ELEC ENG 2CI4: AD3 Homework 3

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2.1 Part 1: RLC Circuit

2.1.a Loop Equation

$v_s(t) - v_R(t) - v_L(t) - v_C(t) = 0$

$$v_s(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\, dt$$
$$\frac{dv_s(t)}{dt} = R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t)$$

2.1.b Homogeneous Version

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$$

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{CL}i(t) = 0$$

2.1.c Finding ω_0

Matching coefficients:

$$\frac{R}{L} = 2\zeta\omega_0, \quad \frac{1}{CL} = \omega_0^2$$

Therefore

$$\omega_0 = \frac{1}{\sqrt{CL}}$$

2.1.d Finding ζ

From c):

$$\frac{R}{L} = 2\zeta\omega_0$$
$$\zeta = \frac{R}{2L\omega_0}$$
$$\zeta = \frac{\sqrt{CL}R}{2L}$$
$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

2.2 Part 2: Overdamped Case (R = 3 kOhms)

2.2.a Theoretical Calculation of Damping Ratio

To determine the nature of the transient response, we calculated the exponential damping ratio (ζ) using the component values provided.

Given Values:

- Resistance $R = 3000\,\Omega$
- Inductance $L = 1.5\,\mathrm{mH} = 1.5 \times 10^{-3}\,\mathrm{H}$
- Capacitance $C = 10\,\mathrm{nF} = 10 \times 10^{-9}\,\mathrm{F}$

Calculation:

The damping ratio for a series RLC circuit is given by:

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Substituting the given values:

$$\zeta = \frac{3000}{2}\sqrt{\frac{10 \times 10^{-9}}{1.5 \times 10^{-3}}}$$

$$\zeta = 1500\sqrt{6.667 \times 10^{-6}}$$

$$\zeta = 1500 \times 0.002582$$

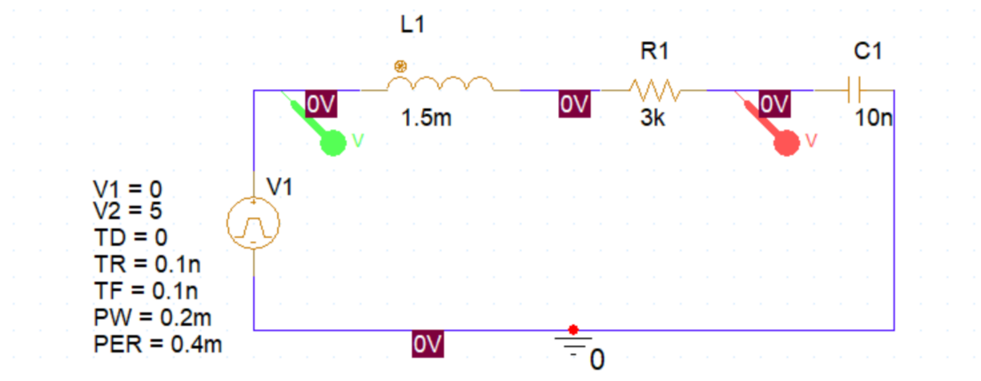
$$\zeta \approx 3.873$$

Conclusion:

Since the damping ratio $\zeta > 1$, the circuit is theoretically overdamped. We expect the output voltage $v_2(t)$ to rise towards the steady-state value of 5V without any oscillation or overshoot.

2.2.b PSpice Simulation

The circuit was simulated in PSpice using the parameters listed above.



