CSCI573000 Assignment 1

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1. Given a random $d \times 1$ vector, the covariance matrix $\Sigma = \mathbb{E}[(x-\mu)(x-\mu)^T]$ of this vector is size $d \times d$ with variance at each ij^{th} point to be $\Sigma_{ij} = \mathbb{E}\big(X_i - \mathbb{E}(X_i)\big)\left(X_j - \mathbb{E}(X_j)\right)$. This matrix Σ is positive semi-definite, such that for any $1 \times d$ vector $x \in \mathbb{R}^d$, $x\Sigma x^T \geq 0$.
2. Given a random variable $X_i = X_1, X_2, \dots, X_n$, $\mathbb{E}[X_i] = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}[X_n]$. If X_i terms are independent, then $var(X_i) = Var(X_1) + Var(X_2) + \cdots + Var(X_n)$, with a covariance matrix of $Var(X) = [\begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$. Given the above, the total variance $var(X_i) = x\Sigma x^T = \sigma_1^2 + \sigma_2^2$.