

Assignment 2 of MATP6610/4820

(Due on Feb-8-2019 in class)

Problem 1

Consider the unconstrained quadratic minimization problem:

$$\underset{\mathbf{x}}{\text{minimize}} f(\mathbf{x}) = \frac{1}{2} \mathbf{x}^\top \mathbf{A} \mathbf{x} - \mathbf{b}^\top \mathbf{x} \quad (\text{QuadMin})$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix, and $\mathbf{b} \in \mathbb{R}^n$.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand two iterations of steepest gradient descent with exact line search.
2. Use the instructor's provided file `quadMin_gd.m` to write a Matlab function `quadMin_gd` with input \mathbf{A} , \mathbf{b} , initial vector $\mathbf{x}0$, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_gd.m` and compare to the instructor's function. Print the results you get.

Problem 2

Let $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$.

1. Given $\mathbf{p}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, find a vector \mathbf{p}_2 with unit 2-norm and \mathbf{A} -conjugate to \mathbf{p}_1 .
2. Give two vectors \mathbf{p}_1 and \mathbf{p}_2 such that they are \mathbf{A} -conjugate and also orthogonal to each other.

Problem 3

Consider again the problem (QuadMin) in Problem 1.

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1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand the conjugate gradient method to find the **exact solution**.
 2. Use the instructor's provided file `quadMin_cg.m` to write a Matlab function `quadMin_gd` with input \mathbf{A} , \mathbf{b} , initial vector \mathbf{x}^0 , and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_cg.m` and compare to the instructor's function. Print the results you get.

Problem 4

Let $f(x) = x^2 + 4\cos x$, $x \in \mathbb{R}$. Start from $x^{(0)} = 1.5$ and do 4 iterations of the Newton's method to find an approximate minimizer of $f(x)$, either by hand or programming. From the solution you just found, give another minimizer without re-computing but by using some property of the function f .