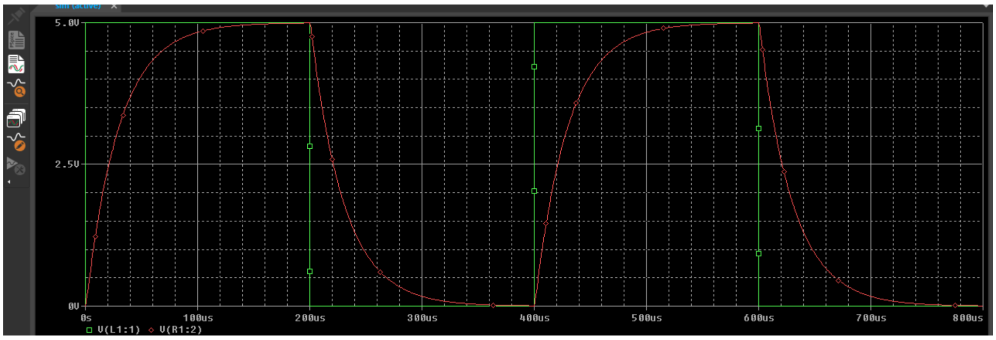
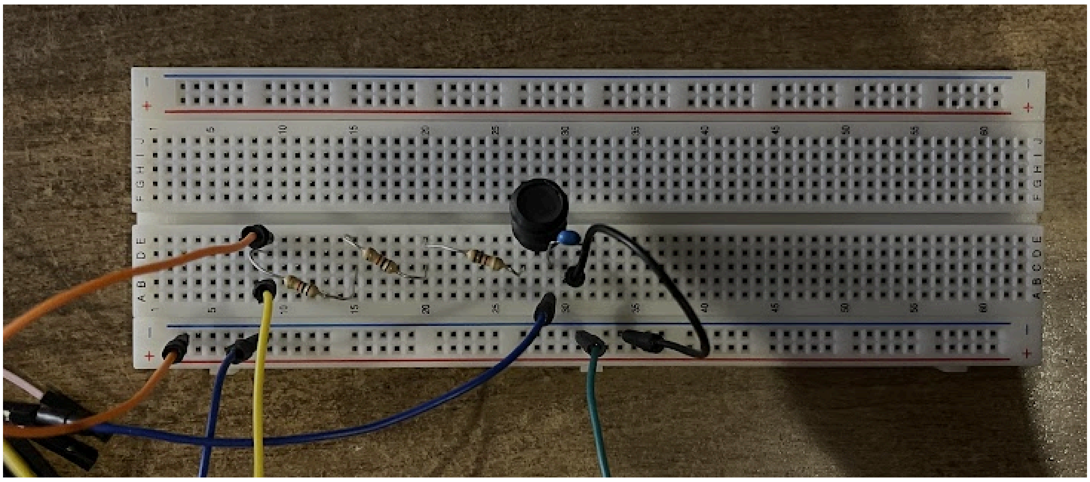


Figure 2.1: PSpice simulation of the Overdamped response (R = 3 kOhms).

Visual Inspection:

The simulation results confirm the theoretical prediction in Part 2(a). The waveform for the capacitor voltage $v_2(t)$ (Red trace) rises smoothly to 5V with a characteristic "shark fin" shape. There is zero overshoot, and the rise time is relatively slow, taking approximately 200 microseconds to reach steady state. This confirms the system is heavily overdamped.

2.2.c AD3 Hardware Implementation



The circuit was constructed on a breadboard. To achieve the required 3000 Ohm resistance using the available kit components, three 1 kOhm resistors were connected in series:

$$R_{total} = 1, \mathrm{k}\Omega + 1, \mathrm{k}\Omega + 1, \mathrm{k}\Omega = 3, \mathrm{k}\Omega$$

