

# Assignment 3 of MATP6960: theory questions

## Problem 1

In class, we used the Hoeffding's inequality to show some properties of the sampling approximation method for solving chance-constrained problems. The inequality is summarized below. **Hoeffding's inequality.** Let  $Y_1, \dots, Y_N$  be i.i.d. variables with  $\text{Prob}(Y_i \in [a_i, b_i]) = 1$  for all  $i = 1, \dots, N$ . Then for any  $t > 0$ ,

$$\text{Prob} \left( \sum_{i=1}^N (Y_i - \mathbb{E}[Y_i]) \geq tN \right) \leq \exp \left( -\frac{2N^2 t^2}{\sum_{i=1}^N (b_i - a_i)^2} \right).$$

Use the Hoeffding's inequality to prove that for any  $\alpha \in [0, 1]$  and any positive integer  $N$ , if  $k < \alpha N$ , it holds

$$\sum_{i=0}^k \binom{N}{k} \alpha^i (1 - \alpha)^{N-i} \leq \exp \left( -\frac{2(\alpha N - k)^2}{N} \right),$$

where  $\binom{N}{k}$  denotes “ $N$  choose  $k$ ”.

## Problem 2

A function  $f(x)$  is called log-concave, if it is positive and  $\log(f(x))$  is a concave function. Prove that if  $\theta : \mathbb{R} \rightarrow \mathbb{R}$  is a log-concave probability density function, then the cumulative distribution function

$$F(x) = \int_{-\infty}^x \theta(t) dt$$

is also log-concave, provided that  $F$  is well-defined for any  $x \in \mathbb{R}$ .