2) $\Delta_1 = \sup_{x \simeq x'} \| f(x) - f(x') \|_1$

a) $x \simeq x' \Leftrightarrow \|x - x'\|_0 = 1 \longrightarrow f(x) = w^T x$

in the worst case the input changes in the biggest coefficient of the weight vector, let's call it w_{max}

$\Rightarrow \| f(x) - f(x') \|_1 = w_{max}$

b) the mean should stay 0 since we're to preserve the original mean

$\Rightarrow z \sim \mathrm{Lap}(0, \frac{\Delta_1}{\epsilon}) = \mathrm{Lap}(0, \frac{w_{max}}{\epsilon})$

c) $\mathrm{Lap}(z; 0, b) = \frac{1}{2b} \exp \frac{-|z|}{b}$

$$\frac{P[M_f(x) \in y]}{P[M_f(x') \in y]} = \prod_{i=1}^{d} \frac{\exp -\epsilon \cdot | f(x)_i - z |}{\exp -\epsilon \, | f(x')_i - z |} = \prod_{i=1}^{d} \exp \epsilon \left(| f(x')_i - z | - | f(x)_i - z | \right)$$

$$\leq \prod_{i=1}^{d} e^{\epsilon | f(x')_i - f(x)_i |} = e^{\epsilon \sum_{i=1}^{d} | f(x')_i - f(x)_i |} =$$

we know the input can change up to 1 bit $\Rightarrow \sum_{i=1}^{d} | f(x')_i - f(x)_i | = 1$

$= e^{\epsilon \cdot 1}$

3) a) Δ_1 with $f' := \mathrm{mean}(f(G_1), \dots f(G_m))$

$\Delta_1 = \sup_{x \simeq x'} \| f(x) - f'(x) \|_1$

assuming $\simeq \longrightarrow x \simeq x' \Leftrightarrow \| x - x' \|_0 = 1$

since only one sample is changed and $f: \mathcal{X}^{\frac{n}{m}} \longrightarrow [a, b]$

suppose $f' = a \lor f' = b$ originally and we change some entry in group G_i such that

$g(f(G_i')) = b \lor g(f(G_i')) = a$ respectively

↳ changed sample belongs to the ith group

$\Rightarrow \Delta_1 = \| f'(x) - f'(x') \|_1 = b - \left(\frac{(m-1)b + a}{m} \right) = \frac{b-a}{m}$

if $f' = b$, a is analogous

(4) we have $e^{-\varepsilon} \leq \frac{P[M_f(x) \in y]}{P[M_f(x') \in y]} \leq e^{\varepsilon}$

$$\rightarrow \frac{P[M'_f(x) \in y]}{P[M'_f(x') \in y]} = \frac{P[M_{f_1}(x) \in y,\ M_{f_2}(x) \in y,\ \dots,\ M_{f_T}(x) \in y]}{P[M_{f_1}(x') \in y,\ M_{f_2}(x') \in y,\ \dots,\ M_{f_T}(x') \in y]}$$

independent samples

$$= \prod_{i=1}^{T} \frac{P[M_{f_i}(x) \in y]}{P[M_{f_i}(x') \in y]} \leq \prod_{i=1}^{T} e^{\varepsilon} = e^{T\varepsilon} \; //$$

$\hookrightarrow$ each M holds $\frac{P[M_{f_i}(x) \in y]}{P[M_{f_i}(x') \in y]} \leq e^{\varepsilon}$