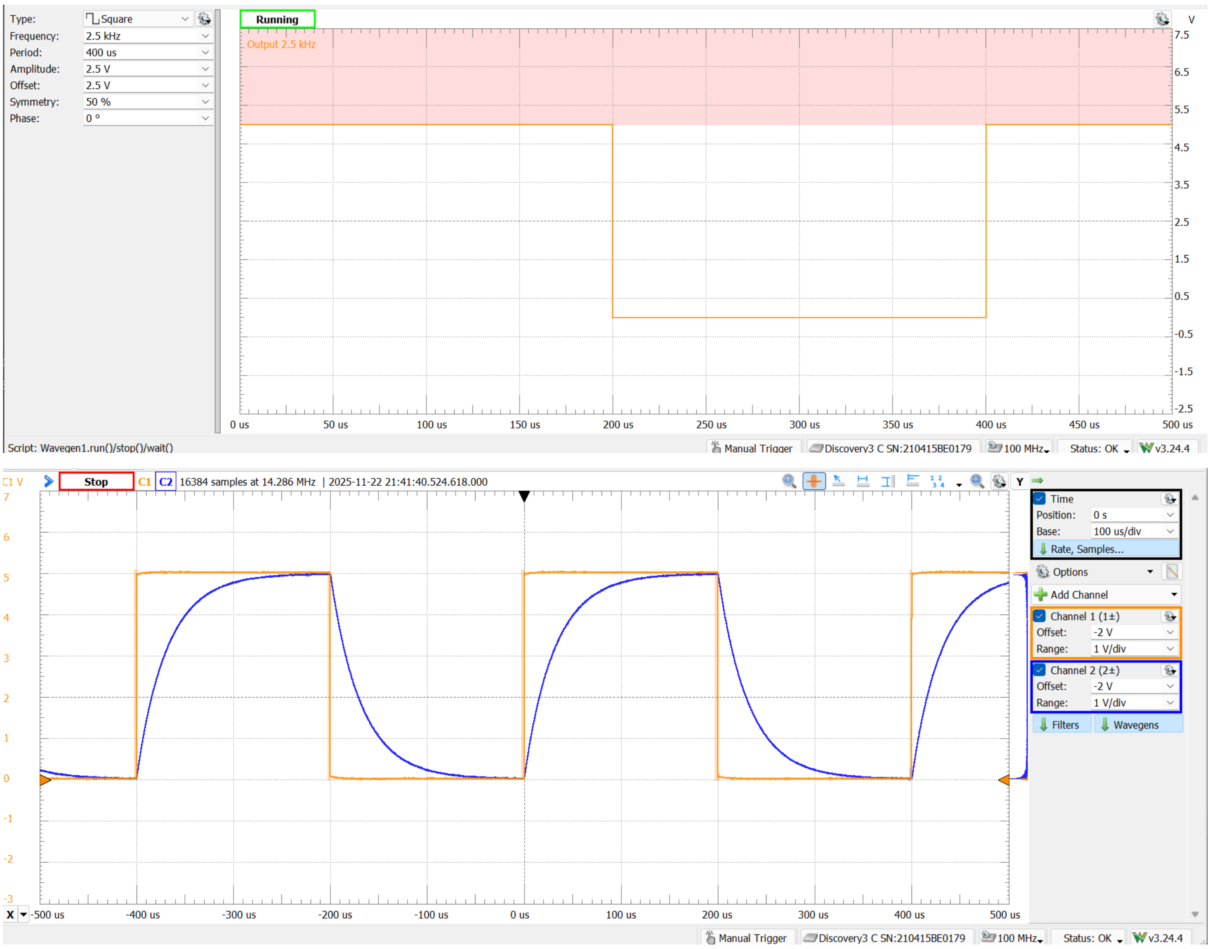


Figure 2.2: AD3 Oscilloscope capture of the Overdamped response.

Comparison and Critical Analysis:

The hardware waveform for $v_2(t)$ (Blue trace) is visually comparable to the PSpice simulation, exhibiting the same smooth, non-oscillatory rise to 5V.

However, a critical comparison reveals that the hardware response rises slightly slower than the ideal simulation. This discrepancy is expected due to parasitic resistance in the hardware setup. The actual total resistance in the circuit includes the explicit resistors (3000 Ohms), the DC Resistance (DCR) of the inductor (typically 5-10 Ohms), and contact resistance from the breadboard. Since the damping ratio is directly proportional to resistance ($\zeta \propto R$), these additional resistances increase the actual ζ slightly above the theoretical 3.873. This results in a slightly more damped (slower) response in the physical circuit compared to the ideal simulation.

