) / np.maximum(np.abs(p50_cal), 1e-6)
band_width = (p90_cal - p10_cal)

bias_med = float(np.median(p50_cal - y_val_abs))
mpe = float(100 * np.mean((p50_cal - y_val_abs) / y_val_abs))

print(f"Sharpness - Avg band width: {np.mean(band_width):,.2f} | Avg relative␣
↪width(%): {np.mean(rel_width):.4f}")
print(f"Bias - Median error: {bias_med:,.2f} | MPE(%): {mpe:.4f}")
```

```
Sharpness - Avg band width: 2,116,247.40 | Avg relative width(%): 1.4305
Bias - Median error: -104,566.44 | MPE(%): -0.0474
```

Tight uncertainty bands without sacrificing target coverage; P50 is effectively unbiased.

Metric	Achieved	Target/Goal	Interpretation
Central-80% coverage	78.85%	80%	Near target → reliable intervals
Avg band width	~$2.12M	—	Tight absolute band (~$2.12M on a ~$150M median week)
Avg relative width	~1.43% of P50	—	Tight relative band (confidence)
Bias (median error)	−$0.10M	$0	No systematic over/under-prediction.
MPE	−0.05%	0%	Near-zero percentage bias

- **Coverage (80% PI):** Coverage **78.85%** (target 80%, **−1.15 pp**).

1.11 Rolling Backtest (Stability)

Repeat the exact same calibration over the last K folds (no retrain) to check: - **Error stability**: RMSE/MAE/WMAPE per fold. - **Reliability**: empirical P10/P50/P90 and central-80% coverage per fold.

Stable folds → robust deployment.

[149]:
```
def rolling_backtest(model, X_all, Y_all_resid, BASE_all, resid_scaler, K=5,
                     a_h=None, b_h=None, off10_h=None, off90_h=None, tau: float␣
↪= 1.0):
    rows, covrows = [], []
    T = len(X_all); K = min(K, T)
    for k in range(K, 0, -1):
        Xk   = X_all[T - k : T - k + 1]
        BASE = BASE_all[T - k]
        yabs = BASE + resid_scaler.inverse_transform(Y_all_resid[T - k].
↪reshape(-1, 1)).ravel()

        preds = model.predict(Xk, verbose=0)
        q_res, _ = extract_quantiles(preds)
        q_abs = resid_scaler.inverse_transform(q_res).ravel().reshape(OUT_LEN,␣
↪3) + BASE[:, None]
        p10, p50, p90 = q_abs[:, 0], q_abs[:, 1], q_abs[:, 2]

        if a_h is not None:     p50 = a_h * p50 + b_h
        if off10_h is not None: p10 = p10 + off10_h
        if off90_h is not None: p90 = p90 + off90_h

        lower = np.minimum.reduce([p10, p50, p90])
        upper = np.maximum.reduce([p50, p90])
        p50   = np.clip(p50, lower, upper)
        p10, p90 = lower, upper

        if tau is not None and tau != 1.0:
            p10, p90 = apply_temperature(p10, p50, p90, tau)
            lower = np.minimum.reduce([p10, p50, p90])
            upper = np.maximum.reduce([p50, p90])
            p50   = np.clip(p50, lower, upper)
            p10, p90 = lower, upper

        rows.append({
            "fold":  k,
            "RMSE":  rmse(yabs, p50),
            "MAE":   mae(yabs, p50),
            "WMAPE (%)": wmape(yabs, p50)
        })
        covrows.append({
            "fold":        k,
            "Emp_P10%":    100*np.mean(yabs <= p10),
            "Emp_P50%":    100*np.mean(yabs <= p50),
            "Emp_P90%":    100*np.mean(yabs <= p90),
            "Central_80%": 100*np.mean((yabs >= p10) & (yabs <= p90))
        })
```

```
    return pd.DataFrame(rows).sort_values("fold"), pd.DataFrame(covrows).
    ↪sort_values("fold")

metrics_roll, calib_roll = rolling_backtest(
    model, X_all, Y_all_resid, BASE_all, resid_scaler,
    K=ROLLING_K, a_h=a_h, b_h=b_h, off10_h=off10_h, off90_h=off90_h, tau=tau
)

m = metrics_roll.copy()

m_overall = pd.DataFrame([{
    "fold": "Overall",
    "RMSE": m["RMSE"].mean(),
    "MAE": m["MAE"].mean(),
    "WMAPE (%)": m["WMAPE (%)"].mean(),
}])
m_plus = pd.concat([m, m_overall], ignore_index=True)

def _hide_index(styler):
    if hasattr(styler, "hide"):
        return styler.hide(axis="index")
    if hasattr(styler, "hide_index"):
        return styler.hide_index()
    return styler

s_metrics = _hide_index(
    m_plus.style.format({
        "fold":        lambda x: f"{x}",
        "RMSE":        lambda x: f"{x:,.0f}",
        "MAE":         lambda x: f"{x:,.0f}",
        "WMAPE (%)":   lambda x: f"{x:.3f}",
    })
)

c = calib_roll.copy()
s_calib = _hide_index(
    c.style.format({
        "fold":        lambda x: f"{x}",
        "Emp_P10%":    lambda x: f"{x:.3f}",
        "Emp_P50%":    lambda x: f"{x:.3f}",
        "Emp_P90%":    lambda x: f"{x:.3f}",
        "Central_80%": lambda x: f"{x:.3f}",
    })
)

display(s_metrics)
display(s_calib)
```

```
<pandas.io.formats.style.Styler at 0x7365927f8e50>
```

```
<pandas.io.formats.style.Styler at 0x7368b6174410>
```

Rolling Metric	Mean	Range (min–max)
WMAPE	**0.297%**	**0.276–0.313%**
Central 80% Coverage	**72.69%**	**65.38–78.85%**
RMSE	**570,878**	**540,122–596,328**
MAE	**439,928**	**409,141–463,789**

- **Material lift vs. a strong seasonal baseline** (**48%** lower WMAPE; **44–48%** lower MAE/RMSE).
- **Intervals are credible** (78.85% near the 80% target).
- **Stable across time** (rolling WMAPE tightly **0.276–0.313%**), indicating reliable production behavior.

1.12 EDA

- **Validation — Calibrated Probabilistic Forecast (P10–P90; =1.282)**
  Full timeline overlay (train, actuals, P50, fan). Shows the calibrated forecast tracks the holdout well with a central band aimed at ~80% coverage.

- **Validation — Baseline vs Calibrated (P10–P90; =1.282)**
  Side-by-side with the seasonal-naive baseline. Highlights the large WMAPE reduction from calibration and that intervals achieve near-target coverage.

- **Validation — Actual vs P50 (Calibrated)**
  Scatter against the 45° line with R^2/ /U . Demonstrates tight alignment and lower error vs baseline on the validation block.

- **Pooled — Actual vs P50 (Calibrated)**
  Hexbin over all recent folds × horizons. Confirms consistency of fit across many points, not just one block.

- **Validation — WMAPE Comparison**
  Bars for Baseline, Uncalibrated, Calibrated. Quantifies accuracy gains (calibrated beats both baseline and uncalibrated).

- **Pooled — Coverage by Horizon (Calibrated)**
  Central 80% coverage vs horizon with 5-week average. Pinpoints horizons under/over-covered relative to the 80% target.

- **Pooled — Quantile Reliability by Horizon (Calibrated)**
  Empirical P10/ P50/ P90 lines + targets. Checks calibration quality; ECE summaries show how close each quantile is to nominal.

- **Pooled — Median Relative Width by Horizon**
  Typical sharpness, i.e., median ((P90−P10)/|P50|). Shows how interval width evolves with horizon and how the validation block compares.

- **Pooled — Horizon-Wise P50 Bias (Post-Calibration)**
  Mean error by horizon (percent or level). Verifies residual bias is small and centered near zero; reports MPE.
- **Rolling Stability by Fold (Older → Newest) — RMSE · MAE · WMAPE**
  Error trends across rolling folds. Demonstrates stability and improvement on newer folds.
- **Recent Folds — Calibrated Fan vs Actual**
  Small multiples for the last 5 folds. Visual check of per-fold fit, interval sharpness, and achieved coverage.
- **Pooled — Mean Absolute Error Improvement by Horizon (Calibrated – Baseline)**
  Δ|error| vs baseline by horizon (blue = better). Shows where calibrated forecasts cut absolute error most—and where they don't.
- **Pooled — Share of Cases Where Calibrated Beats Baseline**
  % of samples with lower |error| than baseline by horizon. Above 50% means calibrated wins more often than not.
- **Validation — Coverage vs**
  Coverage frontier as temperature widens/narrows intervals; vertical marker at chosen *. Used to hit coverage targets.
- **Validation — WMAPE vs**
  Flat across . Confirms only adjusts interval width—P50 (and thus point accuracy) is unchanged.
- **Validation — Sharpness vs**
  Median band width vs . Makes the accuracy–uncertainty trade-off explicit as intervals widen with larger .
- **Validation — Step-Wise Improvement**
  Mean |error| for Baseline → Uncalibrated → Calibrated. Separates gains from the model and from calibration.

[150]:
```python
def r2(y, yhat):
    y = np.asarray(y); yhat = np.asarray(yhat)
    ss_res = np.sum((y - yhat)**2)
    ss_tot = np.sum((y - np.mean(y))**2)
    return float(1 - ss_res / ss_tot) if ss_tot > 0 else float("nan")

def format_ax(ax):
    ax.yaxis.set_major_formatter(FuncFormatter(lambda x, pos: f"{int(x):,}"))
    ax.xaxis.set_major_formatter(DateFormatter("%Y-%m"))
    ax.grid(True, linestyle="--", alpha=0.3)

def _fmt_ax_yint(ax=None):
    ax = ax or plt.gca()
    ax.yaxis.set_major_formatter(FuncFormatter(lambda x, pos: f"{int(x):,}"))
    ax.grid(True, linestyle="--", alpha=0.3)
    return ax
```

```python
def footerize(ax, text, grow=1.25, base_pad=0.12, pad_per_line=0.03, max_pad=0.
↪35):
    fig = ax.figure
    w, h = fig.get_size_inches()
    fig.set_size_inches(w, h * float(grow))
    lines = text.count("\n") + 1
    reserve = min(max_pad, base_pad + pad_per_line * (lines - 1))
    fig.subplots_adjust(bottom=reserve)
    fig.text(
        0.02, reserve * 0.1, text,
        ha="left", va="center", fontsize=10,
        bbox=dict(boxstyle="round", alpha=0.15)
    )

def footerize_fig(fig, text, **kw):
    return footerize(fig.axes[0], text, **kw)

def add_vline_at_tau(ax, tau_value, label=" *"):
    ax.axvline(float(tau_value), ls="--", lw=1.2, alpha=0.6)
    ax.text(float(tau_value), ax.get_ylim()[1], f" {label}={tau_value:.3f}",
            va="top", ha="left", fontsize=9)

def shade_under_target(ax, x, series, target=80.0):
    series = np.asarray(series)
    below = series < target
    if below.any():
        ax.fill_between(x, series, target, where=below, alpha=0.15, color="tab:
↪red", linewidth=0)

def _ensure_baseline_metrics():
    g = globals()
    if all(k in g for k in ["sn_rmse","sn_mae","sn_wmape"]):
        return g["sn_rmse"], g["sn_mae"], g["sn_wmape"]
    assert "y_val_abs" in g and "BASE_val" in g, "Need y_val_abs and BASE_val."
    sn_rmse = rmse(y_val_abs, BASE_val)
    sn_mae  = mae(y_val_abs, BASE_val)
    sn_wmape = wmape(y_val_abs, BASE_val)
    globals().update(sn_rmse=sn_rmse, sn_mae=sn_mae, sn_wmape=sn_wmape)
    return sn_rmse, sn_mae, sn_wmape

def _ensure_val_metrics():
    g = globals()
    cal = g.get("cal", None)
    if cal is None:
        assert "y_val_abs" in g and "p50_cal" in g, "Need y_val_abs and p50_cal.
↪"
```

