Assignment 1 of MATP6610/4820

(Due on Jan-25-2019 in class)

Problem 1

Show that the sequence $x^{(k)} = 1 + 0.5^{2^k}$ is Q-quadratically convergent to 1.

Problem 2

Consider the sequence $\{x^{(k)}\}_{k=0}^{\infty}$ defined by

$$x^{(k)} = \begin{cases} (\frac{1}{4})^{2^k}, & \text{if } k \text{ is even} \\ \frac{1}{k}x^{(k-1)}, & \text{if } k \text{ is odd} \end{cases}$$

Is this sequence Q-superlinearly convergent? Q-quadratically convergent? R-quadratically convergent?

Problem 3

Find all stationary points of $f(x_1, x_2) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$ and verify if they are local minimizers, or local maximizers, or saddle points.

Problem 4

Consider the function $f(x_1, x_2) = (x_1 + x_2^2)^2$. At the point $(1, 0)$, show that $\mathbf{p} = (-1, 1)$ is a descent direction, i.e., $\langle \mathbf{p}, \nabla f(1, 0)\rangle < 0$, where $\langle \cdot\, , \cdot \rangle$ denotes vector inner product. Let $c_1 = 0.1$. Does $\alpha = 1$ satisfy the Armijo's condition?

Problem 5

Consider the function we used in class: $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 - x_1 - x_2$. At the point $\mathbf{x}^{(0)} = (0, 0)$,

1. give the search direction vector $\mathbf{p}$ for the steepest descent method.
2. Let $c_1 = 0.1$, determine $\bar{\alpha} > 0$ such that the Armijo's condition holds for any $\alpha \in (0, \bar{\alpha}]$.
3. Let $c_1 = 0.1$ and $c_2 = 0.5$, determine the values of $\underline{\alpha}$ and $\bar{\alpha}$ such that the Wolfe's conditions hold for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$.