Assignment 6 of MATP6610/4820

(Due on April-02-2019 in class)

Problem 1

Consider the problem

$$\min_{x,y\in\mathbb{R}} f(x,y) = -\frac{1}{2}x^2 - xy + \frac{1}{2}y^2 - 2x - 2y, \text{ s.t. } x+y=1.$$

1. Argue that $(1,0)$ is the unique minimizer.
2. The augmented Lagrangian function with penalty parameter $\beta > 0$ is
$$\mathcal{L}(x,y,v) = -\frac{1}{2}x^2 - xy + \frac{1}{2}y^2 - 2x - 2y + v(x+y-1) + \frac{\beta}{2}(x+y-1)^2.$$
Argue that for any $\beta \in (0,1]$, the augmented Lagrangian method does not work for this instance, i.e., the generated sequence $(x^{(k)}, y^{(k)})$ does not converge to a KKT point.
3. For $\beta = 2$, do by hand two iterations of the augmented Lagrangian method with initial multiplier $v^{(0)} = 0$.
4. (**Only for 6610 students:**) For any $\beta > 1$, does the generated sequence $(x^{(k)}, y^{(k)})$ converge to the unique minimizer $(1,0)$, starting from any $v^{(0)}$? If yes, prove it; if not, give a setting of $\beta > 1$ and $v^{(0)}$ such that the sequence does not converge to $(1,0)$.

Problem 2

Consider the nonnegative quadratic program:

$$\underset{\mathbf{x}\in X}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{Q}\mathbf{x} - \mathbf{c}^\top \mathbf{x}, \text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{b} \tag{1}$$

where $\mathbf{Q}$ is a symmetric and positive semidefinite matrix, and $X = \{\mathbf{x} \in \mathbb{R}^n : x_i \geq 0, i = 1,\ldots,n\}$.

Use the instructor's provided file `alm_qp.m` to write a Matlab function `alm_qp` with input $\mathbf{Q}, \mathbf{c}, \mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, stopping tolerance `tol`, and penalty parameter $\beta > 0$. Also test your function by running the provided test file `test_alm_qp.m` and compare to the instructor's function. Print your code and the results you get.