```
        cal = dict(RMSE=rmse(y_val_abs, p50_cal), MAE=mae(y_val_abs, p50_cal), WMAPE=wmape(y_val_abs, p50_cal))
        globals()["cal"] = cal
    uncal = g.get("uncal", None)
    if uncal is None:
        p50_unc = g.get("p50_abs", None)
        if p50_unc is None:
            p50_unc = p50_cal
        uncal = dict(RMSE=rmse(y_val_abs, p50_unc), MAE=mae(y_val_abs, p50_unc), WMAPE=wmape(y_val_abs, p50_unc))
        globals()["uncal"] = uncal
    return cal, uncal

def _ensure_blocks(OUT_LEN, ROLLING_K):
    g = globals()
    have_full = all(k in g for k in ["Ys_full","BL_full","P10c","P50c","P90c","P10u","P50u","P90u"])
    if have_full:
        return g["Ys_full"], g["BL_full"], (g["P10c"],g["P50c"],g["P90c"]), (g["P10u"],g["P50u"],g["P90u"])

    assert all(k in g for k in [
        "model","X_all","Y_all_resid","BASE_all","resid_scaler", "extract_quantiles","a_h","b_h","off10_h","off90_h","OUT_LEN","ROLLING_K"
    ]), "Missing inputs to rebuild fold×horizon blocks. Run earlier cells."

    T = len(X_all); K = min(ROLLING_K, T)
    Ys, BL = np.zeros((K, OUT_LEN)), np.zeros((K, OUT_LEN))
    P10u = np.zeros((K, OUT_LEN)); P50u = np.zeros((K, OUT_LEN)); P90u = np.zeros((K, OUT_LEN))
    P10c = np.zeros((K, OUT_LEN)); P50c = np.zeros((K, OUT_LEN)); P90c = np.zeros((K, OUT_LEN))

    for i, k in enumerate(range(K, 0, -1)):
        Xk   = X_all[T - k : T - k + 1]
        BASE = BASE_all[T - k]
        yabs = BASE + resid_scaler.inverse_transform(Y_all_resid[T - k].reshape(-1, 1)).ravel()

        preds = model.predict(Xk, verbose=0)
        q_res, _ = extract_quantiles(preds)
        q_abs = resid_scaler.inverse_transform(q_res).ravel().reshape(OUT_LEN, 3) + BASE[:, None]
        p10u, p50u, p90u = q_abs[:,0], q_abs[:,1], q_abs[:,2]
```

```
        p50c = a_h * p50u + b_h
        p10c = p10u + off10_h
        p90c = p90u + off90_h
        lower = np.minimum.reduce([p10c, p50c, p90c])
        upper = np.maximum.reduce([p50c, p90c])
        p50c = np.clip(p50c, lower, upper); p10c, p90c = lower, upper

        if globals().get("tau", 1.0) != 1.0:
            t0 = float(globals().get("tau", 1.0))
            lo = p50c - p10c; hi = p90c - p50c
            p10c = p50c - t0 * lo; p90c = p50c + t0 * hi
            lower = np.minimum.reduce([p10c, p50c, p90c])
            upper = np.maximum.reduce([p50c, p90c])
            p50c = np.clip(p50c, lower, upper); p10c, p90c = lower, upper

        Ys[i,:]  = yabs
        BL[i,:]  = BASE
        P10u[i,:], P50u[i,:], P90u[i,:] = p10u, p50u, p90u
        P10c[i,:], P50c[i,:], P90c[i,:] = p10c, p50c, p90c

    globals().update(Ys_full=Ys, BL_full=BL, P10u=P10u, P50u=P50u, P90u=P90u, P10c=P10c, P50c=P50c, P90c=P90c)
    return Ys, BL, (P10c,P50c,P90c), (P10u,P50u,P90u)

def _ensure_metrics_roll():
    g = globals()
    if "metrics_roll" in g:
        return g["metrics_roll"]
    Ys, BL, (P10c,P50c,P90c), _ = _ensure_blocks(OUT_LEN, ROLLING_K)
    K = Ys.shape[0]
    rows = []
    for i in range(K):
        y = Ys[i,:]; p = P50c[i,:]
        rows.append({"fold": i+1, "RMSE": rmse(y, p), "MAE": mae(y, p), "WMAPE (%)": wmape(y, p)})
    metrics_roll = pd.DataFrame(rows)
    globals()["metrics_roll"] = metrics_roll
    return metrics_roll

Ys_full, BL_full, (P10c,P50c,P90c), (P10u,P50u,P90u) = _ensure_blocks(OUT_LEN, ROLLING_K)
metrics_roll = _ensure_metrics_roll()
x = np.arange(1, OUT_LEN+1)

train_df = df[df["Date"] < val_start][["Date","Sales"]].copy()

# Validation - Calibrated Probabilistic Forecast (P10-P90;  ={tau:.3f})
```

```python
plt.figure(figsize=(16,8))
plt.title(f"Validation - Calibrated Probabilistic Forecast (P10-P90;  ={tau:.3f})")
plt.plot(train_df["Date"], train_df["Sales"], label="Train")
plt.plot(val_dates, y_val_abs, label="Actual")
plt.plot(val_dates, p50_cal, label="Prediction P50 (Calibrated)")
plt.fill_between(val_dates, p10_cal, p90_cal, alpha=0.25, label="P10-P90 (Calibrated +  )")
plt.legend(loc="upper left"); format_ax(plt.gca()); plt.tight_layout(); plt.show()

# Validation - Baseline vs Calibrated (P10-P90;  ={tau:.3f})

sn_rmse, sn_mae, sn_wmape = _ensure_baseline_metrics()
cal, uncal = _ensure_val_metrics()
cov_val = 100.0 * np.mean((y_val_abs >= p10_cal) & (y_val_abs <= p90_cal))
tau = globals().get("tau", 1.0)

fig, ax = plt.subplots(figsize=(12,5))
ax.set_title(f"Validation - Baseline vs Calibrated (P10-P90;  ={tau:.3f})")
ax.plot(val_dates, y_val_abs, label="Actual", linewidth=1.6)
ax.plot(val_dates, BASE_val,  label="Baseline (Lag-52)", linewidth=1.2)
ax.plot(val_dates, p50_cal,   label="P50 (Calibrated)", linewidth=2.0)
ax.fill_between(val_dates, p10_cal, p90_cal, alpha=0.25, label="P10-P90 (Calibrated)")
ax.xaxis.set_major_formatter(DateFormatter("%Y-%m")); _fmt_ax_yint(ax)
ax.legend(loc="upper left"); plt.tight_layout()
footerize(ax,
    f"WMAPE: Cal {cal['WMAPE']:.4f}% | Base {sn_wmape:.4f}% | Uncal {uncal['WMAPE']:.4f}%\n"
    f"Δ vs Base: {100*(sn_wmape-cal['WMAPE'])/sn_wmape:.1f}% | "
    f"Δ vs Uncal: {100*(uncal['WMAPE']-cal['WMAPE'])/uncal['WMAPE']:.1f}%\n"
    f"Coverage (P10-P90): {cov_val:.2f}% (target 80%)"
)
plt.show()

# Validation - Actual vs P50 (Calibrated)

def r2_score_simple(y, yhat):
    y = np.asarray(y, float); yhat = np.asarray(yhat, float)
    ss_res = np.sum((y - yhat)**2)
    ss_tot = np.sum((y - y.mean())**2)
    return 1.0 - ss_res / (ss_tot + 1e-12)

def theils_u2_against(y, yhat, baseline):
    return rmse(y, yhat) / (rmse(y, baseline) + 1e-12)
```

```python
y_flat = y_val_abs.ravel()
p_flat = p50_cal.ravel()
b_flat = BASE_val.ravel()

R2_val  = r2_score_simple(y_flat, p_flat)
r_val   = float(np.corrcoef(p_flat, y_flat)[0,1])
U2_val  = theils_u2_against(y_flat, p_flat, b_flat)

lims = [min(y_flat.min(), p_flat.min()), max(y_flat.max(), p_flat.max())]

fig, ax = plt.subplots(figsize=(6.4, 4.2))
ax.scatter(y_flat, p_flat, s=10, alpha=0.35)
ax.plot(lims, lims, linestyle="--", alpha=0.7)
ax.set_xlim(lims); ax.set_ylim(lims)
ax.set_aspect('equal', adjustable='box')
ax.set_xlabel("Actual"); ax.set_ylabel("P50 (Calibrated)")
ax.set_title(f"Validation - Actual vs P50 (Calibrated) |  R²={R2_val:.3f}, r={r_val:.3f}, U ={U2_val:.3f}")
ax.grid(True, linestyle="--", alpha=0.3)
fig.tight_layout()
footerize(ax, f"RMSE={rmse(y_flat, p_flat):,.0f}  |  MAE={mae(y_flat, p_flat):,.0f}  |  WMAPE={wmape(y_flat, p_flat):.4f}%")
plt.show()

# Pooled - Actual vs P50 (Calibrated)

y_all   = Ys_full.ravel()
p_all   = P50c.ravel()
base_all= BL_full.ravel()

R2_all = r2_score_simple(y_all, p_all)
r_all  = float(np.corrcoef(p_all, y_all)[0,1])
U2_all = theils_u2_against(y_all, p_all, base_all)

lims_all = [min(y_all.min(), p_all.min()), max(y_all.max(), p_all.max())]

fig, ax = plt.subplots(figsize=(6.4, 4.2))
hb = ax.hexbin(y_all, p_all, gridsize=40, mincnt=1, linewidths=0, cmap="Blues", alpha=0.9)
ax.plot(lims_all, lims_all, linestyle="--", alpha=0.7)
ax.set_xlim(lims_all); ax.set_ylim(lims_all)
ax.set_aspect('equal', adjustable='box')
ax.set_xlabel("Actual"); ax.set_ylabel("P50 (Calibrated)")
ax.set_title(f"Pooled - Actual vs P50 (Calibrated) |  R²={R2_all:.3f}, r={r_all:.3f}, U ={U2_all:.3f}")
ax.grid(True, linestyle="--", alpha=0.3)
```

```python
cb = fig.colorbar(hb, ax=ax); cb.set_label("Count")
fig.tight_layout()
footerize(ax, f"RMSE={rmse(y_all, p_all):,.0f}  |  MAE={mae(y_all, p_all):,.0f}␣
↪ |  WMAPE={wmape(y_all, p_all):.4f}%")
plt.show()

