Assignment 5 of MATP6610/4820

(Due on March-27-2020 in LMS)

Bonus for using LaTex: you can earn up to 10% extra credit if you use LaTex to type your solutions and include code and all test results. A template of LaTex is included.

Problem 1

1. Let $f(x) = x^2 + 4\cos x, x \in \mathbb{R}$. Start from $x^{(0)} = 1.5$ and do 4 iterations of the Newton's method to find an approximate minimizer of $f(x)$ by hand (calculator is allowed).

2. Let $f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$. Write the gradient $\nabla f(x_1, x_2)$ and the Hessian matrix $\nabla^2 f(x_1, x_2)$ at any point (x_1, x_2). Program the Newton's algorithm to minimize $f(x_1, x_2)$. Run the algorithm to 10 iterations and report the norm of gradient at each iteration. For the initial point, first use $(1.2, 1.2)$ and then $(-1.2, 1.0)$.

Problem 2

Consider the unconstrained quadratic minimization problem:

$$\underset{\mathbf{x}}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{A}\mathbf{x} - \mathbf{b}^\top \mathbf{x} \tag{QuadMin}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, and $\mathbf{b} \in \mathbb{R}^n$.

Write a solver for (QuadMin) by the DFP method. Use the instructor's provided file `quadMin_DFP.m` to write a Matlab function `quadMin_DFP` with input $\mathbf{A}$, $\mathbf{b}$, initial vector `x0`, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_DFP.m` and compare to the instructor's function. Print your code and the results you get, and print the figures you get.

Problem 3 [bonus question]

In the DFP method for solving the problem $\min_{\mathbf{x}} f(\mathbf{x})$, we denoted $\mathbf{s}^{(k)} = \mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}$ and $\mathbf{y}^{(k)} = \nabla f(\mathbf{x}^{(k+1)}) - \nabla f(\mathbf{x}^{(k)})$. At every update $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha_k \mathbf{B}_k^{-1} \nabla f(\mathbf{x}^{(k)})$, we choose $\alpha_k > 0$ such that the Wolfe's conditions hold. Prove that $(\mathbf{s}^{(k)})^\top \mathbf{y}^{(k)} > 0$ if $\nabla f(\mathbf{x}^{(k)}) \neq \mathbf{0}$. [Recall that we used this result to show positive definiteness of $\mathbf{B}_{k+1}$.]