2.3 Part 3: Underdamped Case

2.4.a Theoretical Calculation

$$
\begin{aligned}
\zeta &= \frac{R}{2}\sqrt{\frac{C}{L}} \\
&= \frac{100}{2}\sqrt{\frac{10\times10^{-9}}{1.5\times10^{-3}}} \\
&= 50\sqrt{6.\overline{6}\times10^{-6}} \\
&= 50\times0.00258199 \\
\zeta &\approx 0.1291
\end{aligned}
$$

We expect the system to be underdamped. The system will oscillate, but we expect it to eventually stabilize.

2.4.b PSpice Simulation

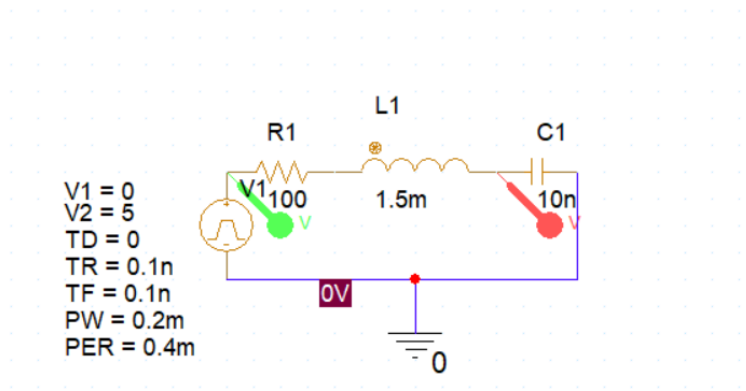


Figure 3.1: PSpice circuit and parameters supplied.

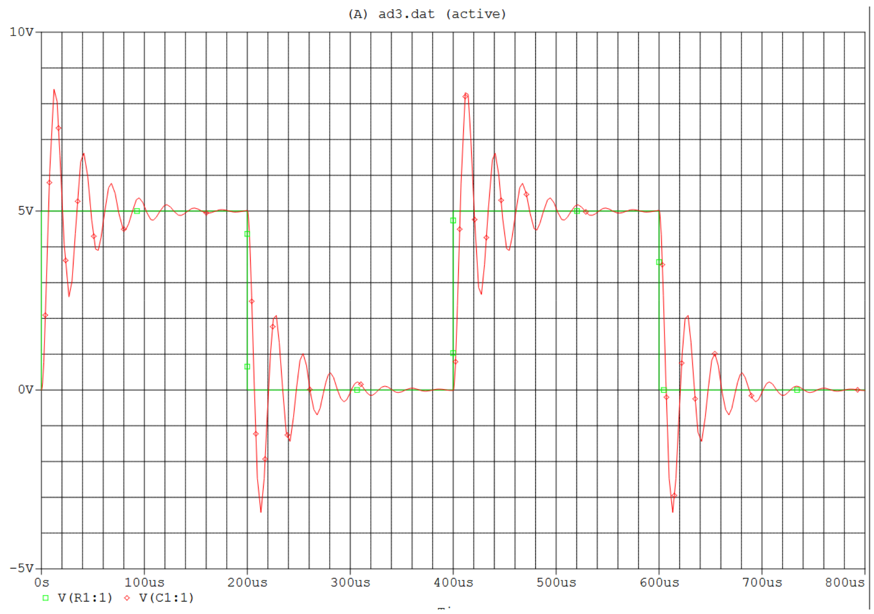


Figure 3.2: PSpice simulation plot readout.

This is suitable for our expectations. The system oscillates before settling at a certain voltage, on par with underdamping. Two periods are shown.