# Validation - WMAPE Comparison

summary_rows = [
    {"Model":"Baseline",      "RMSE":sn_rmse,      "MAE":sn_mae,      "WMAPE%":
↪sn_wmape},
    {"Model":"Uncalibrated",  "RMSE":uncal["RMSE"],"MAE":uncal["MAE"],"WMAPE%":
↪uncal["WMAPE"]},
    {"Model":"Calibrated",    "RMSE":cal["RMSE"],  "MAE":cal["MAE"],  "WMAPE%":
↪cal["WMAPE"]},
]
summary_df = pd.DataFrame(summary_rows).set_index("Model")

fig, ax = plt.subplots(figsize=(6,3.2))
summary_df["WMAPE%"].plot(kind="bar", rot=0, ax=ax)
ax.set_ylabel("WMAPE (%)")
ax.grid(axis="y", linestyle="--", alpha=0.25)
plt.title("Validation - WMAPE Comparison")
for p in ax.patches:
    ax.text(p.get_x()+p.get_width()/2, p.get_height(), f"{p.get_height():.
↪4f}%", ha="center", va="bottom", fontsize=9)

imp_vs_base = 100.0*(sn_wmape - cal["WMAPE"])/sn_wmape
imp_vs_unc  = 100.0*(uncal["WMAPE"] - cal["WMAPE"])/uncal["WMAPE"]
plt.tight_layout()
footerize(ax, f"Cal vs Baseline: {imp_vs_base:.1f}% better\nCal vs Uncal:␣
↪{imp_vs_unc:.1f}% better")
plt.show()

# Pooled - Coverage by Horizon (Calibrated)

cov80_h = 100.0 * np.mean((Ys_full >= P10c) & (Ys_full <= P90c), axis=0)
cov80_h_val = 100.0 * ((y_val_abs >= p10_cal) & (y_val_abs <= p90_cal)).
↪astype(float)

fig, ax = plt.subplots(figsize=(10,3))
ax.plot(x, cov80_h, label="Central 80% Coverage")
ax.plot(x, pd.Series(cov80_h).rolling(5, min_periods=1).mean(),
        linestyle="--", alpha=0.7, label="5w Avg")
ax.axhline(80.0, linestyle="--", label="Target 80%")
shade_under_target(ax, x, cov80_h, target=80.0)
```

```
ax.set_xlabel("Horizon (Weeks)"); ax.set_ylabel("Coverage (%)")
ax.set_title("Pooled - Coverage by Horizon (Calibrated)")
ax.legend(loc="best"); ax.grid(True, linestyle="--", alpha=0.3); plt.
 ↪tight_layout()

footerize(
    ax,
    f"Pooled Mean={np.mean(cov80_h):.2f}%\n"
    f"Validation Mean={np.mean(cov80_h_val):.2f}%"
)
plt.show()

# Pooled - Quantile Reliability by Horizon (Calibrated)

pct_below10_h = 100.0 * np.mean(Ys_full <= P10c, axis=0)
pct_below50_h = 100.0 * np.mean(Ys_full <= P50c, axis=0)
pct_below90_h = 100.0 * np.mean(Ys_full <= P90c, axis=0)
ece10 = abs(np.mean(Ys_full <= P10c) - 0.10)*100
ece50 = abs(np.mean(Ys_full <= P50c) - 0.50)*100
ece90 = abs(np.mean(Ys_full <= P90c) - 0.90)*100

ece10_val = abs(np.mean(y_val_abs <= p10_cal) - 0.10)*100
ece50_val = abs(np.mean(y_val_abs <= p50_cal) - 0.50)*100
ece90_val = abs(np.mean(y_val_abs <= p90_cal) - 0.90)*100

fig, ax = plt.subplots(figsize=(10,3))
ax.plot(x, pct_below10_h, label="Empirical  P10")
ax.plot(x, pct_below50_h, label="Empirical  P50")
ax.plot(x, pct_below90_h, label="Empirical  P90")
for tgt in (10,50,90): ax.axhline(tgt, linestyle="--")
ax.set_xlabel("Horizon (Weeks)"); ax.set_ylabel("Percent (%)")
ax.set_title("Pooled - Quantile Reliability by Horizon (Calibrated)")
ax.legend(loc="best"); ax.grid(True, linestyle="--", alpha=0.3); plt.
 ↪tight_layout()
footerize(
    ax,
    f"Pooled ECE: P10={ece10:.2f}pp | P50={ece50:.2f}pp | P90={ece90:.2f}pp\n"
    f"Validation ECE: P10={ece10_val:.2f}pp | P50={ece50_val:.2f}pp |␣
 ↪P90={ece90_val:.2f}pp"
)
plt.show()

# Pooled - Median Relative Width by Horizon  (P90-P10)/max(|P50|, )

width_abs = P90c - P10c
width_rel = width_abs / np.maximum(np.abs(P50c), 1e-8)
med_rel_pct = np.median(width_rel, axis=0) * 100.0
```

```
pooled_typical = float(np.median(med_rel_pct))

rel_width_val = 100.0 * (p90_cal - p10_cal) / np.maximum(np.abs(p50_cal), 1e-8)
val_mean_rel = float(np.mean(rel_width_val))

fig, ax = plt.subplots(figsize=(10,3))
ax.plot(x, med_rel_pct)
ax.set_xlabel("Horizon (Weeks)")
ax.set_ylabel("Relative Width (%)")
ax.set_title("Pooled - Median Relative Width by Horizon  (P90-P10)/max(|P50|, )")
ax.grid(True, linestyle="--", alpha=0.3); plt.tight_layout()
footerize(ax, f"Pooled Typical: {pooled_typical:.2f}%\nValidation Block: {val_mean_rel:.2f}%")
plt.show()

# Pooled - Horizon-Wise P50 Bias (Post-Calibration)

if BIAS_MODE.lower() == "percent":
    bias_curve_pooled = 100.0 * (Ys_full - P50c) / np.maximum(np.abs(Ys_full), 1e-8)
    val_bias_curve    = 100.0 * (y_val_abs - p50_cal) / np.maximum(np.abs(y_val_abs), 1e-8)
    ylab = "Mean Error (%)"
else:
    bias_curve_pooled = (Ys_full - P50c) / _MILLION
    val_bias_curve    = (y_val_abs - p50_cal) / _MILLION
    ylab = "Mean Error (Millions)"

mean_bias_curve = np.mean(bias_curve_pooled, axis=0)

pooled_mean_abs = float(np.mean(np.mean(np.abs(bias_curve_pooled), axis=0)))
val_mean_abs = float(np.mean(np.abs(val_bias_curve)))
val_mpe = 100.0 * np.mean((p50_cal - y_val_abs) / np.maximum(np.abs(y_val_abs), 1e-8))

if 'x' not in globals(): x = np.arange(1, OUT_LEN+1)
fig, ax = plt.subplots(figsize=(10,3))
ax.axhline(0.0, linestyle="--", alpha=0.7)
ax.plot(x, mean_bias_curve)
ax.set_xlabel("Horizon (Weeks)")
ax.set_ylabel(ylab)
ax.set_title("Pooled - Horizon-Wise P50 Bias (Post-Calibration)")
ax.ticklabel_format(axis="y", style="plain")
ax.grid(True, linestyle="--", alpha=0.3)
plt.tight_layout()
```

```
if BIAS_MODE.lower() == "percent":
    footerize(ax, f"Pooled Mean|bias|: {pooled_mean_abs:.4f}%\n"
                  f"Validation Mean|bias|: {val_mean_abs:.4f}% | MPE: {val_mpe:.
↪4f}%")
else:
    footerize(ax, f"Pooled Mean|bias|: {pooled_mean_abs:.3f} M\n"
                  f"Validation Mean|bias|: {val_mean_abs:.3f} M | MPE: {val_mpe:
↪.4f}%")
plt.show()

# Rolling Stability by Fold (Older → Newest) – RMSE · MAE · WMAPE (K={len(m)})

m = metrics_roll.sort_values("fold")

fig, ax = plt.subplots(figsize=(10,4))

l1, = ax.plot(m["fold"], m["RMSE"], marker="o", linewidth=1.6, label="RMSE")
l2, = ax.plot(m["fold"], m["MAE"],  marker="o", linewidth=1.6, label="MAE")
ax.invert_xaxis()
ax.set_xlabel("Fold (Older → Newest)")
ax.set_ylabel("Error (RMSE / MAE)")
ax.set_title(f"Rolling Stability by Fold (Older → Newest) – RMSE · MAE · WMAPE␣
↪(K={len(m)})")
_fmt_ax_yint(ax)
ax.grid(True, linestyle="--", alpha=0.3)

ax2 = ax.twinx()
l3, = ax2.plot(m["fold"], m["WMAPE (%)"], marker="o", linewidth=1.6,
               color="tab:green", label="WMAPE (%)")
ax2.set_ylabel("WMAPE (%)")
ax2.yaxis.set_major_formatter(FuncFormatter(lambda x, pos: f"{x:.3f}%"))

lines = [l1, l2, l3]
labels = [ln.get_label() for ln in lines]
ax.legend(lines, labels, loc="best")

rmse_mu  = float(m["RMSE"].mean())
mae_mu   = float(m["MAE"].mean())
wmape_mu = float(m["WMAPE (%)"].mean())
oldest   = m.loc[m["fold"].idxmax()]
newest   = m.loc[m["fold"].idxmin()]
d_rmse   = float(newest["RMSE"]        - oldest["RMSE"])
d_mae    = float(newest["MAE"]         - oldest["MAE"])
d_wmape  = float(newest["WMAPE (%)"]   - oldest["WMAPE (%)"])

footerize(
    ax,
```

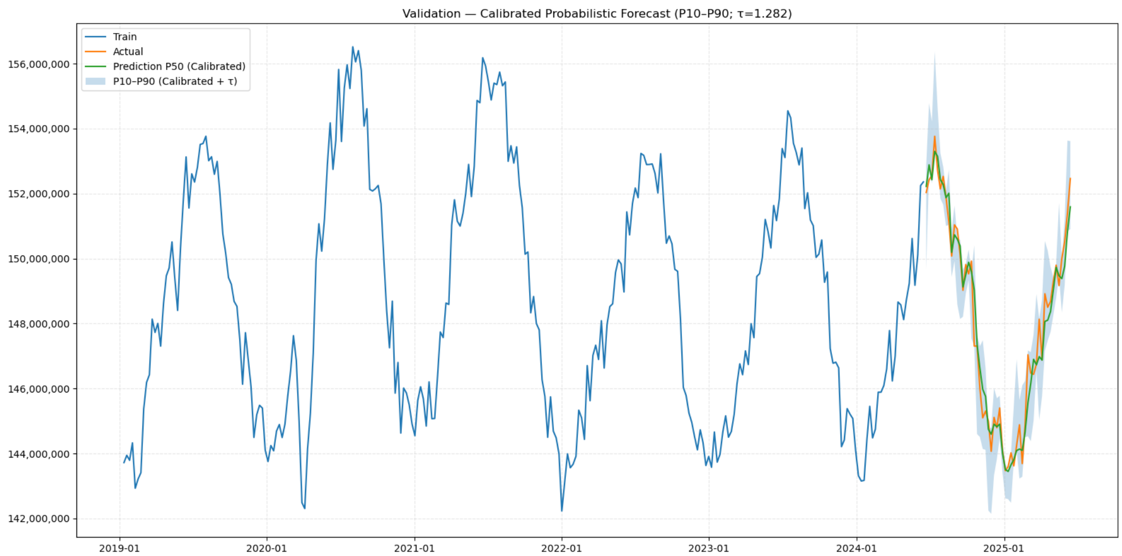
```
    f"Means - RMSE: {rmse_mu:,.0f} | MAE: {mae_mu:,.0f} | WMAPE: {wmape_mu:.
↪3f}%"
)
plt.tight_layout()
plt.show()

