Assignment 2 of MATP6610/4820

(Due on Feb-14-2020 in class)

Problem 1

Consider the function we used in class: $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 - x_1 - x_2$. At the point $\mathbf{x}^{(0)} = (0, 0)$,

1. give the search direction vector $\mathbf{p}$ for the steepest descent method.
2. Let $c_1 = 0.1$, determine $\bar{\alpha} > 0$ such that the Armijo's condition holds for any $\alpha \in (0, \bar{\alpha}]$.
3. Let $c_1 = 0.1$ and $c_2 = 0.5$, determine the values of $\underline{\alpha}$ and $\bar{\alpha}$ such that the Wolfe's conditions hold for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$.

Problem 2

Consider the unconstrained quadratic minimization problem:

$$\underset{\mathbf{x}}{\text{minimize}}\, f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathbf{A}\mathbf{x} - \mathbf{b}^\top \mathbf{x} \qquad \text{(QuadMin)}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix, and $\mathbf{b} \in \mathbb{R}^n$.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand two iterations of steepest gradient descent with exact line search.

2. Use the instructor's provided file `quadMin_gd.m` to write a Matlab function `quadMin_gd` with input $\mathbf{A}$, $\mathbf{b}$, initial vector $\mathbf{x}0$, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_gd.m` and compare to the instructor's function. Print your code and the results you get.

Problem 3 (only for MATP 4820 students)

Let $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$.

1. Given $\mathbf{p}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, find a vector $\mathbf{p}_2$ with unit 2-norm and $\mathbf{A}$-conjugate to $\mathbf{p}_1$.
2. Give two vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ such that they are $\mathbf{A}$-conjugate and also orthogonal to each other.

Problem 4

Consider again the problem (QuadMin) in Problem 1.

1. Let $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$. Set the initial vector $\mathbf{x}^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Do by hand the conjugate gradient method to find the **exact solution**.

2. Use the instructor's provided file `quadMin_cg.m` to write a Matlab function `quadMin_cg` with input $\mathbf{A}$, $\mathbf{b}$, initial vector `x0`, and tolerance `tol`, and with stopping condition $\|\nabla f(\mathbf{x}^k)\| \leq \text{tol}$. Also test your function by running the provided test file `test_cg.m` and compare to the instructor's function. Print your code and the results you get.

Problem 5 (only for MATP 6100 students)

For the conjugate gradient method, prove that $\langle \mathbf{r}^{(k)}, \mathbf{r}^{(i)} \rangle = 0$ for any $i = 1, 2, \ldots, k-1$ and for $k \geq 2$. Recall that we showed, in class, $\langle \mathbf{r}^{(2)}, \mathbf{r}^{(1)} \rangle = 0$. You can use this as an initial step and complete the proof by induction.