2.4.c AD3 Physical Simulation

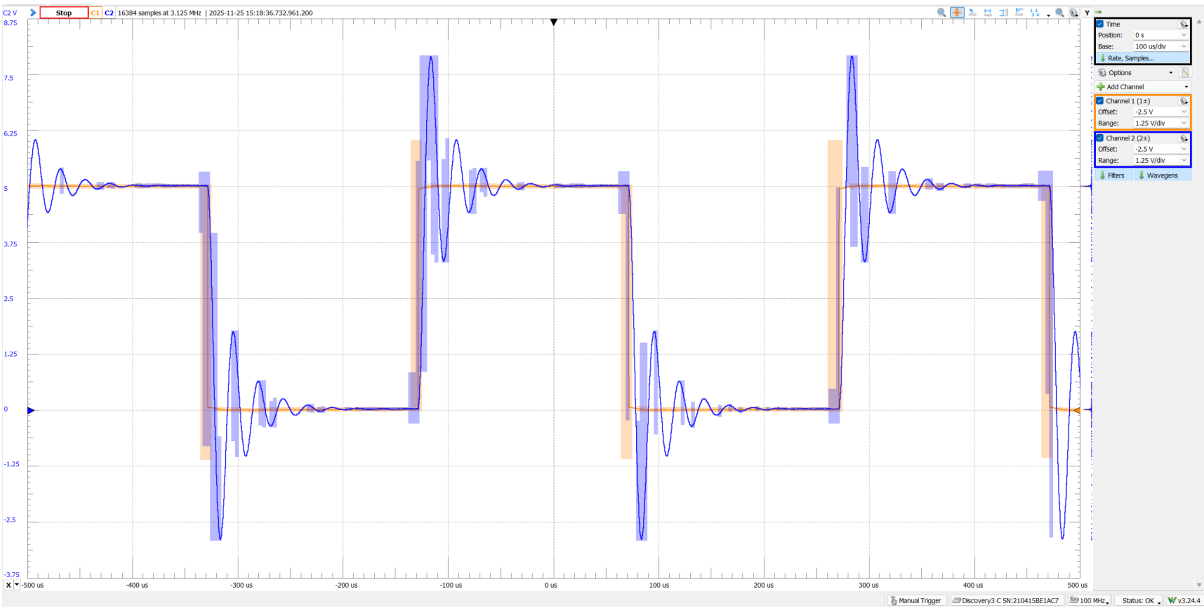


Figure 3.3: AD3 graphed readout.

In comparison with the PSpice readout in Figure 3.2, they look almost identical barring some scaling/offset display differences. This reinforces the finding in a), in that the system is underdamped and oscillates with a decreasing A. There are some slight differences, which can be attributed to the fact this is a physical circuit while PSpice is purely mathematical simulation. This means factors such as contact resistance are in play.

2.4 Part 4: Critically Damped Case

2.4.a Calculation of Critical Resistance

We must determine the resistance value R required to achieve Critical Damping, which is defined as $\zeta = 1$.

Calculation:

Setting $\zeta = 1$ in the damping equation:

$$1 = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Solving for R:

$$R_{crit} = 2\sqrt{\frac{L}{C}}$$

Substituting the component values:

$$R_{crit} = 2\sqrt{\frac{1.5 \times 10^{-3}}{10 \times 10^{-9}}}$$
$$R_{crit} = 2\sqrt{150,000}$$
$$R_{crit} = 2 \times 387.298$$
$$R_{crit} \approx 774.6\,\Omega$$

2.4.b PSpice Simulation

The circuit was simulated in PSpice using the calculated value $R = 774.6\,\Omega$.