# Recent Folds - Calibrated Fan vs Actual

K, H = Ys_full.shape
x_h = np.arange(1, H + 1)

n_show = min(6, K)
order_idx = list(range(K - 1, K - n_show - 1, -1))
ncols = 3
nrows = math.ceil(n_show / ncols)

fig, axes = plt.subplots(nrows, ncols, figsize=(12, 3.2 * nrows), sharex=True)
axes = np.array(axes).reshape(-1)

ymins, ymaxs = [], []

for j, (ax, idx) in enumerate(zip(axes, order_idx)):
    ax.plot(x_h, Ys_full[idx],  lw=1.2, label="Actual")
    ax.plot(x_h, P50c[idx],     lw=1.6, label="P50")
    ax.fill_between(x_h, P10c[idx], P90c[idx], alpha=0.25, label="P10-P90")
    ax.set_title(f"Fold {j+1}")

    wm   = wmape(Ys_full[idx], P50c[idx])
    cov  = 100.0 * np.mean((Ys_full[idx] >= P10c[idx]) & (Ys_full[idx] <=␣
↪P90c[idx]))
    medw = float(np.median(P90c[idx] - P10c[idx]))
    ax.text(
        0.98, 0.02,
        f"WMAPE {wm:.3f}%\nCov {cov:.1f}%\nMedW {medw:,.0f}",
        transform=ax.transAxes, ha="right", va="bottom",
        fontsize=9, bbox=dict(boxstyle="round", alpha=0.15)
    )

    ax.grid(True, linestyle="--", alpha=0.3)
    if j // ncols == nrows - 1:
        ax.set_xlabel("Horizon (Weeks)")
    if j % ncols == 0:
        ax.set_ylabel("Level")

    ylo, yhi = ax.get_ylim()
    ymins.append(ylo); ymaxs.append(yhi)
```

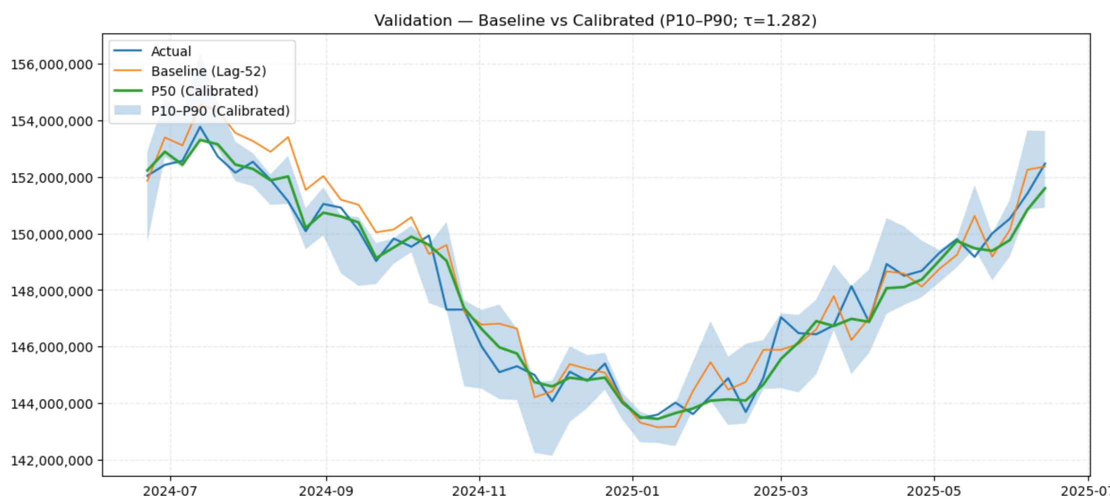
```python
for ax in axes[len(order_idx):]:
    ax.set_visible(False)

ymin, ymax = min(ymins), max(ymaxs)
for ax in axes[:len(order_idx)]:
    ax.set_ylim(ymin, ymax)

fig.suptitle("Recent Folds - Calibrated Fan vs Actual", y=0.98, fontsize=12)

handles, labels = axes[0].get_legend_handles_labels()
fig.legend(handles, labels, loc="lower center", ncol=3, frameon=False,␣
 ↪bbox_to_anchor=(0.5, 0.10))

new_idx = order_idx[0]; old_idx = order_idx[-1]

cov_mat_sel = 100.0 * ((Ys_full[order_idx] >= P10c[order_idx]) &
                       (Ys_full[order_idx] <= P90c[order_idx])).astype(float)

cov_by_fold = cov_mat_sel.mean(axis=1)
cov_mean = float(cov_by_fold.mean())
cov_min  = float(cov_by_fold.min())
cov_max  = float(cov_by_fold.max())

medw_new = float(np.median(P90c[new_idx] - P10c[new_idx]))
medw_old = float(np.median(P90c[old_idx] - P10c[old_idx]))
drift_pct = 100.0 * (medw_new / max(medw_old, 1e-12) - 1.0)

wm_new = float(wmape(Ys_full[order_idx[0]], P50c[order_idx[0]]))
wm_old = float(wmape(Ys_full[order_idx[-1]], P50c[order_idx[-1]]))
delta_pp = wm_new - wm_old
improve_pp = wm_old - wm_new

footer_text = (
    f"Window = Last {len(order_idx)} Folds (Newest → Older as␣
 ↪1…{len(order_idx)})\n"
    f"Coverage (central 80%) - Mean {cov_mean:.2f}% | Min {cov_min:.1f}% | Max␣
 ↪{cov_max:.1f}%\n"
    f"Sharpness - Median Width Oldest vs Newest: {medw_old:,.0f} → {medw_new:,.
 ↪0f} ({drift_pct:+.1f}%)\n"
    f"Accuracy - WMAPE Oldest vs Newest: {wm_old:.3f}% vs {wm_new:.3f}% ␣
 ↪(improvement {improve_pp:+.3f} pp)"
)

footerize(axes[0], footer_text, base_pad=0.24)
plt.tight_layout(rect=[0, 0.14, 1, 0.94])
plt.show()
```

```python
# Pooled - Mean Absolute Error Improvement by Horizon (Calibrated - Baseline)

abs_err_base = np.abs(Ys_full - BL_full)
abs_err_cal  = np.abs(Ys_full - P50c)
impr = abs_err_base - abs_err_cal
mean_impr_h = np.mean(impr, axis=0)
share_better_h = 100.0 * np.mean(impr > 0.0, axis=0)

impr_val_h = np.abs(y_val_abs - BASE_val) - np.abs(y_val_abs - p50_cal)
val_avg_impr = float(np.mean(impr_val_h))
val_share_better = float(100.0 * np.mean(impr_val_h > 0.0))

fig, ax = plt.subplots(figsize=(10,3))
colors = ['tab:blue' if v >= 0 else 'crimson' for v in mean_impr_h]
ax.bar(x, mean_impr_h, color=colors)
ax.set_xlabel("Horizon (Weeks)")
ax.set_ylabel("Δ|error| vs Baseline")
ax.set_title("Pooled - Mean Absolute Error Improvement by Horizon (Calibrated -␣
↪Baseline)")
_fmt_ax_yint(ax); plt.tight_layout()

footerize(
    ax,
    f"Pooled Avg Δ|err| = {np.mean(mean_impr_h):,.0f} | Pooled Share Better =␣
↪{np.mean(impr>0)*100:.1f}%\n"
    f"Validation Avg Δ|err| = {val_avg_impr:,.0f} | Validation Share Better =␣
↪{val_share_better:.1f}%"
)
plt.show()

fig, ax = plt.subplots(figsize=(10,3))
ax.plot(x, share_better_h)
ax.axhline(50, linestyle="--")
ax.set_xlabel("Horizon (Weeks)"); ax.set_ylabel("% Horizons Beating Baseline")
ax.set_title("Pooled - Share of Cases Where Calibrated Beats Baseline")
ax.grid(True, linestyle="--", alpha=0.3); plt.tight_layout(); plt.show()

#  frontier

if all(k in globals() for k in␣
↪["a_h","b_h","off10_h","off90_h","p10_abs","p50_abs","p90_abs"]):
    p50_cal_base = a_h * p50_abs + b_h
    p10_cal_base = p10_abs + off10_h
    p90_cal_base = p90_abs + off90_h
    lower_base = np.minimum.reduce([p10_cal_base, p50_cal_base, p90_cal_base])
    upper_base = np.maximum.reduce([p50_cal_base, p90_cal_base])
    p50_cal_base = np.clip(p50_cal_base, lower_base, upper_base)
```

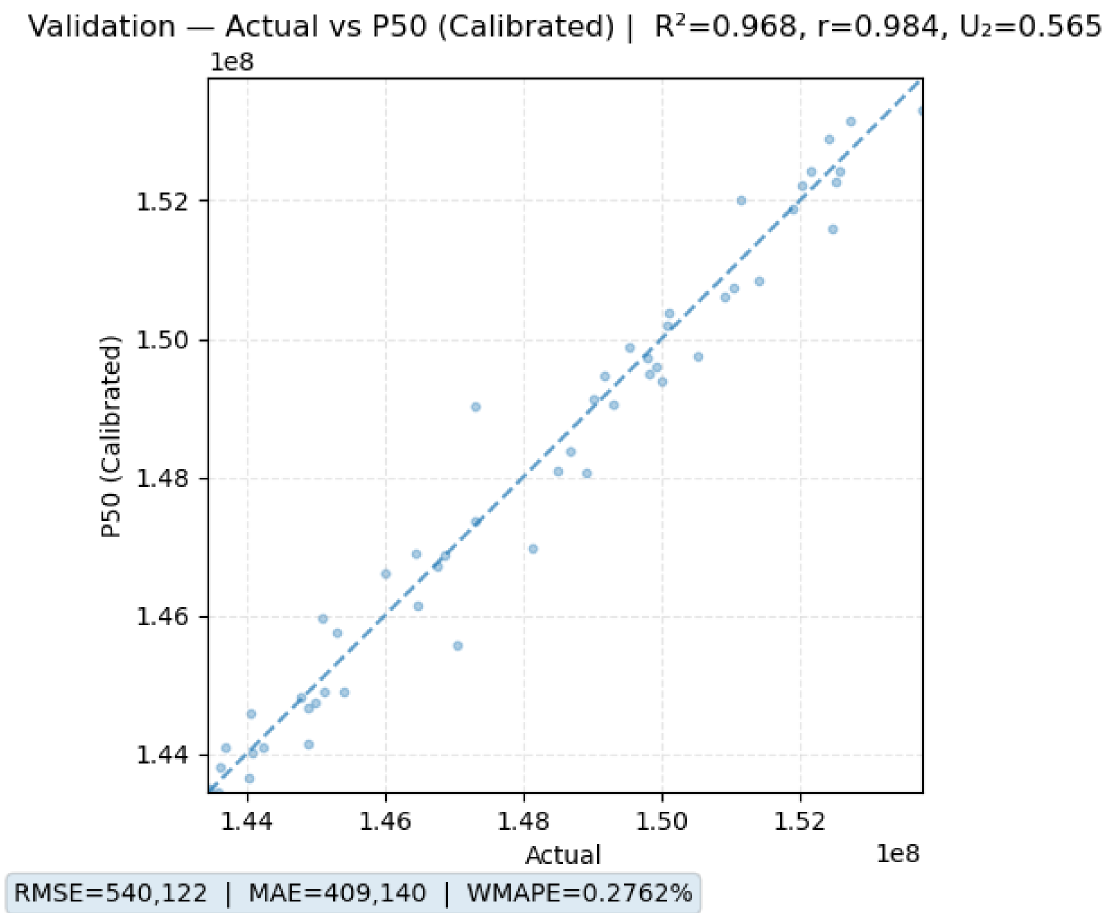
```python
    p10_cal_base, p90_cal_base = lower_base, upper_base
else:
    p50_cal_base = p50_cal.copy(); p10_cal_base = p10_cal.copy(); p90_cal_base = p90_cal.copy()

