Assignment 5 of MATP6610/4820

(Due on March-26-2019 in class)

Problem 1

Consider the problem

$$\min_{x,y\in\mathbb{R}} f(x,y) = \frac{1}{2}x^2 - xy + y^2 - 4x + 6y, \text{ s.t. } x \le 2, y \le 2.$$

1. Does $(0,0)$ satisfy the optimality condition?
2. Note that f is convex. Find the global minimizer.

Problem 2

Consider the problem

$$\begin{aligned}\min_{x,y\in\mathbb{R}} & \; f(x,y) = -x^2 + 2xy + y^2 - x - 2y \\ \text{s.t.} & \; x \ge 0, y \ge 0, x + y \le 4.\end{aligned}$$

In class, we showed that $(1/4, 3/4)$ and $(4,0)$ are two KKT points. Find all other KKT points if there is any.

Problem 3 **(only for 6610 students)**

In class we did the convergence analysis of the quadratic penalty methods for affine equality constrained problems. Now consider the inequality constrained problem

$$\min_{\mathbf{x}\in X} f(\mathbf{x}), \text{s.t. } g_i(\mathbf{x}) \le 0, \; i = 1, \ldots, m. \tag{1}$$

Let $\mu_{k+1} > \mu_k > 0$ for any $k \ge 0$ and $\mu_k \to \infty$ as $k \to \infty$. Generate the sequence $\{\mathbf{x}^k\}$ as

$$\mathbf{x}^k = \arg\min_{\mathbf{x}\in X} f(\mathbf{x}) + \frac{\mu_k}{2}\sum_{i=1}^m [g_i(\mathbf{x})]_+^2,$$

where $[a]_+ = \max(0, a)$ for any $a \in \mathbb{R}$. Suppose that $\{\mathbf{x}^k\}$ has a finite limit point $\bar{\mathbf{x}}$. Show that $\bar{\mathbf{x}}$ is an optimal solution, and also

$$\lim_{k\to\infty} f(\mathbf{x}^k) = f(\mathbf{x}^*), \quad \lim_{k\to\infty}[g_i(\mathbf{x}^k)]_+ = 0, \,\forall\, i,$$

where $\mathbf{x}^*$ is one optimal solution of (1).

Problem 4

Consider the nonnegative quadratic program:

$$\underset{\mathbf{x}\in X}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{Q}\mathbf{x} - \mathbf{c}^\top \mathbf{x}, \text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{2}$$

where $\mathbf{Q}$ is a symmetric and positive semidefinite matrix, and $X = \{\mathbf{x} \in \mathbb{R}^n : x_i \geq 0, i = 1, \ldots, n\}$.

Use the instructor's provided file `penalty_qp.m` to write a Matlab function `penalty_qp` with input $\mathbf{Q}$, $\mathbf{c}$, $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, stopping tolerance `tol`, initial penalty parameter μ_0, and the final μ_1. Also test your function by running the provided test file `test_penalty_qp.m` and compare to the instructor's function. Print your code and the results you get.