Mode function decomposition method

Kang Xu [1] (xukang0527@hotmail.com) and Rui Xin Huang [1, 2]

1 State Key Laboratory of Tropical Oceanography, South China Sea Institute of Oceanology,

Chinese Academy of Science, Guangzhou, China

2 Woods Hole Oceanographic Institute, Woods Hole, Massachusetts, USA

The lowest-order mode (or Mode 0) of the equatorial thermocline is a constant, $f_0 \equiv 1$, representing the mean depth of the equatorial thermocline. Therefore, the thermocline depth deviation from this basin-mean depth can be decomposed into a series of mode functions. The mode functions are defined over the domain [−1, +1], and they are orthogonal to each other, as a fundamental requirement. These modes start from a value of −1 at the left edge of the domain (i.e. $x = -1$) and consist of line segments that vary within the range [−1, +1]. Mode *n* has *n* zero-crossings. As a result, the first mode is a straight line from (−1, −1) to (+1, +1), and all other modes are composed of the straight line segments between −1 and +1, as shown in Fig. A1. The tradition in oceanography is to label depth as a positive value and define the downward direction as positive. This is opposite to the common practice in meteorology; however, in presenting the anomalous signals associated with the thermocline depth, this method can present the signals in a simple and accurate way.

These mode functions are further spatially normalized to a series of functions $f(x)$ satisfying the following unification constraints:

$$\int_{-1}^{1} f_i(x) \cdot f_j(x) dx = \delta_{ij} \quad (i, j \geq 1) \tag{A1}$$

where the subscripts i and j are the mode numbers, and δ is the Kronecker delta function. Regarding the temporal evolution of the decomposed modes, the main equatorial thermocline depth anomalies $D(x, t)$ averaged between 5 °S and 5 °N, defined over the non-dimensional domain $x = [-1, +1]$, can be projected onto these mode function series for any given time t as follows:

$$D(x,t) = \sum_{i=1}^{\infty} a_i(t) \cdot f_i(x); \;\; a_i(t) = \int_{-1}^{1} f_i(x) \cdot D(x,t) dx \tag{A2}$$

where $a_i(t)$ is the time evolution for each mode i.

Figure A1 presents the visual image of the normalized structures of the first six mode functions. The mode functions are orthogonal to each other and have the same amplitude of 1.22; thus, in terms of thermocline depth variability, the amplitude of the contribution from each mode is $1.22 \cdot a_i(t)$. As seen in Fig. A1, the spatial structure of the first mode (M1, depicted by a solid black line), clearly shows a basin-wide east–west-tilted thermocline, with shoaling in the west and deepening in the east, which depicts the basic feature of classical EP El Niño events (Fig. A1). However, the second mode (M2, marked by the V-shape red curve) has a deeper thermocline positioned in the central Pacific Ocean basin and a shallower thermocline on both side boundaries, representing the characteristic of CP El Niño events (Fig. A1). Therefore, this mode function decomposition method may provide a direct physical interpretation of the equatorial thermocline using only a few modes, and the first two leading modes have better capabilities in capturing and distinguishing the characteristics of the two types of El Niño events.

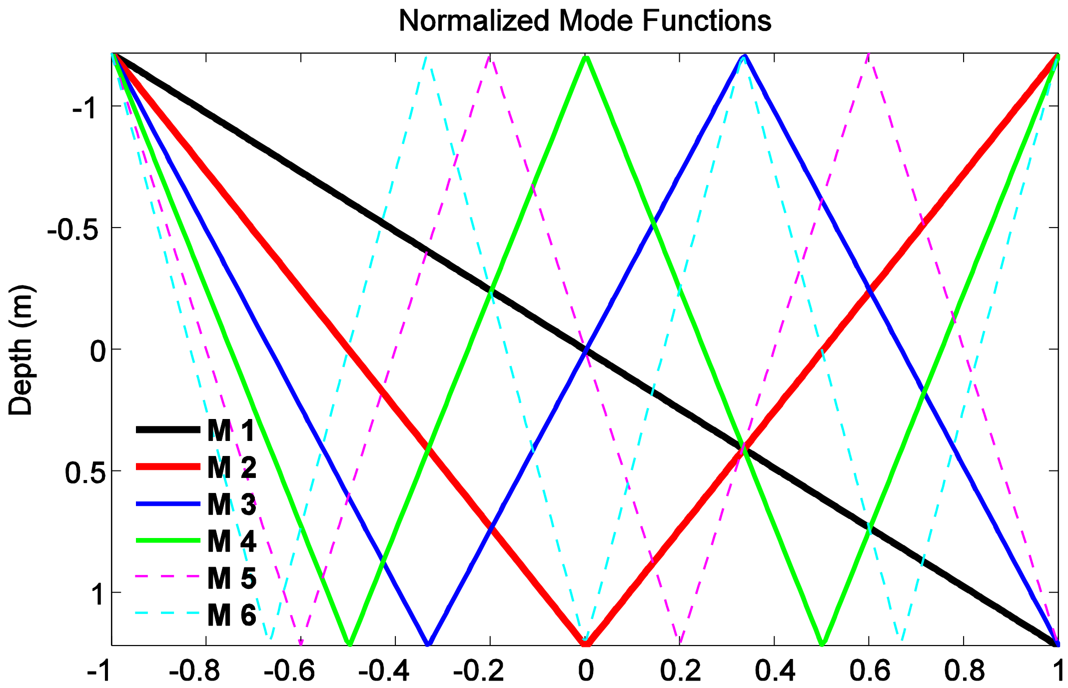

FIG. A1. First six normalized mode functions for the equatorial thermocline. Following the tradition in oceanography, the depth is positive in the downward direction.