p50_final = p50_cal_base

taus = np.linspace(0.80, 1.60, 17)
cov_list, wm_list, width_list = [], [], []

for t0 in taus:
    lo = p50_cal_base - (p50_cal_base - p10_cal_base) * t0
    hi = p50_cal_base + (p90_cal_base - p50_cal_base) * t0
    lo = np.minimum.reduce([lo, p50_cal_base, hi])
    hi = np.maximum.reduce([p50_cal_base, hi])

    cov = 100.0 * np.mean((y_val_abs >= lo) & (y_val_abs <= hi))
    wm  = float(100 * np.sum(np.abs(p50_final - y_val_abs)) / np.sum(np.abs(y_val_abs)))
    w   = float(np.median(hi - lo))

    cov_list.append(cov); wm_list.append(wm); width_list.append(w)

lo = p50_cal_base - (p50_cal_base - p10_cal_base) * tau
hi = p50_cal_base + (p90_cal_base - p50_cal_base) * tau
lo = np.minimum.reduce([lo, p50_cal_base, hi])
hi = np.maximum.reduce([p50_cal_base, hi])

cal, _ = _ensure_val_metrics()
cov_at_tau = 100.0 * np.mean((y_val_abs >= lo) & (y_val_abs <= hi))
width_at_tau = float(np.median(hi - lo))

fig, ax = plt.subplots(figsize=(6,3.2))
ax.plot(taus, cov_list, marker="o"); ax.axhline(80.0, linestyle="--")
add_vline_at_tau(ax, tau, " *")
ax.set_xlabel(" "); ax.set_ylabel("Coverage (%)"); ax.set_title("Validation - Coverage vs  ")
ax.grid(True, linestyle="--", alpha=0.3); plt.tight_layout()
footerize(ax, f"Chosen   = {tau:.3f}\nCoverage @    {cov_at_tau:.2f}%")
plt.show()

fig, ax = plt.subplots(figsize=(6,3.2))
ax.plot(taus, wm_list, marker="o")
add_vline_at_tau(ax, tau, " *")
ax.set_xlabel(" "); ax.set_ylabel("WMAPE (%)"); ax.set_title("Validation - WMAPE vs  ")
ax.grid(True, linestyle="--", alpha=0.3); plt.tight_layout()
```

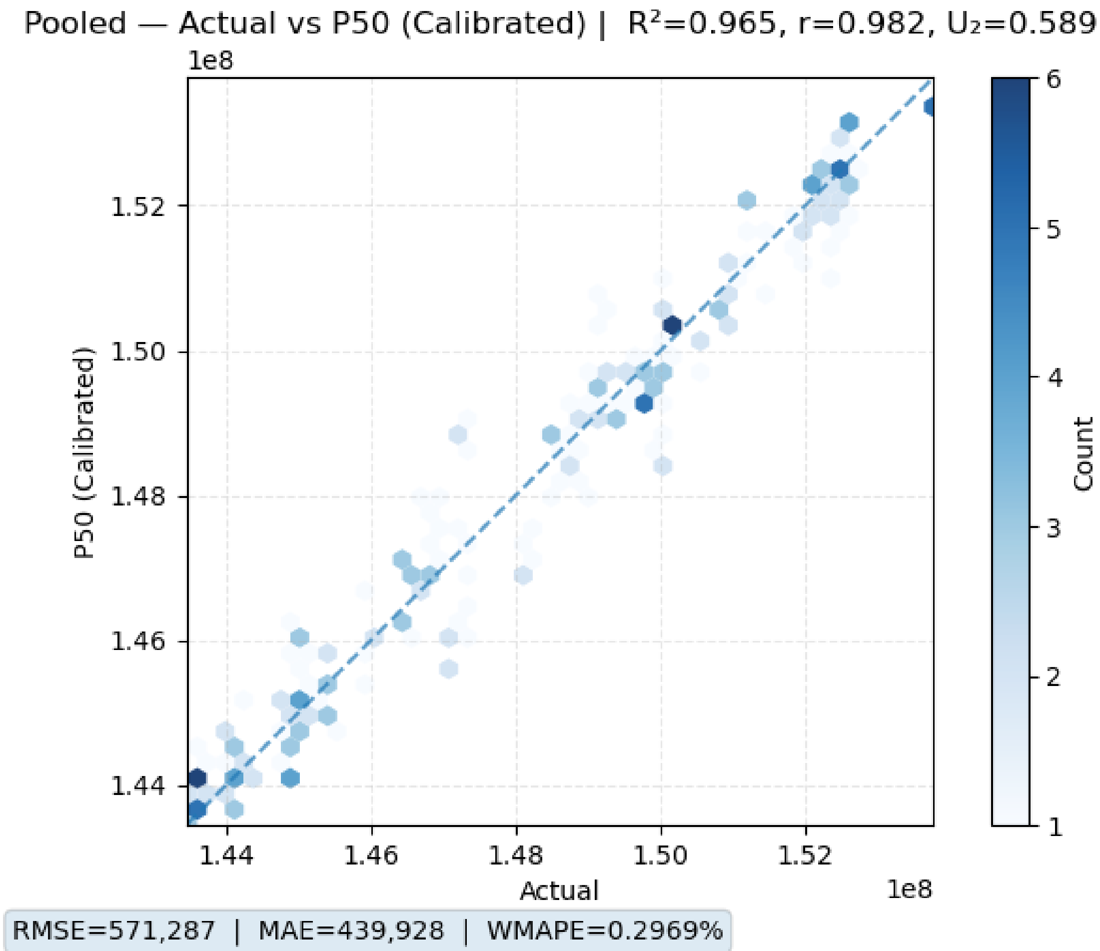
```python
footerize(ax, f" -invariant (P50 Unchanged)\n"f"WMAPE = {cal['WMAPE']:.4f}%")
plt.show()

fig, ax = plt.subplots(figsize=(6,3.2))
ax.plot(taus, width_list, marker="o")
add_vline_at_tau(ax, tau, " *")
ax.set_xlabel(" "); ax.set_ylabel("Median Band Width"); ax.
 ↪set_title("Validation - Sharpness vs  ")
_fmt_ax_yint(ax); plt.tight_layout()
footerize(ax, f"Median Width @  ={tau:.3f}: {width_at_tau:,.0f}")
plt.show()

# Validation - Step-Wise Improvement

assert "p50_abs" in globals(), "p50_abs (uncalibrated P50) missing; computed␣
 ↪earlier in §6."

val_abs_err_base = np.mean(np.abs(y_val_abs - BASE_val))
val_abs_err_unc  = np.mean(np.abs(y_val_abs - p50_abs))
val_abs_err_cal  = np.mean(np.abs(y_val_abs - p50_cal))

fig, ax = plt.subplots(figsize=(6,3))
values = [val_abs_err_base, val_abs_err_unc, val_abs_err_cal]
labels = ["Baseline", "Uncalibrated", "Calibrated"]
ax.bar(labels, values)
for i, v in enumerate(values):
    ax.text(i, v, f"{v:,.0f}", ha="center", va="bottom", fontsize=9)

