Assignment 4 of MATP6610/4820

(Due on March-06-2020 in class)

Problem 1

Consider the unconstrained quadratic minimization problem:

$$\underset{\mathbf{x}}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{A}\mathbf{x} - \mathbf{b}^\top \mathbf{x} \qquad \text{(QuadMin)}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, and $\mathbf{b} \in \mathbb{R}^n$.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand three iterations of the BB method. Your first iteration should be steepest gradient descent with exact line search, and the second and third iterations should use Option-I BB stepsize.

2. Write a solver for (QuadMin) by the BB method. Use the instructor's provided file `quadMin_BB.m` to write a Matlab function `quadMin_BB` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \le$ tol. Also test your function by running the provided test file `test_BB.m` and compare to the instructor's function. Print your code and the results you get.

Problem 2

In class, we derived the proximal gradient method for solving the Lasso problem

$$\underset{\mathbf{x}\in\mathbb{R}^n}{\text{minimize}}\, \frac{1}{2}\|\mathbf{A}\mathbf{x}-\mathbf{b}\|^2 + \lambda\|\mathbf{x}\|_1, \tag{Lasso}$$

where $\mathbf{A} \in \mathbb{R}^{m\times n}$ is a sensing matrix, $\mathbf{b}$ is the measurement vector, and $\lambda > 0$ is the regularization parameter.

1. Write a solver for (Lasso) by the proximal gradient method. Use the instructor's provided file `PG_Lasso.m` to write a Matlab function `PG_Lasso` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, parameter `lam`, and tolerance `tol`, and with a provided stopping condition.

2. Write a solver for (Lasso) by the accelerated proximal gradient method. Use the instructor's provided file `APG_Lasso.m` to write a Matlab function `APG_Lasso` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, parameter `lam`, and tolerance `tol`, and with a provided stopping condition.

3. Test your two solvers by running the provided test file `test_Lasso.m` and compare to the instructor's function. Print your code and the results you get.