

$$③ \quad y = \log \sum_{i=1}^N e^{x_i} = a + \log \sum_{i=1}^N e^{x_i - a}$$

$$\cdot a + \log \sum_{i=1}^N e^{x_i - a} = a + \log \sum_{i=1}^N \frac{e^{x_i}}{e^a} = a + \log \left( \frac{1}{e^a} \sum_{i=1}^N e^{x_i} \right) = a + \log(e^{-a}) + \log \left( \sum_{i=1}^N e^{x_i} \right) = \cancel{a} - \cancel{a}/1 + \log \left( \sum_{i=1}^N e^{x_i} \right) = \log \left( \sum_{i=1}^N e^{x_i} \right) //$$

$$④ \quad \frac{e^{x_i}}{\sum_{i=1}^N e^{x_i}} = \frac{e^{x_i - a}}{\sum_{i=1}^N e^{x_i - a}}$$

$$\cdot \frac{e^{x_i - a}}{\sum_{i=1}^N e^{x_i - a}} = \frac{e^{x_i}}{\cancel{e^a}} \cdot \frac{1}{\cancel{e^a} \sum_{i=1}^N e^{x_i}} = \frac{e^{x_i}}{\sum_{i=1}^N e^{x_i}} //$$