ax.set_ylabel("Mean |error| (Validation)")
ax.set_title("Validation - Step-Wise Improvement")
_fmt_ax_yint(ax); plt.tight_layout()
footerize(ax, f"Δ|err| (Cal - Base): {val_abs_err_cal - val_abs_err_base:,.0f}")
plt.show()
```

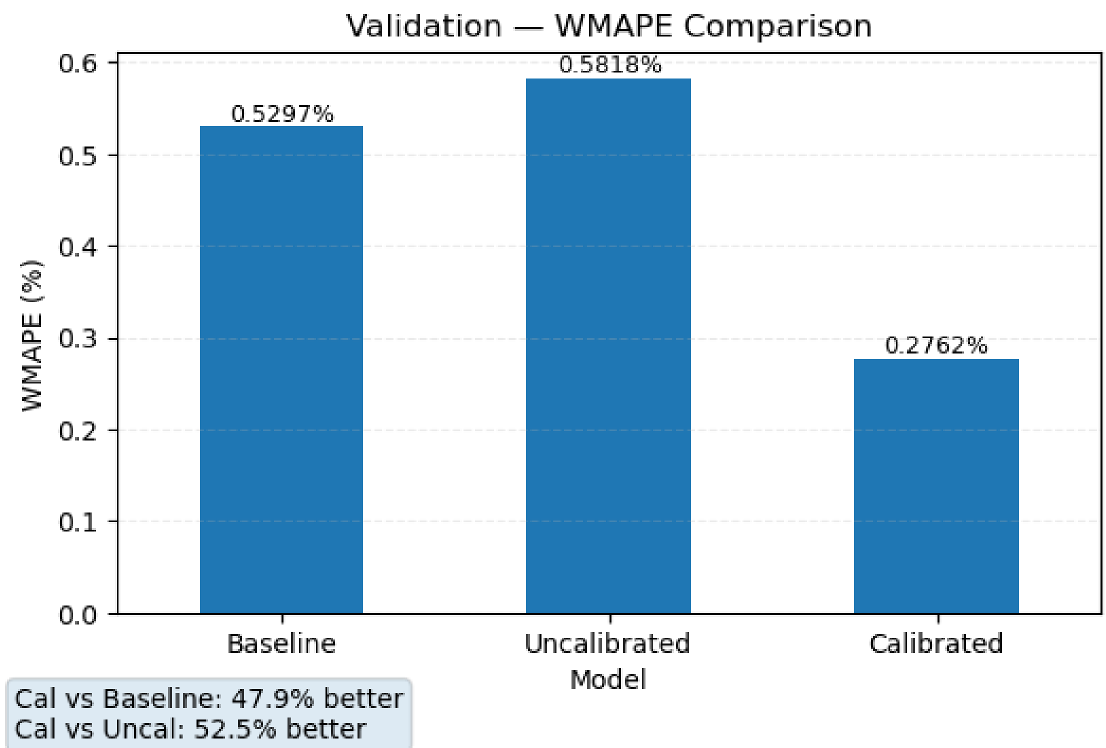




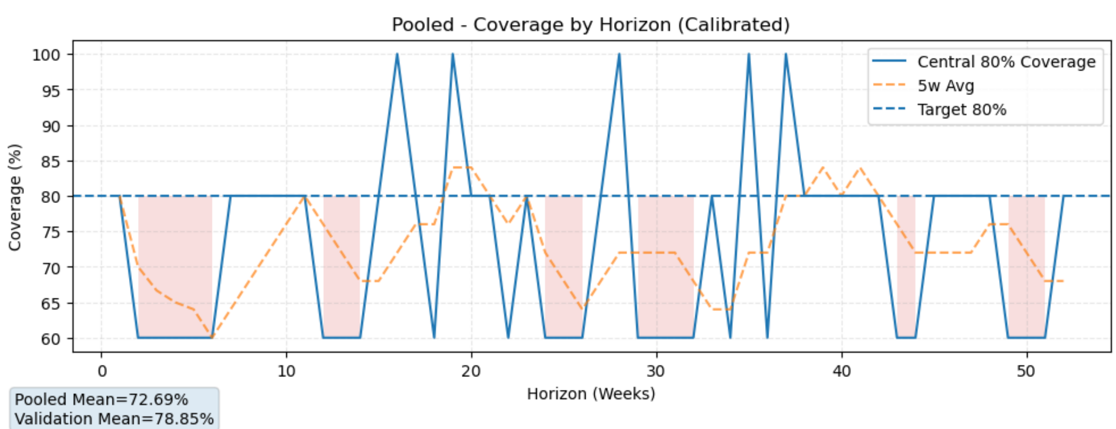
WMAPE: Cal 0.2762% | Base 0.5297% | Uncal 0.5818%
Δ vs Base: 47.9% | Δ vs Uncal: 52.5%
Coverage (P10–P90): 78.85% (target 80%)


Validation — Actual vs P50 (Calibrated) | R²=0.968, r=0.984, U₂=0.565
1e8
1.52
1.50
1.48
1.46
1.44
P50 (Calibrated)
1.44
1.46
1.48
1.50
1.52
Actual
1e8
RMSE=540,122 | MAE=409,140 | WMAPE=0.2762%

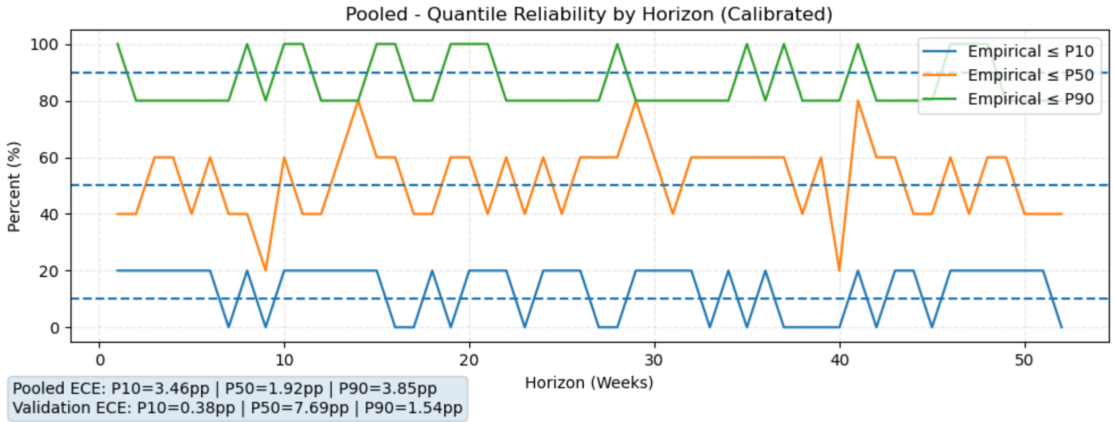

Pooled — Actual vs P50 (Calibrated) | R^2=0.965, r=0.982, U_2=0.589
1e8
1.52
1.50
1.48
1.46
1.44
P50 (Calibrated)
1.44
1.46
1.48
1.50
1.52
Actual
1e8
6
5
4
3
2
1
Count
RMSE=571,287 | MAE=439,928 | WMAPE=0.2969%

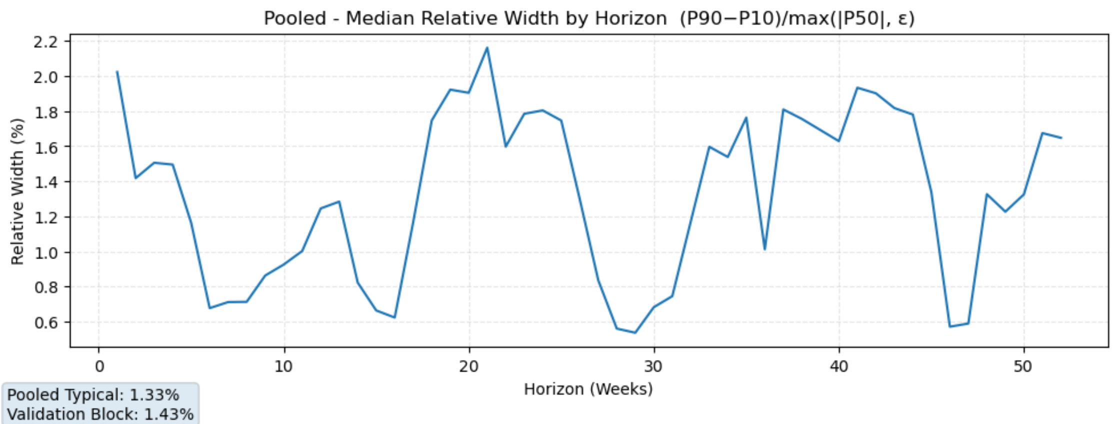


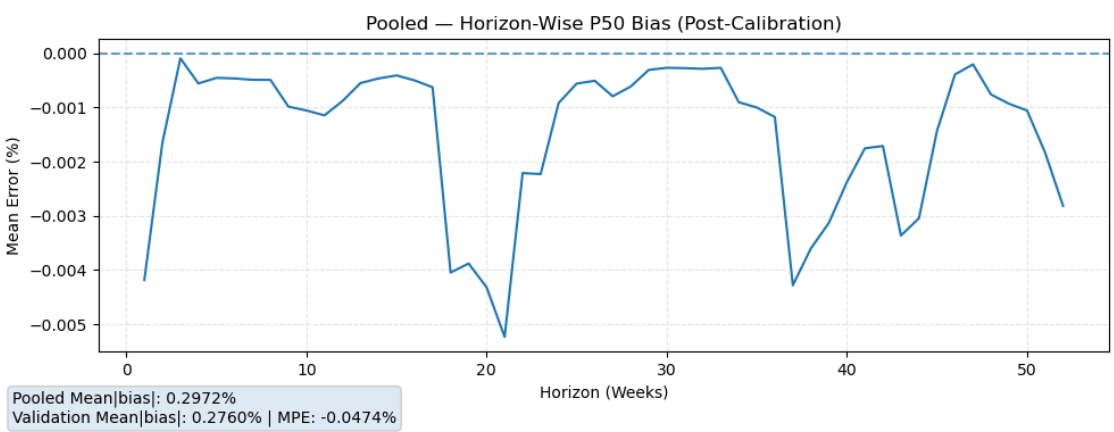
Validation — WMAPE Comparison
WMAPE (%)
0.5297%
0.5818%
0.2762%
Baseline
Uncalibrated Model
Calibrated
Cal vs Baseline: 47.9% better
Cal vs Uncal: 52.5% better

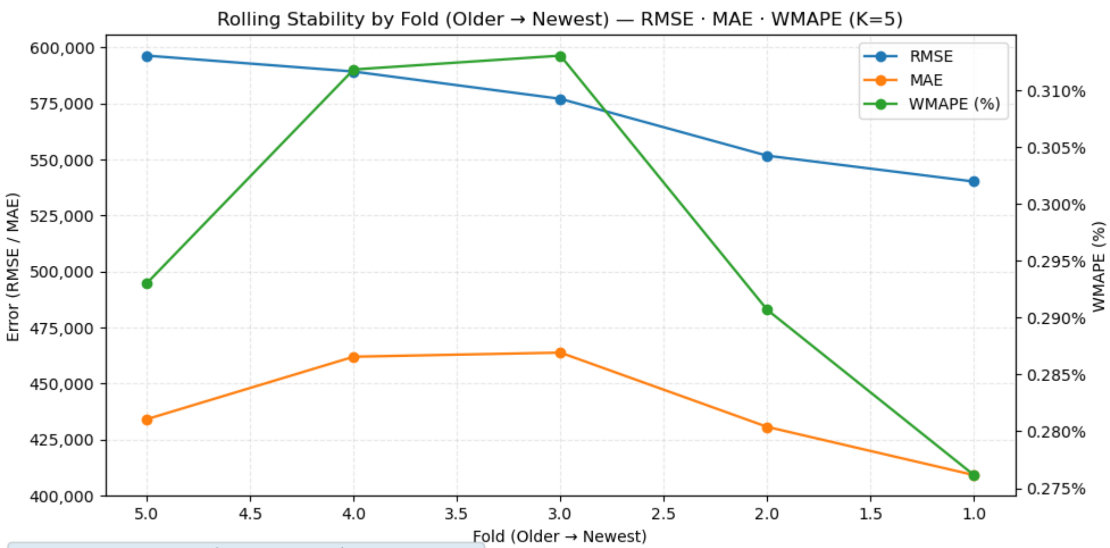


Pooled - Coverage by Horizon (Calibrated)
Central 80% Coverage
5w Avg
Target 80%
Coverage (%)
Horizon (Weeks)
Pooled Mean=72.69%
Validation Mean=78.85%

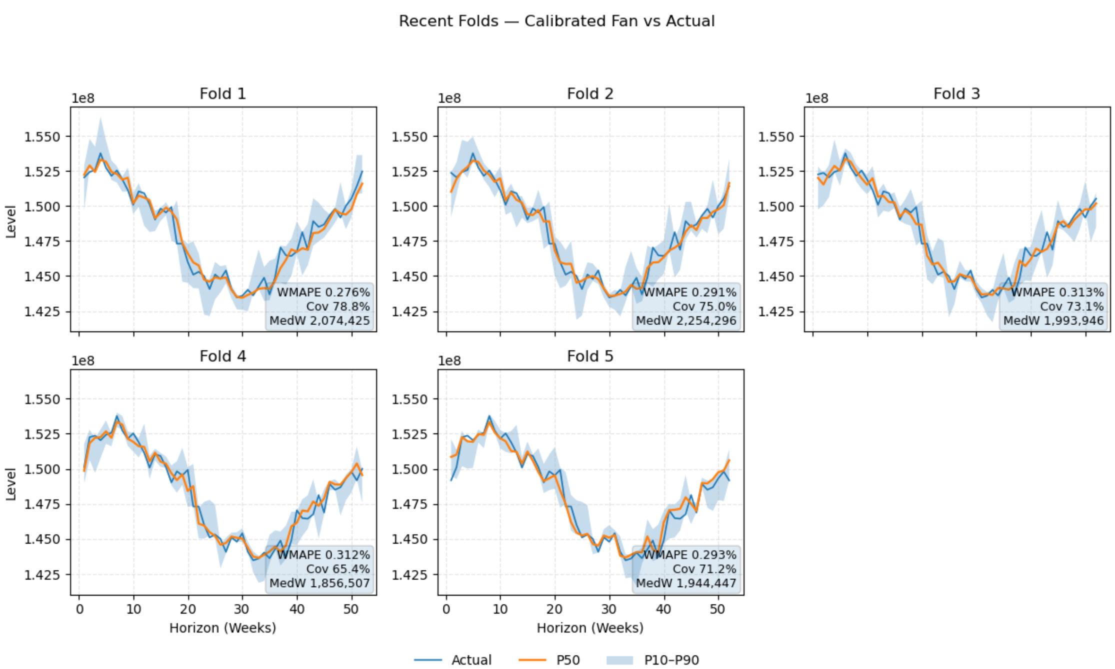

Pooled - Quantile Reliability by Horizon (Calibrated)
Empirical ≤ P10
Empirical ≤ P50
Empirical ≤ P90
Percent (%)
Horizon (Weeks)
Pooled ECE: P10=3.46pp | P50=1.92pp | P90=3.85pp
Validation ECE: P10=0.38pp | P50=7.69pp | P90=1.54pp


Pooled - Median Relative Width by Horizon (P90−P10)/max(|P50|, ε)
Relative Width (%)
Horizon (Weeks)
Pooled Typical: 1.33%
Validation Block: 1.43%

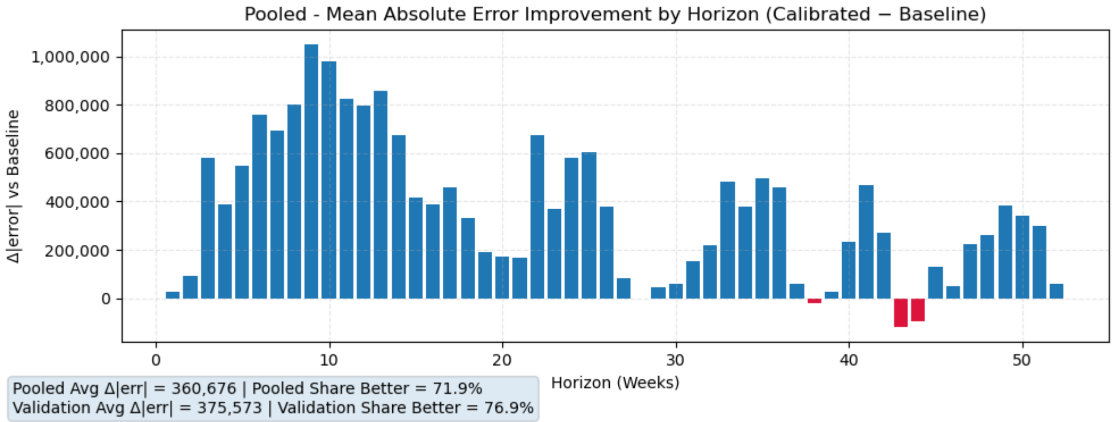

Pooled — Horizon-Wise P50 Bias (Post-Calibration)
Mean Error (%)
Horizon (Weeks)
Pooled Mean|bias|: 0.2972%
Validation Mean|bias|: 0.2760% | MPE: -0.0474%

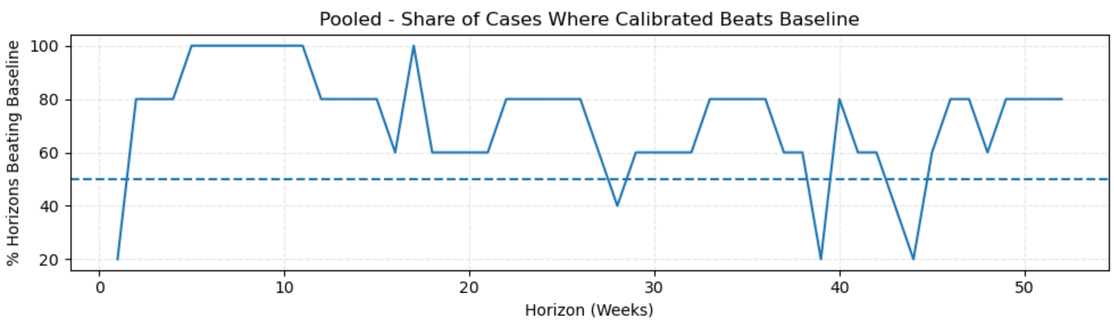


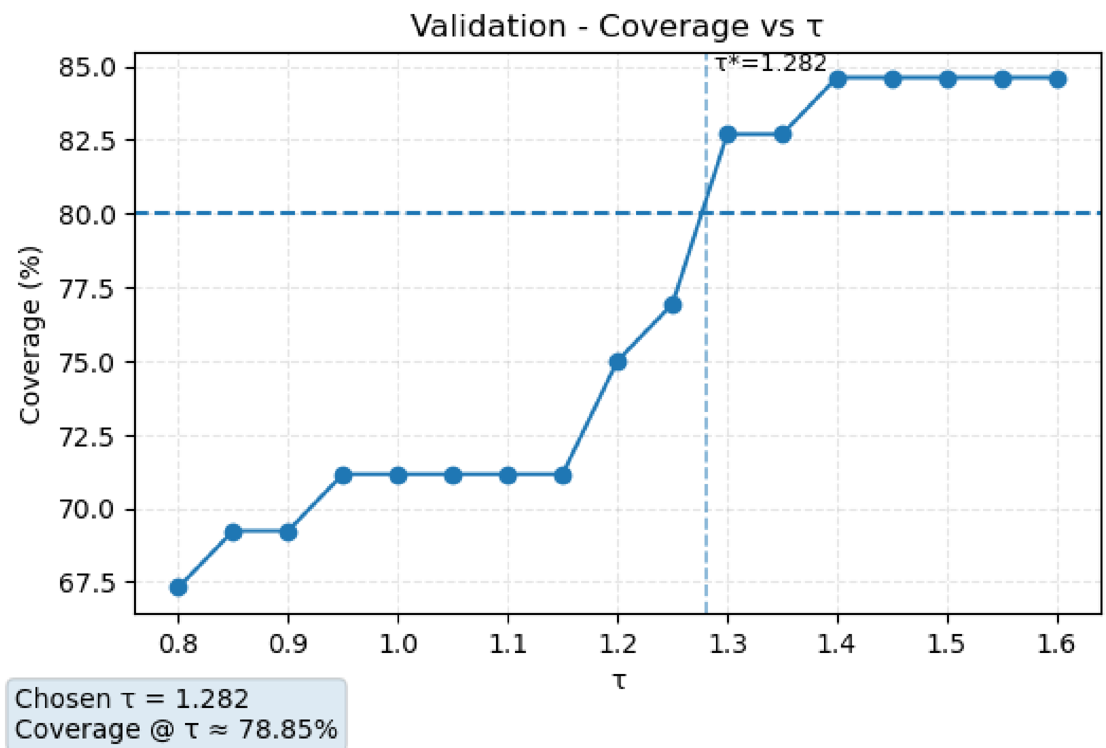