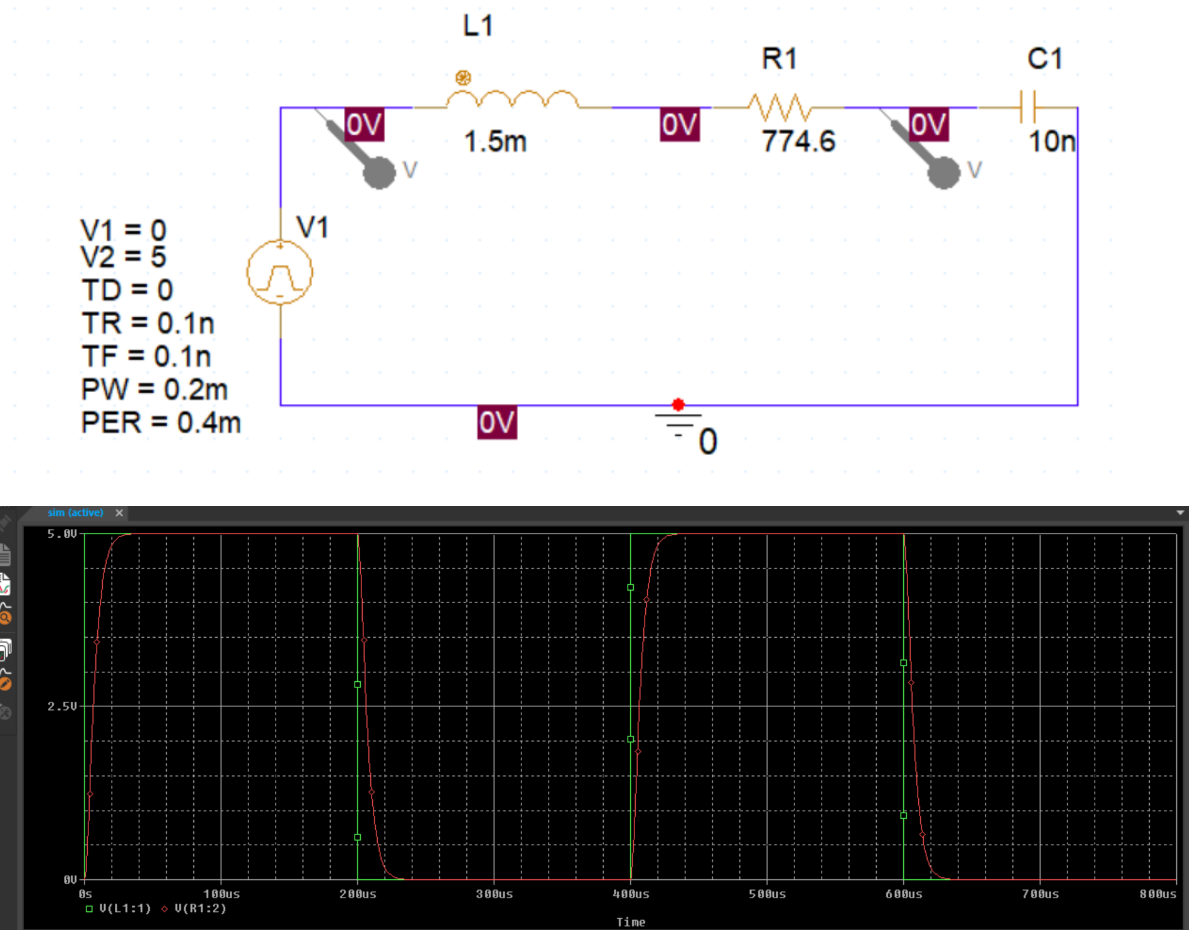


Figure 4.1: PSpice simulation of the Critically Damped response (R = 774.6 Ohms).

Visual Inspection:

The simulation shows the fastest possible rise to 5V without overshoot. Unlike the Overdamped case in Part 2, the "knee" of the curve where it settles at 5V is much sharper, reaching steady state in under 50 microseconds. The voltage does not exceed 5V, confirming the circuit is critically damped.

2.4.c AD3 Hardware Implementation

To approximate the calculated critical resistance of 774.6 Ohms using standard kit components, a series combination of three resistors was used: one 330 Ohm and two 220 Ohm resistors.

