Assignment 1 of MATP6610/4820

(Due on Jan-31-2020 in class)

Problem 1

Show that the sequence $x^{(k)} = (\frac{3}{4})^{2^k} - 2$ is Q-quadratically convergent to -2.

Problem 2

Consider the sequence $\{x^{(k)}\}_{k=1}^{\infty}$ defined by

$$x^{(k)} = \begin{cases} (\frac{1}{2})^{2^k}, & \text{if } k \text{ is odd} \\ \frac{1}{k} x^{(k-1)}, & \text{if } k \text{ is even} \end{cases}$$

Is this sequence Q-superlinearly convergent? Q-quadratically convergent? R-quadratically convergent? Justify your answer.

Problem 3

Find all stationary points of $f(x_1, x_2) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$ and verify if they are local minimizers, or local maximizers, or saddle points.

Problem 4

Consider the function

$$f(x_1, x_2) = \frac{1}{2}\left(2x_1 + \frac{1}{2}x_2^2\right)^2.$$

1. Give the gradient of f at the point $(1, 0)$;
2. Show that $\mathbf{p} = (-1, 2)$ is a descent direction, i.e., $\langle \mathbf{p}, \nabla f(1, 0)\rangle < 0$, where $\langle \cdot\,, \cdot \rangle$ denotes vector inner product.
3. Let $c_1 = 0.1$. Does $\alpha = 1$ satisfy the Armijo's condition?