Means — RMSE: 570,878 | MAE: 439,928 | WMAPE: 0.297%

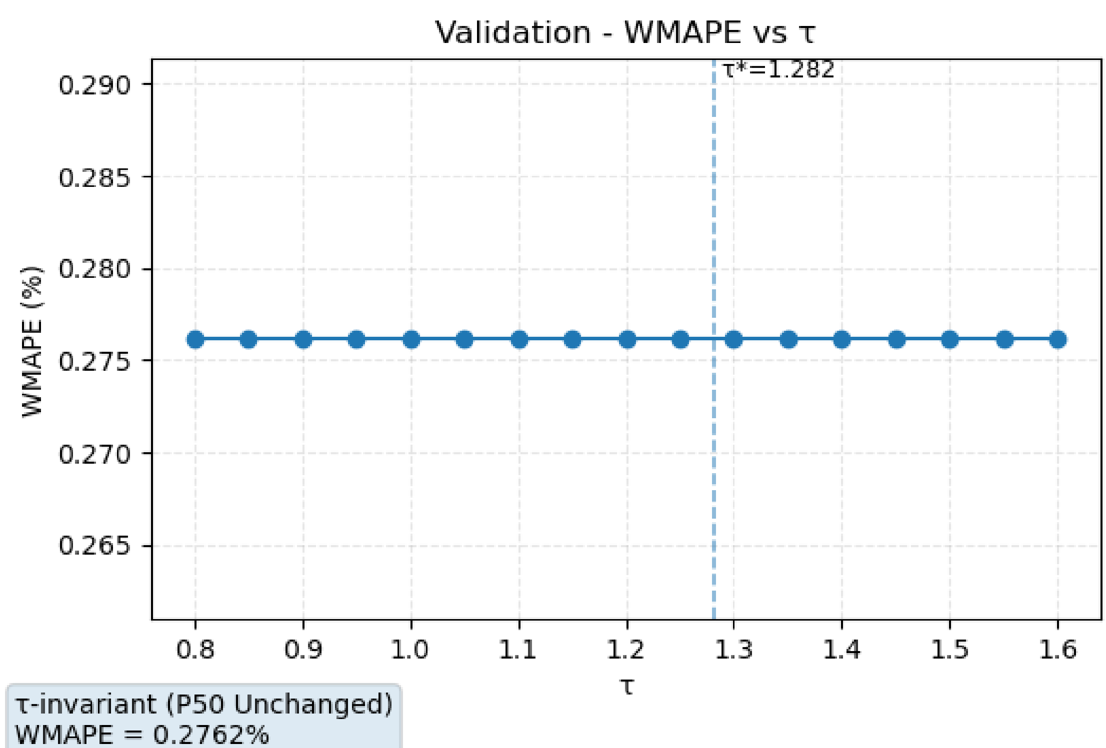


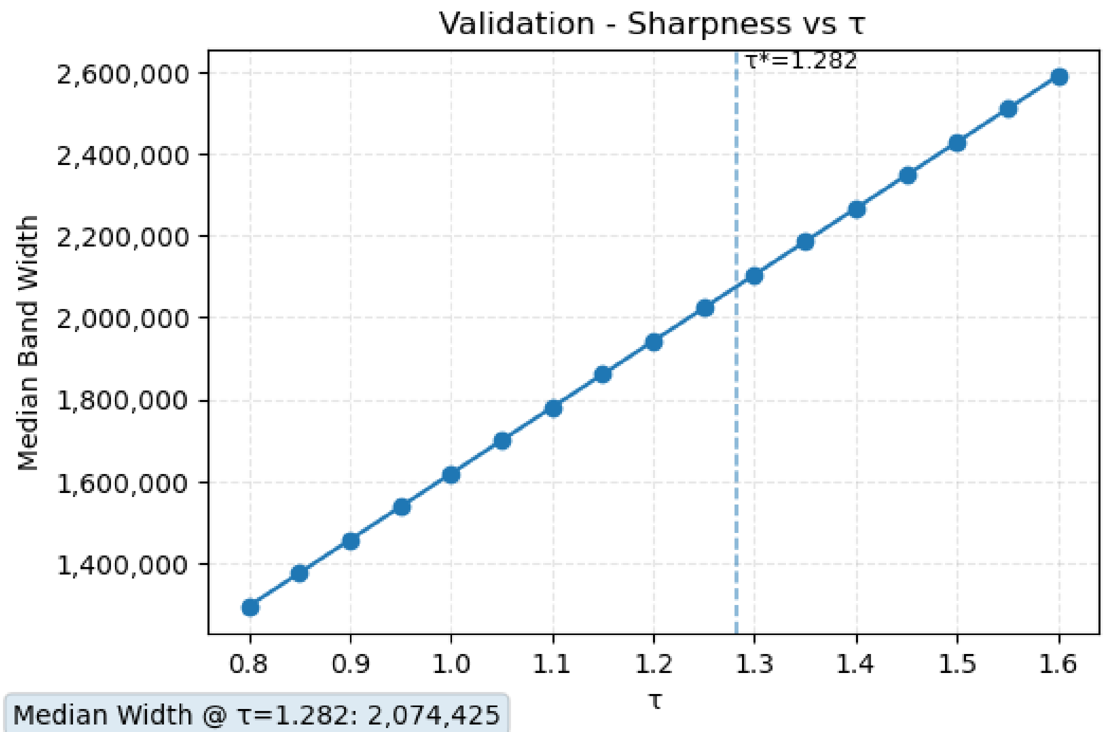
Window = Last 5 Folds (Newest → Older as 1...5)
Coverage (central 80%) — Mean 72.69% | Min 65.4% | Max 78.8%
Sharpness — Median Width Oldest vs Newest: 1,944,447 → 2,074,425 (+6.7%)
Accuracy — WMAPE Oldest vs Newest: 0.293% vs 0.276% (improvement +0.017 pp)

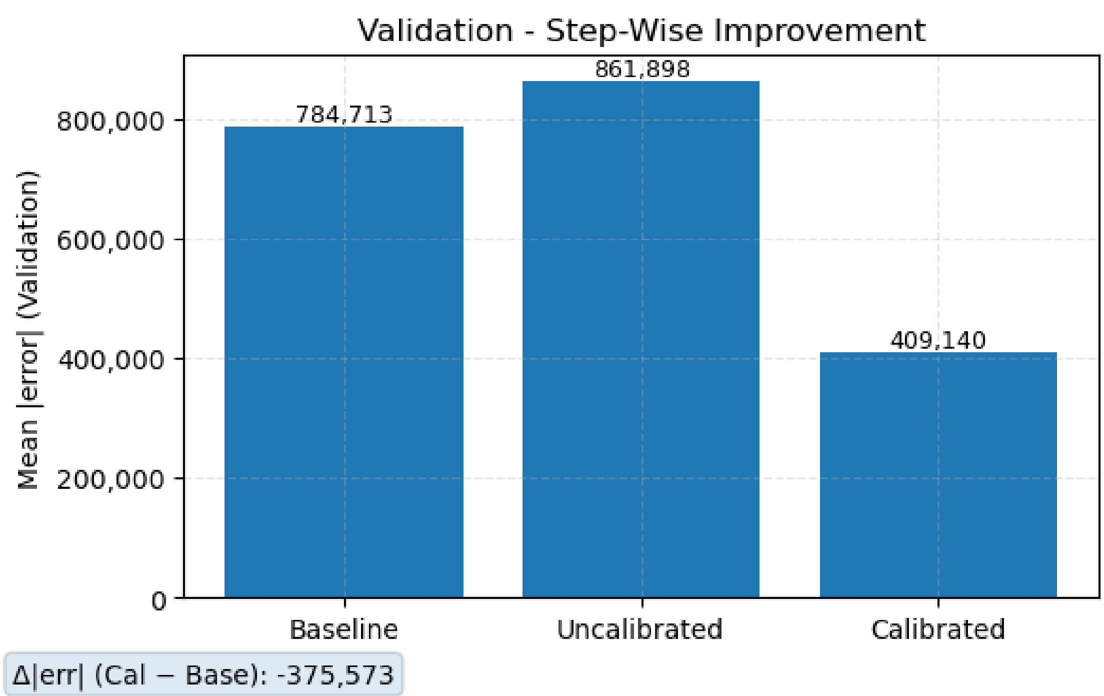

Pooled - Mean Absolute Error Improvement by Horizon (Calibrated − Baseline)
Δ|error| vs Baseline
Horizon (Weeks)
Pooled Avg Δ|err| = 360,676 | Pooled Share Better = 71.9%
Validation Avg Δ|err| = 375,573 | Validation Share Better = 76.9%


Pooled - Share of Cases Where Calibrated Beats Baseline
% Horizons Beating Baseline
Horizon (Weeks)


Validation - Coverage vs τ
τ*=1.282
Coverage (%)
85.0
82.5
80.0
77.5
75.0
72.5
70.0
67.5
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
τ
Chosen τ = 1.282
Coverage @ τ ≈ 78.85%


Validation - WMAPE vs τ
τ*=1.282
WMAPE (%)
0.290
0.285
0.280
0.275
0.270
0.265
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
τ
τ-invariant (P50 Unchanged)
WMAPE = 0.2762%


Validation - Sharpness vs τ
τ*=1.282
Median Band Width
2,600,000
2,400,000
2,200,000
2,000,000
1,800,000
1,600,000
1,400,000
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
τ
Median Width @ τ=1.282: 2,074,425


Validation - Step-Wise Improvement
Mean |error| (Validation)
800,000
600,000
400,000
200,000
0
784,713
861,898
409,140
Baseline
Uncalibrated
Calibrated
Δ|err| (Cal − Base): -375,573

1.13 Future export (multi-year, residual-consistent)

Forecast in **52-week blocks**: 1) Build window at the cursor,
2) Predict residual quantiles → inverse-scale → **add baseline** for that future block,
3) Apply **the same** $(a_h, b_h, \delta_h, \tau)$ calibration,
4) Write the block (P10/P50/P90) and **overwrite timeline P50** so future `lag52` stays consistent.

This preserves the residual framing deep into the horizon.

[151]:
```python
future_df = make_calendar_frame(future_weeks)