Resistor Configuration:

$$R_{constructed} = 330\,\Omega + 220\,\Omega + 220\,\Omega = 770\,\Omega$$

Updated Circuit Diagram:

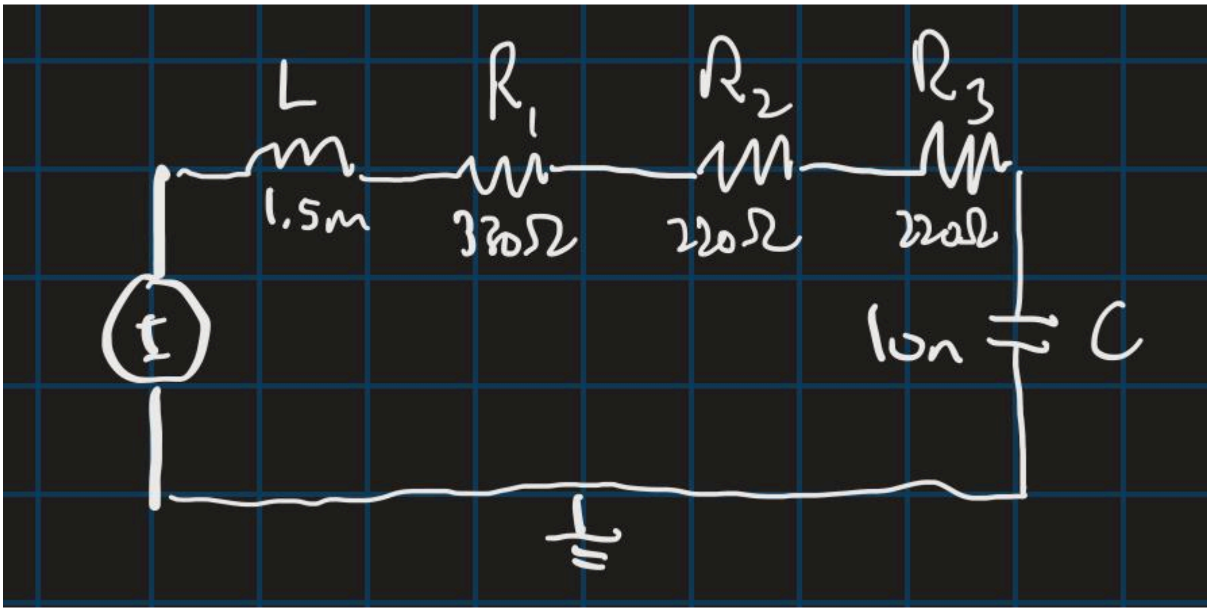


Figure 4.2: Circuit diagram of the resistor configuration used to approximate Critical Resistance.

Breadboard Photograph:

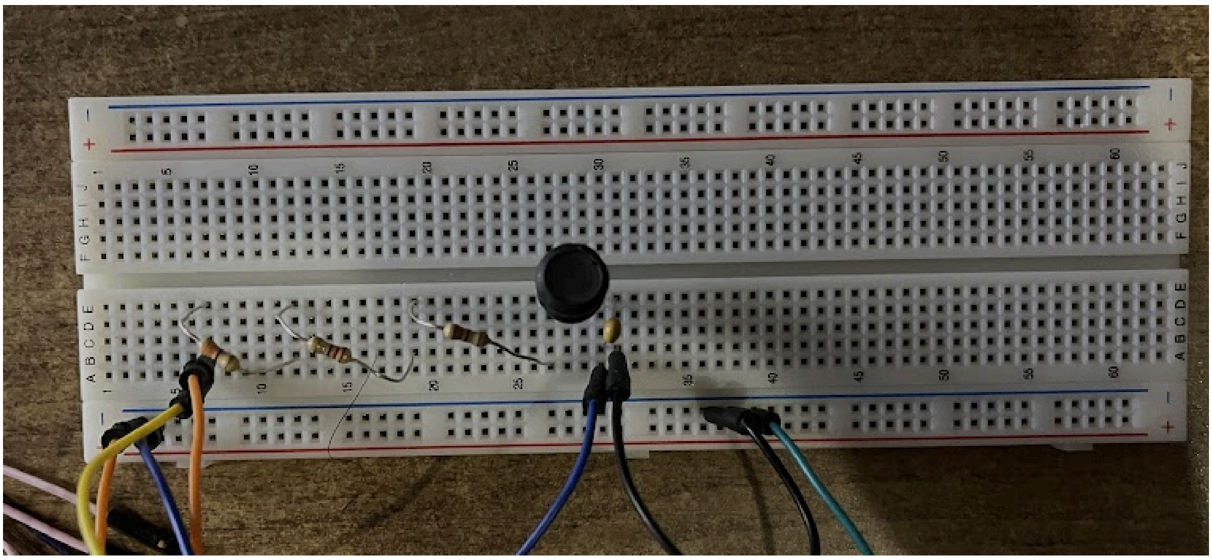


Figure 4.3: Photograph of the hardware implementation showing the series resistor chain.

Oscilloscope Measurement:

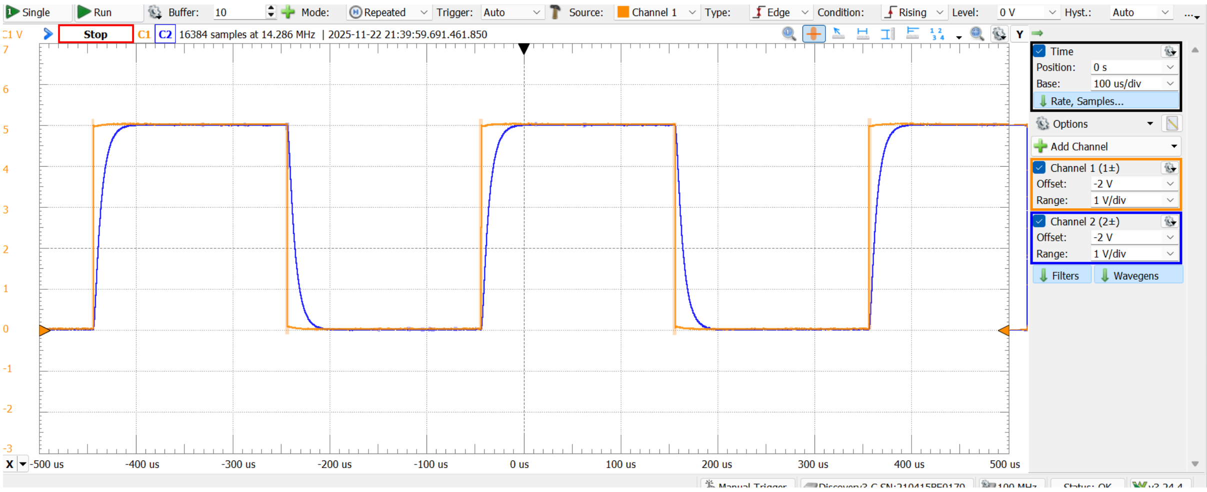


Figure 4.4: AD3 Oscilloscope capture of the hardware response.

Comparison and Critical Analysis:

The AD3 waveform matches the general profile of the PSpice simulation, showing a rapid rise to 5V with no visible ringing or overshoot.

The hardware implementation is remarkably accurate due to the resistor combination chosen.

1. Constructed Resistance: 770 Ohms.
2. Inductor DCR: approx 5-10 Ohms.
3. Total Hardware Resistance: approx 775 - 780 Ohms.
4. Target Critical Resistance: 774.6 Ohms.

Because the total hardware resistance ($780\,\Omega$) is almost exactly equal to the theoretical critical resistance ($774.6\,\Omega$), the actual damping ratio is:

$$\zeta_{actual} \approx \frac{780}{774.6} \approx 1.007$$

Since $\zeta \approx 1$, the hardware response is practically perfectly critically damped. This explains why the "knee" of the voltage curve in the AD3 trace is extremely sharp and matches the ideal PSpice simulation almost perfectly, without the sluggishness seen in the Overdamped case.