Assignment 2 of MATP6610/4820

(Due on Feb-8-2019 in class)

Problem 1

Consider the unconstrained quadratic minimization problem:

$$\underset{\mathbf{x}}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{A}\mathbf{x} - \mathbf{b}^\top \mathbf{x} \qquad \text{(QuadMin)}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, and $\mathbf{b} \in \mathbb{R}^n$.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand two iterations of steepest gradient descent with exact line search.

2. Use the instructor's provided file `quadMin_gd.m` to write a Matlab function `quadMin_gd` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \le \text{tol}$. Also test your function by running the provided test file `test_gd.m` and compare to the instructor's function. Print the results you get.

Problem 2

Let $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$.

1. Given $\mathbf{p}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, find a vector $\mathbf{p}_2$ with unit 2-norm and $\mathbf{A}$-conjugate to $\mathbf{p}_1$.

2. Give two vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ such that they are $\mathbf{A}$-conjugate and also orthogonal to each other.

Problem 3

Consider again the problem (QuadMin) in Problem 1.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand the conjugate gradient method to find the **exact solution**.

2. Use the instructor's provided file `quadMin_cg.m` to write a Matlab function `quadMin_cg` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_cg.m` and compare to the instructor's function. Print the results you get.

Problem 4

Let $f(x) = x^2 + 4\cos x, x \in \mathbb{R}$. Start from $x^{(0)} = 1.5$ and do 4 iterations of the Newton's method to find an approximate minimizer of $f(x)$, either by hand or programming. From the solution you just found, give another minimizer without re-computing but by using some property of the function f.