_tmp = pd.DataFrame({"Date": dfm["Date"], "Sales": dfm["Sales"]})
timeline = pd.merge(_tmp, future_df, on="Date", how="outer").
 ↪sort_values("Date").reset_index(drop=True)

for c in [f"sin_{k}" for k in range(1,7)] + [f"cos_{k}" for k in range(1,7)] +␣
 ↪["is_holiday_week","hol_dist_w","hol_near1w"]:
    timeline[c] = timeline[c].ffill().bfill()

timeline["Sales_scaled"] = np.nan
past_mask = timeline["Date"] <= dfm["Date"].max()
timeline.loc[past_mask, "Sales_scaled"] = (
    sales_scaler.transform(timeline.loc[past_mask, ["Sales"]].
 ↪fillna(method="ffill").to_numpy()).ravel()
)

timeline["lag52"] = timeline["Sales"].shift(52)
timeline["lag26"] = timeline["Sales"].shift(26)
timeline["lag78"] = timeline["Sales"].shift(78)
timeline["rollmean52"] = timeline["Sales"].shift(1).rolling(ROLLING_WINDOW,␣
 ↪min_periods=1).mean()
timeline["ema13"] = timeline["Sales"].ewm(span=13, adjust=False).mean().shift(1)

for c in ["lag52", "lag26", "lag78", "rollmean52", "ema13"]:
    timeline[c] = timeline[c].ffill().bfill().fillna(df["Sales"].median())

for c in ["lag52", "lag26", "lag78", "rollmean52", "ema13"]:
    if c in timeline:
        timeline[f"{c}_scaled"] = sales_scaler.transform(timeline[[c]].
 ↪to_numpy()).ravel()

feature_cols_roll = feature_cols[:]
records = []
cursor = len(dfm)

def build_window(frame: pd.DataFrame, end_idx: int):
    sl = frame.iloc[end_idx - IN_LEN : end_idx].copy()
    Xseq = np.concatenate(
```

```
        [
            sl["Sales_scaled"].values.reshape(-1, 1).astype(np.float32),
            sl[feature_cols_roll].values.astype(np.float32),
        ],
        axis=1,
    )
    return Xseq[np.newaxis, :, :]

while cursor < len(timeline):
    if cursor - 1 >= 0:
        timeline.loc[: cursor - 1, "Sales_scaled"] = (
            sales_scaler.transform(timeline.loc[: cursor - 1, ["Sales"]].
↪fillna(method="ffill").to_numpy()).ravel()
        )

    Xw = build_window(timeline, cursor)
    preds = model.predict(Xw, verbose=0)
    q_res, _ = extract_quantiles(preds)

    base_block = timeline["Sales"].shift(52).iloc[cursor : cursor + OUT_LEN].
↪fillna(method="ffill").to_numpy()

    q_abs = resid_scaler.inverse_transform(q_res).ravel().reshape(OUT_LEN, 3) +␣
↪base_block[:, None]
    p10_next, p50_next, p90_next = q_abs[:, 0], q_abs[:, 1], q_abs[:, 2]

    if APPLY_CAL_ON_EXPORT and DO_POSTHOC_CAL:
        p10_next = p10_next + off10_h
        p50_next = a_h * p50_next + b_h
        p90_next = p90_next + off90_h
        lower = np.minimum.reduce([p10_next, p50_next, p90_next])
        upper = np.maximum.reduce([p50_next, p90_next])
        p50_next = np.clip(p50_next, lower, upper)
        p10_next, p90_next = lower, upper
        p10_next, p90_next = apply_temperature(p10_next, p50_next, p90_next,␣
↪tau)
        lower = np.minimum.reduce([p10_next, p50_next, p90_next])
        upper = np.maximum.reduce([p50_next, p90_next])
        p50_next = np.clip(p50_next, lower, upper)
        p10_next, p90_next = lower, upper

    block_dates = timeline["Date"].iloc[cursor : cursor + OUT_LEN].to_numpy()
    rec = pd.DataFrame({"Date": block_dates, "P10": p10_next, "P50": p50_next,␣
↪"P90": p90_next})
    records.append(rec)

    timeline.loc[cursor : cursor + OUT_LEN - 1, "Sales"] = p50_next
```

```
    timeline.loc[cursor : cursor + OUT_LEN - 1, "Sales_scaled"] = sales_scaler.transform(p50_next.reshape(-1, 1)).ravel()

    timeline["lag52"] = timeline["Sales"].shift(52).ffill().bfill().fillna(df["Sales"].median())
    timeline["lag26"] = timeline["Sales"].shift(26).ffill().bfill().fillna(df["Sales"].median())
    timeline["lag78"] = timeline["Sales"].shift(78).ffill().bfill().fillna(df["Sales"].median())
    timeline["rollmean52"] = (
        timeline["Sales"].shift(1).rolling(ROLLING_WINDOW, min_periods=1).mean().ffill().bfill().fillna(df["Sales"].median())
    )
    timeline["ema13"] = timeline["Sales"].ewm(span=13, adjust=False).mean().shift(1).ffill().bfill().fillna(df["Sales"].median())

    for c in ["lag52", "lag26", "lag78", "rollmean52", "ema13"]:
        timeline[f"{c}_scaled"] = sales_scaler.transform(timeline[[c]].to_numpy()).ravel()

    cursor += OUT_LEN

future_fc = pd.concat(records, ignore_index=True)
future_fc = future_fc[future_fc["Date"].isin(future_weeks)].copy()
future_fc["Forecast Date"] = (latest_date + pd.Timedelta(days=9)).strftime("%Y-%m-%d")
future_fc["Channel"] = f"Carnivore - Kraken ({PROJECT_TAG})"
future_fc["Prediction"] = future_fc["P50"]

pd.DataFrame({
    "Date": val_dates,
    "Actual": y_val_abs,
    "P10": p10_cal,
    "P50": p50_cal,
    "P90": p90_cal,
    "tau": [tau]*len(val_dates)
}).to_csv(os.path.join(ARTIFACTS_DIR, f"Validation_detail_{PROJECT_TAG}_{latest_date.date()}.csv"), index=False)

metrics_roll.to_csv(os.path.join(ARTIFACTS_DIR, f"Rolling_backtest_{PROJECT_TAG}_{latest_date.date()}.csv"), index=False)
calib_roll.to_csv(os.path.join(ARTIFACTS_DIR, f"Rolling_calibration_{PROJECT_TAG}_{latest_date.date()}.csv"), index=False)

xlsx_path = os.path.join(PREDICT_DIR, f"{PROJECT_TAG} - Carnivore - Kraken - {latest_date.date()}.xlsx")
```

```python
future_fc.to_excel(xlsx_path, index=False)
print(f"Forecast saved: {xlsx_path}")
```

```
Forecast saved: /home/linux/Source/Dev/Transformer/Prediction/PatchTST -
Carnivore - Kraken - 2025-06-14.xlsx
```